http://www.newtheory.org ISSN: 2149-1402

Journal of New Theory

Received: 05.11.2017 *Year*: **2017**, *Number*: **18**, *Pages*: **64-73**
Published: 26.11.2017 *Original Article*

C^∞ SOFT MANIFOLDS

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Abstaract — In this paper, we briefly recall several basic notions of soft sets and soft topological spaces and we continue investigating the properties of soft mappings, soft continuous mappings and soft homeomorphisms. We introduce and discuss the properties of the soft topological manifolds of dimension n and define C^∞ soft manifolds which will strengthen the foundations of the theory of soft geometry. We study restriction of a soft mapping and then define submanifolds.

Keywords — *Soft set, soft mapping, soft continuity, soft topological space, C^∞ soft manifold.*

1 Introduction

The concept of soft sets is introduced by Molodtsov [7] which is a completely new mathematical tool to deal with uncertainties while modeling problems in engineering physics, computer science, economics, social sciences and medical sciences.

Soft sets are convenient to be applied in practice and this theory has potential application in many different fields such as smoothness of functions, game theory, Riemann integration, Perron integration, probability theory and measure theory.

Shabir and Naz [10], defined soft topology and studied many properties. Zorlutuna et al. [14] studied some concepts in soft topological spaces such as interior point, interior, neighborhood, continuity, and compactness. In [4], Maji et al. combined fuzzy sets and soft sets and introduced fuzzy soft sets. They described an application of soft set theory to a decision-making problem [5]. Tanay et al. [11] introduced the fuzzy soft topology. Later, Roy et al. [9] and Varol et al. [12] independently modified the definition of fuzzy soft sets and redefined fuzzy soft topology. Research on the soft set theory has been accelerated [1, 2, 3, 6]. In this paper, we introduce the soft topological manifolds of dimension n. Also we define C^∞ soft manifolds and C^∞ soft submanifolds.

2 Preliminary

Definition 2.1. [7] Let X be an initial universe and E be a set of parameters. Let A be a non-empty subset of E. A pair (F, A) is called a soft set over X, where F is a mapping given by $F : A \to P(X)$.

The set of all of soft sets over X, is denoted by $S(X, E)$.

Example 2.2. [4] Zadeh's fuzzy sets [13] may be concidered as a special case of the soft set.

Let $D : X \to [0, 1]$ be a fuzzy set. Let us concider the family of α-level sets for D given by:

$$F(\alpha) = \{x \in X : \ D(x) \geqq \alpha\}, \quad \alpha \in [0, 1].$$

Then we can write

$$D(x) = sup\{\alpha : \alpha \in [0, 1], \ \ x \in F(\alpha)\}.$$

Thus the fuzzy set D may be considered as a soft set $(F, [0, 1])$.

Definition 2.3. [5] Let $(F, A) \in S(X, E)$.

i. The complement of a soft set (F, A) is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, \ A)$ where, $F^c : A \to P(U)$ is a mapping given by $F^c(a) = U - F(a)$, for all $a \in A$.

ii. Let $E = \{e_1, e_2, ..., e_n\}$ be a set of parameters. The NOT set of E denoted by $\rceil E$ is defined by $\rceil E = \{\rceil e_1, \rceil e_2, ..., \rceil e_n\}$ where, $\rceil e_i = note_i$ for all i.

