http://www.newtheory.org ISSN: 2149-1402

Journal of New Theory

Received: 28.10.2017 *Year*:**2017**, *Number*: **17**, *Pages*: **82-97**
Published: 09.11.2017 *Original Article*

A TWO ECHELON SUPPLY CHAIN INVENTORY MODEL FOR A DETERIORATING ITEM WHEN DEMAND IS TIME AND CREDIT SENSITIVE

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Abstaract − While formulating inventory models with trade credit facilities, most of the inventory modelers so far assumed that the up-stream suppliers offer the down-stream retailers a fixed credit period. However, some times the retailers also provide a credit period to the end customers to promote the market competition. In the present paper, a supply chain inventory model of a deteriorating item has been formulated under two levels of trade credit policy, namely by the suppliers to the retailers and also by the retailers to the end customers with default risk consideration. In the proposed model, it has been assumed the demand is dependent linearly on time as well as on credit facility offered by the retailers. The model is solved analytically to find the retailer's optimal credit period and optimal cycle time which maximizes the total average profit. The solution of the model is illustrated with the help of a numerical example. The sensitivity of the optimal solution is also carried out with respect to the changes to the different system parameters.

Keywords − *Inventory, Deterioration, Time and trade credit dependent demand, Supply chain management.*

1 Introduction

In formulating inventory models, two factors are of growing interest among the researchers. One is the deterioration rate of items and another is the variation of demand function over time. Now it is well accepted that deterioration of items should not be neglected for all the cases. Items like chemicals, foodstuff, pharmaceuticals and also electronic components deteriorate significantly. In the first inventory model developed by Harris(1915), the deterioration of item was neglected simplicity. Many inventory models have been developed without considering the deterioration of items. Researchers like Whitin (1957), Shah and Jaiswal (1977), Dave and patel (1981), Dave (1986), Aggarwal(1978), RoyChaudhuri and Chaudhuri(1983), Dave(1979), Bahari-Kashani(1989), Wee (1995), Hariga (1996a, 1996b), Benkherouf (1995, 1997, 1998), Hariga and Al-Alyan (1997), Chang and Dye (1999a, 1999b) assumed that items deteriorate at a constant rate, for simplicity.The assumption of constant deterioration rate was relaxed by Covert and Philip (1973), Philip (1974), Misra (1975), Deb and Chaudhuri (1986), Goswami and Chaudhuri (1992), Giri, Goswami and Chaudhuri (1996), Ghosh and Chaudhuri (2006), Sana (2010), Sarkar(2012), Sarkar and Sarkar (2013a), Sarkar and Sarkar (2013b), Sarkar et al. (2013) etc. They all assumed time-dependent deterioration in their model. On the other hand a production inventory model with probabilistic deterioration in supply chain management was discussed by Sarkar(2013).

In the first EOQ model developed by F.W. Harris in 1915, the demand of items was assumed to be constant. But demand of an item is always a dynamic state. can not be constant throughout the inventory cycle. This unrealistic assumption of constant demand was relaxed by Silver and Meal (1969). They were the first to consider time varying demand. Later, Silver and Meal (1973) developed an approximate solution procedure, known as *Silver-Meal-Heuristic*, for the case of general deterministic time-varying demand pattern. Donaldson (1977) was the first to derive an analytical solution procedure of obtaining the optimal number of replenishments and optimal replenishment times of an EOQ model with linearly time-dependent demand pattern over a finite time horizon. Later, Silver (1979) takes a special case of a positive linear trend and applies his heuristic to the problem of Donaldson (1977). To determine the optimal reorder times, much computational effort was needed in that model. Rather, simpler and less complicated techniques were used to solve the same model by the researchers like Mitra, Cox and Jesse (1984), Henry (1979), Richie (1980, 1984, 1985), etc. Several researchers like Dave and Patel(1981), McDonald(1979), Goyal (1985), Sachan (1984), etc., contributed in this direction.

