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$(\tilde{T}, S)$ - CUBİC HYPER *KU*-İDEALS

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Abstract – It is known that, the concept of hyper KU–algebras is a generalization of KU–algebras. In this paper, the concepts $(\tilde{T}, S)$ – cubic theory of the (s-weak – strong) hyper KU-ideals in hyper KU–algebras are applied and the relations among them are obtained.

Keywords – *KU–algebra, hyper KU–algebra, cubic hyper KU–ideal.*

1. Introduction

Prabpayak and Leerawat [18,19] introduced a new algebraic structure which is called KU–algebras. They studied ideals and congruences in KU-algebras. Also, they introduced the concept of homomorphism of KU–algebra and investigated some related properties. Moreover, they derived some straightforward consequences of the relations between quotient KU–algebras and isomorphism. Mostafa et al. [15] introduced the notion of fuzzy KU–ideals of KU-algebras and then they investigated several basic properties which are related to fuzzy KU-ideals. The hyper structure theory (called also multi–algebras) is introduced in 1934 by Marty [14] at the 8th congress of Scandinvian Mathematiciens. Around the 40's, several authors worked on hyper groups, especially in France and in the United States, but also in Italy, Russia and Japan. Hyper structures have many applications to several sectors of both pure and applied sciences. Since then numerous mathematical papers [2,3,4,5,6,14,16,17,21] have been written investigating the algebraic properties of the hyper BCK/BCI–KU–algebras. Jun and Xin [6] considered the fuzzification of the notion of a (weak, strong, reflexive) hyper BCK–ideal, and investigated the relations among them. Mostafa et al. [16], applied the hyper structures to KU–algebras and introduced the concept of a hyper KU–algebra which is a generalization of a KU–algebra, and investigated some related properties. They also introduced the notion of a hyper KU–ideal, a weak hyper KU–ideal and gave relations between hyper KU–ideals and weak hyper KU–ideals.

On the other hand, triangular norm is a powerful tool in the theory research and application development of fuzzy sets. On the basis of the definition of the intuitionistic fuzzy groups, Li et al . [13], generalized the operators “ ^” and “ $\vee$ ” to T-norm and S-norm and defined the intuitionistic fuzzy groups of (T,S)–norms. as a generalization of the notion of fuzzy set Kim [12] Using t–norm T and s–norm S, they introduced the notion of intuitionistic (T,S) – normed fuzzy subalgebra in BCK/BCI-algebra, and some related properties are investigated. Based on the (interval-valued) fuzzy sets, Jun et al. [7-11] introduced the notion of cubic sub–algebras/ideals in BCK/BCI–algebras, and then they investigated several properties. There are several authors who applied the theory of cubic sets to different algebraic structures for instance, Jun et al. [7-11], Yaqoob et al. [20].

In this paper, we studied the idea of $(\tilde{T}, S)$ –cubic set theory to the (s–weak–strong) hyper KU–ideals in hyper KU–algebras and investigated some of related properties .

2. Preliminaries

Let H be a nonempty set and $P^*(H) = P(H)\setminus\{\phi\}$ the family of the nonempty subsets of H. A multi valued operation (said also hyper operation) "$\circ$" on H is a function, which associates with every pair $(x, y) \in H \times H = H^2$ a non empty subset of H denoted $x \circ y$ .An algebraic hyper structure or simply a hyper structure is a non empty set H endowed with one or more hyper operations.

We shall use the $x \circ y$ instead of $x \circ \{y\}$, $\{x\} \circ y$ or $\{x\} \circ \{y\}$.

Definition 2.1 [16] Let H be a nonempty set and "$\circ$" a hyper operation on H, such that $\circ : H \times H \to P^*(H)$. Then H is called a hyper KU-algebra if it contains a constant "0" and satisfies the following axioms: for all $x, y, z \in H$

(HKU_1) $[(y \circ z) \circ (x \circ z)] << x \circ y$
(HKU_2) $x \circ 0 = \{0\}$
(HKU_3) $0 \circ x = \{x\}$
(HKU_4) *if* $x << y$, $y << x$ *implies* $x = y$.

where x << y is defined by $0 \in y \circ x$ and for every $A, B \subseteq H$, $A << B$ is defined by $\forall a \in A, \exists b \in B$ *such that* $a << b$. In such case, we call "<<" the hyper order in H.

Note that if $A, B \subseteq H$, then by $A \circ B$ we mean the subset $\bigcup_{a\in A, b\in B} a \circ b$ *of* H.

Example 2.2. [16] Let $H = \{0,1,2,3\}$ be a set. Define hyper operation $\circ$ on H as follows:

$\circ$	0	1	2	3
0	{0}	{1}	{2}	{3}
1	{0}	{0}	{1}	{3}
2	{0}	{0}	{0}	{0,3}
3	{0}	{0}	{1}	{0,3}

Then $(H, \circ, 0)$ is a hyper KU-algebra.