Definition 2.4. [5] Let $(F, A), (G, B) \in S(X, E)$.

i. (F,A) is a soft subset of (G,B), denoted by $(F, A)\tilde{\subset}(G, B)$, if $F(e) \subset G(e)$ for each $e \in E$.

ii. (F,A) and (G,B) are said to be soft equal, denoted by $(F, A) = (G, B)$ if $(F, A)\tilde{\subset}(G, B)$ and $(G, B)\tilde{\subset}(F, A)$.

iii. Union of (F, A) and (G, B) is a soft set (H, C), where $C = A \cup B$ and $H(e) = F(e) \cup G(e)$ for each $e \in E$. This relationship is written as $(F, A)\tilde{\cup}(G, B) = (H, C)$.

iv. Intersection of (F, A) and (G, B) is a soft set (H, C), where $C = A \cap B$ and $H(e) = F(e) \cap G(e)$ for each $e \in E$. This relationship is written as: $(F, A)\tilde{\cap}(G, B) = (H, C)$.

v. The difference (H, E) of (F, E) and (G, E), denoted by $(F, E)\backslash(G, E)$, is defined as $H(e) = F(e)\backslash G(e)$ for all $e \in E$.

Definition 2.5. [10]

i. Let (F, E) be a soft set over X and $x \in X$. We say that $x \in (F, E)$ and read as x belongs to the soft set (F, E) whenever $x \in F(a)$ for all $a \in E$.

ii. Let $x \in X$, then (x, E) denotes the soft set over X for which $x(a) = x$, for all $a \in E$.

Definition 2.6. [10] Let (F, E) be a soft set over$X$ and Y be a non-empty subset of X. Then the sub soft set of (F, E) over Y denoted by $({}^{Y}\mathrm{F}, E)$, is defined as follows: ${}^{Y}\mathrm{F}(a) = Y \cap F(a)$, for all $a \in E$. In other words $({}^{Y}\mathrm{F}, E) = \tilde{Y}\tilde{\cap}(F, E)$.

Definition 2.7. [3] Let $S(X, E)$ and $S(Y, K)$ be families of soft sets. Let $u : X \to Y$ and $p : E \to K$ be mappings. Then $f_{pu} : S(X, E) \to S(Y, K)$ is defined as:

i. Let $(F, A) \in S(X, E)$. The image of (F, A) under f_{pu} written as $f_{pu}(F, A) = (f_{pu}(F), p(A))$ is a soft set in $S(Y, K)$ such that:

$$f_{pu}(F)(k) = \begin{cases} \bigcup_{e \in p^{-1}(k) \cap A} u(F(e)) & p^{-1}(k) \cap A \neq \phi, \\ \phi & otherwise \end{cases}$$

ii. Let $(G, B) \in S(Y, K)$. The invers image of (G, B) under f_{pu} written as $f_{pu}^{-1}(G, B) = \left(f_{pu}^{-1}(G), p^{-1}(B)\right)$ is a soft set in $S(X, E)$ such that:

$$f_{pu}^{-1}(G)(e) = \begin{cases} u^{-1}(G(p(e))) & p(e) \in B, \\ \phi & otherwise \end{cases}$$

The soft function f_{pu} is called surjective if p and u are surjective, also is said to be injective if p and u are injective.

Theorem 2.8. [1] Let $S(X, E)$ and $S(Y, K)$ be families of soft sets. For the soft function
$f_{pu} : S(X, E) \to S(Y, K)$, the following statements hold:

i. $f_{pu}^{-1}((G, B)^c) = \left(f_{pu}^{-1}(G, B)\right)^c$, for all $(G, B) \in S(Y, K)$.

ii. $f_{pu}\left(f_{pu}^{-1}(G, B)\right) \tilde{\subset} (G, B)$ for all $(G, B) \in S(Y, K)$. If f_{pu} is surjective, then the equality holds.

iii. $(F, A) \tilde{\subset} f_{pu}^{-1}(f_{pu}(F, A))$ for all $(F, A) \in S(X, E)$. If f_{pu} is injective, then the equality holds.