Some researchers also devoted their attention to inventory replenishment problems with exponentially time-varying demand patterns. Some of the contributions in this direction came from the researchers like Hollier and Mak (1983), Aggarwal and Bahari-Kashani (1991), Hariga and Benkherouf (1994), Wee (1995), etc. Time-dependent quadratic demand function was first used in the model of Khanra and Chaudhuri (2003). Later on Ghosh and Chaudhuri (2006), Manna, Chaudhuri and Chiang (2007), etc. contributed in this direction. The distinct advantage of taking time-dependent quadratic demand function is that it accommodates all the three types of increasing, decreasing or constant demand function depending upon the parameters of the demand function.

During the last few decades, many inventory models have been developed considering trade credit facility in their models. A single-item inventory model under permissible delay in payments was discussed by Goyel(1985). This model was further extended by Aggarwal and Jaggi(1995) considering deterioration of item. The model of Aggarwal and Jaggi(1995) was further extended by Jamal et al.(1997) considering inventory shortages. Later, Teng (2002) suggested that it make economic sense for a well-established retailer to order less quantity and take the benefits of payment delay more frequently considering the difference between selling price and purchasing cost as in the the model of Goyel (1985). Chang et al. (2003) developed an EOQ model for deteriorating items under supplier credits linked to ordering quantity. The up-stream and down-stream trade credits in which the length of down-stream trade credit period is less than or equal to the length of the up-stream trade credit period was discussed by Huang (2003). A more generalized result of the above model was proposed by Teng and Goyal (2007). Khanra et al (2011) developed an EOQ model for a deteriorating item under permissible delay in payment. An EPQ model with time proportional deterioration and permissible delay in payment was discussed by Sarkar et al.(2013). A two-level trade credit policy in the fuzzy sense was discussed by Mahata and Goswami (2007). The model of Huang (2003) was further extended by Huang (2007) model to investigate the two-level trade credit policy in the EPQ frame work. Inventory model under partial delay in payment and partial backordering was discussed by Taleizadeh et al. (2013). Retailer's partial trade policy was discussed by Chen et al. (2014). Taleizadeh and Nematollahi (2014) investigated the effects of time value of money and inflation on the optimal ordering policy for a perishable item over the finite time horizon under backordering and delay in payment payment. Ouyang et al. (2015) discussed an integrated inventory model with the order-size dependent trade credit and a constant demand. A partial up-stream advanced payment and partial downstream delayed payment in a three-level supply chain model was proposed by Lashgari et al. (2015). Inventory model model with backordering under hybrid linked-to-order multiple advance payments and delayed payment was discussed by Zia and Taleizadeh (2015). Several contributions came from the researchers Chang and Teng (2004), Chung (2008), Chung and Liao (2004), Goyal et al. (2007), Huang and Hsu (2008).

The dynamic behavior of demand function was not considered by the above researchers in their model while considering trade credit facility. The dynamic behavior of the demand function as time dependent demand or stock dependent demand were discussed by some researchers in case of trade credit facility. Time dependent demand function was considered in the model of Sarkar (2012) and Teng et al. (2012) in case case of trade credit financing. Soni (2013) developed the optimal replenishment policies under trade credit financing by considering price and stock-sensitive demand. The inventory model with the credit-linked demand were discussed by Jaggi et al. (2008) and Jaggi et al. (2012). Lou and Wang (2012) discussed an inventory model with considering trade credit with a positive correlation of market sales, but with negatively correlated with credit risk. Giri and Maiti(2013) discuss the supply chain model with price and trade credit sensitive demand by considering a retailer's shares as a fraction of the profit earned during the credit period. Wu et al.(2014) studied optimal credit period and lot size model by considering demand dependent delay in payment with default risk for deteriorating items. A deteriorating inventory model with stock-dependent demand and shortages under two-level trade credit was discussed by Annadurai and Uthayakumar (2015). A sustainable trade credit policy with credit -linked demand and credit risk considering the carbon emission constraints was discussed by Dye and Yang (2015).

