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CONTRA-CONTINUITY BETWEEN GRILL-TOPOLOGICAL SPACES

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Abstaract − Recently noted the importance of the concept of grill between topological space. It helps us to measure the things that was difficult to measure and it is also used in many applications such as computer and information systems. Our purpose is to introduce the notation of $e-G-$open, $r-G-$open sets and discuss new class of function named contra $e-G-$continuous function with their various assets, depiction and relationships. Relevance between there new class and other proportion of functions are obtained and several depictions of a new class of functions are discussed.

Keywords − *r-G-open sets, e-G-open sets, e-G-continuous, Contra-e-G-continuous.*

1 Introduction

The concept of grill topological spaces, which is grounded on two operators, is Φ and Ψ. Choquet [1] was the first introduced this concept in 1947. A number of theories and features has been handled in [2, 3, 4, 5, 6, 7]. It helps to expand the topological structure which is used to measure the description rather than quantity, such as love, intelligence, beauty, quality of education and etc. In 1996, Dontchev [8] pass the notation of contra continuous functions. Ekici [9] debate the concepts of contra-e-continuous functions and new class named as e-open sets. Jafari and Noiri [10, 11] exhibited contra-α-continuous and contra-pre-continuous functions. We are working to provide the previous concepts and definitions by using the concept of grill. Some important characteristics and special relations of these concepts are obtained.

2 Preliminary

Definition 2.1. A nonempty subcollection G of a space L which carries topology τ is named grill [1] on this space if the following conditions are true:
(1) $\phi \notin G$,
(2) $A \in G$ and $A \subseteq B \subseteq L \Rightarrow B \in G$,
(3) if $A \cup B \in G$ for $A, B \subseteq L$, then $A \in G$ or $B \in G$.

Since the grill depends on the two mappings Φ and Ψ which is generated a unique grill topological space finer than τ on space L denoted by τ_G on L have been discussion in [3, 5].

A subset B of a space (L, τ, G) is named regular open (resp. regular closed) if $B = Int(Cl(B))$ (resp. $B = Cl(Int(B))$. B is named δ-open [12] if for $x \in B$, there exist a regular open set D such that $x \in D \subset B$. The complement of δ-open set is called δ-closed. A point $x \in L$ is called a δ-derived point of B if $Int(Cl(U)) \cap B \neq \varphi$ for each open set U containing x. The set of all δ-derived points of B is called the δ-closure of B and is denoted by $\delta Cl(B)$ [12]. The set δ-interior of B [12] is the union of all regular open sets of L contained in B and its denoted by $\delta Int(B)$. B is δ-open if $\delta Int(B) = B$. δ-open sets forms a topology τ^{δ}. The collection of all δ-open sets in L is denoted by $\delta O(L)$. A subset B of a space (L, τ) is called e-open [13] (resp. α-open [14], β-open [15]) if $B \subset Cl(\delta Int(B)) \cup Int(\delta Cl(B))$ (resp. $B \subset Int(Cl(Int(B)), B \subset Cl(Int(Cl(B))$. The complement of an e-open set is called is an e-closed set. The intersection of all e-closed sets containing a set B in a topological space (L, τ) is called the e-closure [13]. The union of all e-open sets contained in a set B in a topological space (L, τ) is called the e-interior of B and it is denoted by $e - Int(B)$. A subset B of a topological L is e-regular [16] if it is e-open and e-closed. The family of all e-open (resp. e-closed, e-regular) sets in L will be denoted by $EO(L)$ (resp. $EC(L)$, $ER(L)$. The family of all e-open (resp. e-closed, e-regular) sets which contain x in L will be denoted by $EO(L, x)$ (resp. $EC(L, x)$, $ER(L, x)$).

A function $z : (L, \tau, I) \to (M, \sigma)$ is called contra continuous [8] (resp. contra-e-continuous [17], contra-$e - I-$continuous [18]) if the inverse image of each open set of M is closed (resp.$z^{-1}(V)$ is e-closed in L for every open set V of M, $z^{-1}(V)$ is $e - I$-closed in (L, τ, I) for every open set V of (M, σ))".

