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SOFT IDEALS OVER A SEMIGROUP GENERATED BY A SOFT SET

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Abstract – In this paper, the concept of soft singletons is defined. Consequently, we introduce the soft principal left (right) ideals over a semigroup S. The smallest soft right (left) ideals over S generated by a soft set over S are studied. Some illustrative examples are given**.**

Keywords – *Soft sets, soft semigroups, soft ideals, soft singleton.*

1. Introduction and Preliminaries

The concept of a soft set was first introduced by Molodtsov in [6]. Aktas and Cagman [1] adapted this concept to define soft groups. In [2], the authors introduced the concept of soft semigroups as a collection of subsemigroups of a semigroupand defined soft (left, right, quasi, bi) ideals of a semigroup. Shabir and Ahmad applied soft sets theory of ternary semigroups [7]. Jun and et al introduced concepts of soft ideals over ordered semigroups [5]. Properties of soft Γ-semigroups and soft ideals over a Γ-semigroup were studied in [3]. In Section 2 we introduce the definition of soft singletons andsome basic propositions. In Section 3 we define the soft left (right) ideal generated by a soft set over a semigroup and the soft ideal generated by a softset over a semigroup, and find, as special cases, those soft ideals generated by soft sets over monoids.

Let S be a semigroup. A nonempty subset A of S is called a subsemigroup of Sif $A^2 \subseteq A$, a left (right) ideal of S if $SA \subseteq A$($AS \subseteq A$) and a two-sided ideal (or simply ideal) of S if it is both a left and a right ideal of S.

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Definition 2.1 [1]. Let U be a universal set and let E be a set of parameters. Let $P(U)$ denote the power set of U and let $A \subseteq E$. A pair (F, A) is called a soft set over U if F is a mapping $F: A \rightarrow P(U)$.

Definition 2.2 [5]. Let (F, A) and (G, B) be soft sets over U, then (G, B) is called a soft subset of (F, A), denoted by $(G, B) \sqsubseteq (F, A)$If $B \subseteq A$ and $G(b) \subseteq F(b)$ for all $b \in B$.

Definition 2.3 [2]. Let U be an initial universe set, Ebe the universe set of parameters and $A \subseteq E$.

a) $(F, A)$is called a relative null soft set (with respect to the parameter set$A$), denoted by$(\mathcal{N}, A)$if $F(e) = \emptyset$, for all $e \in A$.
b) We shall denote by $\emptyset_U$ the unique soft set over $U$with an empty parameter set which is called the empty soft set over U.

Definition 2.4 [2]. Let U be an initial universe set, E be the universe set of parameters and $A \subseteq E$. Then (U, A) is said to be an absolute soft set over Uif $U(e) = U$, for all $e \in A$.

Definition 2.3 [2]. Let (F, A) and (G, B) be two soft sets over a common universe U, then

1) The extended intersection of (F, A) and (G, B) denoted by $(F, A) \cap_\varepsilon (G, B)$, is defined as soft set (H, C) where $C = A \cap B, \forall c \in C,$

$$H(c) = \begin{cases} F(c) & \text{if } c \in A \setminus B \\ G(c) & \text{if } c \in B \setminus A \\ F(c) \cap G(c) & \text{if } c \in A \cap B \end{cases}$$

2) The restricted intersection of $(F, A)$and (G, B), denoted by $(F, A) \sqcap (G, B)$, is defined as soft set (H, C) where $C = A \cap B$and $H(c) = F(c) \cap G(c)$ for all $c \in C$.

Definition 2.4 [2]. Let (F, A) and (G, B) be two soft sets over a common universe U, then

1) The extended union of (F, A) and (G, B) denoted by $(F, A) \cup_\varepsilon (G, B)$, is defined as soft set (H, C) where $C = A \cup B, \forall c \in C,$

$$H(c) = \begin{cases} F(c) & \text{if } c \in A \setminus B \\ G(c) & \text{if } c \in B \setminus A \\ F(c) \cup G(c) & \text{if } c \in A \cap B \end{cases}$$

2) The restricted union of (F, A) and (G, B), denoted by $(F, A) \sqcup (G, B)$, is defined as soft set (H, C) where $C = A \cap B$ and $H(c) = F(c) \cup G(c)$ for all $c \in C$.