In case of any supplier-retailer-buyer supply chain models, sometimes the supplier offers the retailer a trade credit of M periods, and the retailer in turn provides a trade credit of N periods to the buyer. This stimulates sales and consequently earns revenue for the supplier as well as the retailer. On the other hand, the reduction in inventory also reduces holding cost for both supplier and retailer. But, when the retailer offers the customers a trade credit period, it also increases opportunity cost (the capital opportunity loss during credit period) and default risk. In a product life cycle, at the maturity stage demand comes to more or less constant and during the growth stage of the product life cycle (especially for newly launched high-tech products), the demand function increases with time. Therefore, at the end of the product life cycle, it is necessary to stimulate the demand of the product and consequently, the proposed result of this paper is suitable for both the growth and maturity stages of a product life cycle. In the proposed paper, an economic order quantity model for the retailer has been derived when the supplier provides an up-stream trade credit and the retailer also offers a down-stream trade credit. The retailer's down-stream trade credit to the buyer increases sales and revenue but at the same time it also increases opportunity cost and default risk. The demand is dependent linearly on time as well as on the trade credit period offered by the retailer. Also it has been assumed that a constant fraction on the on hand inventory deteriorates with time. The retailer's annual profit is maximized to obtain the the optimal replenishment period and the optimal trade credit period offered to the customers by the retailer. The optimal solution of the model is solved analytically and then it is illustrated with the help of a numerical example. The sensitivity of the optimal solution with respect to the changes of different system parameters is also studied.

2 Notations and Assumptions

The inventory model is developed considering the following *notations* which includes *decision variables* and *Parameters* :

Decision Variables

T^* : the optimal replenishment period for the retailer

N^* : the optimal trade credit period offered to the customers by the retailer

Parameters

A: the ordering cost per order for the retailer.

h: the holding cost per unit per year for the retailer, excluding interest charges.

c: the purchasing cost per unit of the retailer.

c_1 the cost of deterioration per unit item.

s: the selling price per unit of the retailer ($s > c$).

I_e: the interest earned per \$ per year by the retailer.

I_p: the interest charged per \$ in stocks per year to the retailer.

M: the retailer's trade credit period offered by supplier in years.

N: the customer's trade credit period offered by retailer in years.

T: the replenishment time period for the retailer.

Q: the order quantity fr the retailer.

$I(t)$: the inventory level at any time t.

θ: the deterioration rate of items.

$D(t, N)$: the annual demand rate, which is a function of time t and trade credit period N.

$\pi(N, T)$: the total profit per year for the retailer.

The following *assumptions* have been used in developing the model:

1. Initially the inventory level is zero.
2. Lead time is zero.
3. The deterioration rate of items is constant over time.
4. The holding cost, ordering cost and shortage cost remain constant over time.
5. The planning horizon is finite.
6. The rate of replenishment is infinite.
7. The supplier provides the trade credit period M to the retailer, and the retailer offers its customers trade credit period N.
8. Demand rate $D(t, N)$ is a function of time t and the credit period N and is given by $D(t, N) = a + bt + \lambda(N), a, b > 0$. Since in real practice, the demand increases with with the increase of trade credit period, we must have $\lambda^{'}(N) > 0$.

7\. There is no repair or replacement of the deteriorated items.

8\. Longer is credit period offered by the retailer to the customer, higher is the default risk to the retailer. It is assumed that the rate of the default risk given the credit period offered by the retailer is $F(N) = 1 - e^{-kN}$, where $k > 0$. This default risk pattern was also used by Lou and Wang (2012) and Zhang et al. (2014).