3 Essential Results

The third section, we define and study some new definitions called $r - G-$open, $e - G-$open and contra $e - G-$continuous. Depiction and basic assets of $e - G-$continuous function are studied.

Definition 3.1. For $B \subseteq L$ which carries topology τ with grill G is called $r - G-$open (resp. $r - G-$closed) if $B = Int(\Psi(B))$ (resp. $B = \Psi(Int(B))$. A point $x \in L$ is called a $\delta - G-$derived point of B if $Int(\Psi(U)) \cap B \neq \varphi$ for each open set U containing x. The family of all $\delta - G-$derived points of B is named the $\delta - G-$closure of B and is denoted by $\delta\Psi(B)$. The set $\delta - G-$interior of B is the union of all $r - G-$open sets of X contained in B and is denoted by $\delta Int_G(B)$. B is said to be $\delta - G-$closed if $\delta\Psi(B) = B$.

Definition 3.2. Let the space L which carries topology τ with grill G then, A subset B of L named $e - G-$open if $B \subset Cl(\delta Int_G(B)) \cup Int(\delta\Psi(B))$ and $e - G-$closed if

$Cl(\delta Int_G(B)) \cap Int(\delta\Psi(B)) \subset B$.

The class of all $e-G-$open sets of an grill with a space L which carries topology τ is denoted by $EGO(L)$.

Theorem 3.3. (1) The union of any family of $e-G-$open sets is an $e-G-$open set;
(2) The intersection of even two $e-G-$open sets need not to be $e-G-$open.

Proof. (1) Let $\{S_\alpha : \alpha \in \triangle\}$ be a family of $e-G-$open set, $S_\alpha \subset Cl(\delta Int_G(S_\alpha)) \cup Int(\delta\Psi(S_\alpha))$. Hence, $\cup_\alpha S_\alpha \subset \cup_\alpha[Cl(\delta Int_G(S_\alpha)) \cup Int(\delta\Psi(S_\alpha))]$
$\subset \cup_\alpha[Cl(\delta Int_G(S_\alpha))] \cup \cup_\alpha[Int(\delta\Psi(S_\alpha))]$
$\subset [Cl(\cup_\alpha(\delta Int_G(S_\alpha))] \cup [Int(\cup_\alpha(\delta\Psi(S_\alpha))]$
$\subset [Cl(\delta Int_G(\cup_\alpha S_\alpha))] \cup [Int(\delta\Psi(\cup_\alpha S_\alpha))]$. Then, $\cup_\alpha S_\alpha$ is $e-G-$open. □

Example 3.4. Let $L = \{1, 2, 3\}$ with $\tau = \{L, \varphi, \{1\}, \{2\}, \{1, 2\}\}$ and $G = P(L) \setminus \{\varphi\}$. Then the set $A = \{1, 3\}$ and $B = \{2, 3\}$ are $e-G-$open sets, but $A \cap B = \{3\}$ is not $e-G-$open.

Definition 3.5. A mapping $z : (L, \tau, G) \to (M, \sigma)$ is said to be contra $e-G-$continuous functions if $z^{-1}(V)$ is $e-G-$closed in (L, τ, G) for every open set V in (M, σ).

Example 3.6. Let $L = M = \{1, 2, 3\}$ and $\tau = \sigma = \{L, \varphi, \{1\}, \{2, 3\}\}$ with $G = \{\{2\}, \{1, 2\}, \{2, 3\}, L\}$. If $z : (L, \tau, G) \to (Y, \sigma)$ is identity function then, we notice that z is contra-e-continuous but not contra-$e-G-$continuous. Since $z^{-1}\{2, 3\} = \{2, 3\}$ is not $e-G-$closed.

Definition 3.7. For the space L which carries topology τ with grill G and $B \subset L$,
(1) Intersection of all open set U containing B is named kernel of B and denoted by $Ker(B)$.
(2) Intersection of all $e-G-$closed in (L, τ, G) containing B is named the $e-G-$closure of B and its denoted by $\Psi_e(B)$.
(3) The $e-G-$interior of B, denoted by $Int_{eG}(B)$, is defined by the union of all $e-G-$open sets contained in B.

