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# PROPERTIES AND APPLICATIONS OF $\theta g^*\alpha$-CLOSED SETS IN TOPOLOGICAL SPACES

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**Abstaract** − In this paper we discussed properties and applications of $\theta g^*\alpha$-closed sets. $\theta g^*\alpha$-closed sets is introduced by Chandrasekar et al. Moreover we analyze some basic properties and applications of neighbourhoods, limit points, border, frontier and exterior of $\theta g^*\alpha$-closed sets.

***Keywords*** − *$\theta g^*\alpha$-closed sets, $\theta g^*\alpha$-neighbourhoods, $\theta g^*\alpha$-limit points, $\theta g^*\alpha$-border, $\theta g^*\alpha$-frontier, $\theta g^*\alpha$-exterior.*

# 1 Introduction

The concepts of $\theta$-closed set, $\delta$-closed set, first introduced by Velicko [16]. $\theta$-closed set have been studied intensively by many authors. Since the advent of these notions, several researches have been done which produced interesting results. $\theta$g-closed set introduced by Dontchev and Maki [7] in 1999. In 1965 Njastad [9] introduced $\alpha$-open sets. In [3], $\theta g^*\alpha$ closed set introduced by Chandrasekar et al. In this paper, we discussed properies and applications of $\theta g^*\alpha$-neighbourhoods, $\theta g^*\alpha$-limit points, $\theta g^*\alpha$- border, $\theta g^*\alpha$-frontier and $\theta g^*\alpha$-exterior.

# 2 Preliminary

Let us recall the following definition, which are useful in the sequel.

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**Definition 2.1.** A subset A of a space $(X, \tau)$ is called:

1. $\alpha$- closed set [9] if $cl(int(cl(A))) \subseteq A$
2. $\theta$-closed [16] if $A = cl_\theta(A)$, where $cl_\theta(A) = \{x \in X : cl(G) \cap A \neq \phi, G \in \tau$ and $x \in G\}$
3. a generalized closed (briefly, g-closed) set [8] if $cl(A) \subseteq G$ whenever $A \subseteq G$and $G$ is open in $(X, \tau)$.
4. a $\theta$-generalized closed (briefly, $\theta g$-closed) set [6] if $cl_\theta(A) \subseteq G$ whenever A $\subseteq G$ and G is open in $(X, \tau)$.
5. $\theta g^*\alpha$-closed set if $\alpha cl(A) \subseteq G$ whenever $A \subseteq G$ and $G$ is $\theta g$-open in $(X, \tau)$.

The complements of the above mentioned closed sets are their respective open sets.

**Definition 2.2.** The union (respectively intersection) of all $\theta g^*\alpha$-open (respectively $\theta g^*\alpha$-closed) sets, each contained in (respectively containing) a set A of X is called the $\theta g^*\alpha$-interior (respectively $\theta g^*\alpha$-closure) of A, which is denoted by $\theta g^*\alpha$-int(A) (respectively $\theta g^*\alpha$-cl(A)).

**Proposition 2.3.** If A and B are subsets of X, then

1. A is $\theta g^*\alpha$-open if and only if $\theta g^*\alpha$-int(A)=A.
2. $\theta g^*\alpha$-int(A) is $\theta g^*\alpha$-open.
3. A is $\theta g^*\alpha$-closed if and only if $\theta g^*\alpha$-cl(A)=A.
4. $\theta g^*\alpha$-cl(A) is $\theta g^*\alpha$-closed.
5. $\theta g^*\alpha$-cl(X∩ A)=X∩ $\theta g^*\alpha$-int(A).
6. $\theta g^*\alpha$-int(X∩ A)=X∩ $\theta g^*\alpha$-cl(A).
7. If A is $\theta g^*\alpha$-open in X and B is open in X, then A  B is $\theta g^*\alpha$-open in X.
8. A point x $\in$ $\theta g^*\alpha$-cl(A) if and only if every $\theta g^*\alpha$-open set in X containing x intersects A.
9. Arbitrary intersection of $\theta g^*\alpha$-closed sets in X is also $\theta g^*\alpha$-closed in X.

# 3 $\theta g^*\alpha$-Neighbourhoods

In this section, we define and study about $\theta g^*\alpha$-neighbourhood, and some of their properties are analogous to those for open sets.

**Definition 3.1.** Let $(X, \tau)$ be a topological space and let x∈ X. A subset N of X is said to be $\theta g^*\alpha$-neighbourhood of a point x∈ X if there exists a $\theta g^*\alpha$-open set G such that x∈G⊂ N

**Definition 3.2.** Let$(X, \tau)$ be a topological space and A be a subset of X .A subset N of X is said to be $\theta g^*\alpha$-neighbourhood of A if there exists a $\theta g^*\alpha$-open set G such that A∈G⊂ N.

The collection of all $\theta g^*\alpha$-neighbourhood of x∈ X is called the $\theta g^*\alpha$-neighbourhood system at x and shall be denoted by $\theta g^*\alpha$-N(x).

It is evident from the above definition that a $\theta g^*\alpha$-open set is a $\theta g^*\alpha$-neighbourhood of each of its points. But a $\theta g^*\alpha$-neighbourhood of a point need not be a $\theta g^*\alpha$-open set. Also every $\theta g^*\alpha$-open set containing x is a $\theta g^*\alpha$-neighbourhood of x.

