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ON NANO πgp-CLOSED SETS

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Abstaract − In this paper, new classes of sets called πgp-closed sets in nano topological spaces is introduced and its properties and studied of nano πgp-closed sets.

Keywords − *Nano π-closed set, nano πg-closed set, nano πgp-closed sets and nano gpr-closed set.*

1 Introduction

Thivagar et al. [4] introduced a nano topological space with respect to a subset X of an universe which is defined in terms of lower approximation and upper approximation and boundary region. The classical nano topological space is based on an equivalence relation on a set, but in some situation, equivalence relations are nor suitable for coping with granularity, instead the classical nano topology is extend to general binary relation based covering nano topological space

Bhuvaneswari et al. [3] introduced and investigated nano g-closed sets in nano topological spaces. Recently, Parvathy and Bhuvaneswari the notions of nano gpr-closed sets which are implied both that of nano rg-closed sets. In 2017, Rajasekaran et.al [7] introduced the notion of nano πgp-closed sets in nano topological spaces. new classes of sets called πgp-closed sets in nano topological spaces is introduced and its properties and studied of nano πgp-closed sets.

2 Preliminaries

Throughout this paper $(U, \tau_R(X))$ (or X) represent nano topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset H of a

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space $(U, \tau_R(X))$, $Ncl(H)$ and $Nint(H)$ denote the nano closure of H and the nano interior of H respectively. We recall the following definitions which are useful in the sequel.

Definition 2.1. *[6] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.*

1. *The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \bigcup_{x \in U}\{R(x) : R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by x.*
2. *The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \bigcup_{x \in U}\{R(x) : R(x) \cap X \neq \phi\}$.*
3. *The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not - X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.*

Property 2.2. *[4] If (U, R) is an approximation space and $X, Y \subseteq U$; then*

1. $L_R(X) \subseteq X \subseteq U_R(X)$;
2. $L_R(\phi) = U_R(\phi) = \phi$ *and* $L_R(U) = U_R(U) = U$;
3. $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$;
4. $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$;
5. $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$;
6. $L_R(X \cap Y) \subseteq L_R(X) \cap L_R(Y)$;
7. $L_R(X) \subseteq L_R(Y)$ *and* $U_R(X) \subseteq U_R(Y)$ *whenever* $X \subseteq Y$;
8. $U_R(X^c) = [L_R(X)]^c$ *and* $L_R(X^c) = [U_R(X)]^c$;
9. $U_R U_R(X) = L_R U_R(X) = U_R(X)$;
10. $L_R L_R(X) = U_R L_R(X) = L_R(X)$.

Definition 2.3. *[4] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then by the Property 2.2, R(X) satisfies the following axioms:*

1. *U and $\phi \in \tau_R(X)$,*
2. *The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$,*
3. *The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.*

That is, $\tau_R(X)$ is a topology on U called the nano topology on U with respect to X. We call $(U, \tau_R(X))$ as the nano topological space. The elements of $\tau_R(X)$ are called as nano open sets and $[\tau_R(X)]^c$ is called as the dual nano topology of $[\tau_R(X)]$.

Remark 2.4. *[4] If $[\tau_R(X)]$ is the nano topology on U with respect to X, then the set $B = \{U, \phi, L_R(X), B_R(X)\}$ is the basis for $\tau_R(X)$.*

Definition 2.5. *[4] If $(U, \tau_R(X))$ is a nano topological space with respect to X and if $H \subseteq U$, then the nano interior of H is defined as the union of all nano open subsets of H and it is denoted by $Nint(H)$.*

That is, $Nint(H)$ is the largest nano open subset of H. The nano closure of H is defined as the intersection of all nano closed sets containing H and it is denoted by $Ncl(H)$.

That is, $Ncl(H)$ is the smallest nano closed set containing H.

Definition 2.6. *A subset H of a nano topological space $(U, \tau_R(X))$ is called*

1. *nano semi open [4] if $H \subseteq Ncl(Nint(H))$.*
2. *nano regular-open [4] if $H = Nint(Ncl(H))$.*
3. *nano π-open [1] if the finite union of nano regular-open sets.*
4. *nano pre-open [4] if $H \subseteq Nint(Ncl(H))$.*

The complements of the above mentioned sets is called their respective closed sets.