3 Formulation and Solution of the Model

The inventory starts with Q units at time $t = 0$, when the retailer orders and receives the order quantity Q units from the supplier. The procurement made at time $t = 0$ is partly used to satisfy the demand and the rest of the procurement accounts for the deterioration in $[0, T]$. The inventory level gradually falls to zero at T. The cycle repeats itself for every cycle.

The instantaneous inventory level $I(t)$ is given by the following differential equation

$$\frac{dI(t)}{dt} + \theta I(t) = -D(t,N) = -a - bt - \lambda(N), \quad 0 \le t \le T \tag{1}$$

with the boundary condition $I(T) = 0$.

The solution of the differential equation (1) is given by

$$I(t) = \frac{b}{\theta^2} - \frac{(a + bt + \lambda(N))}{\theta} + \frac{\theta}{b}(a + bT + \lambda(N))e^{\theta(T-t)}, \qquad 0 \le t \le T \tag{2}$$

Therefore, the retailers order quantity Q is given by

$$Q = I(0) = \frac{b}{\theta^2} - \frac{(a + \lambda(N))}{\theta} + \frac{\theta}{b}(a + bT + \lambda(N))e^{\theta T} \tag{3}$$

To find the retailer's annual profit per cycle, we need to calculate the revenue, ordering cost, purchasing cost, cost od deterioration, holding cost, interest charged and interest earned.

1. Average annual ordering cost is $\frac{A}{T}$.
2. Average annual purchasing cost per cycle is

$$\frac{cQ}{T} = \frac{c}{T}[\frac{b}{\theta^2} - \frac{(a + \lambda(N))}{\theta} + \frac{\theta}{b}(a + bT + \lambda(N))e^{\theta T}]$$

3. Average annual holding cost excluding interest charges per cycle is

$$\begin{aligned}\frac{h}{T}\int_0^T I(t)dt &= \frac{h}{T}\int_0^T \frac{b}{\theta^2} - \frac{(a + bt + \lambda(N))}{\theta} + \frac{\theta}{b}(a + bT + \lambda(N))e^{\theta(T-t)} \\ &= \frac{h}{T}[\{(\frac{b}{\theta^2} - \frac{a}{\theta} - \frac{\lambda(N)}{\theta}) + (\frac{a\theta}{b} + \theta T + \frac{N(\lambda))}{b}e^{\theta T}\}T - \frac{bT^2}{2\theta} \\ &\quad - \frac{(a + bT + \lambda(N))}{b}(e^{-\theta T} - 1)]\end{aligned}$$

4. The sales revenue considering default risk is

$$\frac{se^{-kN}}{T}\int_0^T D(t,N)dt$$
$$= se^{-kN}(a+\frac{b}{2}T+\lambda(N))$$

5. The cost of deteriorated items is

$$c_1\{I(0)-\int_0^T I(t)dt\}$$
$$= c_1[(-\frac{b}{\theta^2}+\frac{a}{\theta}+\frac{\lambda(N)}{\theta})(1-T)+(\frac{a\theta}{b}+\theta T+\frac{\theta\lambda(N)}{b})(e^{\theta T}-T)$$
$$+\frac{bT^2}{2\theta}+\frac{a+bT+\lambda(N)}{b}(e^{-\theta T}-1)]$$

6. The interest earned and interest charged is calculated considering two cases namely $M \leq N$ and $M \geq N$ separately.

Case 1: $M \leq N$

In this case, we have two possible sub cases subcases:(1) $T+N \leq M$ and (2) $T+N \geq M$. Now we discuss two subcases separately
Sub-case 1.1: $T+N \leq M$. In this case the retailer receives the sales revenue at time T + N and is able to pay off the total purchasing cost at time M. Therefore, there is no interest charged. During the period $[N, T+N]$, the retailer can obtain the interest earned on the sales revenues received and on the full sales revenue during the period $[T+N, M]$. Therefore, the annual interest earned is

$$\frac{sI_e}{T}[\int_0^{T+N}\int_N^{t+T} D(u-N,N)dudt+(M-T-N)\int_0^T D(u,N)du]$$
$$= \frac{sI_e}{T}[\frac{(a+\lambda(N))T^2}{2}+\frac{bT^3}{6}+(M-T-N)\{(a+\lambda(N))T+\frac{bT^2}{2}\}] \quad (4)$$