Proposition 2.3. [16] Let H be a hyper KU-algebra. Then for all $x, y, z \in H$, the following statements hold:

(P_1) $A \subseteq B$ implies $A << B$, for all nonempty subsets *A*, *B* of *H*.
(P_2) $0 \circ 0 = \{0\}$.
(P_3) $0 << x$.
(P_4) $z << z$.
(P_5) $x \circ z << z$
(P_6) $A \circ 0 = \{0\}$.
(P_7) $0 \circ A = A$.
(P_8) $(0 \circ 0) \circ x = \{x\}$ and $(x \circ (0 \circ x)) = \{0\}$.
(P_9) $x \circ x = \{x\} \Leftrightarrow x = 0$

Lemma 2.4. [16] In hyper KU-algebra $(X, \circ, 0)$, the following hold:

$$x << y \quad \text{imply} \quad y \circ z << x \circ z \text{ for all } x, y, z \in X$$

Lemma 2.5. [16] In hyper KU-algebra $(X, \circ, 0)$, we have

$$z \circ (y \circ x) = y \circ (z \circ x) \text{ for all } x, y, z \in X .$$

Lemma 2.6. [16] For all $x, y, z \in H$, the following statements hold:

(i) $x \circ y << z \Leftrightarrow z \circ y << x$,
(ii) $0 << A \Rightarrow 0 \in A$,
(iii) $y \in (0 \circ x) \Rightarrow y << x$.

Definition 2.7. [16] Let A be a non-empty subset of a hyper KU-algebra X. Then A is said to be a hyper ideal of X if

(HI_1) $0 \in A$,
(HI_2) $y \circ x << A$ and $y \in A$ imply $x \in A$ for all $x, y \in X$.

Definition 2.8. [16] Let I be a non-empty subset of a hyper KU-algebra H and $0 \in I$. Then,

(1) I is called a weak hyper ideal of H if $y \circ x \subseteq I$ and $y \in I$ imply that $x \in I$, for all $x, y \in H$.
(2) I is called a strong hyper ideal of H if $(y \circ x) \cap I \neq \phi$ and $y \in I$ imply that $x \in I$, for all $x, y \in H$.

Definition 2.9. [16] For a hyper KU-algebras H, a non-empty subset $I \subseteq H$, containing 0 are :

1- A weak hyper KU-ideal of H if $a \circ (b \circ c) \subseteq I$ and $b \in I$ imply $a \circ c \in I$.

2- A hyper KU-ideal of H if $a \circ (b \circ c) << I$ and $b \in I$ imply $a \circ c \in I$.
3- A strong hyper KU-ideal of H if $(\forall x, y \in H)((a \circ (b \circ c) \bigcap I \neq \phi)$ and $b \in I$ imply $a \circ c \in I$.

Example 2.10. [16] Let $H = \{0, a, b, c\}$ be a set with the following Cayley table

∘	0	a	b	c
0	{0}	{a}	{b}	{c}
a	{0}	{0,a}	{0,b}	{b,c}
b	{0}	{0,b}	{0 }	{a}
c	{0}	{0,b}	{0 }	{0,a}

Then H is a hyper KU-algebra. Take $I = \{0, b\}$**,** then I is a weak hyper ideal, however, not a weak hyper KU-ideal of H as $b \circ (b \circ c) \subseteq I,\ b \in I,\ but\ b \circ c = a \notin I$.

Example 2.11.[16]. Let $H = \{0, a, b\}$ be a set with the following Cayley table:

∘	**0**	**a**	**b**
0	$\{0\}$	$\{a\}$	$\{b\}$
a	$\{0\}$	$\{0, a\}$	$\{b\}$
b	$\{0\}$	$\{b\}$	$\{0, b\}$

Then H is a hyper KU-algebra. Take $I = \{0, b\}$. Then I is a hyper ideal, but not a hyper KU-ideal, since $0 \circ (b \circ a) << I$ and $b \in I$ but $a \notin I$

Here $I = \{0, b\}$is also a strong hyper ideal but it is not a strong hyper KU-ideal of H, since $0 \circ (b \circ a) = \{b\} \cap I \neq \phi\ and\ b \in I\ but\ a \notin I$.

Definition 3.12. [12,13] An interval number is $\tilde{a} = [a_L, a_U]$, where $0 \leq a_L \leq a_U \leq 1$. Let D[0, 1] denote the family of all closed subintervals of [0, 1], i.e.,

$$D[0,1] = \{\tilde{a} = [a_L, a_U] : a_L \leq a_U \quad for\ a_L, a_U \in I\}.$$

We define the operations ≤ , ≥ , =,rmin and rmax in case of two elements in D[0, 1]. We consider two elements $\tilde{a} = [a_L, a_U]$ and $\tilde{b} = [b_L, b_U]$ in D[0, 1]. Then

1- $\tilde{a} \leq \tilde{b}$ *iff* $a_L \leq b_L, a_U \leq b_U$;
2- $\tilde{a} \geq \tilde{b}$ *iff* $a_L \geq b_L, a_U \geq b_U$;
3- $\tilde{a} = \tilde{b}$ *iff* $a_L = b_L, a_U = b_U$;
4- $rmim\{\tilde{a}, \tilde{b}\} = [\min\{a_L, b_L\}, \min\{a_U, b_U\}]$;
5- $r\max\{\tilde{a}, \tilde{b}\} = [\max\{a_L, b_L\}, \max\{a_U, b_U\}]$