Lemma 3.8. For a space L which carries topology τ with grill G and $C, B \subset L$,
(1) $x \in Ker(B)$ if and only if $B \cap F \neq \varphi$, where F closed subset of L containing x.
(2) $B \subset Ker(B)$ and $B = Ker(B)$ if A is open in L.
(3) if $C \subset B$, then $Ker(C) \subset Ker(B)$.

Lemma 3.9. For a subset B of a a space L which carries topology τ with grill G, the following properties are holds,
(1) $Int_{eG}(B) = L \setminus \Psi_e(L \setminus B)$.
(2) $x \in \Psi_e(B)$ if and only if $B \cup U \neq \varphi$ for each $U \in EGO(L, x)$.
(3) B is $e-G-$open if and only if $Int_{eG}(B) = B$.
(4) B is $e-G-$closed if and only if $\Psi_e(B) = B$.

Theorem 3.10. Let $z : (L, \tau, G) \to (M, \sigma)$ be a given function then, the next are equivalent:

(1) z is contra $e-G-$continuous,
(2) for each $x \in L$ and each closed set F in M with $z(x) \in F$, there exist $e-G-$open set U containing x such that $z(U) \subset F$,
(3) for each $x \in L$ and each closed set F in Y with $z(x) \in F, z^{-1}(F)$ is $e-G-$open in L ,
(4) $z(\Psi_e(B)) \subset Ker(z(B))$ for every $B \subset L$,
(5) $z(\Psi_e(B)) \subset z^{-1}(Ker(B))$.

Proof. (1) $\Rightarrow$ (2): By the assumption, suppose that $z(x) \in F$ such that $x \in L$ and F be any closed set in M, we have $z^{-1}(M \setminus F) = L \setminus z^{-1}(F)$ is $e-G-$closed in L and so $z^{-1}(F)$ is $e-G-$open. By butting $U = z^{-1}(F)$ containing x, we have$z(U) \subset F$.

(2) $\Rightarrow$ (3): Let $F \subset M$ be any closed set and $x \in z(x)$. Then $z(x) \in F$ and there exists $e-G-$open subset U_x containing x such that $z(U_x) \subset F$. Therefore, we obtain $z^{-1}(F)$ is $e-G-$open in L.
(3) $\Rightarrow$ (1): If U is any open set of Y, then $z^{-1}(Y \setminus U) = L \setminus z^{-1}(U)$ is $e-G-$open in L. Therefore $z^{-1}(U)$ is $e-G-$closed in L.

(3) $\Rightarrow$ (4): Let $B \subset L$ and $y \notin Ker(B)$. Then, by Lemma 3.7, there exists a closed set F of M such that $M \in F$ and $z(B) \cap F = \varphi$. This implies that $B \cap z^{-1}(F)) = \varphi$ and $\Psi_e(B) \cap z^{-1}(F) = \varphi$. Therefore, we obtain $z(\Psi_e(B)) \cap F = \varphi$ and $y \notin z(\Psi_e(B))$. That $z(\Psi_e(B)) \subset Ker(z(B))$.

(4) $\Rightarrow$ (5): Let C be any subset of M. By Lemma 3.7, we have $z(\Psi_e(z^{-1}(C)) \subset Ker(z(z^{-1}(C))) \subset Ker(C)$ and $z(\Psi_e(z^{-1}(C)) \subset z^{-1}(Ker(C))$.

(5) $\Rightarrow$ (1): Let N be any subset of M. By Lemma 3.7, we have $\Psi_e(z^{-1}(N)) \subset z^{-1}(Ker(N)) = z^{-1}(N)$ and $\Psi_e(z^{-1}(N)) = z^{-1}(N)$. This shows that $z^{-1}(N)$ is $e-G-$closed in L. □

Definition 3.11. A mapping $z : (L, \tau, G) \to (M, \sigma)$ is named $e-G-$continuous if $z^{-1}(D)$ is $e-G-$-open in L for every $D \in \sigma$.

