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ON NANO $\alpha^{\star}$-SETS AND NANO $\mathcal{R}_{\alpha^{\star}}$-SETS

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Abstaract − In this paper, nano $\alpha^{\star}$-set and nano $\mathcal{R}_{\alpha^{\star}}$-set are introduced and investigated.

Keywords − *nano α-open sets, nano $\alpha^{\star}$-set, nano $\mathcal{R}_{\alpha^{\star}}$-set.*

1 Introduction

Lellis Thivagar et al [2] introduced a nano topological space with respect to a subset X of an universe which is defined in terms of lower approximation and upper approximation and boundary region. The classical nano topological space is based on an equivalence relation on a set, but in some situation, equivalence relations are nor suitable for coping with granularity, instead the classical nano topology is extend to general binary relation based covering nano topological space.

In this paper, nano $\alpha^{\star}$-set and nano $\mathcal{R}_{\alpha^{\star}}$-set in nano topological spaces and investigate some of their properties.

2 Preliminaries

Throughout this paper $(U, \tau_R(X))$ (or X) represent nano topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset H of a space $(U, \tau_R(X))$, $Ncl(H)$ and $Nint(H)$ denote the nano closure of H and the nano interior of H respectively. We recall the following definitions which are useful in the sequel.

Definition 2.1. *[3] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.*

1. *The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \bigcup_{x \in U}\{R(x) : R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by x.*

2. *The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \bigcup_{x \in U}\{R(x) : R(x) \cap X \neq \phi\}$.*

3. *The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not - X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.*

Property 2.2. *[2] If (U, R) is an approximation space and $X, Y \subseteq U$; then*

1. $L_R(X) \subseteq X \subseteq U_R(X)$;

2. $L_R(\phi) = U_R(\phi) = \phi$ *and* $L_R(U) = U_R(U) = U$;

3. $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$;

4. $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$;

5. $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$;

6. $L_R(X \cap Y) \subseteq L_R(X) \cap L_R(Y)$;

7. $L_R(X) \subseteq L_R(Y)$ *and* $U_R(X) \subseteq U_R(Y)$ *whenever* $X \subseteq Y$;

8. $U_R(X^c) = [L_R(X)]^c$ *and* $L_R(X^c) = [U_R(X)]^c$;

9. $U_R U_R(X) = L_R U_R(X) = U_R(X)$;

10. $L_R L_R(X) = U_R L_R(X) = L_R(X)$.

Definition 2.3. *[2] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then by the Property 2.2, R(X) satisfies the following axioms:*

1. *U and $\phi \in \tau_R(X)$,*

2. *The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$,*

3. *The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.*

That is, $\tau_R(X)$ is a topology on U called the nano topology on U with respect to X. We call $(U, \tau_R(X))$ as the nano topological space. The elements of $\tau_R(X)$ are called as nano open sets and $[\tau_R(X)]^c$ is called as the dual nano topology of $[\tau_R(X)]$.

Remark 2.4. *[2] If $[\tau_R(X)]$ is the nano topology on U with respect to X, then the set $B = \{U, \phi, L_R(X), B_R(X)\}$ is the basis for $\tau_R(X)$.*

Definition 2.5. *[2] If $(U, \tau_R(X))$ is a nano topological space with respect to X and if $H \subseteq U$, then the nano interior of H is defined as the union of all nano open subsets of H and it is denoted by $Nint(H)$.*

That is, $Nint(H)$ is the largest nano open subset of H. The nano closure of H is defined as the intersection of all nano closed sets containing H and it is denoted by $Ncl(H)$.

That is, $Ncl(H)$ is the smallest nano closed set containing H.

Definition 2.6. *[2] A subset H of a nano topological space $(U, \tau_R(X))$ is called*

1. *nano semi-open if $H \subseteq Ncl(Nint(H))$.*

2. *nano regular-open if $H = Nint(Ncl(H))$.*

3. *nano α-open if $H \subseteq Nint(Ncl(Nint(H)))$.*

4. *nano semi pre-open if $H \subseteq Ncl(Nint(Ncl(H)))$.*

The complements of the above mentioned sets is called their respective closed sets.