Definition 2.7. *A subset H of a nano topological space $(U, \tau_R(X))$ is called;*

1. *nano g-closed [2] if $Ncl(H) \subseteq G$, whenever $H \subseteq G$ and G is nano open.*
2. *nano rg-closed set [8] if $Ncl(H) \subseteq G$ whenever $H \subseteq G$ and G is nano regular-open.*
3. *nano πg-closed [7] if $Ncl(H) \subseteq G$, whenever $H \subseteq G$ and G is nano π-open.*
4. *nano gp-closed set [3] if $Npcl(H) \subseteq G$, whenever $H \subseteq G$ and G is nano open.*
5. *nano gpr-closed set [5] if $Npcl(H) \subseteq G$, whenever $H \subseteq G$ and G is nano regular open.*

3 On Nano πgp-closed Sets

Definition 3.1. *A subset H of a space $(U, \tau_R(X))$ is nano πgp-closed if $Npcl(H) \subseteq G$ whenever $H \subseteq G$ and G is nano π-open.*

The complement of nano πgp-open if $H^c = U - H$ is nano πgp-closed.

Example 3.2. *Let $U = \{a, b, c, d\}$ with $U/R = \{\{a, b\}, \{c\}, \{d\}\}$ and $X = \{a, d\}$. Then the nano topology $\tau_R(X) = \{\phi, \{d\}, \{a, b\}, \{a, b, d\}, U\}$.*

1. *then $\{a\}$ is nano πgp-closed set.*

2. *then* $\{b\}$ *is not nano* πgp*-closed set.*

Theorem 3.3. *In a space* $(U, \tau_R(X))$,

1. *If* H *is nano* π*-open and nano* πgp*-closed, then* H *is nano pre-closed and hence nano clopen.*

2. *If* H *is nano semi-open and nano* πgp*-closed, then* H *is nano* πg*-closed.*

Proof. 1. If H is nano π-open and nano πgp-closed, then $Npcl(H) \subseteq H$ and so H is nano pre-closed. Hence H is nano clopen, since nano π-open set is nano open and nano pre-closed open set is nano closed.

2. Let $H \subseteq G$ and G be nano π-open. Since H is nano πgp-closed, $Npcl(H) \subseteq G$. Since H is nano semi-open, $Npcl(H) = Ncl(H) \subseteq G$ and hence H is nano πg-closed.

Remark 3.4. *For a subset of a space* $(U, \tau_R(X))$*, we have the following implications:*

$$
\begin{array}{ccccc}
\textbf{nano closed} & & & & \\
\Downarrow & & & & \\
\textbf{nano } g\textbf{-closed} & \Rightarrow & \textbf{nano } \pi g\textbf{-closed} & \Rightarrow & \textbf{nano } rg\textbf{-closed} \\
\Downarrow & & \Downarrow & & \Downarrow \\
\textbf{nano } gp\textbf{-closed} & \Rightarrow & \textbf{nano } \pi gp\textbf{-closed} & \Rightarrow & \textbf{nano } gpr\textbf{-closed}
\end{array}
$$

None of the above implications are reversible as shown by the following Examples.

Example 3.5. *Let* $U = \{a, b, c\}$ *with* $U/R = \{\{a, c\}, \{b\}\}$ *and* $X = \{c\}$*. Then the nano topology* $\tau_R(X) = \{\phi, \{a, c\}, U\}$*. Then* $\{a, c\}$ *is nano* πgp*-closed set but not nano gp-closed.*

Example 3.6. *In Example 3.2,*

1. *then* $\{a\}$ *is nano* πgp*-closed set but not nano* πg*-closed.*

2. *then* $\{a, d\}$ *is nano gpr-closed set but not nano* πgp*-closed.*

Lemma 3.7. *In a space* $(U, \tau_R(X))$,

1. *every nano open set is nano* πgp*-closed.*

2. *every nano closed set is nano* πgp*-closed.*

Remark 3.8. *The converses of statements in Lemma 3.7 are not necessarily true as seen from the following Examples.*