**Theorem 3.3.** A subset of a topological space is $\theta g^*\alpha$-open if it is a $\theta g^*\alpha$-neighbourhood of each of its points.

*Proof:* Let a subset G of a topological space be $\theta g^*\alpha$-open. Then for every x∈G,x∈G⊂ N and therefore G is a $\theta g^*\alpha$-neighbourhood of each of its points. analogous to those for open sets. The converse of the above Theorem need not be true as seen from the following example.

**Example 3.4.** Let $(X, \tau)$ be topological space and X={a,b,c,d} with topology

$\tau$ ={X,$\phi$ ,{a},{b},{a, b},{b,c},{a,b,c}}.
$\theta g^*\alpha$-Cl(X) = {X, $\phi$ ,{c},{d},{a, d},{b, d},{c, d},{a, c, d},{a ,b ,d},{b, c, d}}
$\theta g^*\alpha$-O(X) = {X,$\phi$ ,{a},{b},{c},{a, b},{b, c},{a ,c},{a, b ,c},{a, b, d}}

the set{a, c, d} is the neighbourhood of {a, c}, since a, c ∈{a, c}⊂ {a, c, d}, and {a, c, d} is the $\theta g^*\alpha$-neighbourhood of each of its points.

**Theorem 3.5.** Let $(X, \tau)$ be a topological space. If A is a $\theta g^*\alpha$-closed subset of X and x∈ X\ A, then there exists a $\theta g^*\alpha$-neighbourhood N of x such that N ∩A=$\phi$

*Proof:* Since A is $\theta g^*\alpha$-closed, then X\ A is $\theta g^*\alpha$-open set in $(X, \tau)$ . By the above Theorem 3.3, X\ A contains a $\theta g^*\alpha$-neighbourhood of each of its points. Hence there exists a $\theta g^*\alpha$-neighbourhood N of x, such that N⊂ X\ A.That is, no point of N belongs to A and hence N∩A=$\phi$ .

**Theorem 3.6.** Let $(X, \tau)$ be a topological space and A⊆X. Then x∈ $\theta g^*\alpha$-cl(A) if and only if for any $\theta g^*\alpha$-neighbourhood N of x in $(X, \tau)$,A∩ N≠$\phi$.

*Proof:* Suppose x∈ $\theta g^*\alpha$-cl(A). Let us assume that there is a$\theta g^*\alpha$-neighbourhood N of the point x in $(X, \tau)$ such that N ∩ A=$\phi$ . Since N is a $\theta g^*\alpha$-neighbourhood of x in $(X, \tau)$ by definition of $\theta g^*\alpha$-neighbourhood there exists an $\theta g^*\alpha$-open set G of x such that x∈G⊂ N. Therefore we have G ∩ A=$\phi$ and so A ⊆ $H^c$. Since X\G is an $\theta g^*\alpha$-closed set containing A.We have by definition of $\theta g^*\alpha$-closure, $\theta g^*\alpha$-cl(A)⊆ X\G and therefore x∉$\theta g^*\alpha$-cl(A), which is a contradiction to hypothesis x∈ $\theta g^*\alpha$-cl(A). Therefore A∩ N≠$\phi$ . Conversely, Suppose for each $\theta g^*\alpha$-neighbourhood N of x in $(X, \tau)$ .A ∩ N≠$\phi$ . Suppose that x ∈ $\theta g^*\alpha$-cl(A). Then by definition of $\theta g^*\alpha$-cl(A), there exists a $\theta g^*\alpha$-closed set G of $(X, \tau)$ such that A⊆ G and x∉G. Thus x∈ X\G and X\ G is $\theta g^*\alpha$-open in $(X, \tau)$ and hence X\G is a $\theta g^*\alpha$-neighbourhood of x in $(X, \tau)$ .But A∩ (X\ G)= $\phi$ which a contradiction.Hence x∈ $\theta g^*\alpha$-cl(A).

**Theorem 3.7.** Let $(X, \tau)$ be a topological space and p$\in$ X. Let $\theta g^*\alpha$-N(p) be the collection of all $\theta g^*\alpha$-neighbourhoods of p. Then

1. $\theta g^*\alpha$-N(p)$\neq\phi$ and p each member of $\theta g^*\alpha$-N(p).
2. The intersection of any two members of $\theta g^*\alpha$-N(p) is again a member of $\theta g^*\alpha$-N(p).
3. If N $\in$ $\theta g^*\alpha$-N(p) and M$\subseteq$N, then M $\in\theta g^*\alpha$-N(p).
4. Each member N$\in$ $\theta g^*\alpha$ -N(p) is a superset of a member G$\in$ $\theta g^*\alpha$-N(p) where G is a $\theta g^*\alpha$-open set.