Theorem 2.9. [3] Let $\{(F_i, E)\}_{i \in I} \subseteq S(X, E)$ and $\{(G_i, k)\}_{i \in I} \subseteq S(Y, K)$. Then for a soft mapping $f_{pu} : S(X, E) \to S(Y, K)$, the following are true.

i. If $(F_1, E) \tilde{\subseteq} (F_2, E)$, then $f_{pu}(F_1, E) \tilde{\subseteq} f_{pu}(F_2, E)$

ii. $(G_1, K) \tilde{\subseteq} (G_2, K)$ then $f_{pu}^{-1}(G_1, K) \tilde{\subseteq} f_{pu}^{-1}(G_2, K)$

iii. $f_{pu}(\tilde{\cup}_i (F_i, E)) = \tilde{\cup}_i f_{pu}(F_i, E)$

iv. $f_{pu}^{-1}((G_1, K) \tilde{\cap} (G_2, K)) = f_{pu}^{-1}(G_1, K) \tilde{\cap} f_{pu}^{-1}(G_2, K)$.

v. $f_{pu}^{-1}((G_1, K) \tilde{\cup} (G_2, K)) = f_{pu}^{-1}(G_1, K) \tilde{\cup} f_{pu}^{-1}(G_2, K)$.

Definition 2.10. [10] Let τ be the collection of soft sets over X, then τ is said to be a soft topology on X if

i. ϕ, X belong to τ

ii. the union of any number of soft sets in τ belongs to τ

iii. the intersection of any two soft sets in τ belongs to τ .

The triplet (X, τ, E) is called a soft topological space over X.

The members of τ are said to be soft open sets in X.

Definition 2.11. [14]

i. The soft set $(F, A) \in (X, \tau, E)$ is called a soft point in X, denoted by e_F, if for the element $e \in A, F(e) \neq \phi$ and $F(e') = \phi$ for all $e \in A\{e\}$.

ii. The soft point e_F is said to be in the soft set (G, A), denoted by $e_F \tilde{\in} (G, A)$, if for the element $e \in A,\ F(e) \subseteq G(e)$.

Definition 2.12. Let (X, τ, E) be a soft topological space and $(F, E) \in S(X, E)$.

i. [10] The soft closure of (F, E), denoted by $\overline{(F, E)}$ is the intersection of all closed soft supersets of (F, E) is the smallest closed soft set over X which contains (F, E), i.e.

$$\overline{(F, E)} = \tilde{\cap}\{(H, E) : (H, E) \in \tau', (F, E)\tilde{\subseteq}(H, E)\}.$$

ii. [14] The soft interior of (F, E), denoted by $(F, E)^{\circ}$ is the union of all open soft subsets of (F,E). Clearly (F,E) is the largest open soft set over X which contained in (F,E), i.e.

$$(F, E)^{\circ} = \tilde{\cup}\{(H, E) : (H, E) \in \tau \ and\ (H, E)\tilde{\subset}(F, E)\}.$$

iii. [2] The soft boundary of (F, E) is the soft set

$$\partial (F, E) = \overline{(F, E)}\tilde{\cap}\overline{(F, E)^c}$$

Theorem 2.13. [14] Let $f_{pu} : (X, \tau, E) \to (Y, \upsilon, K)$ be a soft mapping. Then the following statements are equivalent:

i. f_{pu} is soft continuous;

ii. $f_{pu}^{-1}(G, K) \in \tau', \forall (G, K) \in \upsilon'$;

iii. $\overline{f_{pu}^{-1}(G, K)}\tilde{\subseteq} f_{pu}^{-1}\overline{(G, K)}, \forall (G, K) \in S(Y, K)$;

iv. $\partial \left(f_{pu}^{-1}(G, K)\right) \tilde{\subseteq} f_{pu}^{-1}(\partial (G, K)), \forall (G, K) \in S(Y, K)$;

v. $f_{pu}(\partial (F, E)) \tilde{\subseteq} \partial (f_{pu}(F, E)), \forall (F, E) \in S(X, E)$

vi. $f_{pu}\left(\overline{(F, E)}\right) \tilde{\subseteq} \overline{f_{pu}(F, E)}, \forall (F, E) \in S(X, E)$

vii. $f_{pu}^{-1}((G, K)^{\circ}) \tilde{\subseteq} \left(f_{pu}^{-1}(G, K)\right)^{\circ}, \forall (G, K) \in S(Y, K)$

Definition 2.14. A family β of members of τ is called a basis of soft topological space (X, τ), if each element of τ is a union of members of β .