Here we consider that $\tilde{0} = [0,0]$ as least element and $\tilde{1} = [1,1]$as greatest element. Let $\tilde{a}_i \in D[0,1]$,where $i \in \Lambda$ .We define

$$r\inf_{i\in\Lambda} \tilde{a}_i = \left[\inf_{i\in\Lambda}(a_i)_L, \inf_{i\in\Lambda}(a_i)_U\right] \text{ and } r\sup_{i\in\Lambda} \tilde{a}_i = \left[\sup_{i\in\Lambda}(a_i)_L, \sup_{i\in\Lambda}(a_i)_U\right]$$

An interval valued fuzzy set (briefly, i-v-f-set) $\tilde{\mu}$ on a set X is defined as

$$\tilde{\mu} = \left\{\left\langle x\ ,\ [\mu^L(x), \mu^U(x)],\ x \in X \right\rangle\right\}$$

where $\tilde{\mu} : X \to D[0,1]$ and $\mu^L(x) \leq \mu^U(x)$, for all $x \in X$. Jun et al. [7-11], introduced the concept of cubic sets defined on a non-empty set *X* as objects having the form:

$$A = \{\langle x, \tilde{\mu}_A(x), \lambda_A(x)\rangle : x \in X\},$$

which is briefly denoted by $A = \langle \tilde{\mu}_A, \lambda_A \rangle$, where the functions $\tilde{\mu}_A : X \to D[0,1]$ and $\lambda_A : X \to [0,1]$.

Definition 2.12. [12,13] A triangular norm (t-norm) is a function $T : [0,1]\times[0,1] \to [0,1]$ that satisfies following conditions:

(T_1) boundary condition :T(x, 1) = x,
(T_2) commutativity condition: T(x, y) = T(y, x),
(T_3) associativity condition :T(x, T(y, z)) = T(T(x, y), z),
(T_4) monotonicity:T(x, y) ≤ T(x, z) ,whenever y ≤ z for all x, y, z ∈ [0, 1].

A simple example of such defined *t*-norm is a function *T* (*α, β*) = min*{α, β}*. In the general case *T* (*α, β*) ≤ min*{α, β}* and *T* (*α,* 0) = 0 for all *α, β* ∈ [0, 1].

Definition 2.13 [12,13] A triangular conorm (t-conorm S) is a mapping $S : [0,1]\times[0,1] \to [0,1]$ that satisfies following conditions:

(S1) S(x, 0) = x,
(S2) S(x, y) = S(y, x),
(S3) S(x, S(y, z)) = S(S(x, y), z),
(S4) S(x, y) ≤ S(x, z) , whenever y ≤ z for all x, y, z ∈ [0, 1].

A simple example of such definition s-norm S is a function S(x, y) = max{x, y}. Every S-conorm S has a useful property: max{α, β}≤ S (α, β) for all α, β ∈ [0, 1].

Definition 2.14. [1] An interval valued triangular norm (interval valued t-norm) is a function $\tilde{T} : D[0,1]\times D[0,1] \to D[0,1]$ that satisfies following conditions:

(T_1) interval valued boundary condition : $\tilde{T}(\tilde{x}, \tilde{1}) = \tilde{x}$,

(T_2)) interval valued commutativity condition: $\tilde{T}(\tilde{x}, \tilde{y}) = \tilde{T}(\tilde{y}, \tilde{x})$,

(T_3) interval valued associativity condition : $\tilde{T}(\tilde{x},\tilde{T}(\tilde{y},\tilde{z}))=\tilde{T}(\tilde{T}(\tilde{x},\tilde{y}),\tilde{z})$,
(T_4) interval valued monotonicity: $\tilde{T}(\tilde{x},\tilde{y})\leq\tilde{T}(\tilde{y},\tilde{z})$,whenever $\tilde{y}\leq\tilde{z}$ for all $\tilde{x},\tilde{y},\tilde{z}\in D[0,1]$.

A simple example of such defined interval valued *t*-norm is a function $\tilde{T}(\tilde{\alpha},\tilde{\beta})=r\min\{\tilde{\alpha},\tilde{\beta}\}$.

In the general case $\tilde{T}(\tilde{\alpha},\tilde{\beta})\leq r\min\{\tilde{\alpha},\tilde{\beta}\}$ and $\tilde{T}(\tilde{\alpha},\tilde{0})=\tilde{0},\forall\tilde{\alpha},\tilde{\beta}\in D[0,1]$.

Definition 2.15.3 [1] An interval valued triangular conorm (interval valued t-conorm $\tilde{S}$) is a mapping $\tilde{S}$: $D[0,1]\times D[0,1]\rightarrow D[0,1]$ that satisfies following conditions:

(S1) $\tilde{S}(\tilde{x},\tilde{0})=\tilde{x}$,
(S2) $\tilde{S}(\tilde{x},\tilde{y})=\tilde{S}(\tilde{y},\tilde{x})$,
(S3) $\tilde{S}(\tilde{x},\tilde{S}(\tilde{y},\tilde{z}))=\tilde{S}(\tilde{S}(\tilde{x},\tilde{y}),\tilde{z})$
(S4) interval valued monotonicity : $\tilde{S}(\tilde{x},\tilde{y})\leq\tilde{S}(\tilde{y},\tilde{z})$,whenever $\tilde{y}\leq\tilde{z}$ for all $\tilde{x},\tilde{y},\tilde{z}\in D[0,1]$.

