http://www.newtheory.org ISSN: 2149-1402

Journal of New Theory

Received: 19.10.2017 *Year*: **2017**,*Number*: **18**, *Pages*: **22-38**
Published: 22.11.2017 *Original Article*

CHARACTERIZATIONOF SOFT STRONG SEPARATION AXIOMS IN SOFT BI TOPOLOGICAL SPACES

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Abstract – The main aim of this article is to introduce Soft strong separation axioms in soft bi topological spaces. We discuss soft strong separation axioms in soft bi topological spaces with respect to ordinary point and soft points. Further study the behavior of soft strong T_0 space, soft strong $T_1$and soft strong T_2spaces at different angles with respect to ordinary points as well as with respect to soft points. Hereditary properties are also talked over.

Keywords – *Soft sets, soft open sets, soft points, soft bi-topological space, soft strong* T_i *spaces* $(i = 0,1,2)$ *spaces in soft bi topology.*

1. Introduction.

In real life condition the problems in economics, engineering, social sciences, medical science etc we cannot beautifully use the traditional classical methods because of different types of uncertainties presented in these problems. To overcome these difficulties, some kinds of theories were put forwarded like theory of Fuzzy set, intuitionistic fuzzy set, rough set and bi polar fuzzy sets, inwhich we can safely use a mathematical technique for businessing with uncertainties. But, all these theories have their inherent difficulties. To overcome these difficulties in the year 1999, Russian scientist Molodtsov [4], originated the notion of soft set as a new mathematical technique for uncertainties, which is free from the above complications. Mololdtsov [4] and Ahmad [5], successfully applied the soft set theory in different directions, such as smoothness of functions, game theory, operation research, Riemann integration, perron integration, probability, theory of measurement and so on.

After presentation of the operations of soft sets [6], the properties and applications of the soft set theory have been studied increasingly [7,8,6]. Xiao et al. [9] and Pei and Maio [10] discussed the linkage between soft sets and information systems. They showed that soft sets are class of special information system. In the recent year, many attention-grabbing applications of soft sets theory have been extended by embedding the ideas of fuzzy sets [11,12,13,14,15,16,17,18,20,21,22] industrialized soft set theory, the operations of the soft sets are redefined and indecision making method was constructed by using their new operations [23].

Recently, in 2011, Shabir and Naz [23] launched the study of soft Topological spaces; they beautiful defined soft Topology as a collection of τ of soft sets over X. They also defined the basic conceptionof soft topological spaces such as open set and closed soft sets, soft nbd of a point, soft separation axiom, soft regular and soft normal spaces and published their several behaviors. Min in [25] scrutinized some belongings of these soft separation axioms. Kandil et al. [26] introduced some soft operations such as semi open soft, pre-open soft, α-open soft and β-open soft and examined their properties in detail. Kandil et al. [27] introduced the concept of soft semi – separation axioms, in particular soft semi- regular spaces. The concept of soft ideal was discussed for the first time by Kandil et al. [28] .They also introduced the concept of soft local function. These ideas are discussed with a view to find new soft topological from the original one, called soft topological spaces with soft ideal(X, τ, E, I).

Applications of different zone were further discussed by Kandil et al. [28,29,30,32,33, 34,35]. The notion of super soft topological spaces was initiated for the first time by El-Sheikh and Abd-e-Latif [36]. They also introduced new different types of sub-sets of supra soft topological spaces and study the dealings between them in great detail. Bin Chen [41] introduced the concept of semi open soft sets and studied their related properties, Hussain [42] discussed soft separation axioms. Mahanta [39] introduced semi open and semi closed soft sets. Lancy and Arockiarani [40], On Soft β-Separation Axioms, Arockiarani and Arokialancy in [43] generalized soft g β closed and soft gs β closed sets in soft topology are exposed. Analogous to [44] and based on the concept of b-open sets in topological spaces, the notion of quasi-b-open set in bi topological spaces is introduced and discovered. [43] discussedbi topological strong separation axioms, pair wise intersection property and pair wise strong ly regular property is also studied.

In this present paper we introduce the soft strong separation axioms, soft strong pair wise regularity and soft strong pair wise normal. Concept of soft strong T_0, soft strong T_1 and soft strong T_2 spaces in Soft bi topological spaces is introduced with respect to soft points. Many mathematicians discussed soft separation axioms in soft topological spaces at full length with respect to soft open set, soft b-open set, soft semi-open set, soft α-open set and soft β-open set. They also worked over the hereditary properties of different soft topological structures in soft topology. In this present work hand is tried and work is encouraged over the gap that exists in soft bi-topology. Related to soft strong T_0, soft strong T_1 and soft strong T_2 spaces, some Proposition in soft bi topological spaces are discussed with respect to ordinary points as well as with respect to soft points. When we talk about the distance between the points in soft topology then the concept of soft separation axioms will automatically come in play. That is why these structures are catching our attention. We hope that these results will be valuable for the future study on soft bi topological spaces to accomplish general framework for the practical applications and to solve the most intricate problems containing scruple in economics, engineering, medical, atmosphere and in general mechanic systems of various kinds.

2. Preliminaries

The following Definitions which are pre-requisites for present study.

Definition 1 [4]. Let X be an initial universe of discourse and E be a set of parameters. Let P(X) denotes the power set of X and A be a non-empty sub-set of E. A pair (F, A) is called a soft set over U, where F is a mapping given by $F: A \to P(X)$

In other words, a set over X is a parameterized family of sub set of universe of discourse X For $e \in A, F(e)$ may be considered as the set of e-approximate elements of the soft set $(F, A)$and if$e \notin A$ then$F(e) = \phi$, that is$F_{A=}\{F(e): e \in A \subseteq E, F: A \to P(X)\}$ the family of all these soft sets over X denoted by$SS(X)_A$.

Definition 2 [4]. Let$F_{A,}G_B \in SS(X)_E$ then F_A is a soft subset of G_B denoted by$F_A \sqsubseteq G_B$, if

1. $A \subseteq B$ and
2. $F(e) \subseteq G(e), \forall \in A$

In this case F_A is said to be a soft subset of G_B and G_B is said to be a soft super setF_A, $G_B \sqsupseteq F_A$.

Definition 3 [6].Two soft subsets F_A and G_B over a common universe of discourse set X are said to be equal if F_A is a soft subset of G_B and G_B is a soft subset of F_A.

Definition 4 [6].The complement of soft subset(F, A)ddenoted by $(F, A)^C$ is defined by $(F, A)^C = (F^C, A)F^C \to P(X)$is a mapping given by $F^C(e) = U - F(e)\forall e \in A$ and $F^C$is called the soft complement function of$F$. Clearly $(F^C)^C$ is the same as F and $((F, A)^C)^C = (F, A)$.

Definition 5 [7]. The difference between two soft subset (G, E) and (G, E) over common of universe discourse X denoted by $(F, E) - (G, E)$ is the soft set(H, E) where for all $e \in E. \overline{\emptyset} or\ \emptyset_A$if $\forall e \in A, F(e) = \emptyset$.

Definition 6 [7]. Let (G, E) be a soft set over$X$ and $x \in X$ We say that $x \in (F, E)$ and read as x belong to the soft set(F, E) whenever $x \in F(e)\forall e \in E$ The soft set(F, E) over$X$such that $F(e) = \{x\}\forall e \in E$ is called singleton soft point and denoted by $x_E, or\ (x, E)$.

Definition 7 [6]. A soft set (F, A) over X is said to be Null soft set denoted by $\overline{\emptyset}\ or\ \emptyset_A$ if $\forall e \in A, F(e) = \emptyset$.

Definition 8 [6]. A soft set(F, A) over X is said to be an absolute soft denoted by $\bar{A}\ or\ X_A$ if $\forall e \in A, F(e) = X$.

Clearly, we have. $X_A^C = \emptyset_A\ and\ \emptyset_A^C = X_A$.

Definition 9 [42]. Let(G, E) be a soft set over$X$and$e_G \in X_A$, we say that $e_G \in (F, E)$ and read as e_G belong to the soft set(F, E)whenever $e_G \in F(e) \forall e \in E$. the soft set$(F, E)$ over$X$such that $F(e) = \{e_G\}$, $\forall e \in E$ is called singleton soft point and denoted bye_G, $or\ (e_G, E)$.

A soft point is an element of a soft set F_A. the class of all soft sets over U is denoted by$S(U)$.

For example, $U = \{u_1, u_2, u_3\}, E = \{x_1, x_2, x_3\}$, $A = \{x_1, x_2\}$ and $F_A = \{(x_1, \{u_1, u_2\})\}$, $\{(x_2, \{u_2, u_3\})\}$.