Example 2.15. Let $(\mathbb{R}^n, \tau_{\epsilon n})$ be the fuzzy topology induced by $\beta = \{B(p, \epsilon, r) \mid p \in \mathbb{R}^n,\ \epsilon \in \mathbb{R}^+,\ r \in [0, 1]\}$ which $B(p, \epsilon, r)$ is a fuzzy subset that equals to zero outside the sphere $B(p, \epsilon)$ and equals to r inside $B(p, \epsilon)$. Since by 1.2 each fuzzy subset $B(p, \epsilon, r)$ can be considered as a soft set over $\mathbb{R}^n$ then β can be considered as a basis of a soft topology called soft Euqleadian space denoted by $(\mathbb{R}^n, \tau_{\epsilon n}, [0, 1])$.

Example 2.16. Let $\beta = \{B(p, q, r) \mid p \in \mathbb{R}^n,\ q \in \mathbb{Q}^+,\ r \in \mathbb{Q}^+ \cap [0, 1]\}$ which $B(p, q, r)$ is a fuzzy subset that equals to zero outside the sphere $B(p, q)$ and equals to r inside $B(p, q)$. Since $\mathbb{Q}$ is dence in $\mathbb{R}$, we can easily prove that the soft topology induced by β equels to $(\mathbb{R}^n, \tau_{\epsilon n}, [0, 1])$.

Example 2.17. Let E be a set of parameters and

$$\beta = \{A \mid A : E \to \prod_{i=1}^{n}(a_i, b_i)\ is\ \ surjective\ and\ \forall i \in I,\ a_i, b_i \in \mathbb{R}\}.$$

We call the soft topology τ induced by β, the natural soft topology over $\mathbb{R}^n$. So we have the natural soft topological space $(\mathbb{R}^n, \tau, E)$.

Example 2.18. Let $E = \{1\}$. Then $\{1\}$ is the single nonempty subset of E. We can consider

$$\beta = \{(F, \{1\}) \mid F : \{1\} \to B(p, \epsilon)\ is\ surjective\ and\ p \in \mathbb{R}^n,\ \epsilon \in \mathbb{R}^+\}$$

as a basis of soft Euqleadian space denoted by $(\mathbb{R}^n, \tau_\epsilon, \{1\})$. As in ordinary topology, this space is equel to the natural soft topological space.

3 Soft Topological Manifolds

Definition 3.1. A soft topological space (X, τ, E) is a soft topological space of dimention n if for any $x \in X$ there exists a soft open set (F, A) over X containing x and soft homeomorphic to a soft open set (G, B) of natural soft topology over $(\mathbb{R}^n,\ \tau_\epsilon,\ K)$.

Remark 3.2. When we write $f_{pu} : (F, A) \to (G, B)$, is a soft homeomorphism, it means that there is a soft homeomorphism $f_{pu} : (Z, \tau_Z, E) \to (Y,\ \tau_{\epsilon_Y}, K)$ where $Z = F(A),\ Y = G(B)$. So there is a homeomorphism $u : Z \to Y$ and a bijective map $p : E \to K$.
The triple (F, A, f_{up}) is called a soft local coordinate neighborhood of each $q \in (F, A)$ and we assign to q the n soft coordinates $x_1(q), ..., x_n(q)$, of its image $u(q)$ in $\mathbb{R}^n$.

Proposition 3.3. With the above notations and soft local coordinate neighborhood (F, A, f_{up}), we have $f_{up}(A)(k) = u(A(e))$ where $p^{-1}(k) = \{e\}$.