A simple example of such definition interval valued s-norm S is a function $\tilde{S}(\tilde{\alpha},\tilde{\beta})=r\max\{\tilde{\alpha},\tilde{\beta}\}$.

In the general case $\tilde{S}(\tilde{\alpha},\tilde{\beta})\leq r\max\{\tilde{\alpha},\tilde{\beta}\}$ and $\tilde{S}(\tilde{\alpha},\tilde{1})=\tilde{1},\forall\tilde{\alpha},\tilde{\beta}\in D[0,1]$

Example.2.16. Here the set H = {0, 1, 2, 3,4} in which $\circ$ is defined by the following table

$\circ$	0	1	2	3	4
0	{0}	{1}	{2}	{3}	{4}
1	{0}	{0}	{2}	{3}	{4}
2	{0}	{1}	{0}	{3}	{3}
3	{0}	{0}	{2}	{0}	{2}
4	{0}	{0}	{0}	{0}	{0}

Then $(H,*,0)$ is a hyper KU-algebra . The function $\tilde{T}_m$ defined by

$$\tilde{T}_m(\tilde{\alpha},\tilde{\beta})=r\max\{\tilde{\alpha}+\tilde{\beta}-\tilde{1},\tilde{0}\},\forall\tilde{\alpha},\tilde{\beta}\in D[0,1]$$

By routine calculations, we known that a fuzzy set $\tilde{\mu}$ in H defined by $\tilde{\mu}$ (1) =[0.3,0.5] and $\tilde{\mu}$(0) = $\tilde{\mu}$ (2) = $\tilde{\mu}$ (3) = $\tilde{\mu}$ (4) = [0.3 0.9] is interval valued $\tilde{T}_m$ -fuzzy KU-ideal of H, which is interval valued $\tilde{T}_m$ -fuzzy KU-ideal because $\tilde{\mu}_A(x)\geq\tilde{T}\{\tilde{\mu}_A(z*(y*x)),\tilde{\mu}_A(y*z)\}$.

3. $(\tilde{T}, S)$ - Cubic hyper *KU*–ideals

Now some fuzzy logic concepts are reviewed .A fuzzy set μ in a set H is a function $\mu : H \to [0,1]$. A fuzzy set μ in a set H is said to satisfy the inf (resp. sup) property if for any subset T of H there exists $x_0 \in T$ such that $\mu(x_0) = \inf_{x \in T} \mu(x)$ (resp. $\mu(x_0) = \sup_{x \in T} \mu(x)$). For a fuzzy set μ in X and a $\in$ [0, 1] the set U(μ ; a):={x $\in$ H , μ (x) $\geq$ a}, which is called a level set of μ .

Definition 3.1. A fuzzy set μ in H is said to be a fuzzy hyper KU-subalgebra of H if it satisfies the inequality:

$$\inf_{z \in x \circ y} \mu(z) \geq \min\{\mu(x), \mu(y)\} \ \forall x, y \in H .$$

Proposition 3.2. Let μ be a fuzzy hyper KU-sub-algebra of H. Then $\mu(0) \geq \mu(x)$ for all x $\in H$.

Proof. Using Proposition **2.3** (P_9), we see that 0 $\in$ x $\circ$ x for all x $\in H$. Hence

$$\inf_{0 \in x \circ x} \mu(0) \geq \min\{\mu(x), \mu(x)\} = \mu(x) \text{ for all x } \in H .$$

Example 3.3. Let $H = \{0, a, b\}$ be a set. Define hyper operation $\circ$ on H as follows:

$\circ$	0	a	b
0	$\{0\}$	$\{a\}$	$\{b\}$
a	$\{0\}$	$\{0,a\}$	$\{a,b\}$
b	$\{0\}$	$\{0,a\}$	$\{0,a,b\}$

Then $(H, \circ, 0)$ is a hyper KU-algebra. Define a fuzzy set μ : $H \to$[0, 1] by μ(0) = μ(a)=α_1>α_2= μ(b). Then μ is a fuzzy hyper sub-algebra of H .

A fuzzy set ν : $H \to$[0, 1] defined by ν(0) = 0.7, ν(a)=0.5 and ν(b)=0.2 is also a fuzzy Hyper sub-algebra of H .

Definition3.4[17] Let H be nonempty set .A cubic set A in H is Structure $A = \{\langle x, \tilde{\mu}_A(x), \lambda_A(x) \rangle, x \in X\}$ which is briefly denoted by $A = \langle \tilde{\mu}_A(x), \lambda_A(x) \rangle$, where $\tilde{\mu}_A(x) = [\mu_A^L, \mu_A^U]$ is an interval value fuzzy set in H and λ_A is an fuzzy set in X.