2. Principle Soft Ideals

In the rest of this paper, S is a semigroup and S^1 denotes the monoid generated by S.

Definition 2.1. [2]. Let (F,A) and (G,B) be two soft sets over a semigroup S. The restricted product of (F,A) and (G,B) denoted by $(F,A) \tilde{\circ} (G,B)$ is defined as the soft set $(H;C)$ where $C = A \cap B$ and $H(c) = F(c)G(c)$ for all $c \in C$.

Definition 2.2. [2]. A soft set (F,A) over a semigroup S is called a soft semigroup if by $(\mathcal{N},A) \neq (F,A) \neq \emptyset_S$ and $(F,A) \tilde{\circ} (F,A) \subseteq (F,A)$.

It is shown that (F,A) is a soft semigroup over S if and only if $\forall x \in A,\ F(x) \neq \emptyset$is a subsemigroup of S [2].

Definition 3.3. [2]. A soft set $(\mathcal{N},A) \neq (F,A) \neq \emptyset_S$ over a semigroup S is called a soft left (right) ideal over S, if $(S,A) \tilde{\circ} (F,A) \subseteq (F,A)$ $((F,A) \tilde{\circ} (S,A) \subseteq (F,A))$ Where (S,A) is an absolute soft set over S. A soft set over S is a soft ideal if it is both a soft left and a soft right ideal over S.

It is shown that a soft set (F,A) over S is a soft ideal over S ifand only if $F(a) \neq \emptyset$ is an ideal of S [2].

Definition 2.3. Let $x \in S$. A soft set (x,A) over a semigroup S is called a soft singleton if $x(a) = \{x\}$ for all $a \in A$.

Definition 2.3. For a soft singleton (x,A) and a soft set (F,A) over S, we say(x,A) belongs to (F,A), denoted by $(x,A) \Subset (F,A)$, if $x \in F(a)$, for all $a \in A$.

Example 2.4. Let $S = (N,+)$ be the semigroup of natural numbers. Define $F : A = \{1,2,3\} \rightarrow P(N)$ by $F(1) = \{2,3,4,\ldots\}$, $F(2) = \{3,4,5,\ldots\}$ and $F(3) = \{4,5,6,\ldots\}$. It is obvious that $(4,A) \Subset (F,A)$ because $4 \in F(a)$ for all $a \in A$ while (x,A) does not belong to (F,A) for all $x \in A$.

Proposition 2.5. Let (F,A) be a soft set over a semigroup S. If (F,A) is a soft semigroup, then $(x,A) \tilde{\circ} (y,A) \Subset (F,A)$ for any $(x,A),(y,A) \Subset (F,A)$.

Proof. Assume that(F,A) is a soft semigroup, then for all $a \in A, F(a)$is a subsemigroup of S. Let$(x,A),(y,A) \Subset (F,A) \Rightarrow (x,A) \tilde{\circ} (y,A) = (xy,A) \Subset (F,A)$ because $xy \in F(a)$ for all $a \in A$. □

Proposition 2.6. If(F,A) is a soft left (right) ideal over a semigroup S, then $(S,A) \tilde{\circ} (x,A) \sqsubseteq (F,A)$, $((x,A) \tilde{\circ} (S,A) \sqsubseteq (F,A))$ for all $(x,A) \Subset (F,A)$.

Proof. Suppose that (F,A) is a soft left (right) ideal over, then for all $a \in A, F(a)$ is a left (right) ideal of S. Let $x \in F(a)$ for all $a \in A$, then $Sx \subseteq F(a)$ $(xS \subseteq F(a))$ for all $a \in A$. Thus $(S,A) \tilde{\circ} (x,A) \sqsubseteq (F,A)$,$((x,A) \tilde{\circ} (S,A) \sqsubseteq (F,A))$ for all $(x,A) \Subset (F,A)$. □

Generally, the opposite direction of the above proposition is not true. Also, it is not necessary that a soft set (F,A) equals union of all soft singletons belonging to it. This fact is depicted in the following example.