*Proof:*

1. Since X is a $\theta g^*\alpha$-open set containing p, it is a $\theta g^*\alpha$-neighbourhood of every p$\in$ X. Hence there exists at least one $\theta g^*\alpha$-neighbourhood namely X for each p $\in$X. Here $\theta g^*\alpha$-N(p)$\neq\phi$. Let N $\in\theta g^*\alpha$-N(p), N is a $\theta g^*\alpha$-neighbourhood of p.Then there exists a $\theta g^*\alpha$-open set G such that p$\in$ G $\subseteq$N.So p$\in$ N.Therefore p $\in$ every member N of $\theta g^*\alpha$-N(p).
2. Let N $\in$ $\theta g^*\alpha$-N(p) and M$\in\theta g^*\alpha$-N(p). Then by definition of $\theta g^*\alpha$-neighbourhood, there exists $\theta g^*\alpha$-open sets G and F such that p$\in$ N and p$\in$ F $\subseteq$ M. Hence p$\in$ G$\cap$F$\subseteq$ M $\cap$N, Note that G $\cap$ F is a $\theta g^*\alpha$-open set since intersection of $\theta g^*\alpha$-open sets is $\theta g^*\alpha$-open.Therefore it follows that N $\cap$ M is a $\theta g^*\alpha$ - neighbourhood of p. Hence N$\cap$ M $\theta g^*\alpha$-N(p).
3. If N$\in$ $\theta g^*\alpha$-N(p) then there is an $\theta g^*\alpha$-open set G such that p$\in$ G$\subseteq$ N.Since M $\cap$ N, M is a $\theta g^*\alpha$- neighbourhood of p. Hence M $\in$ $\theta g^*\alpha$-N(p).
4. Let N $\in$ $\theta g^*\alpha$-N(p).Then there exist an $\theta g^*\alpha$-open set G such that p$\in$ G$\subseteq$ N.Since G is $\theta g^*\alpha$-open and p $\in$ G, G is $\theta g^*\alpha$-neighbourhood of p. Therefore G $\in$ $\theta g^*\alpha$-N(p) and also G$\subseteq$ N.

# 4 $\theta g^*\alpha$-limit Points

In this section, we define and study about $\theta g^*\alpha$-limit point and $\theta g^*\alpha$-derived set of a set and show that some of their properties.

**Definition 4.1.** Let$(X, \tau)$ be a topological space and A be a subset of X.Then a point x$\in$ X is called a $\theta g^*\alpha$-limit point of A if and only if every $\theta g^*\alpha$-neighbourhood of x contains a point of A distinct from x. That is [N\x]$\cap$ A$\neq\phi$ for each $\theta g^*\alpha$-neighbourhood N of x.

Also equivalently if and only if every $\theta g^*\alpha$-open set G containing x contains a point of A other than x.

In a topological space $(X, \tau)$ the set of all $\theta g^*\alpha$-limit points of a given subset A of X is called a $\theta g^*\alpha$-derived set of A and it is denoted by $\theta g^*\alpha$-d(A).

**Theorem 4.2.** Let A and B be subsets of a topological space$(X, \tau)$. Then

1. $\theta g^*\alpha$-D($\phi$) = $\phi$
2. $\theta g^*\alpha$-D(A) ⊂ D(A) where D(A) is the derived set of A.
3. If A\B, then a $\theta g^*\alpha$-D (A)⊆ $\theta g^*\alpha$-D(B),
4. If x∈$\theta g^*\alpha$-D(A), then x∈ $\theta g^*\alpha$-D[A\{x}],
5. $\theta g^*\alpha$-D(A∪ B)⊃ $\theta g^*\alpha$-D(A)∪ $\theta g^*\alpha$-D(B),
6. $\theta g^*\alpha$-D(A∩ B)⊆$\theta g^*\alpha$-D(A) ∩ $\theta g^*\alpha$-D(B).

*Proof:*

1. For all $\theta g^*\alpha$ open set U and for all x∈X, U∩ {$\phi$\x} =$\phi$ . Hence $\theta g^*\alpha$-D($\phi$) = $\phi$.
2. Since every open set is $\theta g^*\alpha$-open, the proof follows.
3. If X∈$\theta g^*\alpha$-D(A), that is if x is $\theta g^*\alpha$-limit point of A, then by Definition 4.1[G\{x}]A∩ $\phi$ . for every $\theta g^*\alpha$-open set G containing x. Since A⊆ B implies [G\{x}]∩ A ⊆ [G\{x}]∩ B. Thus if x is a $\theta g^*\alpha$-limit point of A it is also a $\theta g^*\alpha$-limit point of B, that is x ∈ $\theta g^*\alpha$-D(B). Hence $\theta g^*\alpha$-D(A)⊆ $\theta g^*\alpha$-D(B).
4. If x∈ $\theta g^*\alpha$-D(A), that is x is a $\theta g^*\alpha$-limit point of A. Then by Definition 4.1 every $\theta g^*\alpha$- open set G containing x contains at least one point other than x of A \ {x}. That is G∩(A \{x})≠ $\phi$ Hence x is a $\theta g^*\alpha$-limit point of A\{x} and as such it belongs to $\theta g^*\alpha$-D[A\{x}].Therefore x∈ $\theta g^*\alpha$-D(A) ⇒ x∈ $\theta g^*\alpha$-D[A\ {x}].
5. Since A⊆ A∪B and B ⊆ A∪ B,it follows from (2)$\theta g^*\alpha$-D(A)⊆ $\theta g^*\alpha$-D(A∪ B) and $\theta g^*\alpha$-D(B)⊆ $\theta g^*\alpha$-D(A∪ B) and hence $\theta g^*\alpha$-D(A) ∪ $\theta g^*\alpha$-D(B)⊆ $\theta g^*\alpha$-(A∪ B).To prove the other way that is $\theta g^*\alpha$-D(A∪B)⊆ $\theta g^*\alpha$-d(A) ∪ $\theta g^*\alpha$-d(B).If x∈ $\theta g^*\alpha$-d(A)∪ $\theta g^*\alpha$-d(B),then x∈ $\theta g^*\alpha$-d(A) and x∈ $\theta g^*\alpha$-d(B),that is x is neither a $\theta g^*\alpha$-limit point of A nor a $\theta g^*\alpha$-limit point of B. Hence there exist $\theta g^*\alpha$-neighbourhoods $G_1$ and $G_2$ of x such that $G_1$ ∩ (A\{x}) = $\phi$ and $G_2$∩ (B\x)=$\phi$ Since $G_1$∩ $G_2$ is a $\theta g^*\alpha$-neighbourhood of x,we have ($G_1$∩ $G_2$)∩ [(A∪B)\{x}] =$\phi$ . Therefore x∈ $\theta g^*\alpha$-D(A∪ B).Thus $\theta g^*\alpha$-D(A∪ B) ⊆ $\theta g^*\alpha$-D(A) $\theta g^*\alpha$-(B). Hence $\theta g^*\alpha$-D(A∪ B) = $\theta g^*\alpha$-D(A)∪ $\theta g^*\alpha$-D(B).
6. Since A ⊆ B∪ A and A ⊆ B ∪ A, by (2)$\theta g^*\alpha$-D(A∩ B) ⊆ $\theta g^*\alpha$-D(A) and $\theta g^*\alpha$-D(A∩ B) ⊆ $\theta g^*\alpha$-D(B). Consequently $\theta g^*\alpha$-D(A∩ B) ⊆ $\theta g^*\alpha$-D(A)∩ $\theta g^*\alpha$-D(B).