Then

$F_{A_1} = \{(x_1, \{u_1\})\}, F_{A_2} = \{(x_1, \{u_2\})\}, F_{A_3} = \{(x_1, \{u_2, u_2\})\}$,
$F_{A_4} = \{(x_2, \{u_2\})\}$,$F_{A_5} = \{(x_2, \{u_3\})\}$,
$F_{A_6} = \{(x_2, \{u_1, u_3\})\}, F_{A_7} = \{(x_1, \{u_2\}, (x_{2,} \{u_2\})\}$,
$F_{A_8} = \{(x_1, \{u_1\}, (x_2, \{u_3\})\}$,
$F_{A_9} = \{(x_1, \{u_1\})\}, (x_2, \{u_2, u_3\})\}, F_{A_{10}} = \{(x_1, \{u_2\})\}, (x_2, \{u_2\})\}$,
$F_{A_{11}} = \{(x_1, \{u_2\}), (x_{2,} \{u_3\})\}, F_{A_{12}} = \{(x_1, \{u_2\}), (x_{2,} \{u_2, u_3\}, F_{A_{13}} =$
$\{(x_1, \{u_1, u_2\}), (x_2, \{u_2\})\}, F_{A_{14}} = \{(x_1, \{u_1, u_2\}), (x_2, u_3\}, F_{A_{15}} = F_A, F_{A_{16}} = F_\emptyset$,

are all soft sub sets of F_A

Definition 10 [42].The soft set $(F, A) \in SSX_A$is called a soft point inX_A, denoted by e_F, if for the element $e \in A, F(e) \neq \{x\}$ and $F(e^/) = \phi$ if for all $e^/ \in A - \{e\}$

Definition 11 [42]. The soft point e_F is said to be in the soft set(G, A), denoted by $e_F \in (G, A)$ if for the element $e \in A, F(e) \subseteq G(e)$.

Definition 12 [42].Two soft sets $(G, A), (H, A)$ in SSX_A are said to be soft disjoint, written $(G, A) \cap (H, A) = \emptyset_A$If $G(e) \cap H(e) = \emptyset \forall\ e \in A$.

Definition 13 [42]. The soft point $e_G, e_H \in X_A$ are disjoint, written $e_G \neq e_{H,}$ if their corresponding soft sets (G, A) and (H, A) are disjoint.

Definition 14 [6]. The union of two soft sets(F, A) and (G, B) over the common universe of discourse X is the soft set(H, C), where, $C = AUB \forall e \in C$

$$H(e) = \begin{cases} F(e) & if\ e\ \in A - B \\ G(e) & if\ e\ \in (B - A) \\ F(e), & if\ e\ \in A \cap B \end{cases}$$

Written as $(F, A) \cup (G, B) = (H, C)$

Definition 15 [6]. The intersection$(H.C)$of two soft sets$(F,A)$and (G,B)over common universe X, denoted $(F,A) \bar{\cap} (\mathrm{G},\mathrm{B})$ is defined as $C = A \cap B$and $H(e) = F(e) \cap G(e), \forall e \in C$.

Definition 16 [2]. Let(F,E) be a soft set over X and Y be a non-empty sub set of X. Then the sub soft set of (F,E)over Y denoted by (Y_F,E),is defined as follow$Y_{F(\alpha)} = Y \cap F(\alpha), \forall \in E$ in other words

$$(Y_F, E) = Y \cap (F,E).$$

Definition 17 [2]. Let τ be the collection of soft sets over X , then τ is said to be a soft topology on X, if

1. $\emptyset,\ X \epsilon\tau$
2. The union of any number of soft sets in τ belongs to τ
3. The intersection of any two soft sets in τ belong to τ

The triplet (X,F,E) is called a soft topological space.

Definition 18 [1]. Let (X,τ,E) be a soft topological space over X then the member of τ are said to be soft open sets in X.

Definition 19 [1]. Let(X,τ,E) be a soft topological space over$X$.A soft set(F,A) over X is said to be a soft closed set in X , if its relative complement $(F,E)^C$ belong to τ.

Definition 20 [3]. Let(X,τ_1,E) and (X,τ_2,E) be two different soft topologies on X. Let $\tau_1 v \tau_2$be the smallest soft topology on X that contains$\tau_1 \cup \tau_2$.

Example: Suppose there are three Houses in the universe $U = \{h_1,h_2,h_3\}$ under observation, and that $E = \{e_1,e_2\}$ is a decision parameters which stands for "beautiful", and "in green surrounding". In this case to define a soft set means to point out beautiful house and in green surrounding house.

Let$\tau_1 = \{\emptyset, \widetilde{X,}(F_1,E), (F_2,E)\}$ and
$\{\tau_1 =$
$\emptyset, \widetilde{X,}(G_1,E),(G_2,E),(G_3,E),(G_4,E)\}$ where
$(F_1,E),(F_1,E),(G_1,E),(G_2,E),(G_3,E),(G_4,E)$ are soft sets over$X$, defined as follow:

$F_1(e_1) = \{h_1\}, F_1(e_2) = \{h_1,h_2\}, F_2(e_1) = \{h_1,h_3\}, F_2(e_2) = X, G_1(e_1) = \{h_2\}, G_1(e_2) = \{h_2\}, G_2(e_1) = \{h_1,h_2\}, G_2(e_2) = \{h_2\}, G_3(e_1) = \{h_2\}, G_3(e_2) = \{h_2\}, G_4(e_1) = \{\ \ \}, G_4(e_2) = \{h_2\}$.

Then τ_1 and τ_2 are soft topology on X.Now,

$$\tau_1 v \tau_2 = \{\emptyset, X, (F_1,E), (F_2,E), (G_1,E), (G_2,E), (G_3,E), (G_4,E), (H_1,E)\}.$$

Where, $H_1(e_1) = \{h_1,h_2\}, H_1(e_2) = \{h_1,h_2\}$. Thus $(X,\tau_1 v \tau_2,E)$ is the smallest soft topological space over X that contains $\tau_1 \cup \tau_2$.

3. Separation Axioms of Soft Topological Spaces With Respect to Ordinary Points as Well as Soft Points

Definition 21 [1]. Let (X, τ, A) be a soft topological space over X and $x, y \in X$ such that $x \neq y$ if there exist at least one soft open set (F_1, A) OR (F_2, A) such that $x \in (F_1, A), y \notin (F_1, A)ory \in (F_2, A), x \notin ((F_2, A)$then (X, τ, A) is called a soft T_0 $space$.

Definition 22 [1]. Let (X, τ, A) be a soft topological spaces over X and $x, y \in X$ such that $x \neq y$ if there exist soft open sets (F_1, A) and (F_2, A) such that $x \in (F_1, A), y \notin (F_1, A)$ and $y \in (F_2, A), x \notin ((F_2, A)$ then (X, τ, A) is called a soft T_1 $space$.

Definition 23 [1]. Let (X, τ, A) b e a soft Topological space over X and $x, y \in X$ such that $x \neq y$ if there exist soft open set $(F_1, A)$and (F_2, A) such that $x \in (F_1, A)$, and $y \in (F_2, A)$ and $F_1 \cap F_2 = \varphi$

Then (X, τ, A) is called soft T_2 spaces.

Definition 24 [42].Let (X, τ, A) be a soft topological space over X and $e_G, e_H \in X_A$ such that $e_G \neq e_H$ if we can search at least one soft open set (F_1, A) or (F_2, A) such that $e_G \in (F_1, A), e_H \notin (F_1, A)$or $e_H \in (F_2, A), e_G \notin ((F_2, A)$then (X, τ, A) is called a soft T_0 space.

Definition 25 [42]. Let (X, τ, A) be a soft topological spaces over X and $e_G, e_H \in X_A$ such that $e_G \neq e_H$ if we can search soft open sets $(F_1, A)$and (F_2, A) such that $e_G \in (F_1, A), e_H \notin (F_1, A)$ and $e_H \in (F_2, A), e_G \notin ((F_2, A)$ then (X, τ, A) is calleda soft T_1 $space$.

Definition 26 [42]. Let (X, τ, A) b e a soft topological space over X and $e_G, e_H \in X_A$ such that $e_G \neq e_H$ if we can search soft open set $(F_1, A)$and (F_2, A) such that $e_G \in (F_1, A)$, and $e_H \in (F_2, A)$

$(F_1, A) \cap (F_2, A) = \phi_A$ Then (X, τ, A) is called soft T_2 space.

4. Soft Strong Separation Axioms of Soft Bi Topological Spaces

Let X is an initial set and E be the non-empty set of parameter. In [1] soft bi topological space over the soft set X is introduced. Soft separation axioms in soft bi topological spaces were introduced by Basavaraj and Ittanagi [1]. In this section we introduced the concept of soft b T_3 and bT_4 spaces in soft bi topological spaces with respect to ordinary as well as soft points and some of its basic properties are studied and applied to different results in this section.

Definition 27 [1]. Let(X, τ_1, E) and (X, τ_2, E) be two different soft topologies on X. Then (X, τ_1, τ_2, E) is called a *soft bi topological space.* The two soft topologies (X, τ_1, E) and (X, τ_2, E) are independently satisfy the axioms of soft topology. The members ofτ_1 are called τ_1*soft open*. And complement ofτ_1. Soft open set is called τ_1*soft closed* set.

Similarly, the member of τ_2 are called τ_2 *soft open* sets and the complement of τ_2 soft open sets are called τ_2*soft closed* set.