Example 2.7. Let $S = \{1,2,3,4,5\}$ be a semigroup defined by the following table

.	1	2	3	4	5
1	1	2	3	4	5
2	2	2	2	2	2
3	3	2	3	3	2
4	4	2	4	4	2
5	5	5	5	5	5

For $A = \{1,2\} \subset S$, define the soft set $(F,A)$by $F(1) = \{4,5\}$ and $F(2) = \{4\}$. Clearly, $(4,A)$ is the only soft singleton belonging to (F,A). Moreover, $(4,A) \tilde{\circ} (4,A) \tilde{\in} (F,A)$ but (F,A) is not a soft semigroup over S because $F(1) = \{4,5\}$ is not subsemigroup of S. It is obvious that (F,A) is not the union of its soft singletons. Let (G,A) be a soft set over S defined as $G(1) = \{1,2,4\}$ and $G(2) = \{2,4\}$. The soft singletons belonging to (G,A) are $(2,A)$ and $(4,A)$. Easily, one can show that $(x,A) \tilde{\circ} (S,A) \sqsubseteq (G,A)$ for all $(x,A) \tilde{\in} (G,A)$ but (G,A) is not a soft right ideal over S because $G(1) = \{1,2,4\}$ is not an ideal of S.

Definition 2.8. The smallest soft right (left) ideal over S containing (x,A) is called the principal soft right (left) ideal generated by (x,A). The smallest soft ideal over S containing (x,A) is called the principal soft ideal generated by (x,A).

By definition, $(x,A) \tilde{\circ} (S^1,A) = (H,A)$ such that $H(a) = xS^1 = \{x\} \cup xS$. That is, $(x,A) \tilde{\circ} (S^1,A)$ is a soft set over S with a constant value equals the principal right ideal of S generated by $\{x\}$.

Lemma2.9. $(x,A) \tilde{\circ} (S^1,A)$ is the principal soft right ideal over Sgenerated by (x,A).

Proof. Clearly, $(x,A) \tilde{\circ} (S^1,A)$ is a soft right ideal over S and $(x,A) \tilde{\in} (x,A) \tilde{\circ} (S^1,A)$. Let (G,A) be a soft right ideal over S containing (x,A), then

$$xS^1 \subseteq G(a)S^1 = G(a) \cup G(a)S \subseteq G(a)$$

hence $(x,A) \tilde{\circ} (S^1,A) \sqsubseteq (G,A)$. Then $(x,A) \tilde{\circ} (S^1,A)$ is the principal soft right ideal over Sgenerated by (x,A). □

Similarly, we get the dual result.

Lemma2.9. $(S^1,A) \tilde{\circ} (x,A)$ is the principal soft left ideal over S generated by (x,A).

Lemma2.10. $(S^1,A) \tilde{\circ} (x,A) \tilde{\circ} (S^1,A)$ is the principal soft ideal over S generated by (x,A).

Proof. Since $x = 1x1 \subseteq S^1xS^1$, then $(x,A) \Subset (S^1,A) \tilde{\circ} (x,A) \tilde{\circ} (S^1,A)$. Obviously, $(S^1,A) \tilde{\circ} (x,A) \tilde{\circ} (S^1,A)$ is a soft ideal over S. Suppose that (G,A) be a soft ideal over S containing (x,A), then

$$S^1xS^1 \subseteq S^1G(a)S^1 = G(a) \cup G(a)S \cup SG(a) \cup SG(a)S \subseteq G(a)$$

thus $(S^1,A) \tilde{\circ} (x,A) \tilde{\circ} (S^1,A) \sqsubseteq (G,A)$. Then $(S^1,A) \tilde{\circ} (x,A) \tilde{\circ} (S^1,A)$ is the principal soft ideal over S generated by (x,A). □

Lemma 2.12. (Principle soft left Ideal Lemma). Let $x, y \in S$, then the following statements are equivalent;

1) $(S^1,A) \tilde{\circ} (x,A) \sqsubseteq (S^1,A) \tilde{\circ} (y,A)$,
2) $(x,A) \Subset (S^1,A) \tilde{\circ} (y,A)$,
3) $x = y$or$x = sy$ for some $s \in S$.

Proof. Straightforward.

Lemma 2.13. (Principle Soft Right Ideal Lemma). Let $x,y \in S$, then the following statements are equivalent;

1) $(x,A) \tilde{\circ} (S^1,A) \sqsubseteq (y,A) \tilde{\circ} (S^1,A)$,
2) $(x,A) \Subset (y,A) \tilde{\circ} (S^1,A)$,
3) $x = y$ or $x = ys$ for some $s \in S$.