**Example 4.3.** Let X = {a, b, c,d} and $\tau$ ={$\phi$, {a, b}, X}. Then $\beta$ *O( $\tau$ )=( P(X) \ {a}, {b}, {a, b}). Let A={a, b, d} and B ={c}. Then D$\beta$ * (A∪ B) = {a, b} and D$\beta^*$(A)=$\phi$, D$\beta$ *(B)=$\phi$.

**Theorem 4.4.** Let $(X, \tau)$ be a topological space and A be subset of X. If A is $\theta g^*\alpha$-closed, then $\theta g^*\alpha$-D(A)⊆A.

*Proof:* Let A be to $\theta g^*\alpha$-closed, Now we will show that $\theta g^*\alpha$-D(A) $\subseteq$ A. Since A is $\theta g^*\alpha$-closed, X \A is $\theta g^*\alpha$-open. To each x$\in$ X\ A there exists $\theta g^*\alpha$-neighbourhood G of x such that G$\subset$ X\A. Since A $\cap$ (X\A) =$\phi$ , the $\theta g^*\alpha$-neighbourhood G contains no point of A and so x is not a $\theta g^*\alpha$-limit point of A. Thus no point of X \ A can be $\theta g^*\alpha$-limit point of A that is, A contains all its $\theta g^*\alpha$-limit points. That is $\theta g^*\alpha$-D(A)$\subseteq$ A.

# 5 $\theta g^*\alpha$-Border of a Set

**Definition 5.1.** For any subset A of X,The border of A is defined by Bd(A)=A \int(A).

**Definition 5.2.** For any subset A of X, $\theta g^*\alpha$-border of A is defined by $\theta g^*\alpha$-Bd(A)=A \ $\theta g^*\alpha$-int(A).

**Theorem 5.3.** In a topological space $(X, \tau)$, for any subset A of X, the following statements hold.

1. $\theta g^*\alpha$-Bd($\phi$ ) = $\theta g^*\alpha$-Bd(X) =$\phi$ .
2. $\theta g^*\alpha$-Bd(A) $\subseteq$ Bd(A).
3. A=$\theta g^*\alpha$-int(A)$\cup$ $\theta g^*\alpha$-Bd(A).
4. $\theta g^*\alpha$-int(A)$\cap\theta g^*\alpha$-Bd(A) = $\phi$ .
5. $\theta g^*\alpha$-int(A) = A \$\theta g^*\alpha$-Bd(A).
6. $\theta g^*\alpha$-int($\theta g^*\alpha$-Bd(A)) =$\phi$ .
7. A is $\theta g^*\alpha$-open if and only if $\theta g^*\alpha$-Bd(A) =$\phi$ .
8. $\theta g^*\alpha$-Bd($\theta g^*\alpha$-int(A)) = $\phi$ .
9. $\theta g^*\alpha$-Bd($\theta g^*\alpha$-Bd(A)) = $\theta g^*\alpha$-Bd(A).
10. $\theta g^*\alpha$-Bd(A) = A$\cap$ $\theta g^*\alpha$-cl(X\A).