Definition 28 [1]. Let (X, τ_1, τ_2, E)be a soft topological space over X and Y be a non-empty subset of X. Then$\tau_{1Y} = \{(Y_E, E): (F, E) \in \tau_1)\}$ and $\tau_{2Y} = \{(G_E, E): (G, E) \in \tau_2)\}$ are said to be the relative topological on Y. Then $(Y, \tau_{1Y}, \tau_{2Y}, E)$ is called relative soft bi-topological space of (X, τ_1, τ_2, E).

4.1 Soft Strong Separation Axioms of Soft Bi Topological Spaces with Respect to Ordinary Points.

In this section we introduced soft strong separation axioms in soft bi topological space with respect to ordinary points and discussed some results with respect to these points in detail.

Definition 29. In a *soft bi topological* space (X, τ_1, τ_2, E)

1) τ_1 said to be *soft strong* T_0 space with respect to τ_2 if for each pair of distinct points $x, y \in X$ there exists τ_1 soft open set (F, E) and a τ_2 soft open set (G, E) such that $x \in \tau_2 int(F, E)$ and $y \notin \tau_2 int(F, E)$ or $y \in \tau_1 int(G, E)$ and $x \notin \tau_1 int(G, E)$ similarly τ_2 is said to be *soft strong* T_0 space with respect to τ_1 if for each pair of distinct points $x, y \in X$ there exists τ_2*soft open set*(F, E) and a $\tau_1$soft open set (G, E) such that $x \in \tau_1\, int(F, E)$ and $y \notin \tau_1 int(F, E)$ or $y \in \tau_2 int(G, E)$ and $x \notin \tau_2 int(G, E)$. *Soft bi topological spaces*(X, τ_1, τ_2, E) is said to be *pair wise soft strong* T_0 space if τ_1 is *soft strong* T_0 *space* with respect to τ_2 and τ_2 is *soft strong* T_0 space with respect to τ_1.

2) τ_1 is said to be soft strong T_1 space with respect to τ_2 if for each pair of distinct points $x, y \in X$ there exists τ_1 *soft open set* (F, E) and τ_2 soft open set (G, E) such that $x \in \tau_2\mathrm{int}\,(F, E)$ and $y \notin \tau_2\mathrm{int}\,(F, E)$ and $y \in \tau_1 int(G, E)$ and $x \notin \tau_1 int(G, E)$. Similarly,$\tau_2$ is said to be soft strong T_1 space with respect to τ_1 if for each pair of distinct points $x, y \in X$there exist a τ_2 soft open set (F, E) and a τ_1 soft open set (G, E) such that $x \in \tau_1 int(F, E)$ and $y \notin \tau_1 int(F, E)$ and $y \in \tau_2 int(G, E)$ and $x \notin \tau_2 int(G, E)$. *Soft bi topological spaces* (X, τ_1, τ_2, E) is said to be *pair wise soft strong* T_1 space if τ_1 is *soft strong* T_1 *space*with respect to $\tau_2$and τ_2 is *soft strong* T_1space with respect to τ_1.

3) τ_1 said to be soft strong T_2 space with respect to $\tau_2$if for each pair of distinct points $x, y \in X$ there exists a τ_1 soft open set (F, E) and a τ_2 soft open set (G, E) such that $x \in \tau_2 int(F, E)$and $y \in \tau_1\mathrm{int}\,(G, E)$ and$(F, E) \cap (G, E) = \phi$. Similarly,$\tau_2$ is said to be *soft strong* T_2space with respect to τ_1 if for each pair of distinct points $x, y \in X$ there exists a τ_2*soft open* set (F, E) and a τ_1*soft open*set(G, E) such that $x \in \tau_1 int(F, E)$ and $y \in \tau_2 int(G, E)$ and $(F, E) \cap (G, E) = \phi$. The soft bi topological space (X, τ_1, τ_2, E) is said to be pair wise soft strong T_2 space if τ_1 is soft strong T_2 space with respect to τ_2 and τ_2 is soft strong T_2 space with respect toτ_1.

Proposition 1. Let (X, τ_1, τ_2, E) be a *bi soft topological* space over X . If (X, τ_1, τ_2, E) is a *pair wise soft strong* T_0 space then $(X, \tau_1 v \tau_2, E)$ is a *soft strong* T_0 space.

Proof. A *soft bi topological* space (X, τ_1, τ_2, E) is called *pair wise soft strong* T_0 space if τ_1 is a *soft strong* T_0 space with respect to τ_2 and τ_2 is *soft strong* T_0 space with respect toτ_1. If $x, y \in X$, $x \neq y$. Then, since $\tau_1$is *soft strong* T_0 space with respect to τ_2 so there exists τ_1 *soft open* set (F, E) and τ_2 *soft open* set (G, E) such that $x \in \tau_2\mathrm{int}(F, E)$ and $y \notin \tau_2 int(F, E)$ or $y \in \tau_1 int(G, E)$ and $x \notin \tau_1 int(G, E)$ and since $\tau_2$is *soft* T_0 space with respect to τ_1 so there existsτ_2 *soft open* set$(G,E)$and $\tau_1$soft openset(F,E) such that and $x \in \tau_2 int(F,E)$ and $y \notin \tau_2 int(F,E)$ or $y \in \tau_1 int(G,E)$ and $x \notin \tau_1 int(G,E)$. In either case $(F,E),(G,E) \in (X,\tau_1 v \tau_2,E)$. Hence $(X,\tau_1 v \tau_2,E)$ is a soft strong T_0 *space.*

Proposition 2. Let (X,τ_1,τ_2,E) be a *soft bi topological* space over X and Y be a non-empty subset of X. if (X,τ_1,τ_2,E) is *pair wise soft strong* T_0 space. Then$(X,\tau_{1Y},\tau_{2Y},E)$ is *pair wise soft strong* T_0 space.

Proof. Let (X,τ_1,τ_2,E) be a *soft bi topological* space over X $x,y \in X$ such that$x \neq y$. If(X,τ_1,τ_2,E) is *pair wise soft strong* T_0 *space*. Then there exist τ_1 *soft open* set (F,E) and τ_2 s*oft open* set (G,E). Such that $x \in \tau_2\text{int}(F,E)$ and $y \notin \tau_2\text{int}(F,E)$ or $y \in \tau_1\text{int}\,(G,E)$ and$x \notin \tau_1 int(G,E)$. Now, $x \in Y$ and $x \in \tau_2\text{int}\,(F,E)$. Hence, where $x \in Y \cap (F,E) = (Y_F,E)$ where $(F,E) \in \tau_1$. Consider $y \notin \tau_2\text{int}(F,E)$, this means that $\alpha \in E$ for some $\alpha \in E. y \notin Y \cap (F,E) = (Y_E,E)$ There fore τ_{1Y} is *soft strong* T_0 *space* with respect τ_{2Y}. Similarly, can proved thatτ_{2Y} is *soft strong* T_0 *space* with respect to τ_{1Y}, that is $y \in \tau_1\text{int}(G,E)$ and $x \notin \tau_1\text{int}(G,E)$ then $y \in (Y_{G,}E)$ and $x \notin \left(Y_{G,}E\right)$. Thus $(Y,\tau_{1Y},\tau_{2Y},E)$ is *pair wise soft strong* T_0 *space.*

Proposition 3. Let(X,τ_1,τ_2,E) be a *soft bi topological* space over X . Then (X,τ_1,τ_2,E) is a *pair wise soft strong* T_1 space $\Leftrightarrow$ then(X,τ_1,E) and (X,τ_2,E) are *soft strong* T_1 space.

Proof. Suppose (X,τ_1,E) and (X,τ_2,E) are *soft strong* T_1 spaces. Let $x,y \in X, x \neq y$ then

1). $\tau_1$is a soft strong T_1 space with respect to τ_2. So there exists τ_1 s*oft open* set (F,E) and τ_2 *soft open* set (G,E) Such that $x \in \tau_2\text{int}(F,E)$, $y \notin \tau_2\text{int}(F,E)$ and $y \in \tau_1\text{int}(G,E)$, $x \notin \tau_1\text{int}(G,E)$.

2). $\tau_2$is a *soft strong* T_1 space with respect to τ_1So there exists $\tau_2$soft open set (G,E) and τ_1*soft open* set (F,E) such that $x \in \tau_1\text{int}(G,E)$ and $y \notin \tau_1\text{int}(G,E)$ and $y \in \tau_2\text{int}(F,E)$ and $x \notin \tau_2\text{int}(F,E)$. In either case we obtained the requirement and so (X,τ_1,τ_2,E) is a *pair wise soft strong* T_1space. Conversely, we suppose that (X,τ_1,τ_2,E) is a pair *wise soft strong* T_1 space. Then

1). There exists some *soft open set* $(F,E) \in \tau_1$ with respect to *soft open* set $(G,E) \in \tau_2$ such $x \in \tau_2\text{int}(F,E)$, $y \notin \tau_2\text{int}(F,E)$ and $y \in \tau_1\text{int}(G,E)$, $x \notin \tau_1\text{int}(G,E)$.

2). There exists some *soft open* set$(G,E) \in \tau_2$with respect to *soft open* set $(F,E) \in \tau_1$such that $x \in \tau_1\text{int}(G,E)$ and $y \notin \tau_1\text{int}(G,E)$ and $y \in \tau_2\text{int}(F,E)$ and $x \notin \tau_2\,\text{int}(F,E)$ Thus(X,τ_1,E) and (X,τ_2,E) are *soft strong* T_1 Spaces.