Proof. Let $(H, D) \in \tau_Z$, then by definition 1.6 we have $f_{pu}(H, D) = (f_{pu}(H), p(D))$.

$$f_{pu}(H)(k) = \begin{cases} \bigcup_{e \in p^{-1}(k) \cap H} u(H(e)) & p^{-1}(k) \cap H \neq \phi \\ \phi & otherwise \end{cases}$$

for each $k \in D$. Since p is bijective, there is exactly one element of A such that $p^{-1}(k) = \{e\}$. So we have $f_{up}(H)(k) = u(H(e))$. If we set $H = A$ then we have $f_{up}(A)(k) = u(A(e))$ where $p^{-1}(k) = \{e\}$. □

Definition 3.4. A soft topological space (X, τ, E) is called a soft topologica manifold of dimension n if satisfies the following axoims:

i. X is a a soft topological space of dimention n,

ii. X is a T_2-space,

iii. X has a countable soft basis of soft open sets.

Definition 3.5. Let $\mathfrak{B} = \{(F_i,\ A_i,\ f_{u_ip_i}) :\ i \in I\}$ be a countable collection of soft local coordinate neighborhoods such that $\tilde{X} = \tilde{\bigcup}_{i\in I}(F_i,\ A_i)$. Since $f_{u_ip_i}$ is a soft homeomorphism for all $i \in I$, then

$$f_{u_ip_i}\, f^{-1}_{u_jp_j} :\ f_{u_jp_j}((F_i,\ A_i)\tilde{\cap}(F_j,\ A_j)) \to f_{u_ip_i}((F_i,\ A_i)\tilde{\cap}(F_j,\ A_j))$$

is a soft homeomorphism for all $i, j \in I$ whenever $(F_i,\ A_i)\tilde{\cap}(F_j,\ A_j) \neq \phi$ that is called a soft transition function.

Let $f_{u_ip_i} :\ (F_i,\ A_i) \to (G_i,\ B_i)$, be a soft homeomorphism for each $i \in I$, then

$$f_{u_i,p_i} : (Z_i, \tau_{Z_i}, E) \ \to\ (Y_i, \sigma_{Y_i}, K)$$

is a soft homeomorphism where $Y_i = G_i(B_i),\ \ Z_i = F_i(A_i)$.

Now for each $(H, D) \in \tau_{Z_i \cap\, Z_j}$ and $k \in D$, we have $f_{u_jp_j}(H)(k) = u_j(H(e))$ where $p_j^{-1}(k) = e$. Therefore

$$f_{u_ip_i}\, f^{-1}_{u_jp_j}\left(f_{u_jp_j}(H)(k)\right) = f_{u_ip_i}\, f^{-1}_{u_jp_j}\left(u_j(H(e))\right) = u_i(H(e))$$

Since $u_i : Z_i \to Y_i$ is a homeomorphism for each $i \in I$, hence for all $q \in H(e)$,

$$u_i\ u_j^{-1}(u_j(q)) = u_i\ u_j^{-1}(x_1^j, x_2^j,\ ...,\ x_n^j) = (x_1^i, x_2^i,\ ...,\ x_n^i) = u_i(q)$$

Proposition 3.6. With the above notations and soft local coordinate neighborhood (F, A, f_{up}), we have $f_{up}(A)(k) = u(A(e))$ where $p^{-1}(k) = \{e\}$.

Proof. Let $(H, D) \in \tau_Z$, then by definition 1.6 we have $f_{pu}(H, D) = (f_{pu}(H), p(D))$.

$$f_{pu}(H)(k) = \begin{cases} \bigcup_{e\in p^{-1}(k)\cap H} u(H(e)) & p^{-1}(k) \cap H \neq \phi \\ \phi & otherwise. \end{cases}$$

for each $k \in D$. Since p is bijective, there is exactly one element of A such that $p^{-1}(k) = \{e\}$. So we have $f_{up}(H)(k) = u(H(e))$. If we set $H = A$ then we have $f_{up}(A)(k) = u(A(e))$ where $p^{-1}(k) = \{e\}$. □

Definition 3.7. A soft topological space (X, τ, E) is called a soft topologica manifold of dimension n if satisfies the following axoims:

i. X is a a soft topological space of dimention n,

ii. X is a T_2-space,

iii. X has a countable soft basis of soft open sets.