Definition3.5 . A cubic set $A = \langle \tilde{\mu}_A(x), \lambda_A(x) \rangle$ in H is called a $(\tilde{T}, S)$ -cubic hyper ideal of H, if it satisfies the following conditions:

K_1 : $x << y$ *implies* $\tilde{\mu}(x) \geq \tilde{\mu}(y)$ and $\lambda_A(x) \leq \lambda_A(y)$

$$K_2 : \tilde{\mu}(z) \geq \tilde{T}\left\{ \inf_{u\in((y\circ z)} \tilde{\mu}(u), \tilde{\mu}(y) \right\}, \quad \lambda_A(x) \leq S\left\{ \sup_{a\in(y\circ x)} \mu(a), \mu(y) \right\}$$

Definition3.6. For a hyper KU-algebra H , a cubic $A = \langle \tilde{\mu}_A(x), \lambda_A(x) \rangle$" in H is called:

(I) $(\tilde{T}, S)$ -Cubic hyper KU-ideal of H, if

$K_1 :\ x << y\ \ implies\ \ \tilde{\mu}(x) \geq \tilde{\mu}(y), \lambda_A(x) \leq \lambda_A(y)$

$$\tilde{\mu}_A(x \circ z) \geq \tilde{T}\left\{ \inf_{u\in(x\circ(y\circ z)} \tilde{\mu}_A(u)\ ,\ \tilde{\mu}_A(y) \right\} \text{and } \lambda_A(x \circ z) \leq S\left\{ \sup_{a\in x\circ(y\circ z)} \mu(a), \mu(y) \right\}$$

(II) $(\tilde{T}, S)$ - Cubic weak hyper KU-ideal of H" if, for any x; y; z $\in H$

$$\tilde{\mu}_A(0) \geq \tilde{\mu}_A(x \circ z) \geq \tilde{T}\left\{ \inf_{u\in(x\circ(y\circ z)} \tilde{\mu}_A(u), \tilde{\mu}_A(y) \right\}$$

$$\lambda_A(0) \leq \lambda_A(x \circ z) \leq S\left\{ \sup_{a\in x\circ(y\circ z)} \lambda_A(a), \lambda_A(y) \right\}$$

(III) $(\tilde{T}, S)$ -Cubic hyper KU-ideal of H" if, x << y implies

$K_1 :\ x << y\ \ implies\ \ \tilde{\mu}(x) \geq \tilde{\mu}(y)$ and $\lambda_A(x) \leq \lambda_A(y)$ for any x; y; z $\in H$

$$\tilde{\mu}_A(x \circ z) \geq \tilde{T}\left\{ \inf_{u\in(x\circ(y\circ z))} \tilde{\mu}_A(u), \tilde{\mu}_A(y) \right\}, \ \lambda_A(0) \leq \lambda_A(x \circ z) \leq S\left\{ \sup_{a\in x\circ(y\circ z)} \mu(a), \mu(y) \right\}$$

(IV) $(\tilde{T}, S)$ -Cubic strong hyper KU-ideal of H" if, for any x; y; z $\in H$

$$\inf_{u\in x\circ(y\circ z)} \tilde{\mu}_A(u) \geq \tilde{\mu}_A(z) \geq \tilde{T}\left\{ \inf_{u\in x\circ(y\circ z)} \tilde{\mu}_A(u), \tilde{\mu}_A(y) \right\},$$

$$\sup_{u\in x\circ(y\circ z)} \lambda_A(u) \leq \lambda_A(z) \leq S\left\{ \sup_{u\in x\circ(y\circ z)} \lambda_A(u), \lambda_A(y) \right\}.$$

Ending the proof.

Example 3.7. Let $H = \{0,1,2,3\}$ be a set. The hyper operations $\circ$ on H are defined as follows.

$\circ_1$	0	1	2	3
0	{0}	{1}	{2}	{3}
1	{0}	{0}	{1}	{3}
2	{0}	{0}	{0}	{0,3}
3	{0}	{0}	{1}	{0,3}

Then $(H,\circ,0)$ is hyper KU-algebras.Define $\tilde{\mu}_A(x)$, as follows:

$$\tilde{\mu}_A(x)=\begin{cases}[0.2,0.9] & if\ x=\{0,1\}\\ [0.1,0.4] & otherwise\end{cases}$$

H	0	1	2	3
$\lambda_A(x)$	0.2	0.2	0.6	0.7

The function $\tilde{T}_m$ defined by $\tilde{T}_m(\tilde{\alpha},\tilde{\beta})=r\max\{\tilde{\alpha}+\tilde{\beta}-\tilde{1},\tilde{0}\}\forall\tilde{\alpha},\tilde{\beta}\in D[0,1]$, $S_m:[0,1]\times[0,1]\to[0,1]$ be a function defined by $S_m(\alpha,\beta)=\min\{1-(\alpha+\beta),1\}$. It is easy to check that

$$\tilde{\mu}_A(x\circ z)\geq\tilde{T}\left\{\inf_{u\in(x\circ(y\circ z)}\tilde{\mu}_A(u)\ ,\ \tilde{\mu}_A(y)\right\}\text{ and }\lambda_A(x\circ z)\leq S\left\{\sup_{a\in x\circ(y\circ z)}\mu(a),\mu(y)\right\}$$

Then $A=\langle\tilde{\mu}_A(x),\lambda_A(x)\rangle$ is $(\tilde{T},S)$-cubic hyper KU-ideal of H.

Theorem 3.8. Any $(\tilde{T},S)$- cubic (weak, strong) hyper KU-ideal is $(\tilde{T},S)$-cubic (weak, strong) hyper ideal.