Lemma 3.12. The next declarations are equivalent for a mapping $z : (L, \tau, G) \to (M, \sigma)$
(1) z is $e-G-$continuous,
(2) for each open set V of Y and each $x \in X$ with $z(x) \in V$, there exist $U \in EGO(L, x)$ such that $z(U) \subset V$.

Theorem 3.13. Let $z : (L, \tau, G) \to (M, \sigma)$ be a contra $e-G-$continuous functions such that M is regular set, then z is $e-G-$continuous functions.

Proof. Let N be an open set of M containing $z(x)$ for each $x \in L$. Since M is regular, there exists an open set C in Y containing $z(x)$ such that $Cl(C) \subset N$. Since z is contra $e-G-$continuous, then there exists $U \in EGO(L)$ containing x such that $z(U) \subset Cl(C)$. Then $f(U) \subset Cl(C) \subset N$. that, z is $e-G-$continuous. □

Definition 3.14. An grill with the space Y which carries topology τ is named $e-G-$connected if Y not equal the union of two disjoint non-null $e-G-$open subsets of Y.

Theorem 3.15. Let $z:(L,\tau,G)\rightarrow(M,\sigma)$ be a contra $e-G-$continuous functions from a $e-G-$connected onto any space M, then M is not a discrete space.

Proof. Let M be a discrete. Suppose that A be an appropriate non-null clopen set in M. Then $z^{-1}(A)$ is appropriate non-null $e-G-$clopen subset of L, which conflict with L is $e-G-$connected. □

Theorem 3.16. If $z:(L,\tau,G)\rightarrow(M,\sigma)$ is a contra $e-G-$continuous surjection functions and X is $e-G-$connected, then Y is connected.

Proof. Let $z:(L,\tau,G)\rightarrow(M,\sigma)$ be a contra $e-G-$continuous functions from a $e-G-$connected L onto any space M. Suppose that M is disconnected. Then $M=A\cup B$, where A and B are non-null clopen sets in M with $A\cap B=\varphi$. Because of z is contra $e-G-$continuous, that $z^{-1}(A)$ and $z^{-1}(B)$ are $e-G-$open non-null sets in L with $z^{-1}(A)\cup z^{-1}(B)=f^{-1}(A\cup B)=f^{-1}(M)=L$ and $z^{-1}(A)\cap z^{-1}(B)=z^{-1}(A\cap B)=z^{-1}(\varphi)=\varphi$. This leads to L is not $e-G-$connected, this is a contradiction. Then M is connected. □

Definition 3.17. A function $z:(L,\tau,G)\rightarrow(M,\sigma)$ is called almost- $e-G-$continuous if $z^{-1}(V)\in EGO(X)$ for every regular open set V of M.

Definition 3.18. A function $z:(L,\tau,G)\rightarrow(M,\sigma)$ is called pre- $e-G-$open if $z^{-1}(V)$ is $e-G-$open for every $e-$open set V of M.

Definition 3.19. The $e-G-$boundary of a subset A of a space L denoted by $e-G-B(A)$, is defined as $e-G-B(A)=\Psi_e(A)\cap\Psi_e(L\setminus A)$.

Theorem 3.20. For each $x\in L$ at $z:(L,\tau,G)\rightarrow(M,\sigma)$ is not contra-$e-G-$continuous is identical with the union of $e-G-$boundary of the inverse images of closed sets of M containing $z(x)$.