Proposition 4. Let (X,τ_1,τ_2,E) be a *soft bi topological* space over X. if (X,τ_1,τ_2,E) is *pair wise soft strong* T_1 space then $(X,\tau_1 v\, \tau_2,E)$ is also *soft strong* T_1 space.

Proof. Let$x,y \in X$ such that $x \neq y$.then exists *soft open* set $(F,E) \in \tau_1$with respect to soft open $(G,E) \in \tau_2$ such that $x \in \tau_2\text{int}(F,E)$, $y \notin \tau_2\text{int}(F,E)$ and $y \in \tau_1\text{int}(G,E)$ and $x \notin \tau_1\text{int}(G,E)$.Similarly ,There exists *soft open* set$(G,E) \in \tau_2$ with respect to τ_1 *soft open* set$(F,E) \in \tau_1$ such that $x \in \tau_1\text{int}(G,E)$ and $y \notin \tau_1\text{int}(G,E)$ and $y \in \tau_2\text{int}(F,E)$ and$x \notin \tau_2\text{int}(F,E)$ So $(F,E),(G,E) \in \tau_1 v \tau_2$ and thus $(X,\tau_1 v\, \tau_2,E)$ is *soft strong* T_1 space.

Proposition 5. Let(X,τ_1,τ_2,E) be a *soft bi topological* space over X and Y be non-empty sub set of X . If (X,τ_1,τ_2,E) is *pair wise soft strong* T_1 space then $(X,\tau_{1Y},\tau_{2Y},E)$ is *pair wise soft strong* T_1 space.

Proof. Let (X,τ_1,τ_2,E) be a *soft bi topological space* over X and $x,y \in Y, x \neq y$. If (X,τ_1,τ_2,E) is *pair wise soft* T_1 then there exists τ_1 *soft open set*(F,E) and τ_2 *soft open* set (G,E) such that $x \in \tau_2\text{int}(F,E)$ and $y \notin \tau_2\text{int}(F,E)$ and $y \in \tau_1\text{int}(G,E)$ and $x \notin \tau_1\text{int}(G,E)$. Now $x \in Y$and $x \in (F,E)$. Hence$x \in Y \cap (F,E) = (Y_F,E)$. Then $y \notin Y \cap F(\alpha)$ for some $\alpha \in E$. This means that $\alpha \in E$ then $y \notin Y \cap F(\alpha)$ for some $\alpha \in E$. Therefore $y \notin Y \cap (F,E) = (Y_F,E)$ Now $y \in Y$ and $y \in (G,E)$.

Hence $y \in Y \cap (G,E) = (Y_G,E)$ where $(G,E) \in \tau_2$. Consider$x \notin (G,E)$ this means that $\alpha \in E$ then$x \notin Y \cap G(\alpha)$ for some $\alpha \in E$. Therefore. $x \notin Y \cap (G,E) = (Y_G,E)$ hence τ_{1Y} is bT_1 space with respect (Y_G,E) to τ_{2Y}. Similarly it can be provide that τ_{2_Y} is *soft strong* T_1 space with respect to τ_{1Y}, that is $y \in \tau_1\text{int}(G,E)$ and $x \notin \tau_1\text{int}(G,E)$. Then $y \in (Y_G,E)$ and $x \notin (Y_G,E)$ .Thus $(X,\tau_{1Y},\tau_{2Y},E)$ is *pair wise soft strong* T_1space.

Proposition 6. Every *pair wise soft strong* T_1 space is *pair wise sot strong* T_0 space.

Proof. Let (X,τ_1,τ_2,E) be a *soft bi topological* space over X and $x,y \in X$ such that $x \neq y$ If (X,τ_1,τ_2,E) is *pair wise soft strong* T_1 space. That is, (X,τ_1,τ_2,E) is *pair wise soft strong* T_1 space with respect toτ_2 and τ_2 is s*oft strong* T_1 space with respect to τ_1 .If τ_1 is *soft strong* T_1 space with respect to τ_2 then there exists a τ_1*soft open* set (F,E) and a τ_2 *soft open* set (G,E) such that $x \in \tau_2\text{int}(F,E)$ and $y \notin \tau_2\text{int}(F,E)$ and $y \in \tau_1\text{int}(G,E)$ and $x \notin \tau_1\text{int}(G,E)$.

Obviously $x \in \tau_2\text{int}(F,E)$ and $y \notin \tau_2\text{int}(F,E)$ or $y \in \tau_1\text{int}(G,E)$ and $x \notin \tau_1\text{int}(G,E)$. Therefore τ_1 is *soft strong* T_0 space with respect to τ_2. Similarly, ifτ_2 is a *soft strong* T_1 space with respect to τ_1 then, there exists τ_2*soft open* set $(F,E)$and τ_1*soft open* set (G,E) such that $x \in \tau_2\text{int}(F,E)$ and $y \notin \tau_2\text{int}(F,E)$ and $y \in \tau_1\text{int}(G,E)$ and$x \notin \tau_1\text{int}(G,E)$. Obviously $x \in \tau_2\,\text{int}(F,E)$ and $y \notin \tau_2\text{int}(F,E)$ or $y \in (G,E)$ and $x \notin \tau_1\text{int}(G,E)$. Therefore$\tau_2$ soft strong T_0 space with respect to τ_1. Thus (X,τ_1,τ_2,E) is a *pair wise* soft strong T_0 space.

Proposition 7. Let (X,τ_1,τ_2,E) be a *soft bi topological* space over X. if (X,τ_1,τ_2,E) is a pair wise soft strong T_2 space over X then $(X,\tau_{1e},\tau_{2e},E)$ is a *pair wise soft strong* T_2 space for each$e \in E$.

Proof. Suppose that (X,τ_1,τ_2,E) is a *pair wise soft strong* T_2 space over X. For any $e \in E$.

$$\tau_{1e} = \{F(e),(F,E) \in \tau_1$$
$$\tau_{2e} = \{G(e),(G,E) \in \tau_2$$

Let $x,y \in X$ such that $x \neq y$ then there *exists soft open set* $(F,E) \in \tau_1$ and *soft open* set $(G,E) \in \tau_2$ such that $x \in \tau_2\text{int}(F,E), y \in \tau_1\text{int}(G,E)$ and $(F,E) \cap (G,E) = \emptyset$. This implies that $x \in F(e) \in \tau_{1e}, y \in G(e) \in \tau_{2e}$for each $e \in E$. Similarly, for the other case. Thus $(X,\tau_{1e},\tau_{2e},E)$ is *a pair wise strong* T_2 space for each $e \in E$.

Proposition 8. Let (X, τ_1, τ_2, E) be a *soft bi topological* space over$X$. If (X, τ_1, τ_2, E) is *Pair wise soft strong* T_2 space. Then $(X, \tau_1 v \tau_2, E)$ also a *soft strong* T_2 space.

Proof. Let $x, y \in X$ such that$x \neq y$. Since (X, τ_1, τ_2, E) be a soft *bi topological* space over $X(X, \tau_1, \tau_2, E)$ is a *pair wise soft strong* T_2space if τ_1 is *soft strong* T_2space with respect τ_2 and τ_2 is *soft strong* T_2 space with respect to τ_1. If τ_1 be *soft strong* T_2 space with respect to τ_2 then there exists a τ_1 *soft open* set $(F, E)$and τ_2*soft open* set(G, E) such that $x \in \tau_2\text{int}(F, E), y \in \tau_2\text{int}(G, E)$ and $(F, E) \cap (G, E) = \emptyset$. Obviously $x \in \tau_2\text{int}(F, E)$ and $y \notin \tau_2\text{int}(F, E)$ and $y \in \tau_1\text{int}(G, E)$ and $x \notin \tau_1\text{int}(G, E)$. Therefore τ_1 is *soft strong* T_1 space with respect to τ_2.

Similarly, τ_2 is soft strong T_2 space with respect to τ_1 then there exists a τ_2 *soft open* set (F, E) and τ_1

Soft open set (G, E) such that $x \in \tau_1\text{int}(F, E), y \in \tau_2\text{int}(G, E)$ and$(F, E) \cap (G, E) = \emptyset$.

Obviously $x \in \tau_1\text{int}(F, E)$ and $y \notin \tau_1\text{int}(F, E)$ and $y \in \tau_2\text{int}(G, E)$ and $x \notin \tau_2\text{int}(G, E)$. Therefore τ_2 soft strong T_2 space with respect to τ_1 thus in either case $(F, E), (G, E) \in \tau_1 v \tau_2$. Hence $(X, \tau_1 v \tau_2, E)$ is *soft strong* T_2 space over X.

Proposition 9. Let (X, τ_1, τ_2, E) be a *soft bi topological* space over X and Y be a non-empty sub set of X. if (X, τ_1, τ_2, E) *is pairwise soft strong* T_2 space then $(Y, \tau_{1Y}, \tau_{2Y}, E)$ is *pair wise soft strong* T_2 space.