*Proof:* (1) , straight forward. (2) , (3) , (4) and (5) follows From the Definition 5.2 To prove(6) if possible let x$\in\theta g^*\alpha$-int($\theta g^*\alpha$-Bd(A)).Then x$\in$ $\theta g^*\alpha$-Bd(A),since $\theta g^*\alpha$-Bd(A)$\subseteq$A, x$\in\theta g^*\alpha$-int($\theta g^*\alpha$-Bd(A)) $\subseteq$ $\theta g^*\alpha$-int(A). Therefore x$\in$ $\theta g^*\alpha$-int(A)$\cap$ $\theta g^*\alpha$-Bd(A) which is a contradiction to (4). Thus (6)is proved. A is $\theta g^*\alpha$-open if and only if $\theta g^*\alpha$-int(A)=A . But $\theta g^*\alpha$-Bd(A)= A\ $\theta g^*\alpha$-int(A) implies $\theta g^*\alpha$-Bd(A)= $\phi$ . This proves (7) and (8).When A = $\theta g^*\alpha$-Bd(A),Definition 5.2 becomes $\theta g^*\alpha$-Bd($\theta g^*\alpha$-Bd(A))= $\theta g^*\alpha$-Bd(A)\$\theta g^*\alpha$-int($\theta g^*\alpha$-Bd(A)). Using (8), we get (9). To prove (10).$\theta g^*\alpha$-Bd(A)=A\$\theta g^*\alpha$-int(A)= A$\cap$ (X\ $\theta g^*\alpha$-int(A))= A$\cap$ $\theta g^*\alpha$-cl(X\A). Hence (10)is proved.

**Example 5.4.** Let X={a,b,c,d} with topology $\tau$ ={X,$\phi$ ,{a},{b},{c},{a,b},{a,c},{b,c}, {a,b,c}}. In this topological space$(X, \tau)$, $\theta g^*\alpha$-O(X) = {X, $\phi$ ,{a},{b},{c},{a, b},{b, c},{a ,c}{a, b ,c}} Let A = {a, c}, then $\theta g^*\alpha$-Bd(A) = {a, c} -{a, c} = $\phi$ and Bd-(A) = {a, c}\{a} = {c}. Therefore Bd-(A) $\nsubseteq$ $\theta g^*\alpha$-Bd(A)

## 6 $\theta g^*\alpha$- Frontier of a Set

**Definition 6.1.** For any subset A of X,The frontier of A is defined by Fr(A)=cl(A) \ int(A).

**Definition 6.2.** For any subset A of X, its $\theta g^*\alpha$-Frontier is defined by $\theta g^*\alpha$-Fr(A)= $\theta g^*\alpha$-cl(A) \ $\theta g^*\alpha$-int(A).

**Theorem 6.3.** For any subset A of X, in a topological space$(X, \tau)$,the following statements hold.

(1). $\theta g^*\alpha$-Fr( $\phi$ )= $\theta g^*\alpha$-Fr(X)= $\phi$ .

(2). $\theta g^*\alpha$-cl(A)= $\theta g^*\alpha$-int(A) $\cap$ $\theta g^*\alpha$-Fr(A).

(3). $\theta g^*\alpha$-int(A) $\cap$ $\theta g^*\alpha$-Fr(A)=$\phi$ .

(4). $\theta g^*\alpha$-bd(A) $\subseteq$ $\theta g^*\alpha$-Fr(A) $\subseteq$ $\theta g^*\alpha$-cl(A).

(5). If A is $\theta g^*\alpha$-closed, then A=$\theta g^*\alpha$-int(A) $\cup$ $\theta g^*\alpha$-Fr(A).

(6). $\theta g^*\alpha$-Fr(A)= $\theta g^*\alpha$-cl(A)$\cap\theta g^*\alpha$-cl(X\A).

(7). A point x$\in\theta g^*\alpha$-Fr(A), if and only if every $\theta g^*\alpha$-open set containing x intersects both A and its complement X\A.

(8). $\theta g^*\alpha$-cl($\theta g^*\alpha$-Fr(A))= $\theta g^*\alpha$-Fr(A), i.e, $\theta g^*\alpha$-Fr(A) is $\theta g^*\alpha$-closed.

(9). $\theta g^*\alpha$-Fr(A) = $\theta g^*\alpha$-Fr(X\A).

(10). A is $\theta g^*\alpha$-closed if and only if $\theta g^*\alpha$-Fr(A) = $\theta g^*\alpha$-bd(A), i.e, A is $\theta g^*\alpha$-closed if and only if A contains its $\theta g^*\alpha$-frontier.

(11). $\theta g^*\alpha$-Fr($\theta g^*\alpha$-int(A))$\subseteq$ $\theta g^*\alpha$-Fr(A).

(12). $\theta g^*\alpha$-Fr($\theta g^*\alpha$-cl(A))$\subseteq$ $\theta g^*\alpha$-Fr(A).

(13). $\theta g^*\alpha$-Fr($\theta g^*\alpha$-Fr(A))$\subseteq$ $\theta g^*\alpha$-Fr(A).

(14). X=$\theta g^*\alpha$-int(A)$\cup$ $\theta g^*\alpha$-int(X\A)$\cup$ $\theta g^*\alpha$-Fr(A).

(15). $\theta g^*\alpha$-int(A) = A\$\theta g^*\alpha$-Fr(A).

(16). If A is $\theta g^*\alpha$-open, then A $\cap$ $\theta g^*\alpha$-Fr(A) =$\phi$ , i.e, $\theta g^*\alpha$-Fr(A) $\subseteq$ X\A.

*Proof:* (1), (2), (3) and (4) follows from Definition 6.2 (5) follows from (2) and Proposition2.3 (2). (6) follows from Proposition2.3 (5). (7) can be proved using (6)and Proposition2.3 (8). From (6), we can prove (8) by applying the results of Proposition2.3 (3) and (9). Proof of (9)is similar. To prove (10): If A is $\theta g^*\alpha$-closed, then A = $\theta g^*\alpha$-cl(A).Hence Definition 6.2 reduces to $\theta g^*\alpha$-Fr(A)= A\ $\theta g^*\alpha$-int(A) = $\theta g^*\alpha$-bd(A). Conversely, suppose that $\theta g^*\alpha$-Fr(A) = $\theta g^*\alpha$-bd(A),using Definition 6.2 and 6.1, we get $\theta g^*\alpha$-cl(A) = A, which proves the sufficient part.