Proof. Straightforward.

Theorem 2.14. Let $\mathcal{L}$,$\mathcal{R}$ be relations on a semigroup S defined by

1) $x\mathcal{L}y$ if and only if $(S^1,A) \tilde{\circ} (x,A) = (S^1,A) \tilde{\circ} (y,A)$,
2) $x\mathcal{R}y$ if and only if $(x,A) \tilde{\circ} (S^1,A) = (y,A) \tilde{\circ} (S^1,A)$.

Then $\mathcal{L}[\mathcal{R}]$ is aright [left] congruence relation.

Proof. $x\mathcal{L}x$ $(x\mathcal{R}x)$ because $S^1x = S^1x$ $(xS^1 = xS^1)$. It is clear that$\mathcal{L}$ and are symmetric and transitive relations. Then $\mathcal{L}$ and $\mathcal{R}$ are equivalence relations. To show that $\mathcal{L}$ $[\mathcal{R}]$ is a right [left] congruence, assume $x\mathcal{L}y$ $[x\mathcal{R}y]$ and $s \in S$ then

$$(S^1,A) \tilde{\circ} (x,A) = (S^1,A) \tilde{\circ} (y,A)\ [(x,A) \tilde{\circ} (S^1,A) = (y,A) \tilde{\circ} (S^1,A)]$$

that is,

$$S^1x = S^1y \Rightarrow S^1xs = S^1ys\ [xS^1 = yS^1 \Rightarrow sxS^1 = syS^1\ ].$$

Hence

$$\begin{aligned}(S^1,A) \tilde{\circ} (x,A) \tilde{\circ} (s,A) = (S^1,A) \tilde{\circ} (y,A) \tilde{\circ} (s,A)[(s,A) \tilde{\circ} (x,A) \tilde{\circ} (S^1,A) \\ = (s,A) \tilde{\circ} (y,A) \tilde{\circ} (S^1,A)],\end{aligned}$$

This implies that $xs\mathcal{L}ys$ $[sx\mathcal{R}sy]$. Thus $\mathcal{L}$ $[\mathcal{R}]$ is a right [left] congruence. □

Corollary 2.15. For $x, y \in S$, we have

- $x\mathcal{L}y \Leftrightarrow \exists\, s, t \in S^1$ such that $(s, A) \tilde{\circ} (y, A) = (x, A)$ and $(t, A) \tilde{\circ} (x, A) = (y, A)$.
- $x\mathcal{R}y \Leftrightarrow \exists\, s, t \in S^1$ such that $(y, A) \tilde{\circ} (s, A) = (x, A)$ and $(x, A) \tilde{\circ} (t, A) = (y, A)$.

Proof. Let $x\mathcal{L}y \Leftrightarrow$ if $(S^1, A) \tilde{\circ} (x, A) = (S^1, A) \tilde{\circ} (y, A) \Leftrightarrow (S^1, A) \tilde{\circ} (x, A) \sqsubseteq (S^1, A) \tilde{\circ} (y, A)$ and $(S^1, A) \tilde{\circ} (y, A) \sqsubseteq (S^1, A) \tilde{\circ} (x, A) \Leftrightarrow x = sy$ and $y = tx$ for some $t, s \in S \Leftrightarrow (s, A) \tilde{\circ} (y, A) = (x, A)$ and $(t, A) \tilde{\circ} (x, A) = (y, A)$, by lemma 3.7. For $x\mathcal{R}y$, the result comes directly by a similar argument.□

Definition 2.16. We define the equivalence relation $\mathcal{H} = \mathcal{L} \cap \mathcal{R}$. For $x \in S$, we define L_x to be the $\mathcal{L}$-class of x; R_x to be the $\mathcal{R}$-class of x and H_x is the $\mathcal{H}$-class of x.

Example 2.17. Let $x, y \in S = (N, +)$, then

$$x\mathcal{L}y \Leftrightarrow (N^1, A) \tilde{\circ} (y, A) = (N^1, A) \tilde{\circ} (y, A) \Leftrightarrow N^1 + x = N^1 + y \Leftrightarrow x = y.$$

Thus $\mathcal{L} = \mathcal{R} = \mathcal{H} = \{(x, x) : \forall x \in N\}$ and then $L_x = R_x = H_x = \{x\}$, for all $x \in N$.