Proof. Firstly, let z be not satisfy the condition of contra-$e-G-$continuous at a point $x\in L$. Then there exists a closed set F of M containing $z(x)$ such that $z(U)\cap(M\setminus F)\neq\varphi,\forall U\in EGO(L,x)$, which leads to $U\cap z^{-1}(M\setminus F)\neq\varphi$, that by Theorem 3.9. Therefore, $x\in\Psi_e(z^{-1}(M\setminus z))=\Psi_e(L\setminus z^{-1}(F))$. Also, since $x\in z^{-1}(F)$, we get $x\in\Psi_e(z^{-1}(F))$ and so, $x\in e-G-B(z^{-1}(F))$. Secondly, let $x\in e-G-B(z^{-1}(F))$ for some closed set F of M containing $z(x)$ and z is contra-$e-G-$continuous at a point x. Then there exists $U\in EGO(L,x)$ such that $z(U)\subset F$. So, $x\in U\subset z^{-1}(F)$ and then $x\in Int_{eG}(z^{-1}(F))\subset L\backslash e-G-B(z^{-1}(F))$, which is a contradiction. So z is not contra-$e-G-$continuous at x.

□

Theorem 3.21. If $z:(L,\tau,G)\rightarrow(M,\sigma)$ is pre-$e-G-$open, contra-$e-G-$continuous. Then, its almost-$e-G-$continuous.

Proof. Let $x \in L$ and V be an open set containing $z(x)$. Because of z is contra-$e-G-$continuous, then by Theorem 3.9, there exists $U \in EGO(L,x)$ such that $z(U) \subset Cl(V)$. Also, since z is pre-$e-G-$open, $z(U)$ is $e-\sigma-$open in M. Therefore, $z(U) = Int_{eG}(z(U))$ and hence $z(U) \subset Int_{eG}(Cl(z(U))) \subset Int_{eG}(Cl(V))$. So z is almost-$e-G-$continuous. □

Definition 3.22. A mapping $z : (L,\tau,G) \to (M,\sigma)$ is named almost weakly-$e-G-$continuous if for each $x \in L$ and for each open set V of M containing $z(x)$, there exist $U \in EGO(L,x)$ such that $z(U) \subset Cl(V)$.

Theorem 3.23. If $z : (L,\tau,G) \to (M,\sigma)$ is contra $e-G-$continuous then, z is almost weakly-$e-G-$continuous.

Proof. For any open set V of M, $Cl(V)$ is closed in M. Since z is contra $e-G-$continuous, $z^{-1}(Cl(V))$ is $e-G-$open set in L. We take $U = z^{-1}(Cl(V))$, then $z(U) \subset Cl(V)$. Hence z is almost weakly-$e-G-$continuous. □

Example 3.24. Let $L = \{1,2,3\}$ with a topology $\tau = \{L, \varphi, \{1\}, \{2\}, \{1,2\}\}$ and a grill $G = \{\{1,2\}, L\}$. If $z : (L,\tau,G) \to (L,\tau)$ is the identity function. We notice that z is almost weakly-$e-G-$continuous. But z is not contra $e-G-$continuous since $z^{-1}(V)$ is not $e-G-$closed at $V = \{1,2\}$.

Definition 3.25. An a space L which carries topology τ with grill G is called $e-G-T_1$ if for each $x, y \in L$ such that $x \neq y$, there exist $e-G-$open sets U and V containing x and y, respectively, such that $y \notin U$ and $x \notin V$.

Definition 3.26. An a space L which carries topology τ with grill G is called $e-G-T_2$ if for each $x, y \in L$ such that $x \neq y$, there exist $e-G-$open sets U and V containing x and y, respectively, such that $U \cap V = \varphi$.

Theorem 3.27. Let $z : (L,\tau,G) \to (M,\sigma)$ be a contra $e-G-$continuous injection. If M is a Urysohn space, then L is $e-G-T_2$.

Proof. If $x, y \in L$ such that $x \neq y$, then $z(x) \neq z(y)$. Since M is a Urysohn space, there exist open sets U and V of m such that $z(x) \in U, z(y) \in V$ and $Cl(U) \cap Cl(V) = \varphi$. Since z is contra $e-G-$continuous at x and y, there exist $e-G-$open sets A and B in L such that $x \in A, y \in B$ and $z(A) \subset Cl(U), z(B) \subset Cl(V)$. Then, $z(A) \cap z(B) = \varphi$, so $A \cap B = \varphi$ and L is $e-G-T_2$.