(11) and (12) Since $\theta g^*\alpha$-int(A) is $\theta g^*\alpha$-open, (11) holds. Similarly (12) can also be proved. Since $\theta g^*\alpha$-Fr(A) is $\theta g^*\alpha$-closed, invoking (10), (13) can be proved. since X=$\theta g^*\alpha$-cl(A) (X\\$\theta g^*\alpha$-cl(A)), but from (2) $\theta g^*\alpha$-cl(A) = $\theta g^*\alpha$-int(A) $\theta g^*\alpha$-Fr(A). Also X\\$\theta g^*\alpha$-cl(A)=$\theta g^*\alpha$-int(X\A). Hence X = $\theta g^*\alpha$-int(A) $\theta g^*\alpha$-Fr(A) $\theta g^*\alpha$-int(X\ A). Thus (14) is proved. Proof of (15) is obvious. If A is $\theta g^*\alpha$-open, A= $\theta g^*\alpha$-int(A). (16) follows from (3).

**Theorem 6.4.** If a subset A of X is $\theta g^*\alpha$-open or $\theta g^*\alpha$-closed in $(X, \tau)$, then $\theta g^*\alpha$-Fr($\theta g^*\alpha$-Fr(A))=$\theta g^*\alpha$-Fr(A).

*Proof:* By Theorem 6.3 (6), we have $\theta g^*\alpha$-Fr($\theta g^*\alpha$-Fr(A))= $\theta g^*\alpha$-cl($\theta g^*\alpha$-Fr(A)) ∩ $\theta g^*\alpha$-cl(X \ $\theta g^*\alpha$-Fr(A))= $\theta g^*\alpha$-Fr(A)∩$\theta g^*\alpha$-cl(X\\$\theta g^*\alpha$-Fr(A))= $\theta g^*\alpha$-cl(A)\\$\theta g^*\alpha$-cl(X∩ A)∩ $\theta g^*\alpha$-cl(X∩ $\theta g^*\alpha$-Fr(A)).If A is $\theta g^*\alpha$-open in X, by Theorem 6.3 (16), we have $\theta g^*\alpha$-Fr(A) ∩A =$\phi$ . Therefore A⊆ X ∩ $\theta g^*\alpha$-Fr(A). Hence $\theta g^*\alpha$-cl(A) $\theta g^*\alpha$-cl(X∩ $\theta g^*\alpha$-Fr(A)).i.e,$\theta g^*\alpha$-cl(A)\ $\theta g^*\alpha$-cl(X∩ $\theta g^*\alpha$-Fr(A))=$\theta g^*\alpha$-cl(A). If A is $\theta g^*\alpha$-closed in X,then X ∩ A is $\theta g^*\alpha$-open and hence From the above case, we have $\theta g^*\alpha$-cl(X\A)\ $\theta g^*\alpha$-cl(X∩$\theta g^*\alpha$-Fr(X\A))= $\theta g^*\alpha$-cl(X\A). In both the cases using Theorem 6.3(6), we get $\theta g^*\alpha$-Fr($\theta g^*\alpha$-Fr(A))=$\theta g^*\alpha$-cl(A)∉$\theta g^*\alpha$-cl(X \A)= $\theta g^*\alpha$-Fr(A).

**Theorem 6.5.** If A is any subset of X, then $\theta g^*\alpha$-Fr($\theta g^*\alpha$-Fr($\theta g^*\alpha$-Fr(A)))= $\theta g^*\alpha$-Fr($\theta g^*\alpha$-Fr(A)).

*Proof:* It follows From Theorem 6.3 (8) and Theorem 6.4.

**Theorem 6.6.** If A and B are subsets of X such that A∩B=$\phi$, where A is $\theta g^*\alpha$-open in X, then A∩$\theta g^*\alpha$ cl(B)=$\phi$.

*Proof:* If possible, let x∈A∩$\theta g^*\alpha$-cl(B). Then A is a $\theta g^*\alpha$-open set containing x and also x∈ $\theta g^*\alpha$-cl(B).By Proposition2.3 (8)A∩B =$\phi$, which is a contradiction. Thus A∩$\theta g^*\alpha$-cl(B)=$\phi$.

**Theorem 6.7.** If A and B are subsets of X such that A B and B is $\theta g^*\alpha$-closed in X, then $\theta g^*\alpha$-Fr(A)⊆ B.

*Proof:* $\theta g^*\alpha$-Fr(A)= $\theta g^*\alpha$-cl(A) ∉ $\theta g^*\alpha$-int(A) $\theta g^*\alpha$-cl(B)\\$\theta g^*\alpha$-int(A)= B \\$\theta g^*\alpha$-int(A) ⊆ B.

**Theorem 6.8.** If A and B are subsets of X such that A ∩ B=$\phi$, where A is $\theta g^*\alpha$-open in X, then A ∉$\theta g^*\alpha$-Fr(B)=$\phi$.