Proof. Let $A=\langle\tilde{\mu}_A(x),\lambda_A(x)\rangle$ be $(\tilde{T},S)$- cubic hyper KU-ideal of *H*, we get for any

x; y; z $\in H$, $\tilde{\mu}_A(x\circ z)\geq\tilde{T}\left\{\inf_{u\in x\circ(y\circ z)}\tilde{\mu}_A(u),\tilde{\mu}_A(y)\right\}$ Put x = 0 , we get

$$\tilde{\mu}_A(0\circ z)\geq\tilde{T}\left\{\inf_{u\in 0\circ(y\circ z)}\tilde{\mu}_A(u)\ ,\ \tilde{\mu}_A(y)\right\}$$

which gives,

$$\tilde{\mu}_A(z)\geq\tilde{T}\left\{\inf_{u\in(y\circ z)}\tilde{\mu}_A(u)\ ,\ \tilde{\mu}_A(y)\right\}\text{and }\lambda_A(x\circ z)\leq S\left\{\sup_{u\in x\circ(y\circ z)}\lambda_A(u),\lambda_A(y)\right\}$$

Take x = 0 , we get $\lambda_A(0\circ z)\leq S\left\{\sup_{u\in 0\circ(y\circ z)}\lambda_A(u)\ ,\ \lambda_A(y)\right\}$, which gives,

$\lambda_A(z)\leq S\left\{\sup_{u\in(y\circ z)}\lambda_A(u)\ ,\ \lambda_A(y)\right\}$. Ending the proof.

Definition 3.9. A cubic set $A=\langle\tilde{\mu}_A(x),\lambda_A(x)\rangle$ in H is called a $(\tilde{T},S)$- cubic s-weak hyper KU-ideal of H if

(i) $\tilde{\mu}_A(0)\geq\tilde{\mu}_A(x)$, $\lambda_A(0)\leq\lambda_a(x)$ $\forall x\in H$,
(ii) for every $x,y,z\in H$ there exists $a\in x\circ(y\circ z)$ such that $\tilde{\mu}_A(x\circ z)\geq\tilde{T}\{\tilde{\mu}_A(a),\tilde{\mu}_A(y)\}$.

(iii) $\lambda_A(x \circ z) \leq S\left\{ \sup_{a \in x \circ (y \circ z)} \lambda_A(a), \lambda_A(y) \right\}$.

Proposition 3.10. Let $A = \langle \tilde{\mu}_A(x), \lambda_A(x) \rangle$ be a $(\tilde{T}, S)$ -cubic weak hyper KU-ideal of *H*. If $A = \langle \tilde{\mu}_A(x), \lambda_A(x) \rangle$ satisfies the inf-sup property, then $A = \langle \tilde{\mu}_A(x), \lambda_A(x) \rangle$ is a $(\tilde{T}, S)$ -cubic fuzzy s-weak hyper KU -ideal of *H*.

Proof. Since $A = \langle \tilde{\mu}_A(x), \lambda_A(x) \rangle$ satisfies the inf property, there exists $a_0 \in x \circ (y \circ z)$, such that $\tilde{\mu}_A(a_0) = \inf_{a_0 \in x \circ (y \circ z)} \tilde{\mu}_A(a_0)$. It follows that

$$\tilde{\mu}_A(x \circ z) \geq \tilde{T}\left\{ \inf_{a \in x \circ (y \circ z)} \tilde{\mu}_A(a) \, , \, \tilde{\mu}_A(y) \right\}$$

And since $A = \langle \tilde{\mu}_A(x), \lambda_A(x) \rangle$ satisfies the sup property, there exists $b_0 \in x \circ (y \circ z)$, such that $\lambda_A(b_0) = \sup_{b_0 \in x \circ (y \circ z)} \lambda_A(a_0)$ It follows that

$$\lambda_A(x \circ z) \leq S\left\{ \sup_{b \in x \circ (y \circ z)} \lambda_A(b) \, , \, \lambda_A(y) \right\}.$$

Ending the proof.

Note that, in a finite hyper KU-algebra, every $(\tilde{T}, S)$ - cubic set satisfies (inf -sup) property. Hence the concept of $(\tilde{T}, S)$ -cubic weak hyper KU -ideals and $(\tilde{T}, S)$ -cubic s-weak hyper KU-ideals coincide in a finite hyper KU -algebra.

Proposition 3.11. Let $A = \langle \tilde{\mu}_A(x), \lambda_A(x) \rangle$ be a $(\tilde{T}, S)$ -cubic strong hyper KU-ideal of *H* and let x; y; z ∈ *H*. Then

(i) $\tilde{\mu}_A(0) \geq \tilde{\mu}_A(x)$, $\lambda_A(0) \leq \lambda_A(x)$
(ii) x << y implies $\tilde{\mu}_A(x) \geq \tilde{\mu}_A(y)$.
(iii) $\tilde{\mu}_A(x \circ z) \geq \tilde{T}\{\tilde{\mu}_A(a), \tilde{\mu}_A(y)\}, \forall a \in x \circ (y \circ z)$
(v) x << y implies $\lambda_A(x) \leq \lambda_A(y)$
(iv) $\lambda_A(x \circ z) \leq S\left\{ \sup_{b \in x \circ (y \circ z)} \lambda_A(b) \, , \, \lambda_A(y) \right\}$

Proof. (i) Since $0 \in x \circ x \ \forall x \in H$,we have $\mu(0) \geq \inf_{a \in x \circ x} \mu(a) \geq \mu(x)$, $\lambda(0) \leq \sup_{b \in x \circ x} \lambda(b) \leq \lambda(x)$, which proves (i).