□

Definition 3.28. A mapping $z : (L,\tau,G) \to (L,\sigma,\jmath)$ is named be $e-G-$irresolute if $z^{-1}(B) \in EGO(L)$ for each $B \in EGO(M)$.

Theorem 3.29. If $z : (L,\tau,G) \to (M,\sigma,\jmath)$ and $g : (M,\sigma,\jmath) \to (N,\zeta)$ is two maps then,the following are holds:
(1) If z is contra $e-G-$continuous function and g is a continuous function, then $g \circ z$ is contra $e-G-$continuous.
(2) If z is $e-G-$irresolute function and g is a contra $e-G-$continuous function, then $g \circ z$ is contra $e-G-$continuous.

Proof. (1) For $x \in L$, let W be any closed set of N containing $(g \circ z)(x)$. Since g is continuous, $V = g^{-1}(W)$ is closed in M. Also, since z is contra $e - G-$continuous, there exists $U \in EGO(L, x)$ such that $z(U) \subset V$. Therefore $(g \circ z(U)) \subset W$. Hence, $g \circ z$ is is contra $e - G-$continuous.

(2) For $x \in L$, let W be any closed set of N containing $(g \circ z)(x)$. Since g is a contra $e - G-$continuous, there exists $V \in E\jmath O(M, z(x))$ such that $g(V) \subset W$. Also, since z is $e - G-$irresolute there exist $U \in EGO(L, x)$ such that $z(U) \subset V$. This shows that $(g \circ z(U)) \subset W$. Hence, $g \circ z$ is contra $e - G-$continuous. □

Definition 3.30. A space (L, τ, G) is named $e - G-$normal if each pair of nonempty disjoint closed sets can be separated by disjoint $e - G-$open sets. Also, it is called ultra normal if each pair of non-null disjoint closed sets can be separated by disjoint clopen sets.

Theorem 3.31. If $z : (L, \tau, G) \to (M, \sigma, \jmath)$ is a contra $e - G-$continuous, closed injection and M is ultra normal, then L is $e - G-$normal.

Proof. Let C_1 and C_2 be disjoint closed subsets of L. Since z is closed and injective, $z(C_1)$ and $z(C_2)$ are separated by disjoint clopen sets V_1 and V_2, respectively. Since z is contra $e - G-$continuous, $z^{-1}(V_1)$ and $z^{-1}(V_2)$ are $e - G-$open, with $C_1 \subset z^{-1}(V_1), C_2 \subset z^{-1}(V_2)$ and $z^{-1}(V_1) \cap z^{-1}(V_2) = \varphi$. Hence, L is $e - G-$normal. □

Definition 3.32. A topological space (L, τ) is ultra Hausdorff [19] if for each pair of distinct points x and y of L there exist closed sets U and V such that $x \in U$ and $y \in V$ and $U \cap V = \varphi$. A topological space (L, τ) is weakly Hausdorff [20]if each element of L is the intersection of regular closed sets of L.

Theorem 3.33. If $z : (L, \tau, G) \to (M, \sigma, \jmath)$ is a contra $e - G-$continuous injection and M is ultra Hausdorff, then L is $e - G - T_2$.

Proof. Let $x, y \in L$ where $x \neq y$. Then, since z is an injection and M is ultra Hausdorff, $z(x) \neq z(y)$ and there exist disjoint closed sets U and V containing $z(x)$ and $z(y)$ respectively. Also, since f is contra $e - G-$continuous, $z^{-1}(U) \in EGO(L, x)$ and $z^{-1}(V) \in EGO(L, y)$ with $z^{-1}(U) \cap z^{-1}(V) = \varphi$. This shows that L is $e - G - T_2$. □

4 Conclusion

The concept of grill with topological space generating finer topological space τ_G.vIt helps us to measure the things that was difficult to measure and it is also used in many applications such as computer and information systems. Also, $e-G$-continuous and contra $e-G-$continuous functions is very important so, we have introduced the definition of $e - G-$continuous and contra $e - G-$continuous functions. Characterizations and basic properties of contra $e - G-$continuous functions are discussed.

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