Definition 3.8. Let $\mathfrak{B} = \{(F_i,\ A_i,\ f_{u_ip_i}):\ i \in I\}$ be a countable collection of soft local coordinate neighborhoods such that $\tilde{X} = \tilde{\bigcup}_{i\in I}(F_i,\ A_i)$. Since $f_{u_ip_i}$ is a soft homeomorphism for all $i \in I$, then

$$f_{u_ip_i}\, f^{-1}_{u_jp_j}:\ f_{u_jp_j}((F_i,\ A_i)\tilde{\cap}(F_j,\ A_j)) \to f_{u_ip_i}((F_i,\ A_i)\tilde{\cap}(F_j,\ A_j))$$

is a soft homeomorphism for all $i, j \in I$ whenever $(F_i,\ A_i)\tilde{\cap}(F_j,\ A_j) \neq \phi$ that is called a soft transition function.

Let $f_{u_ip_i}:\ (F_i,\ A_i) \to (G_i,\ B_i)$, be a soft homeomorphism for each $i \in I$, then

$$f_{u_i,p_i} : (Z_i, \tau_{Z_i}, E)\ \to\ (Y_i, \sigma_{Y_i}, K)$$

is a soft homeomorphism where $Y_i = G_i(B_i),\ \ Z_i = F_i(A_i)$.

Now for each $(H, D) \in \tau_{Z_i\cap\, Z_j}$ and $k \in D$, we have $f_{u_jp_j}(H)(k) = u_j(H(e))$ where $p_j^{-1}(k) = e$. Hence

$$f_{u_ip_i}\, f^{-1}_{u_jp_j}\left(f_{u_jp_j}(H)(k)\right) = f_{u_ip_i}\, f^{-1}_{u_jp_j}\left(u_j(H(e))\right) = u_i(H(e))$$

Since $u_i : Z_i \to Y_i$ is a homeomorphism for each $i \in I$, hence for all $q \in H(e)$,

$$u_i\, u_j^{-1}(u_j(q)) = u_i\, u_j^{-1}(x_1^j, x_2^j,\ ...,\ x_n^j) = (x_1^i, x_2^i,\ ...,\ x_n^i) = u_i(q)$$

4 C^∞ Soft Manifolds

Definition 4.1. With the above notations, we shall say that $((F_i,\ A_i),\ f_{u_ip_i})$ is C^∞-compatible with $((F_j,\ A_j),\ f_{u_jp_j})$ when $(F_i,\ A_i)\tilde{\cap}(F_j,\ A_j) \neq \phi$ if $u_i\ u_j^{-1}$ and $u_j\ u_i^{-1}$ changing the soft coordinates are C^∞ functions or we say that $f_{u_ip_i}\, f^{-1}_{u_jp_j}$ is a soft deffeomorphism of soft open subsets (G_j, B_j) and (G_i, B_i) of $\mathbb{R}^n$.

Definition 4.2. A soft differetiable or C^∞ structure on a soft topological manifold (X, τ, E) is a family $\mathfrak{A} = \{(F_i, A_i, f_{u_ip_i}),\ i \in I\}$ of soft coordinate neighborhoods s.t.

i. $\tilde{X} = \tilde{\bigcup}_{i\in I}(F_i,\ A_i)$;

ii. Each triple $((F_i,\ A_i,\ f_{u_ip_i})$ and $((F_j,\ A_j,\ f_{u_jp_j})$ are C^∞ soft compatible for all $i, j \in I$.

iii. Any soft coordinate neighborhood $((H,\ D,\ h_{up})$ that is fuzzy compatible with every $(F_i,\ A_i,\ f_{u_ip_i}),;i \in I$, is itself in $\mathfrak{A}$.