Proof. Let (X, τ_1, τ_2, E) be *soft bi topological* space over X $x, y \in X$ such that $x \neq y$ such that (X, τ_1, τ_2, E) is *pair wise soft strong* T_2 space. Then there exists a τ_1 *soft open* set(F, E) and a τ_2 *soft open* (G, E) such that $x \in \tau_2\,\text{int}(F, E), y \in \tau_1\text{int}(G, E)$ and $(F, E) \cap (G, E) = \emptyset$. So for each $\alpha \in E$, $x \in F(\alpha), y \in G(\alpha)$ and $F(\alpha) \cap G(\alpha) = \emptyset$. This implies that $x \in Y \cap F(\alpha), y \in Y \cap G(\alpha)$ and $F(\alpha) \cap G(\alpha) = \emptyset$. Hence $x \in (Y_F, E), y \in (Y_G, E)$ $(Y_F, E) \cap (Y_G, E) = \emptyset$. Where (Y_F, E) is *soft open* set in τ_{1Y}, and (Y_G, E) is *soft open* set in τ_{2Y} therefore τ_{1Y} is *soft* P_2 space with respect to τ_{2Y}. Similarly, it can be proved that τ_{2Y} is *soft* T_2 space with respect to τ_{1Y}. Thus $(Y, \tau_{1Y}, \tau_{2Y}, E)$ is *pair wise soft strong* T_2 space.

Proposition 10. Every *pair wise soft strong* T_1 space is *pair wise soft strong* T_1 space.

Proof. Let (X, τ_1, τ_2, E) be a *soft bi topological* space over X and $x, y \in X$ such that $x \neq y$ If (X, τ_1, τ_2, E) is *pair wise soft strong* T_2 space. That is (X, τ_1, τ_2, E) is *pair wise soft strong* T_2 space if τ_1 is *soft* T_2 space with respect τ_2 then there exists a τ_1 *soft open* set (F, E) and τ_2 *soft open* set(G, E) such that $x \in \tau_2\text{int}(F, E), y \in \tau_1\text{int}(G, E)$ and $(F, E) \cap (G, E) = \emptyset$. Obviously, $x \in \tau_2\text{in}(F, E)$ and $y \notin \tau_2\text{in}(F, E)$ and $y \in \tau_1\text{in}(G, E)$ and $x \notin \tau_1\text{in}((G, E)$. Therefore τ_1 is *soft* strong T_1 space with respect to τ_2. Similarly, if τ_2 is *soft strong* T_2 space with respect to τ_1 then there exists a τ_2 *soft open* set (F, E) and τ_1 *soft open* set(G, E) such that $x \in \tau_2\text{int}(F, E), y \in \tau_1\text{int}(G, E)$ and $(F, E) \cap (G, E) = \emptyset$. Obviously, $x \in \tau_2\text{int}(F, E)$ and $y \notin \tau_1\text{int}(F, E)$ and $y \in \tau_1\text{int}(G, E)$ and $x \notin \tau_1\text{int}(G, E)$. Therefore τ_2 is *soft strong* T_1space with respect to τ_1 thus $(X, \tau_1, \tau_2, E)$is *pair wise soft strong* T_1 space.

4.2 Soft Strong Separation Axioms of Soft Bi Topological Spaces with Respect to Soft Points

In this section, we introduced soft strong separation axioms in soft topology and in soft bi topology with respect to soft points. With the application of these soft strong separation axioms different results are discussed. Soft point is beautifully defined in Definition 9 [42].

Definition 30. In a *soft bi topological* space (X, τ_1, τ_2, E)

1) τ_1 said to be *soft strong* T_0 space with respect to τ_2 if for each pair of distinct points $e_G, e_H \in X_A$ there happens τ_1 soft open set (F, E) and a τ_2 soft open set (G, E) such that $e_G \in \tau_2\text{int}(F, E)$ and $e_H \notin \tau_2\text{int}(F, E)$or $e_H \in \tau_1\text{int}(G, E)$ and $e_G \notin \tau_1\text{int}(G, E)$, Similarly, τ_2 is said to be *soft strong* T_0 space with respect to τ_1 if for each pair of distinct points $e_G, e_H \in X_A$ there happens τ_2 *soft open set*(F, E) and a $\tau_1$soft open set (G, E) such that $e_G \in \tau_1\text{int}(F, E)$ and $e_H \notin \tau_1\text{int}(F, E)$ or $e_H \in \tau_2\text{int}(G, E)$ and $e_G \notin \tau_2\text{int}(G, E)$. *Soft bi topological spaces* (X, τ_1, τ_2, E) is said to be *pair wise soft strong* T_0 space if τ_1 is *soft strong* T_0 *space* with respect to τ_2 andτ_2 is soft strong T_0 space with respect to τ_1.

2) τ_1 is said to be soft strong T_1 space with respect to τ_2 if for each pair of distinct points $e_G, e_H \in X_A$ there happens a τ_1*soft open set* (F, E) and τ_2 soft open set (G, E) such that $e_G \in \tau_2\text{int}(F, E)$ and $e_H \notin \tau_2\text{int}(F, E)$ and $e_H \in \tau_1\text{int}(G, E)$ and $e_G \notin \tau_2\text{int}(G, E)$. Similarly, τ_2 is said to be soft strong T_1 space with respect to τ_1 if for each pair of distinct points $e_G, e_H \in X_A$ there exist a τ_2 soft open set (F, E) and a τ_1 soft open set (G, E) such that $e_G \in \tau_1\text{int}(F, E)$ and $e_H \notin \tau_1\text{int}(F, E)$ and $e_H \in \tau_2\text{int}(G, E)$ and $e_G \notin \tau_2\text{int}(G, E)$. *Soft bi topological spaces* (X, τ_1, τ_2, E) is said to be *pair wise soft strong* T_1 space if τ_1 is *soft strong* T_1 *space* with respect to τ_2 and τ_2 is soft strong T_1 spaces with respect to τ_1.

3) τ_1 is said to be soft strong T_2 space with respect to τ_2, if for each pair of distinct points $e_G, e_H \in X_A$ there happens a τ_1 softopen set (F, E) and a τ_2 soft open set (G, E) such that $e_G \in \tau_2\text{int}(F, E)$ and $e_H \in \tau_1\text{int}(G, E)$ and $(F, E) \cap (G, E) = \phi$. Similarly, τ_2 is aid to be *soft strong* T_2 space with respect to τ_1 if for each pair of distinct points $e_G, e_G \in X_A$there happens a τ_2 *soft open* set (F, E) and a τ_1 *soft open* set(G, E) such that $e_G \in \tau_1\text{int}(F, E)$ and $e_H \in \tau_2\text{int}(G, E)$ and $(F, E) \cap (G, E) = \phi$. The soft bi topological space (X, τ_1, τ_2, E) is said to be pairwise soft strong T_2 space if τ_1 is soft strong T_2 space with respect to τ_2 and τ_2 is soft strong T_2 space with respect to τ_1.

Proposition 11. Let (X, τ_1, τ_2, E) be a *soft bi topological* space over X . If (X, τ_1, E) and (X, τ_2, E) is a *Soft strong* T_0space. Then (X, τ_1, τ_2, E) is a *pair wise soft strong* T_0 space.

Proof. Let$e_G, e_H \in X$, $e_G \neq e_H$and suppose that $(X, \tau_1, E)$is a *soft strong* T_0 *space* with respect to (X, τ_2, E). Then, according to definition, there exists τ_1 *softopen* set $(F, E)$and τ_2 *soft open* set(G, E)such that $e_G \in \tau_2\text{int}(F, E)$ and $e_H \notin \tau_2\text{int}(F, E)$ or$e_H \in \tau_1\text{int}(G, E)$ and $e_G \notin \tau_1\text{int}(G, E)$. Similarly, let $e_G, e_H \in X$, $e_G \neq e_H$ and suppose that (X, τ_2, E) is a *soft strong* T_0 *space* with respect to (X, τ_1, E)then, according to definition there exists τ_2 *soft open* set (G, E) and τ_1 *soft open* set (F, E) such that $e_G \in \tau_2\text{int}(F, E)$ and $e_H \notin \tau_2\text{int}(F, E)$ or $e_H \in \tau_1\text{int}(G, E)$ and $e_G \notin \tau_1\text{int}(G, E)$. Hence (X, τ_1, τ_2, E) is a *pair wise soft strong* T_0 space.

Proposition 12. Let (X, τ_1, τ_2, E) be a *soft bi topological* space over X . If (X, τ_1, τ_2, E) is a *pair wise soft strong* T_0 space then $(X, \tau_1 v, \tau_2, E)$ is a *soft strong* T_0 space.