Definition 2.7. *[1] A subset H of a space $(U, \tau_R(X))$ is called nano t-set if*

$$Nint(H) = Nint(Ncl(H))$$

3 Nano $\alpha^\star$-sets and Nano $\mathcal{R}_{\alpha^\star}$-sets

Definition 3.1. *A subset H of a space $(U, \tau_R(X))$ is called;*

1. *an nano $\alpha^\star$-set if $Nint(Ncl(Nint(H))) = Nint(H)$.*

2. *nano $\mathcal{R}_{\alpha^\star}$-set if $H = P \cap Q$, where P is nano open and Q is nano $\alpha^\star$-set.*

The family of all nano $\alpha^\star$-sets (resp. nano $\mathcal{R}_{\alpha^\star}$-sets) of a space $(U, \tau_R(X))$ will be denoted by $N\alpha^\star(U, \tau_R(X))$ (resp. $N\mathcal{R}_{\alpha^\star}(U, \tau_R(X))$).

Example 3.2. *Let $U = \{1, 2, 3\}$ with $U/R = \{\{1\}, \{2, 3\}\}$ and $X = \{1\}$. Then the nano topology $\tau_R(X) = \{\phi, \{1\}, U\}$.*

1. *then $\{2\}$ is nano $\alpha^\star$-set.*

2. *then $\{1\}$ is nano $\mathcal{R}_{\alpha^\star}$-set.*

Proposition 3.3. *For a subset H of a space $(U, \tau_R(X))$, the following are equivalent:*

1. *$H \in N\alpha^\star(U, \tau_R(X))$.*

2. *H is nano semi-pre closed.*

3. *$Nint(H)$ is nano regular open.*

Proof. The proof is obvious.

Proposition 3.4. *In a space* $(U, \tau_R(X))$.

1. *If* H *is a nano* t*-set, then* $H \in N\alpha^\star(U, \tau_R(X))$.
2. *nano semi-open set* H *is a nano* t*-set* $\iff$ $H \in N\alpha^\star(U, \tau_R(X))$.
3. H *is an nano* α*-open set and* $H \in N\alpha^\star(U, \tau_R(X))$ $\iff$ H *is nano regular-open.*

Proof. (1) Let H be a nano t-set, then

$$Nint(H) = Nint(Ncl(H))$$

and

$$Nint(Ncl(Nint(H))) = Nint(Ncl(H)) = Nint(H)$$

Therefor H is an nano $\alpha^\star$-set.

(2). Let H be nano semi-open and $H \in N\alpha^\star(U, \tau_R(X))$. Since H is nano semi-open,

$$Ncl(Nint(H)) = Ncl(H)$$

and hence

$$Nint(Ncl(H)) = Nint(Ncl(Nint(H))) = Nint(H)$$

Therefore, H is a nano t-set.

(3). Let H be an nano α-open set and $H \in N\alpha^\star(U, \tau_R(X))$. By Proposition 3.3 and the Definition of an nano α-open set, we have

$$Nint(Ncl(Nint(H))) = H$$

and hence

$$Nint(Ncl(H)) = Nint(Ncl(Nint(H))) = H$$

The converse is obvious.

Remark 3.5. *In a space* $(U, \tau_R(X))$*, the union of two nano* $\alpha^\star$*-sets but not nano* $\alpha^\star$*-set.*

Example 3.6. *Let* $U = \{a, b, c, d\}$ *with* $U/R = \{\{a\}, \{c\}, \{b, d\}\}$ *and* $X = \{a, b\}$*. Then the nano topology* $\tau_R(X) = \{\phi, \{a\}, \{b, d\}, \{a, b, d\}, U\}$*. Then* $H = \{a, b\}$ *and* $Q = \{d\}$ *is nano* $\alpha^\star$*-sets. Clearly* $H \cup Q = \{a, b, d\}$ *is but not nano* $\alpha^\star$*-set.*

Remark 3.7. *In a space* $(U, \tau_R(X))$*, the union of two nano* $\alpha^\star$*-sets is nano* $\alpha^\star$*-set.*

Example 3.8. *In Example 3.6, then* $H = \{a\}$ *and* $Q = \{b\}$ *is nano* $\alpha^\star$*-sets. Clearly* $H \cup Q = \{a, b\}$ *is nano* $\alpha^\star$*-set.*