Therefore, the retailer's annual total profit $\pi_1(N,T)$ is given by $\Pi_1(N,T)$= Net revenue after default risk-annual ordering cost - annual purchasing cost - annual holding cost + annual interest earned - interest charged- cost of deterioration

Therefore

$$\Pi_1(N,T) = se^{-kN}(a+\frac{b}{2}T+\lambda(N)) + \frac{sI_e}{T}[\frac{(a+\lambda(N))T^2}{2}+\frac{bT^3}{6}$$
$$+(M-T-N)\{(a+\lambda(N))T+\frac{bT^2}{2}\}] - \frac{A}{T} - \frac{c}{T}[\frac{b}{\theta^2} - \frac{(a+\lambda(N))}{\theta}$$
$$+\frac{\theta}{b}(a+bT+\lambda(N))e^{\theta T}] - \frac{h}{T}[\{(\frac{b}{\theta^2}-\frac{a}{\theta}-\frac{\lambda(N)}{\theta})+(\frac{a\theta}{b}+\theta T+\frac{N(\lambda))}{b}e^{\theta T}\}T - \frac{bT^2}{2\theta}$$
$$-\frac{(a+bT+\lambda(N))}{b}(e^{-\theta T}-1)] - c_1[(-\frac{b}{\theta^2}+\frac{a}{\theta}+\frac{\lambda(N)}{\theta})(1-T)$$
$$+(\frac{a\theta}{b}+\theta T+\frac{\theta\lambda(N)}{b})(e^{\theta T}-T)+\frac{bT^2}{2\theta}+\frac{a+bT+\lambda(N)}{b}(e^{-\theta T}-1)] \quad (5)$$

Sub-case 1.2: $M \le T+N$. During the interval $[M, T+N]$, the retailer must pay the interest for the items unsold. Therefore, have interest charged per $ in stocks per year to the retailer as

$$\frac{cI_p}{T}\int_M^{T+N} I(t-N)dt = \frac{cI_p}{T}\int_{M-N}^{T} I(t,N)dt$$
$$= \frac{cI_p}{T}[\frac{a+\lambda(N)}{2}\{T-(M-N)\}^2 + \frac{bT^2}{2}\{T-(M-N)\} - \frac{b}{6}\{T^3-(M-N)^3\}] \quad (6)$$

Also the retailer can earn interest from the delayed payment during period [N, M]. Therefore, we have the interest earned by the retailer is

$$\frac{sI_e}{T}\int_N^M\int_N^{t+N} D(u-N,N)dudt = \frac{sI_e}{T}[\frac{a+\lambda(N)}{2}(M-N)^2+\frac{b}{6}(M-N)^3] \quad (7)$$

As in *subcase 1.1*, the retailer's annual total profit is

$$\Pi_2(N,T) = se^{-kN}(a+\frac{b}{2}T+\lambda(N)) + \frac{sI_e}{T}[\frac{a+\lambda(N)}{2}(M-N)^2+\frac{b}{6}(M-N)^3]$$
$$-\frac{cI_p}{T}[\frac{a+\lambda(N)}{2}\{T-(M-N)\}^2 + \frac{bT^2}{2}\{T-(M-N)\} - \frac{b}{6}\{T^3-(M-N)^3\}]$$
$$-\frac{A}{T} - \frac{c}{T}[\frac{b}{\theta^2} - \frac{(a+\lambda(N))}{\theta} + \frac{\theta}{b}(a+bT+\lambda(N))e^{\theta T}]$$
$$-\frac{h}{T}[\{(\frac{b}{\theta^2}-\frac{a}{\theta}-\frac{\lambda(N)}{\theta})+(\frac{a\theta}{b}+\theta T+\frac{N(\lambda))}{b}e^{\theta T}\}T - \frac{bT^2}{2\theta}$$
$$-\frac{(a+bT+\lambda(N))}{b}(e^{-\theta T}-1)] - c_1[(-\frac{b}{\theta^2}+\frac{a}{\theta}+\frac{\lambda(N)}{\theta})(1-T)$$
$$+(\frac{a\theta}{b}+\theta T+\frac{\theta\lambda(N)}{b})(e^{\theta T}-T)+\frac{bT^2}{2\theta}+\frac{a+bT+\lambda(N)}{b}(e^{-\theta T}-1)] \quad (8)$$