*Proof:* Since $\theta g^*\alpha$-Fr(B) ⊆ $\theta g^*\alpha$-cl(B), proof is obvious From Theorem 6.6.

**Theorem 6.9.** If A , B⊆ X such that $\theta g^*\alpha$-Fr(A)∩Fr(B) =$\phi$ and Fr(A)∩$\theta g^*\alpha$-Fr(B) =$\phi$, then $\theta g^*\alpha$-int(A ∪ B)=$\theta g^*\alpha$-int(A) ∪$\theta g^*\alpha$-int(B).

*Proof:* We know that $\theta g^*\alpha$-int(A) ∪$\theta g^*\alpha$-int(B) ⊆ $\theta g^*\alpha$-(A ∪ B). Let x∈$\theta g^*\alpha$-int(A ∪ B). i.e, x∈ U⊆ A ∪ B, U is a $\theta g^*\alpha$-open set. Thus either x∈$\theta g^*\alpha$-Fr(A) ∉Fr(B), since $\theta g^*\alpha$-Fr(A) ∩ Fr(B) = $\phi$. Hence x∈int(B). i.e, x∉cl(B). Since x∈ int(B) ⊆ $\theta g^*\alpha$-int(B), x⊆ $\theta g^*\alpha$-int(B). Moreover since x ∉cl(B), there exists an open set V containing x which is disjoint From B, i.e, V ⊆ X \B. So x∈ U∩V⊆ A. Hence

U∩V is a $\theta g^{*}\alpha$-open subset of A containing x.(By Proposition2.3 (7)). i.e, x∈$\theta g^{*}\alpha$-int(A). Thus x∈$\theta g^{*}\alpha$-int(A) ∪$\theta g^{*}\alpha$-int(B). If x∉ $\theta g^{*}\alpha$-Fr(A), x∈$\theta g^{*}\alpha$-int(A) or x ∉ $\theta g^{*}\alpha$-cl(A). If x ∉ $\theta g^{*}\alpha$-cl(A), there exists a $\theta g^{*}\alpha$-open set W containing x which is disjoint From A, i.e, W⊆ X\A. i.e, x∈ U∩W⊆ B $\theta g^{*}\alpha$-cl(B). i.e, x∈$\theta g^{*}\alpha$-Fr(B).Hence From the above case, we get x∈$\theta g^{*}\alpha$-int(A) ∪$\theta g^{*}\alpha$-int(B). So $\theta g^{*}\alpha$-int(A∪B)⊆ $\theta g^{*}\alpha$-int(A) ∪$\theta g^{*}\alpha$-int(B). Thus $\theta g^{*}\alpha$-int(A ∪ B)=$\theta g^{*}\alpha$-int(A)∪$\theta g^{*}\alpha$-int(B).

# 7 $\theta g^{*}\alpha$ -Exterior of a Set

**Definition 7.1.** For any subset A of X,The exterior of A is defined by Ext(A)=int(X\A).

**Definition 7.2.** For any subset A of X, its $\theta g^{*}\alpha$-Exterior is defined by $\theta g^{*}\alpha$-Ext(A)= $\theta g^{*}\alpha$-int(X\A).

**Theorem 7.3.** For any subset A of X, in a topological space$(X, \tau)$,the following statements hold.

(1). $\theta g^{*}\alpha$-Ext$\phi$) = $\theta g^{*}\alpha$-Ext(X)=$\phi$.

(2). Ext(A) ⊂ $\theta g^{*}\alpha$-Ext(A) where Ext(A) denote the exterior of A.

(3). If A ⊆ B, then $\theta g^{*}\alpha$-Ext(B)⊆ $\theta g^{*}\alpha$-Ext(A).

(4). $\theta g^{*}\alpha$-Ext(A) is $\theta g^{*}\alpha$-open.

(5). A is $\theta g^{*}\alpha$-closed if and only if $\theta g^{*}\alpha$-Ext(A)= X\ A.

(6). $\theta g^{*}\alpha$-Ext(A) = X\$\theta g^{*}\alpha$-cl(A).

(7). $\theta g^{*}\alpha$-Ext($\theta g^{*}\alpha$-Ext(A))=$\theta g^{*}\alpha$-int($\theta g^{*}\alpha$-cl(A)).

(8). Ext(A) ⊆ $\theta g^{*}\alpha$-Ext(A) where Ext(A) denote the exterior of A.

(9). $\theta g^{*}\alpha$-Ext(A)= $\theta g^{*}\alpha$-Ext(X\$\theta g^{*}\alpha$-Ext(A)).

(10). $\theta g^{*}\alpha$-int(A) $\theta g^{*}\alpha$-Ext($\theta g^{*}\alpha$-Ext(A)).

(11). X=$\theta g^{*}\alpha$-int(A) ∪$\theta g^{*}\alpha$-Ext(A) ∪ $\theta g^{*}\alpha$-Fr(A).

(12). $\theta g^{*}\alpha$-Ext(A ∪B)⊆ $\theta g^{*}\alpha$-Ext(A)∩$\theta g^{*}\alpha$-Ext(B)

(13). $\theta g^{*}\alpha$-Ext(A ∪B) ⊆ $\theta g^{*}\alpha$-Ext(A) ∪$\theta g^{*}\alpha$-Ext(B)r(A) ⊆X\A.