A C^∞ soft manifold is a soft topological manifold with a soft C^∞ structure on it.

Example 4.3. Let $E = \{1\}$. Then $\{1\}$ is the single nonempty subset of E. We can consider the soft Euqleadian space $(\mathbb{R}^n, \tau_\epsilon, \{1\})$. Now let $X = \mathcal{M}_{m\times n}(\mathbb{R})$. Since there is a bijection $\psi : X \to \mathbb{R}^{mn}$:

$$\psi(a_{ij}) = (a_{11},\ ...,\ a_{1n};\ ...;\ a_{m1},\ ...,\ a_{mn}),$$

we can define a natural soft topology τ on X as follows:

$$\tau = \{\phi, X\} \cup \{(F, \{1\}) : F(\{1\})\ is\ an\ ordinary\ open\ set\ of\ X\}.$$

Also we can cover $(X, \tau, \{1\})$ by a single soft coodinate neighborhood $(F, \{1\}, f_{up})$ where$F(\{1\}) = X,\ \ u = \psi,\ \ p = id$. Hence $(X, \tau, \{1\})$ is a C^∞ soft manifold of dimention mn.

Definition 4.4. (Soft open submanifolds) Let (X, τ, E) be an C^∞ soft manifold and Z be a soft open subset of X. If $\mathfrak{A} = \{(F_i,\ A_i,\ f_{u_ip_i}),\ i \in I\}$ is a C^∞ structure on X, then $(Z,\ \tau_Z, E)$ is a C^∞ soft manifold with soft differentiable structure consisting of the soft coordinate neighborhoods $(F_i|_{A_i\cap Z},\ A_i \cap Z,\ f_{(u_i|_{F(A_i\cap Z)})p_i})$.

Example 4.5. Since $Z = Gl(n, \mathbb{R})$ is an open subset of $X = \mathcal{M}_{n\times n}(\mathbb{R})$, then $(Z,\ \tau_Z, E)$ is an C^∞ soft submanifold of (X, τ, E).

Example 4.6. Let $X = S^2$, the unit sphere and $E = 1$. As in example 3.3 we have a natural soft topology τ on X. We prove that $(X, \tau, \{1\})$ is a C^∞ soft manifold of dimention 2. Let $I = \{1, 2, 3\}$. We define six soft open subsets covering $X,\ F_i^\pm : \{1\} \to \mathbb{R}^3,\ i \in I$ by:

$$F_i^\pm(\{1\}) = \{(x_1, x_2, x_3)|\ \|x\| = 1,\ \pm x_i > 1\}.$$

Then we show that all soft sets $(F_i^\pm, \{1\})$ are homeomorphic to the soft open subset $(G, \{1\})$ which $G\{1\} = \{(y_1, y_2)\ |\|y\| < 1\}$, with six soft homeomorphisms $f_{u_i\pm p}$ where $p = id,\ u_i^\pm\ (F_i^\pm(\{1\})) \to G(\{1\}),\ \forall i \in I$ is defined by:

$$u_1^\pm(x_1, x_2, x_3) = (x_2, x_3)\ , \quad (u_1^\pm)^{-1}(y_1, y_2) = (\pm\sqrt{1 - y_1^2 - y_2^2}\ , y_1, y_2)$$

$$u_2^\pm(x_1, x_2, x_3) = (x_1, x_3)\ , \quad (u_2^\pm)^{-1}(y_1, y_2) = (y_1, \pm\sqrt{1 - y_1^2 - y_2^2}, y_2)$$

$$u_3^\pm(x_1, x_2, x_3) = (x_1, x_2)\ , \quad (u_3^\pm)^{-1}(y_1, y_2) = (y_1, y_2,\ \pm\sqrt{1 - y_1^2 - y_2^2})$$