Case 2: $N \geq M$. Since $M \leq N$, there is no interest earned. The retailer must finance all the purchasing cost from $[M, N]$ and pay off the loan from $[N, T+N]$. Therefore, the interest charged per cycle is

$$\begin{aligned}
&\frac{cI_p}{T}[(N-M)Q + \int_N^{T+N} I(t-N)dt] \\
&\frac{cI_p}{T}[(N-M)(\frac{b}{\theta^2} - \frac{(a+\lambda(N))}{\theta} + \frac{\theta}{b}(a+bT+\lambda(N))e^{\theta T}) \qquad (9)
\end{aligned}$$

Therefore, the retailer's average total profit is given by

$$\begin{aligned}
\Pi_3(N,T) &= se^{-kN}(a+\frac{b}{2}T+\lambda(N)) - \frac{A}{T} - \frac{c}{T}[\frac{b}{\theta^2} - \frac{(a+\lambda(N))}{\theta} + \frac{\theta}{b}(a+bT+\lambda(N))e^{\theta T}] \\
&-\frac{h}{T}[\{(\frac{b}{\theta^2} - \frac{a}{\theta} - \frac{\lambda(N)}{\theta}) + (\frac{a\theta}{b} + \theta T + \frac{N(\lambda))}{b}e^{\theta T}\}T - \frac{bT^2}{2\theta} - \frac{(a+bT+\lambda(N))}{b}(e^{-\theta T}-1)] \\
&-c_1[(-\frac{b}{\theta^2} + \frac{a}{\theta} + \frac{\lambda(N)}{\theta})(1-T+(\frac{a\theta}{b} + \theta T + \frac{\theta\lambda(N)}{b})(e^{\theta T}-T) + \frac{bT^2}{2\theta} \\
&+\frac{a+bT+\lambda(N)}{b}(e^{-\theta T}-1)] - \frac{cI_p}{T}[(N-M)(\frac{b}{\theta^2} - \frac{(a+\lambda(N))}{\theta} + \frac{\theta}{b}(a+bT+\lambda(N))e^{\theta T}) \qquad (10)
\end{aligned}$$

Therefore, the objective is to determine optimal customer's trade credit period N^* offered by retailer in years and the optimal replenishment time period T^* for the retailer such that the annual total profit $\Pi_i(N,T)$ for $i = 1,2,3$ is maximized.

Treating N and T as decision variables, the necessary conditions for the minimization of the average system cost are

$$\frac{\partial \Pi_i}{\partial N} = \frac{\partial \Pi_i}{\partial T} = 0, i = 1,2,3. \qquad (11)$$

The optimal values N^* of N and T^* of T are obtained by solving equations (11). The sufficient conditions that these values minimize $\Pi_i(N,T)$ are

$$\frac{\partial^2(\Pi_i)}{\partial N^{*2}}\frac{\partial^2(\Pi_i)}{\partial T^{*2}} - \left(\frac{\partial^2(\Pi_i)}{\partial N^*\partial T^*}\right)^2 > 0 \qquad (12)$$

$$\frac{\partial^2(\Pi_i)}{\partial N^{*2}} > 0, \quad \frac{\partial^2(\Pi_i)}{\partial T^{*2}} > 0, i = 1,2,3. \qquad (13)$$

Equations (11) can only be solved with the help of a computer oriented numerical technique for a given set of parameter values. Once N^* and T^* are obtained, we get ${\Pi_i}^*, i = 1,2,3$ from (5), (8) and (10) respectively.