(ii) Let x; y ∈ *H* be such that x << y. Then $0 \in y \circ x \ \forall x, y \in H$ and so $\inf_{b \in (y \circ x)} \tilde{\mu}_A(b) \leq \tilde{\mu}_A(0)$, it follows from (i) that ,

$$\tilde{\mu}_A(x) \geq \tilde{T}\left\{\inf_{a\in(y\circ x)} \tilde{\mu}_A(a), \tilde{\mu}_A(y)\right\} \geq \tilde{T}\{\tilde{\mu}_A(0), \tilde{\mu}_A(y)\} = \tilde{\mu}_A(y),$$

$$\lambda_A(x) \leq S\left\{\sup_{b\in(y\circ x)} \lambda_A(b), \lambda_A(y)\right\} \leq S\{\lambda_A(0), \lambda_A(y)\} = \lambda_A(y).$$

(iii) $\tilde{\mu}_A(x\circ z) \geq \tilde{T}\left\{\inf_{a\in x\circ(y\circ z)} \tilde{\mu}_A(a), \tilde{\mu}_A(y)\right\} \geq \tilde{T}\{\tilde{\mu}_A(a), \tilde{\mu}_A(y)\}, \forall a \in x\circ(y\circ z),$

$$\lambda(x\circ z) \leq S\left\{\sup_{b\in x\circ(y\circ z)} \lambda(b), \lambda(y)\right\} \leq S\{\mu(b), \mu(y)\}, \forall b \in x\circ(y\circ z)$$

we conclude that (iii), (v), (iv) are true. Ending the proof.

Corollary 3.12. Every $(\tilde{T}, S)$-cubic strong hyper KU-ideal is both a $(\tilde{T}, S)$-cubic s-weak hyper KU-ideal and a $(\tilde{T}, S)$-cubic hyper KU-ideal.

Proof. Straight forward.

Proposition 3.13. Let $A = \langle \tilde{\mu}_A(x), \lambda_A(x)\rangle$ be $(\tilde{T}, S)$- cubic hyper KU-ideal of H and let $x, y, z \in H$. Then, (i) $\tilde{\mu}_A(0) \geq \tilde{\mu}_A(x)$, $\lambda_A(0) \leq \lambda_A(x)$

(ii) if $A = \langle \tilde{\mu}_A(x), \lambda_A(x)\rangle$ satisfies the inf-sup property, then

$$\tilde{\mu}_A(x\circ z) \geq \tilde{T}\{\tilde{\mu}_A(a), \tilde{\mu}_A(y)\}, \forall a \in x\circ(y\circ z), \lambda_A(x\circ z) \leq S\left\{\sup_{b\in x\circ(y\circ z)} \lambda_A(b), \lambda_A(y)\right\}$$

Proof. (i) Since 0 << x for each $x \in H$; we have $\tilde{\mu}_A(0) \geq \tilde{\mu}_A(x)$, $\lambda_A(0) \leq \lambda_A(x)$ by Definition 3.6(I) and hence (i) holds.

(ii) Since $A = \langle \tilde{\mu}_A(x), \lambda_A(x)\rangle$ satisfies the inf property, there is $a_0 \in x\circ(y\circ z)$, such that $\tilde{\mu}(a_0) = \inf_{a\in x\circ(y\circ z)} \tilde{\mu}_A(a)$. Hence $\tilde{\mu}_A(x\circ z) \geq \tilde{T}\left\{\inf_{a\in x\circ(y\circ z)} \tilde{\mu}_A(a), \tilde{\mu}_A(y)\right\} = \tilde{T}\{\tilde{\mu}_A(a_0), \tilde{\mu}_A(y)\}$
Since $A = \langle \tilde{\mu}_A(x), \lambda_A(x)\rangle$ satisfies the sup- property, there is $b_0 \in x\circ(y\circ z)$, such that $\lambda(b_0) = \sup_{b\in x\circ(y\circ z)} \lambda_A(b)$, Hence $\lambda_A(x\circ z) \leq S\left\{\sup_{b\in x\circ(y\circ z)} \lambda_A(b), \lambda_A(y)\right\} = S\{\lambda_A(b_0), \lambda_A(y)\}$

which implies that (ii) is true. The proof is complete.

Corollary 3.13. (i) Every $(\tilde{T}, S)$ - cubic hyper KU -ideal of *H* is a $(\tilde{T}, S)$-cubic weak hyper KU –ideal of *H*.
(ii) If $A = \langle \tilde{\mu}_A(x), \lambda_A(x)\rangle$ is a $(\tilde{T}, S)$-cubic hyper KU -ideal of H satisfying inf-sup property, then $A = \langle \tilde{\mu}_A(x), \lambda_A(x)\rangle$ is a $(\tilde{T}, S)$-cubic s-weak Hyper KU -ideal of *H*.

Proof. Straightforward.