3. Soft Ideals Generated by Soft Sets

Authors in [2], showed that $(F, A) \cap_{\mathcal{R}} (G, B)$ for any soft ideals (F, A) and (G, B) over S is a soft ideal. Hence the restricted intersection of all soft ideals over S containing the soft set (H, A) is the soft ideal over S generated by (H, A).

Definition 3.1. The smallest soft right (left) ideal over S containing (F, A) is called the soft right (left) ideal generated by (F, A), denoted by $([F], A)$ $((\langle F \rangle, A))$. The smallest soft ideal over S containing (F, A) is called the soft ideal generated by (F, A), denoted by $((F), A)$.

Theorem 3.2. Let (F, A) be a soft set over S, then

$$(\langle F \rangle, A) = (F, A) \sqcup (S, A) \tilde{\circ} (F, A).$$

Proof. Let $\{(F_i, A) : i \in I\}$ the family of all soft left ideals over S containing (F, A), then $F_i(a)$ is a left ideal of S for all $i \in I, a \in A$. Since $SF(a) \subseteq SF_i(a) \subseteq F_i(a)$ for each $i \in I$, $a \in A$, then

$$(S, A) \tilde{\circ} (F, A) \sqsubseteq \sqcap_{i \in I} \{(F_i, A)\}.$$

As a result, $(F, A) \sqcup (S, A) \tilde{\circ} (F, A) \sqsubseteq \sqcap_{i \in I} \{(F_i, A)\}$. We notice that $(\langle F \rangle, A)$ is a soft left ideal over S^1 because $\langle F \rangle(a)$ is the left ideal of S generated by $F(a)$ for all $a \in A$. This follows that we have $\sqcap_{i \in I} \{(F_i, A)\} \sqsubseteq (\langle F \rangle, A)$. By definition, we get

$$\sqcap_{i \in I} \{(F_i, A)\} = (\langle F \rangle, A).$$

Similarly, we prove the following result.

Theorem 3.3. Let (F,A) be a soft set over S, then

$$([F],A) = (F,A) \sqcup (F,A) \tilde{\circ} (S,A).$$

Theorem 3.4. Let (F,A) be a soft set over S^1, then

$$\langle F\rangle(a) = \bigcup_{x\in S^1} xF(a)$$

Proof. Since for all $a \in A, F(a) = 1F(a) \subseteq \langle F\rangle(a)$, then $(F,A) \sqsubseteq (\langle F\rangle,A)$. The soft set $(\langle F\rangle,A)$ is a soft left ideal over S^1. Indeed, by definition $(S^1,A) \tilde{\circ} (\langle F\rangle,A) = (H,A)$ where

$$H(a) = S^1\langle F\rangle(a) = S^1\left(\bigcup_{x\in S^1} xF(a)\right) = \bigcup_{x\in S^1} S^1xF(a) \subseteq \bigcup_{x\in S^1} xF(a) = \langle F\rangle(a)$$

Thus $H(a) \subseteq \langle F\rangle(a)$ for all $a \in A$. As a result, $(\langle F\rangle,A)$ is a soft left ideal over S^1. Let (G,A) be a soft left ideal over S^1 containing (F,A), then

$$\langle F\rangle(a) = \bigcup_{x\in S^1} xF(a) \subseteq \bigcup_{x\in S^1} xG(a) \subseteq G(a).$$

Hence $(\langle F\rangle,A) \sqsubseteq (G,A)$. By definition, we conclude that $(\langle F\rangle,A) = (G,A)$. This ends the proof. □

Similarly, we prove the following result.