4 Numerical Examples

Example 1: Let a = 100 units/year, b = 0.2/year, d = 1/year, θ = 0.01, u = 0.1/year, s = \$20/unit, k = 0.2/year, A = \$10/order, M = 0.5/year, h = \$5/year, c =\$10/unit, I_e = 0.09, I_p= 0.14. Using software Mathematica 7.0, we have the optimal solution as follow:

$N_1{}^*$ =0.0 years, $T_1{}^*$ =0.1487 years ; and $\Pi_1{}^*(N.T)$ = \$994.33

$N_2{}^*$ =0.0 years, $T_2{}^*$ =0.2332 years ; and $\Pi_2{}^*(N.T)$ = \$973.54

$N_1{}^*$ =0.5 years, $T_1{}^*$ =0.3177 years ; and $\Pi_1{}^*(N.T)$ = \$813.87

Therefore, the retailer's optimal solution is $N_1{}^*$ =0.0 years, $T_1{}^*$ =0.1487 years ; and $\Pi_1{}^*(N.T)$ = \$994.33.

Example 2 Taking the same parameter values as in Example 1,except d =5/year, u =5/year, s = \$30/unit, k = 0.5/year, the optimal solution can be obtained as follows.

$N_1{}^*$ =0.4327 years, $T_1{}^*$ =0.1883 years ; and $\Pi_1{}^*(N.T)$ = \$2444.71

$N_2{}^*$ =0.0 years, $T_2{}^*$ =0.5 years ; and $\Pi_2{}^*(N.T)$ = \$2573.53

$N_1{}^*$ =1.4421 years, $T_1{}^*$ =0.0678 years ; and $\Pi_1{}^*(N.T)$ = \$27793.53

Therefore, the retailer's optimal solution is $N_1{}^*$ =1.4421 years, $T_1{}^*$ =0.0678 years; and $\Pi_1{}^*(N.T)$ = \$27793.53

5 Conclusion

From the first development of the inventory model by Harris in 1915, Inventory modellers and researchers are still very much curious about the nature of demand function. In the inventory model developed by Harris, the demand of item was assumed to be constant. This was taken only because to make the model simple. But in reality, there is hardly available product in market whose demand remains constant over time. When it was realized that this inventory model has very little real applications, inventory modellers tried to investigate which are the parameters on which the demand function is dependent. Numerous inventory models have been developed considering linear, exponential, quadratic and general nonlinear time dependent demand functions. Some researchers also understood that a display of large quantity of goods may increase its sales. This means that in this case the demand is dependent on the stock of goods. This dynamic behavior of demand was discussed by several researchers. Now most of the inventory management researches believe that demand is affected by several factors such as price, time, inventory level, and delay in payment period etc. In today's high-tech products demand rate increases significantly during the growth stage. Moreover, the marginal influence of the credit period on sales is associated with the unrealized potential market demand. In this paper, we have developed an EOQ model of an deteriorating item with under two-level trade credit financing involving default risk by considering demand to a credit-sensitive and linear non-decreasing function of time. Our objective is to find optimal customer's trade credit period offered by retailer in years and optimal replenishment time period for the retailer. The model is solved analytically and the optimal solution is illustrated with the help of numerical examples.

This model can be extended by several ways, for example, we may extend the model to allow for shortages and partially backlogging. Also the effect of inflation rates on the economic order quantity can also be considered.

Acknowledgement

The author is grateful for infrastructure based support from Kazi Nazrul University, Asansol, West Bengal, Pin-713340.

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