Theorem 3.15 . If $A = \langle \tilde{\mu}_A(x), \lambda_A(x) \rangle$ is a $(\tilde{T}, S)$ -cubic strong hyper KU-ideal of H , then the set $\mu_{t,s} = \{x \in H, \tilde{\mu}_A(x) \geq \tilde{t}, \lambda_A(x) \leq s\}$ is a $(\tilde{T}, S)$ -strong hyper KU-ideal of H ,when $\mu_{t,s} \neq \Phi,\ for\ \tilde{t} \in D[0,1], s \in [0,1]$.

Proof. Let $A = \langle \tilde{\mu}_A(x), \lambda_A(x) \rangle$ be a $(\tilde{T}, S)$ -cubic strong hyper KU-ideal of H and $\mu_{t,s} \neq \Phi,\ for\ \tilde{t} \in D[0,1], s \in [0,1]$ .Then there $a \in \mu_{t,s}$ and so $\tilde{\mu}_A(a) \geq \tilde{t}, \lambda_A(a) \leq s$. By Proposition 3.13 (i), $\tilde{\mu}_A(0) \geq \tilde{\mu}_A(a) \geq \tilde{t}, \lambda(0) \geq \lambda(a) \leq s$ and so $0 \in \mu_{t,s}$. Let $x, y, z \in H$ such that $x \circ (y \circ z) \cap \mu_{t,s} \neq \Phi\ and\ y \in \mu_{t,s}$, Then there exist $a_0 \in x \circ (y \circ z) \cap \mu_{t,s}\ and\ hence\ \tilde{\mu}_A(a_0) \geq \tilde{t}, \lambda_A(a_0) \leq s$. By Definition 3.6 (iv), we have $\tilde{\mu}_A(x \circ z) \geq \tilde{T}\left\{ \inf_{a \in x \circ (y \circ z)} \tilde{\mu}_A(a), \tilde{\mu}_A(y) \right\} \geq \tilde{T}\{\tilde{\mu}_A(a), \tilde{\mu}_A(y)\} \geq \tilde{T}\{\tilde{t}, \tilde{t}\} = \tilde{t}$, and

$$\lambda_A(x \circ z) \leq S\left\{ \sup_{a \in x \circ (y \circ z)} \lambda_A(a) \ , \ \lambda_A(y) \right\} = S\{ \ \lambda_A(a_0) \ , \ \lambda_A(y) \ \} = S\{ \ s \ , \ s) \ \} = s \ .$$

So $(x \circ z) \in \mu_{t,s}$. It follows that $\mu_{t,s}$ is a strong hyper KU -ideal of H .

Ending the proof.

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Conflicts of Interest

State any potential conflicts of interest here or “The author declare no conflict of interest”.

Conclusion

In this paper concepts $(\tilde{T}, S)$ -cubic theory of the (s-weak – strong) hyper KU-ideals in hyper KU-algebras are applied and the relations among them are obtained .Example is given in support of the definition of $(\tilde{T}, S)$ -cubic fuzzy ideals. Some theorems are proved. And also introduced $(\tilde{T}, S)$ -cubic fuzzy ideal extension. The main purpose of our future work is to investigate the following:

- Folding theory applied to some types of $(\tilde{T}, S)$ -cubic positive implicative hyper KU-ideals in hyper KU-algebras

- Homomorphism and quotient of $(\tilde{T}, S)$ -cubic fuzzy KU-hyper-ideals.
- On implicative $(\tilde{T}, S)$ -cubic hyper ku-ideals of hyper KU-algebras.
- Intuitionistic fuzziness of $(\tilde{T}, S)$ -cubic strong hyperKU-ideals.
- Filter theory on $(\tilde{T}, S)$ -cubic hyper KU-algebras.

- On $(\tilde{T}, S)$-cubic intuitionistic Fuzzy Implicative Hyper KU-Ideals of Hyper KU-algebras.
- On $(\tilde{T}, S)$-cubic intuitionistic fuzzy commutative hyper KU-ideals.
- On $(\tilde{T}, S)$- interval-valued intuitionistic fuzzy Hyper KU-ideals of hyper KU-algebras.
- (T, S) Bipolar fuzzy implicative hyper KU-ideals in hyper KU-algebras.

Algorithm for hyper KU-algebras

Input (X : set, $\circ$ hyper operation)
Output (" X is a hyper KU-algebra or not")
Begin
If $X = \phi$ then go to (1.);
End If
If $0 \notin X$ then go to (1.);
End If
Stop: =false;
$i := 1$;
While $i \leq |X|$ and not (Stop) do
If $0 \notin x_i \circ x_i$ then
Stop: = true;
End If
$j := 1$
While $j \leq |X|$ and not (Stop) do
If $0 \notin x_i \circ (y_j \circ x_i)$ or $0 \in x_i \circ y_j$ *and* $0 \in (y_j \circ x_i)$ and $x_i \neq y_j$,then
Stop: = true;
End If
End If
$k := 1$
While $k \leq |X|$ and not (Stop) do
If $0 \notin (x_i * y_j) \circ ((y_j * z_k) \circ (x_i * z_k))$ then
Stop: = true;
    End If
  End While
 End While
End While
If Stop then
(1.) Output (" X is not hyper KU-algebra")
Else
  Output (" X is hyper KU-algebra")
    End If
End.

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