Theorem 3.5. Let (F,A) be a soft set over S^1, then

$$\langle F\rangle(a) = \bigcup_{x\in S^1} F(a)x.$$

Example 3.6. Consider the non-commutative semigroup $S = \{1, a, b, c\}$

.	1	a	b	c
1	1	a	b	c
a	a	a	a	a
b	b	b	b	b
c	c	b	a	c

For $A = \{1\} \subset S$, define a soft set (F,A) over $S$by $F(1) = \{b\}$. By definition, $(S,A) \tilde{\circ} (F,A) = (H,A)$ such that $H(1) = SF(1) = S\{b\} = \{a,b\}$. Then

$$\langle F\rangle(1) = F(1) \cup SF(1) = \{a, b\}.$$

That is, $(\langle F\rangle, A) = (F, A) \sqcup (S, A) \tilde{\circ} (F, A)$ is a soft left ideal over S containing (F, A). Let (G, A)be a soft left ideal over S containing (F, A). Then $\{b\} = F(1) \subseteq G(1) = \{a, b, c\}$ or $G(1) = \{a, b\}$. For all cases, $(\langle F\rangle, A) \sqsubseteq (G, A)$. Therefore, $(\langle F\rangle, A)$ is the soft left ideal over S containing (F, A).

Let (F, A)be a soft set over Sdefined by $F(1) = \{c\}$. By definition, $(F, A) \tilde{\circ} (S, A) = (H, A)$ such that $H(1) = F(1)S = \{c\}S = \{a, b, c\}$. Then

$$[F](1) = F(1) \cup F(1)S = \{a, b, c\}.$$

That is, $([F], A) = (F, A) \sqcup (F, A) \tilde{\circ} (S, A)$ is a soft right ideal over S containing (F, A). Let (G, A) be a soft right ideal over Scontaining (F, A). Then $\{c\} = F(1) \subseteq G(1) = \{a, b, c\}$ is the only right ideal of S that contains $F(1)$. Thus $([F], A) \sqsubseteq (G, A)$. Therefore, $([F], A)$ is the soft right ideal over$S$containing (F, A). □

Theorem 3.7. Let (F, A) be a soft set over S, then

$$((F), A) = (F, A) \sqcup (F, A) \tilde{\circ} (S, A) \sqcup (F, A) \tilde{\circ} (S, A) \sqcup (S, A) \tilde{\circ} (F, A) \tilde{\circ} (S, A).$$

Proof. Let $\{(F_i, A): i \in I\}$ the family of all soft ideals over S containing (F, A), then $F_i(a)$ is an ideal of S for all $i \in I, a \in A$. By the same way as in theorem, we show that

$$(S, A) \tilde{\circ} (F, A) \sqsubseteq \sqcap_{i \in I} \{(F_i, A)\},$$
$$(F, A) \tilde{\circ} (S, A) \sqsubseteq \sqcap_{i \in I} \{(F_i, A)\}$$

and

$$(S, A) \tilde{\circ} (F, A) \tilde{\circ} (S, A) \sqsubseteq \sqcap_{i \in I} \{(F_i, A)\}$$

for each $i \in I, a \in A$. Hence $((F), A) \sqsubseteq \sqcap_{i \in I} \{(F_i, A)\}$. Because

$$(F)(a) = F(a) \cup SF(a) \cup F(a)S \cup SF(a)S$$

is the ideal of S generated by $F(a)$ for all $a \in A$. Thus we have $\sqcap_{i \in I} \{(F_i, A)\} \sqsubseteq ((F), A)$. By definition, we get $\sqcap_{i \in I} \{(F_i, A)\} = ((F), A)$.□

Theorem 3.8. Let (F, A) be a soft set over S^1, then

$$([\langle F\rangle], A) = ((F), A) = (\langle [F]\rangle, A).$$

Proof. By definition, $([\langle F\rangle], A)$ is a soft right ideal over S^1. Also $([\langle F\rangle], A)$ is a soft left ideal over S^1. Indeed, we have

$$S^1[\langle F\rangle](a) = S^1(\bigcup_{x \in S^1} \langle F\rangle(a)x) = \bigcup_{x \in S^1} S^1\langle F\rangle(a)x \subseteq \bigcup_{x \in S^1} \langle F\rangle(a)x = [\langle F\rangle](a).$$

So $([\langle F\rangle],A)$ is a soft ideal over S^1 containing (F,A). Let (G,A) be a soft ideal over S^1 containing (F,A), then$(\langle F\rangle,A) \sqsubseteq (G,A)$ and $([\langle F\rangle],A) \sqsubseteq (G,A)$. This means $([\langle F\rangle],A)$ is a soft ideal over S^1 generated by (F,A), hence $([\langle F\rangle],A) = \big((F),A\big)$. Similarly, we can show that $\big((F),A\big) = (\langle[F]\rangle,A)$. This completes the proof. □

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