Proof. A *soft bi topological* space (X, τ_1, τ_2, E) is called *pair wise soft strong* T_0 space if τ_1 is a *soft strong* T_0 space with respect to τ_2 and τ_2 is *soft strong* T_0 space with respect toτ_1. If$e_G, e_H \in X$, $e_G \neq e_H$ then since $\tau_1$is *soft strong* T_0 space with respect to τ_2 so there exists τ_1 *soft open* set (F, E) and τ_2 *soft open* set (G, E) such that $e_G \in \tau_2\text{int}(F, E)$ and $e_H \notin \tau_2\text{int}(F, E)$ or$e_H \in \tau_1\text{int}(G, E)$ and $e_G \notin \tau_1\text{int}(G, E)$ and since $\tau_2$is *soft strong* T_0 space with respect to τ_1 so there existsτ_2 *soft open* set$(G, E)$and $\tau_1$soft open set(F, E) such that and $e_G \in \tau_2\text{int}(F, E)$ and $e_H \notin \tau_2\text{int}(F, E)$ or $e_H \in \tau_1\text{int}(G, E)$ and $e_G \notin \tau_1\text{int}(G, E)$. In either case $(F, E), (G, E) \in (X, \tau_1 v, \tau_2, E)$. Hence $(X, \tau_1 v, \tau_2, E)$ is a *soft strong* T_0 *space.*

Proposition 13. Let (X, τ_1, τ_2, E) be a *soft bi topological* space over X and Y be a non-empty subset of X. If (X, τ_1, τ_2, E) is *pair wise soft strong* T_0 space. Then $(Y, \tau_{1Y}, \tau_{2Y}, E)$ is *pair wise soft strong* T_0 space.

Proof. Let (X, τ_1, τ_2, E) be a *soft bi topological* space over X, $e_G, e_H \in X$ such that$e_G \neq e_H$. If (X, τ_1, τ_2, E) is *pair wise soft strong* T_0 *space*. Then there exist τ_1 *soft open* set (F, E) and τ_2 *Soft open* set(G, E) such that $e_G \in \tau_2\text{int}(F, E)$ and $e_H \notin \tau_2\text{int}(F, E)$ or $e_H \in \tau_1\text{int}(G, E)$ and$e_G \notin \tau_1\text{int}(G, E)$. Now, $e_G \in Y$ and $e_G \in \tau_2\text{int}(F, E)$. Hence where $e_G \in Y \cap \tau_2\text{int}((F, E) = (Y_F, E)$ where $(F, E) \in \tau_1$. Consider$e_H \notin \tau_2\text{int}(F, E)$ this means that $\alpha \in E$ for some $\alpha \in E. y \notin Y \cap \tau_2\text{int}(F, E) = (Y_E, E)$ There fore τ_{1Y} is *soft strong* T_0 *space* with respect τ_{2Y}. Similarly, can proved that τ_{2Y} is *soft strong* T_0 *space* with respect to τ_{1Y}, that is $e_H \in \tau_1\text{int}(G, E)$ and $e_G \notin \tau_1\text{int}(G, E)$ then $e_H \in (Y_{G,}E)$ and $e_G \notin (Y_{G,}E)$. Thus $(X, \tau_{1Y}, \tau_{2Y}, E)$ is *pair wise soft strong* T_0 *space.*

Proposition 14. Let (X, τ_1, τ_2, E) be a *soft bi topological* space over X . Then (X, τ_1, τ_2, E) is a *pair wise soft strong* T_1 space if and only if (X, τ_1, E) and (X, τ_2, E) are *soft strong* T_1 space.

Proof. Suppose (X, τ_1, E) and (X, τ_2, E) are *soft strong* T_1 spaces. Let $e_G, e_H \in X, e_G \neq e_H$ then

1) $\tau_1$is a *soft strong* T_1 space with respect to τ_2. So there exists τ_1 *soft open* set (F, E) and τ_2 *soft open* set (G, E) Such that $e_G \in \tau_2\text{int}(F, E)$ and $e_H \notin \tau_2\text{int}(F, E)$ and $e_H \in \tau_1\text{int}(G, E)$ and $e_G \notin \tau_1\text{int}(G, E)$.

2) $\tau_2$is a *soft strong* T_1 space with respect to τ_1So there exists $\tau_2$softopen set (G, E) and τ_1 *soft open* set (F, E) such that $e_G \in \tau_1\text{int}(G, E)$ and $e_H \notin \tau_1\text{int}(G, E)$ and $e_H \in \tau_2\text{int}(F, E)$ and $e_G \notin \tau_2\text{int}(F, E)$. In either case we obtained the requirement and so (X, τ_1, τ_2, E) is a *pair wise soft strong* T_1 space. Conversely, we suppose that (X, τ_1, τ_2, E) is a *pair wise soft strong* T_1 space. Then

1) There exists some *soft open set* $(F, E) \in \tau_1$ with respect to *soft open* set $(G, E) \in \tau_2$such $e_G \in \tau_2\text{int}(F, E)$ and $e_H \notin \tau_2\text{int}(F, E)$ and $e_H \in \tau_1\text{int}(G, E)$ and $e_G \notin \tau_1\text{int}(G, E)$.

2) There exists some *soft open* set$(G, E) \in \tau_2$with respect to *soft open* set $(F, E) \in \tau_1$such that $e_G \in \tau_1\text{int}(G, E)$ and $e_H \notin \tau_1\text{int}(G, E)$ and $e_H \in \tau_2\text{int}(F, E)$ and $e_G \notin \tau_2\text{int}(F, E)$ Thus(X, τ_1, E) and (X, τ_2, E) are *soft strong* T_1 Space.

Proposition 15. Let (X, τ_1, τ_2, E) be a *soft bi topological* space over X. If (X, τ_1, τ_2, E) is *pair wise soft strong* T_1 space then $(X, \tau_1 v\, \tau_2, E)$ is also *a soft strong* T_1 space.

Proof. Let$e_G, e_H \in X$, $e_G \neq e_H$. Then exists *soft open* set$(F, E) \in \tau_1$with respect to soft open $(G, E) \in \tau_2$ such that $e_G \in \tau_2\text{int}(F, E)$ and $e_H \notin \tau_2\text{int}(F, E)$ and $e_H \in \tau_1\text{int}(G, E)$ and $e_G \notin \tau_1\text{int}(G, E)$. Similarly, There exists *soft open* set$(G, E) \in \tau_2$ with respect to τ_1 *soft open* set$(F, E) \in \tau_1$ such that $e_G \in \tau_1\text{int}(G, E)$ and $e_H \notin \tau_1\text{int}(G, E)$ and $e_H \in \tau_2\text{int}(F, E)$ and $e_G \notin \tau_2\text{int}(F, E)$ so $(F, E), (G, E) \in \tau_1 v \tau_2$ and thus $(X, \tau_1 v\, \tau_2, E)$ is *soft strong* T_1 space.

Proposition 16. Let (X, τ_1, τ_2, E) be a *soft bi topological* space over X and Y be non-empty sub set of X . If(X, τ_1, τ_2, E) is *pair wise soft strong* T_1 space then $(X, \tau_{1Y}, \tau_{2Y}, E)$ is *pair wise soft strong* T_1 space.

Proof. Let (X, τ_1, τ_2, E) be a *soft bi topological space over* X , $e_G, e_H \in Y, e_G \neq e_H$, If (X, τ_1, τ_2, E) is *pair wise soft strong* T_1 then there exists τ_1 *soft open set*(F, E) andτ_2 *soft open* set (G, E) such that $e_G \in \tau_2\text{int}(F, E)$ and $e_H \notin \tau_2\text{int}(F, E)ande_H \in \tau_1\text{int}(G, E)$ and $e_G \notin \tau_1\text{int}(G, E)$. Now$e_G \in Y$and $e_G \in \tau_2\text{int}(F, E)$. Hence $e_G \in Y \cap \tau_2\text{int}(F, E) = (Y_F, E)$ Then $e_H \notin Y \cap F(\alpha)$ for some $\alpha \in E$. This means that $\alpha \in E$ then $e_H \notin Y \cap F(\alpha)$ for some $\alpha \in E$. Therefore, $e_H \notin Y \cap \tau_2\text{int}(F, E) = (Y_F, E)$. Now $e_H \in Y$ and $e_H \in (G, E)$. Hence $e_H \in Y \cap \tau_1\text{int}(G, E) = (Y_G, E)$ where $(G, E) \in \tau_2$. Consider $e_G \notin (G, E)$ this means that $\alpha \in E$ then $e_G \notin YG(\alpha)$ for some $\alpha \in E$. Therefore, $e_G \notin Y \cap \tau_1\text{int}(G, E) = (Y_G, E)$ hence τ_{1Y} is *soft strong* T_1 space with respect (Y_G, E) to τ_{2Y}. Similarly, it can be provide that τ_{2Y} is *soft strong* T_1 space with respect to τ_{1Y}, that is $e_H \in \tau_1\text{int}(G, E)$ and $e_G \notin \tau_1\text{int}(G, E)$. Then $e_H \in (Y_G, E)$ and $e_G \notin (Y_G, E)$ thus $(X, \tau_{1Y}, \tau_{2Y}, E)$ is *pair wise soft strong* T_1 space.

Proposition 17. Every *pair wise soft strong* T_1 space is *pair wise soft strong* T_0 space.