Also it is seen that $u_j^\pm \circ (u_i^\pm)^{-1}$ is infinitly differentiable for all $i,\ j \in I$. For example:

$$u_2^\pm \circ (u_1^\pm)^{-1}(y_1, y_2) = u_2^\pm(\pm\sqrt{1 - y_1^2 - y_2^2}, y_1, y_2) = (\pm\sqrt{1 - y_1^2 - y_2^2}, y_2)$$

Therefor each triple $(F_i^\pm, \{1\}, f_{u_i^\pm p})$ and $(F_j^\pm, \{1\}, f_{u_j^\pm p})$ are C^∞ soft compatible for all $i,\ j \in I$.

Example 4.7. Let $E = \mathbb{R}^n$ and $X = T\mathbb{R}^n = \bigcup_{p\in\mathbb{R}^n} T_p\mathbb{R}^n$. We define the soft topology τ as follows:

$$\tau = \{(F, U) \mid F(U) = \bigcup_{p\in U} T_p\mathbb{R}^n,\ U\ is\ an\ open\ subset\ of\ \mathbb{R}^n\}.$$

We define soft homeomorphism $f_{up} : (F, E) \to (G, E)$, where $F(E) = X,\ G(E) = \mathbb{R}^{2n},\ p = id$ and $u(v_p) = (x_1,\ ...,\ x_n,\ v_1,\ ...,\ v_n),\ \forall p = (x_1,\ ...,\ x_n) \in \mathbb{R}^n, v = (v_1,\ ...,\ v_n) \in T_p\mathbb{R}^n$. Thus (X, τ, E) is an C^∞ soft manifold of dimension $2n$ with single soft coordinate neighborhood (F, E, f_{up})

Theorem 4.8. Let (M, σ) be an C^∞ topological manifold of dimension n and $E = \mathbb{R}^n$. Let $X = TM = \bigcup_{p\in M} T_pM$ and $\mathfrak{B} = \{(U_i,\ \psi_i,),\ i \in I\}$ be an C^∞ structure on M. We define the soft topology $\tau = \{(F, U) \mid F(U) = \bigcup_{p\in U} T_pM,\ U \in \sigma\}$. Then (X, τ, E) is an C^∞ soft manifold of dimension $2n$.

Proof. One can easily prove that τ is an soft topology on X. If $\psi_i(p) = (x_1^i, x_2^i, \ ..., \ x_n^i)$, $\forall p \in U_i$, then we define soft homeomorphisms:

$$f_{u_ip} : (F_i, U_i) \to (G, U_i), \quad F_i(U_i) = \bigcup_{p \in U_i} T_pM, \quad G(U_i) = U_i \times \mathbb{R}^n, \ p = id$$

$$u_i(v_p) = (x_1^i, \ ..., \ x_n^i, \ v_1^i, \ ..., \ v_n^i), \quad \forall p \in U_i, \quad \forall v_p \in T_pM$$

Note that $v_p = \sum_{k=1}^n v_k^i \ E_{ip}$ where $E_{ip} = \psi_{i*}^{-1}(\dfrac{\partial}{\partial x_k^i})$. Since $\psi_i \circ \psi_j^{-1}$ is C^∞ for all $i, j \in I$, then $u_i u_j^{-1}$ and then $f_{u_ip} \circ f_{u_jp}^{-1}$ are C^∞ for all $i, j \in I$. Therefore we have C^∞ soft structure:

$$\mathfrak{A} = \{(F_i, \ U_i, \ f_{u_ip}), \ i \in I\}.$$

□

5 Conclusion

In the present study, we have continued to study the properties of soft continuous, soft open and soft closed mappings between soft topological spaces.We obtain new characterizations of these mappings and investigate preservation properties. We expect that results in this paper will be basis for further applications of soft manifolds in soft sets theory.

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