*Proof:* (1) ,(2)and (3)can be proved From Definition 7.2. Since $\theta g^{*}\alpha$-int(A) is $\theta g^{*}\alpha$-open, Proof of (4) is obvious. Since $\theta g^{*}\alpha$-int(X\A)= X\$\theta g^{*}\alpha$-cl(A),(5) follows From Definition 7.2.Similarly (6) and (7)can be proved.

To Prove (8), $\theta g^{*}\alpha$-Ext(X\ $\theta g^{*}\alpha$-Ext(A))=$\theta g^{*}\alpha$-Ext(X\ $\theta g^{*}\alpha$-int(X\A))= $\theta g^{*}\alpha$-int(X\(X\ $\theta g^{*}\alpha$- int(X\A)))=$\theta g^{*}\alpha$-int($\theta g^{*}\alpha$-int(X\A))=$\theta g^{*}\alpha$- int(X\A)= $\theta g^{*}\alpha$-Ext(A). Hence (8)is proved. Since A ⊆ $\theta g^{*}\alpha$-cl(A), using (6), (9)can be proved. (10).follows From Theorem 6.3 (14) and Definition 7.2. Proof of (11),(12)and (13)are obvious.

**Example 7.4.** LetX={a,b,c,d}with topology $\tau$ ={X,$\phi$ ,{a},{b},{a,b}, {a,b,c}}. In this topological space$(X, \tau)$, $\theta g^{*}\alpha$-O(X) ={X,$\phi$ ,{a},{b},{c},{a, b},{b, c}, {a ,c},{a, b ,c},{a,b,d}} Let A = {a,b.d}, then $\theta g^{*}\alpha$-Ext (A) = {c} and Ext-(A)=$\phi$. Therefore $\theta g^{*}\alpha$-Ext-(A) $\nsubseteq$ Ext (A)

# 8 Conclusion

Every year many topologist introduced diffrent type of closed sets.we introduced $\theta g^{*}\alpha$-closed sets in topological spaces. In this paper, we discussed properies and applications of $\theta g^{*}\alpha$-neighbourhoods,$\theta g^{*}\alpha$-limit points, $\theta g^{*}\alpha$- border, $\theta g^{*}\alpha$-frontier and $\theta g^{*}\alpha$-exterior.This shall be extended in the future Research with some applications

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# References

[1] M. Caldas, S. Jafari and M. M. Kovar, Some Properties of $\theta$-open Sets, *Divulg. Mat.* 12 (2004) 161-169.

[2] J. Cao, M. Ganster, Reilly and M. Steiner, $\theta$-closure, $\theta$-closure and Generalized Closed Sets, *Appl. Gen. Topology, UnivesidadPolotecnica de Valencia* 6 (2005) 79-89.

[3] S. Chandrasekar, V. Banupriya and M. Suresh, $\theta g^{*}\alpha$ closed sets Topological spaces International Journal of Pure and applied Mathematics (In Press).

[4] S. Chandrasekar, M. Suresh, T. R. Kannan, On Quasi Soft sg-open and Quasi Soft sg-closed Functions in soft topological spaces,Int. Journal of Mathematics And its Applications, 5/2-B (2017) 239-246.

[5] S. Chandrasekar, M. Suresh, T. R. Kannan, Nano sg-interior and Nano sg-closure in nano topological spaces Int.J.Mathematical Archive 8(4) 2017, 94-100.

[6] J. Dontchev and M. Ganster, On $\delta$-generalised Closed Sets and T3/4-Spaces, *Mem.Fac.Sci. Kochi Univ. Ser. A Math.* 17 (1996) 15-31.

[7] J. Dontchev and H. Maki, On $\theta$-generalised Closed Sets, *Int. J. Math. and Math. Sci.* 2 (1999) 239-249.

[8] N. Levine, Generalised Closed Sets in Topology, *Rend. Circ. Mat. Palermo* 19 (1970) 89-96.

[9] O. Njastad, On some classes of nearly open sets, *Pacific J. Math.* 15 (1965) 961-970.

[10] O. Ravi, S. Ganesan and S. Chandrasekar, Almost$\alpha$gs-closed functions and separation axioms, *Bulletin of Mathematical Analysis and Applications* 3 (2011) 165-177.

[11] O. Ravi, S. Chandrasekar and S. Ganesan, $(\tilde{g},s)$-Continuous Functions between Topological Spaces, *Kyungpook Math. J.* 51 (2011) 323-338.

[12] O. Ravi, S. Chandrasekar and S. Ganesan, ($\widetilde{g}$, s)-Continuous Functions between Topological Spaces , Kyungpook Math. J. 51 (2011) 323-338.

[13] O. Ravi, S. Ganesan and S. Chandrasekar, On Totally sg-Continuity, Strongly sg-Continuity and Contra sg-Continuity Gen. Math. Notes, Vol. 7, No 1, Nov (2011), pp. 13-24.

[14] O. Ravi, S. Chandrasekar and S. Ganesan, On Weakly $\pi$G-Closed Sets In Topological Spaces,Italian Journal Of Pure And Applied Mathematics 36 (2016) 651-666.

[15] O. Ravi, G. Ramkmar and S. Chandrasekar, On Quasi sg-open and Quasi sg-closed Functions Advances in Applied Mathematical Analysis 4 (2009) 73-78.

[16] N. V. Velicko, H-Closed Topological Spaces, *Amer. Math. Soc. Transl.* 78 (1968) 13-18.