Proof. Let (X, τ_1, τ_2, E) be a *soft bi topological* space over X and $e_G, e_H \in X$ such that $e_G \neq e_H$. If (X, τ_1, τ_2, E) is *pair wise soft strong* T_1 space, that is (X, τ_1, τ_2, E) is *pair wise soft strong* T_1 space with respect toτ_2 and τ_2 is *soft strong* T_1 space with respect to τ_1 . If τ_1 is *soft strong* T_1 space with respect to τ_2 then there exists a τ_1 *soft open* set (F, E) and a τ_2 *soft open* set(G, E) such that $e_G \in \tau_2\text{int}(F, E)$ and $e_H \notin \tau_2\text{int}(F, E)$ and $e_H \in \tau_1\text{int}(G, E)$ and $e_G \notin \tau_1\text{int}(G, E)$ Obviously $e_G \in \tau_2\text{int}(F, E)$ and $e_H \notin \tau_2\text{int}(F, E)$ or $e_H \in \tau_1\text{int}(G, E)$ and $e_G \notin \tau_1\text{int}(G, E)$. Therefore τ_1 is *soft strong* T_0 space with respect to τ_2. Similarly if τ_2 is a *soft strong* T_1 space with respect to τ_1 then their exists τ_2 *soft open* set (F, E) and τ_1*soft open* set(G, E) such that such that $e_G \in \tau_1\text{int}(F, E)$ and $e_H \notin \tau_1\text{int}(F, E)$ and $e_H \in \tau_2\text{int}(G, E)$ and $e_G \notin \tau_2\text{int}(G, E)$. Obviously $e_G \in \tau_1\text{int}(F, E)$ and $e_H \notin \tau_1\text{int}(F, E)$ or $e_H \in \tau_2\text{int}(G, E)$ and $e_G \notin \tau_2\text{int}(G, E)$. Therefore τ_2 soft strong T_0 space with respect to τ_1. Thus (X, τ_1, τ_2, E) is a *pair wise soft strong* T_0.

Proposition 18. Let (X, τ_1, τ_2, E) *be soft bi topological* space over X. If (X, τ_1, τ_2, E) is a pair wise soft strong T_2 space over X then $(X, \tau_{1e}, \tau_{2e}, E)$ is a *pair wise soft strong* T_2 space for each $e \in E$

Proof. Suppose that (X, τ_1, τ_2, E) is a *pair wise soft strong* T_2 space over X. For any$e \in E$

$$\tau_{1e} = \{F(e), (F, E) \in \tau_1$$
$$\tau_{2e} = \{G(e), (G, E) \in \tau_2$$

Let$e_G, e_H \in X$ such that $e_G \neq e_H$

Case 1) then there *exists open set* $(F, E) \in \tau_1$ and *soft open* set $(G, E) \in \tau_2$ such that $e_G \in \tau_1\text{int}(F, E), e_H \in \tau_2\text{int}(G, E)$ and $(F, E) \cap (G, E) = \emptyset$. This implies that $e_G \in F(e) \in \tau_{1e}, e_H \in G(e) \in \tau_{2e}$ for each $\in E$ and $F(e) \cap G(e) = \emptyset$. Similarly,

Case 2) then there *exists open set* $(F, E) \in \tau_2$ and *soft open* set $(G, E) \in \tau_1$ such that $e_G \in \tau_2\text{int}(F, E), e_H \in \tau_1\text{int}(G, E)and(F, E) \cap (G, E) = \emptyset$. This implies that $e_G \in F(e) \in \tau_{2e}, e_H \in G(e) \in \tau_{1e}$ for each $\in E$and$F(e) \cap G(e) = \emptyset$. This implies that $e_G \in F(e) \in \tau_{2e}, e_H \in G(e) \in \tau_{1e}$ for each $\in E$ and $F(e) \cap G(e) = \emptyset$ thus $(X, \tau_{1e}, \tau_{2e}, E)$ is *a pair wise soft strong* T_2 space for each$e \in E$.

Proposition 19. Let (X, τ_1, τ_2, E) be a *soft bi topological* space over$X$. If (X, τ_1, τ_2, E) is p*air wise soft strong* T_2space. Then$(X, \tau_1 v \tau_2, E)$ also a *soft strong* T_2 space.

Proof. Let $e_G, e_H \in X$ such that$e_G \neq e_H$. Since (X, τ_1, τ_2, E) be a soft *bi topological* space over X. (X, τ_1, τ_2, E) is a *pair wise soft strong* T_2 space if τ_1 is *soft strong* T_2 space with respect τ_2 and τ_2 is *soft strong* T_2 space with respect to τ_1. If τ_1 be *soft strong* T_2 space with respect to τ_2, then there exists a τ_1*soft open* set (F, E) and τ_2 *soft open* set (G, E) such that $e_G \in \tau_2\text{int}(F, E), e_H \in \tau_1\text{int}(G, E)$ and $(F, E) \cap (G, E) = \emptyset$. Obviously $e_G \in \tau_2\text{int}(F, E)$ and $e_H \notin \tau_2\text{int}((F, E)$ and $e_H \in \tau_1\text{int}(G, E)$ and $e_G \notin \tau_1\text{int}(G, E)$. Therefore τ_1 is *soft strong* T_1 space with respect to τ_2.

Similarly, τ_2 is soft strong T_2 space with respect to τ_1 then there exists a τ_2 *soft open* set (F, E) and τ_1 s*oft open* set (G, E) such that

$$e_G \in \tau_2\text{int}(F, E), e_H \in \tau_1\text{int}(G, E) \; and \; (F, E) \cap (G, E) = \emptyset.$$

Obviously $e_G \in \tau_2\text{int}(F, E)$ and $e_H \notin \tau_2\text{int}(F, E)$, $e_H \in \tau_1\text{int}(G, E)$ and $e_G \notin \tau_1\text{int}(G, E)$. Therefore τ_2 soft strong T_1 space with respect to τ_1. Thus in either case $(F, E), (G, E) \in \tau_1 v \tau_2$. Hence $(X, \tau_1 v \tau_2, E)$ is *soft strong* T_2 space over X.

Proposition 20. Let (X, τ_1, τ_2, E) be a *soft bi topological* space over X and Y be a non-empty sub set of X. if (X, τ_1, τ_2, E)*is pair wise soft strong* T_2 space. Then$(Y, \tau_{1Y}, \tau_{2Y}, E)$ is *pair wise soft strong* T_2 space $e_G, e_H \in X$ and $e_G \neq e_H$.

Proof. Let (X, τ_1, τ_2, E) be *soft bi topological* space over$X$ $e_G, e_H \in X$ such that $e_G \neq e_H$ is *pair wise soft* strong T_2 space then there exist a τ_1 *soft open* set (F, E) and a τ_2 *soft open*(G, E) such that $e_G \in \tau_2\text{int}(F, E), e_H \in \tau_1\text{int}(G, E)$, $(F, E) \cap (G, E) = \emptyset$ so for each $\alpha \in E e_G \in F(\alpha), e_H \in G(\alpha)$ and $F(\alpha) \cap (G(\alpha) = \emptyset$. This implies that $e_G \in Y \cap F(\alpha), e_H \in Y \cap G(\alpha)$ and $F \in (\alpha) \cap G(\alpha) = \emptyset$. Hence $(Y_F, E) \cap (Y_G, E) = \emptyset$. Where (Y_F, E) is *soft open* set in τ_{1Y}, and (Y_G, E) is *soft open* set in τ_{2Y}. Therefore, τ_{1Y} is *soft strong* T_2 space with respect to τ_{2Y}. Similarly, it can be proved that $\tau_{2Y}$is soft strong T_2 space with respect to τ_{1Y}. Thus $(Y, \tau_{1Y}, \tau_{2Y}, E)$ is *pair wise soft strong* T_2 space.

Proposition 21. Every *pair wise soft strong* T_2 space is *pair wise soft strong* T_1 space.

Proof. Let (X, τ_1, τ_2, E) be a *soft bi topological* space over X and $e_G, e_H \in X$ such that $e_G \neq e_H$. If (X, τ_1, τ_2, E) is *pair wise soft strong* T_2 space. That is (X, τ_1, τ_2, E) is *pair wise soft strong* T_2 space. If τ_1 is *soft strong* T_2 space with respect τ_2 then there exists a τ_1*soft open* set (F, E) and τ_2*soft open* set(G, E)such that $e_G \in \tau_2\mathrm{int}(F, E), e_H \in \tau_1\mathrm{int}(G, E)$ and $(F, E) \cap (G, E) = \emptyset$. Obviously, $e_G \in \tau_2\mathrm{int}(F, E)$ and $e_H \notin \tau_2\mathrm{int}(F, E)$ and $e_H \in \tau_1\mathrm{int}(G, E)$ and $e_G \notin \tau_1\mathrm{int}(G, E)$. Therefore τ_1 is *soft strong* T_1 space with respect to τ_2. Similarly, if τ_2 is *soft strong* T_2 space with respect to τ_1 then there exists a τ_2*soft open* set $(F, E)$and τ_1*soft open* set (G, E)such that$e_G \in \tau_1\mathrm{int}(F, E), e_H \in \tau_2\mathrm{int}(G, E)$ and $(F, E) \cap (G, E) = \emptyset$. Obviously $e_G \in \tau_1\mathrm{int}(F, E)$ and $e_H \notin \tau_1\mathrm{int}(F, E)$ and $e_H \in \tau_2\mathrm{int}(G, E)$ and $e_G \notin \tau_2\mathrm{int}(G, E)$. Therefore τ_2 is *soft strong* T_1 space with respect to τ_1 thus (X, τ_1, τ_2, E) is *pair wise soft strong* T_1 space.