Remark 3.9. *In a space* $(U, \tau_R(X))$*, the intersection of two nano* $\alpha^\star$*-sets are nano* $\alpha^\star$*-set.*

Example 3.10. *In Example 3.6, then* $H = \{a, b\}$ *and* $Q = \{b, c\}$ *is nano* $\alpha^\star$*-sets. Clearly* $H \cap Q = \{b\}$ *is nano* $\alpha^\star$*-set.*

Proposition 3.11. *In a space $(U, \tau_R(X))$, then $N\alpha^\star(U, \tau_R(X)) \subseteq N\mathcal{R}_{\alpha^\star}(U, \tau_R(X))$ and $\tau_R(X) \subseteq N\mathcal{R}_{\alpha^\star}(U, \tau_R(X))$.*

Proof. Since $U \in \tau_R(X) \cap N\alpha^\star(U, \tau_R(X))$, the inclusions are obvious.

Example 3.12. *In Example 3.2, then $\{1\}$ is a nano $\mathcal{R}_{\alpha^\star}$-set but not a nano $\alpha^\star$-set and $\{2, 3\}$ is a nano $\mathcal{R}_{\alpha^\star}$-set but not nano open.*

Lemma 3.13. *In a space $(U, \tau_R(X))$. If either H, P is nano semi-open, then*

$$Nint(Ncl(H \cap P)) = Nint(Ncl(H)) \cap Nint(Ncl(P))$$

Proof. For any subset $H, P \subseteq U$, we generally have

$$Nint(Ncl(H \cap P)) \subseteq Nint(Ncl(H)) \cap Nint(Ncl(P))$$

Assume that H is nano semi-open. Then we have $Ncl(H) = Ncl(Nint(H))$. Therefore

$$\begin{aligned}
Nint(Ncl(H)) \cap Nint(Ncl(P)) &= Nint(Ncl(Nint(Ncl(H)) \cap Nint(Ncl(P))))\\
&\subseteq Nint(Ncl(Ncl(H) \cap Nint(Ncl(P))))\\
&= Nint(Ncl(Ncl(Nint(H)) \cap Nint(Ncl(P))))\\
&\subseteq Nint(Ncl(Nint(H) \cap Ncl(P)))\\
&\subseteq Nint(Ncl(Nint(H) \cap P))\\
&\subseteq Nint(Ncl(H \cap P)
\end{aligned}$$

Proposition 3.14. *A subset H is nano open in a space $(U, \tau_R(X)) \iff$ it is a nano α-open set and a nano $\mathcal{R}_{\alpha^\star}$-set.*

Proof. It is obvious that every nano open set is a nano α-open set and a nano $\mathcal{R}_{\alpha^\star}$-set. let H be a nano α-open set and a nano $\mathcal{R}_{\alpha^\star}$-set. Since H is a nano $\mathcal{R}_{\alpha^\star}$-set, there exist $G \in \tau_R(X)$ and $Q \in N\alpha^\star(U, \tau_R(X))$ such that $H = G \cap Q$. Since H is a nano α-open set, by using Lemma 3.13, we have

$$\begin{aligned}
H \subseteq Nint(Ncl(Nint(H))) &= Nint(Ncl(Nint(G \cap Q)))\\
&= Nint(Ncl(G)) \cap Nint(Ncl(Nint(Q)))\\
&= Nint(Ncl(G)) \cap Nint(Q)
\end{aligned}$$

and hence

$$\begin{aligned}
H &= G \cap H\\
&\subseteq G \cap (Nint(Ncl(G)) \cap Nint(Q))\\
&= G \cap Nint(Q)\\
&\subseteq H
\end{aligned}$$

Consequently, we obtain $H = G \cap Nint(H)$ and H is nano open.

Remark 3.15. *The following example shows that the concepts of nano α-open set and nano $\mathcal{R}_{\alpha^\star}$-set are independent of each other.*

Example 3.16. *In a Example 3.2,*

1. *then $\{1, 2\}$ is nano α-open set but not nano $\mathcal{R}_{\alpha^\star}$-set.*

2. *then $\{2, 3\}$ is nano $\mathcal{R}_{\alpha^\star}$-set but not nano α-open set.*

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