5. Conclusions

Topology is the most important and major area of mathematics and it can make a marriage between other scientific area and mathematical structures beautifully. Recently, many researchers have studied the soft set theory which is initiated by Molodtsov [4] and safely applied to many problems which contain uncertainties in our social life. Shabir and Naz in [23] introduced and deeply studied the concept of soft topological spaces. They also studied topological structures and exhibited their several properties with respect to ordinary points.

In the present work, we have continued to study the properties of soft separation axioms in soft bi topological spaces with respect to soft points as well as ordinary points of a soft topological space. We defined soft strong T_0, T_1, T_2 and spaces with respect to soft points and studied their behaviors in soft bi topological spaces. We also extended these axioms to different results. These soft separation axioms would be useful for the growth of the theory of soft topology to solve complex problems, comprising doubts in economics, engineering, medical etc. We also beautifully discussed some soft transmissible properties with respect to ordinary as well as soft points. We hope that these results in this paper will help the researchers for strengthening the toolbox of soft topology. In the next study, we extend the concept of soft semi open, α- open, Pre-open and b^{**}open soft sets in soft bi topological spaces with respect to ordinary as well as soft points.

References

[1] B. M. Ittanagi, Soft Bi topological spaces international journal of Computer application (0975 8887), v107-No. 7. December 2014.

[2] S. A. El-Sheikh, R. A. Hosny and A. M. Abd-e-Latif, Characterization of b soft separation Axioms in soft topological spaces in Inf. Sci. Lett.4 (3) (2015) 125-133.

[3] M. Naz, M. Shabir and, M. I. Ali, Separation axiom in bi soft topological spaces, Arxiv. 1850 of .00866 vl [Math, Gn] 29 August 2015.

[4] D. A. Molodtsov, Soft set theory first results, Comp. Math. Appl., 37, (1999), 19-31.

[5] B. Ahmad and A. Kharal, Mapping of soft classes, New Math. Nat. Comp., 7 (3) (2011) 471-481.

[6] P. K. Maji, R, Biswas and A. R. Roy, Soft set theory, Comp. Math. Appl., 45, (2003), 555-562.
[7] M. I. Ali, F. Feng, X. Liu, W. K. Min and M. Shabir, On some new operations in soft set theory, Comp. Math. Appl., 57, (2009), 1547-1553.
[8] D. V. Kovkov, V.M.Kolbanov and D.A.Moldtsov, Soft set theory based optimization, Journal of Computer and System, 46 (6) (2007), 872-880.
[9] Z. Xiao, L. Chen, B. Zhong and S. Ye, Recognition for soft information based on the theory of soft sets, J. Chen (Ed) Proceeding of ICSSM – 05, 2, IEEE, 2005,Appl. 1404 -1106.
[10] D. Pei and D. Miao, From soft sets to information systems, in :X, Hu, Q liu, A skowron , T. Y. Lin, R. R Yager, B. Zhang (Eds), Proceedings of Granular computing in IEEE, Vol , 21,2005,pp,617-621.
[11] B. Ahmad and A. Kharal, On fuzzy soft sets, Adv, Fuzzy syst, 2009, Avt, ID 586507, 6PP.
[12] H. Aktas and N. Cagman, Soft sets and soft groups, Information Sciences, 1(77) (2007) 2726-2735.
[13] N. Cagman, F. Citak and S. Enginoglu, Fuzzy parameterized fuzzy soft sets theory and its application, Turkish Journal of Fuzzy systems, 1 (1) (2010)21-35.
[14] A. Kandil, O. A.E Tantawy , S. A. El-Sheikh and A.M Abd El-e-Latif , Fuzzy soft semi connection properties in fuzzy soft topological spaces math, Sci, Lett, 4(2015)171-179.
[15] N. Levine, Semi open sets and semi continuity in topological spaces, Amer. Math. Monthly 70 (1963) 36-41.
[16] P. K. Maji, R. Biswas and A. R. Roy, Fuzzy soft sets, Journal of Fuzzy Mathematics, 9(3) (2001) 589-602.
[17] P. K. Maji, R. Biswas, and A. R. Roy, Intuitionistic fuzzy soft sets, Journal of fuzzy Mathematics 9(3) (2001) 677 – 691.
[18] P. Majumdar and S. K. Samanta, Gernalized fuzzy soft sets, Comp, Math, Appl. 59 (2010) 1425–1432.
[19] T. T. Buhaescu, Some observations Intuitionistic fuzzy relations, Itimerat Seminar on Functional Equations, 111-118.
[20] D. Andrijevic, On b-open sets, Mat.Vesnik, 48 (1996) 24-32.
[21] Y. Zou and Z.Xiao, Data analysis approaches of soft sets under incomplete information, Knowledge Bond systems 21 (2008) 941-945.
[22] C. L. Chang, Fuzzy topological spaces, J. Math. Anal. Appl. 24 (1968) 182-190.
[23] M.Shabir and M.Naz, On soft topological spaces, Compt, Math, Appl. 61(2011)1786 – 1799.
[24] W. K. Min, A note on soft topological spaces, Compt, Math, Appl., 62 (2011) 3524-3528.
[25] A. Kandil, O. A.E. Tantawy , S.A. El-Sheikh and A. M. Abd-e- Latif, R- Operation and decomposition of some forms of soft continuity in soft topological spaces , FMI 7(2014)181-196.
[26] A. Kandil, O. A. E. Tantawy , S. A. EI-Sheikh and A.M Abd-e-Latif, Soft semi-separation axioms and irresolute softfunction, FMI, 8(2) (2014) 305- 318.
[27] A. Kandil, O. A. E. Tantawy, S.A El-Sheikh and A.M.Abd-e-Latif, Soft ideal theory, soft local function and generated soft topological spaces, Appl. Math. Inf. Sci., 8 (4) (2014) 1595-1603.
[28] A. Kandil, O. A. E. Tantawy, S. A. El- Sheikh and A. M Abd-e-Latif, Operation and decomposition of some forms of soft continuity of soft topological spaces via soft ideal, FMI , (3)(2015) 385- 420.

[29] A. Kandil, O.A. E. Tantawy, S.A El-Sheikh and A.M.Abd-e-Latif, Soft connected ness via soft ideals, Journal of New Results in Science, 4 (2014) 90-108.

[30] A. Kandil, O. A.E. Tantawy, S.A. El-Sheikh and A.M.Abd-e-Latif, Soft regularity and normally based on semi – open soft sets and soft ideal, Appl. Math. Inf. Sci. let. 3(2015) 47-55.

[31] A. Kandil, O. A.E.Tantawy, S.A. El-Sheikh and A.M. Abd-e-Latif,Soft semi compactness via soft ideal Appl. Math. Inf. Sci, 8 (5) (2014) 2297- 2306.

[32] A. Kandil, O. A. E. Tantawy, S. A. El-Sheikh and A. M.abd-e- Latif, Soft semi soft ideal, South Asian j. Math. 4 (6) (2014) 265- 284.

[33] A. Kandil, O. A. E.Tantawy, S. A. El- Sheikh and A. M. Abd El-e-Latif, Supra generalized closed soft sets with respect to an soft ideal in supra soft topological spaces Appl. Math.inf . Sci. (4) (2014) 1731- 1740.

[34] L. A. Zadeh, Fuzzy sets, Information and control 8(1965) 338-353.

[35] I. Zorlutana, N. Akdag, W.K. Min, S.Atmaca, Remarks on soft topological spaces, FMI 3 (2) (2012) 171-185.

[36] S.A. El -Sheikh and A.M.Abd-e-Latif, Decomposition of some type of supra soft sets and soft continuity, International Journal of Mathematics Trends and Technology 9 (1) (2014) 37 -56.

[37] Q. M. Sun, Z.L. Zhang, J. Liu, Soft sets and soft modules, Proceeding of Rough Sets and Knowledge Technology , Third International Conference, RSKT 2008, 17-19 May, Chengdu, China, PP. 403- 409, 2008.

[38] C. H. Park, Y.B. Jun, M. A. Ozturk, Soft W.S- Algebras, Communications of the Korean Mathematical society 23(3) (2008) 313- 324.

[39] J. Mahanta, P. K. Das, On soft topological spaces via semi open and semi close soft sets are XIV: 1203, 1203.4133 VI P [Math, GN] 16 Mar 2012.

[40] A. Lancy and Arockiarani, On Soft β-Separation Axioms, IJMRS, 1 (5), 2013.

[41] B. Chen, Soft Semi open sets and related properties in soft topological spaces, App, Math. Inf. Sci., 7.No.1.287-294 (2013).

[42] S. Hussain, B. Ahmad, Soft separation axioms in soft topological space submitted to Elsvier 28 September 2015.

[43] I. Arockiarani, A. Arokialancy, Generalized soft g β closed sets and soft gs β closed sets in soft topological spaces. International Journal of Mathematical Archive 4(2) (2013) 1-7.