Example 3.9. *In Example 3.2,*

1. *then* $\{a, c, d\}$ *is nano* πgp*-closed set but not nano open.*

2. *then* $\{b, c, d\}$ *is nano* πgp*-closed set but not nano closed.*

Theorem 3.10. *Let* H *be nano* πgp*-closed. Then* $Npcl(H) - H$ *does not contain any non-empty nano* π*-closed set.*

Proof. Let K be a nano π-closed set such that $K \subseteq Npcl(H) - H$. Then $H \subseteq U - K$. Since H is nano πgp-closed and $U - K$ is nano π-open, $Npcl(H) \subseteq U - K$, i.e. $K \subseteq U - Npcl(H)$. Hence $K \subseteq Npcl(H) \cap (U - Npcl(H)) = \phi$. This shows that $K = \phi$.

Corollary 3.11. *Let H be nano πgp-closed. Then H is nano pre-closed $\iff$ $Npcl(H) - H$ is nano π-closed $\iff$ $H = Npcl(Nint(H))$.*

Theorem 3.12. *In a space $(U, \tau_R(X))$, the union of two nano πgp-closed sets is nano πgp-closed.*

Proof. Let $H \cup Q \subseteq G$, then $H \subseteq G$ and $Q \subseteq G$ where G is nano π-open. As H and Q are πgp-closed, $Ncl(H) \subseteq G$ and $Ncl(Q) \subseteq G$. Hence $Ncl(H \cup Q) = Ncl(H) \cup Ncl(Q) \subseteq G$.

Example 3.13. *In Example 3.2, then $H = \{a\}$ and $Q = \{c\}$ is nano πgp-closed. Clearly $H \cup Q = \{a, c\}$ is nano πgp-closed.*

Theorem 3.14. *In a space $(U, \tau_R(X))$, the intersection of two nano πgp-open sets are nano πgp-open.*

Proof. Obvious by Theorem 3.12.

Example 3.15. *In Example 3.2, then $H = \{a, d\}$ and $Q = \{b, d\}$ is nano πgp-open. Clearly $H \cap Q = \{d\}$ is nano πgp-open.*

Remark 3.16. *In a space $(U, \tau_R(X))$, the union of two nano πgp-closed sets but not nano πgp-closed.*

Example 3.17. *In Example 3.2, then $H = \{a\}$ and $Q = \{b\}$ is nano πgp-closed. Clearly $H \cup Q = \{a, b\}$ is but not nano πgp-closed.*

Remark 3.18. *In a space $(U, \tau_R(X))$, the intersection of two nano πgp-open sets but not nano πgp-open.*

Example 3.19. *In Example 3.2, then $H = \{a, c, d\}$ and $Q = \{b, c, d\}$ is nano πgp-open sets. Clearly $H \cup Q = \{c, d\}$ is but not nano πgp-open.*

Corollary 3.20. *If H ia nano πgp-closed and nano regular open and P is nano pre-closed in U, then $H \cap P$ is nano πgp-closed.*

Proof. Let $H \cap P \subseteq G$ and G is nano π-open in H. Since P is nano pre-closed in U, $H \cap P$ is nano pre-closed in H and so $Npcl_H(H \cap P) = H \cap P$. That is $Npcl_H(H \cap P) \subseteq G$. Then $H \cap P$ is nano πgp-closed in the nano πgp-closed and nano regular open set H and hence $H \cap P$ is nano πgp-closed in U.

Theorem 3.21. *If H is nano πgp-closed in U and $H \subseteq P \subseteq Npcl(H)$, then P is nano πgp-closed.*

Proof. Let $P \subseteq G$ and G be nano π-open in U. Since $H \subseteq G$ and H is nano πgp-closed, $Npcl(H) \subseteq G$ and then $Npcl(P) = Npcl(H) \subseteq G$. hence P is nano πgp-closed.

Theorem 3.22. *A set H of a space $(U, \tau_R(X))$ is called nano πgp-open $\iff$ if $K \subseteq Npint(H)$ whenever K is nano π-closed and $K \subseteq H$.*

Proof. Obvious.

Theorem 3.23. *A subset H of U is nano πgp-open $\iff$ $G = U$ whenever G is nano π-open and $Npint(H) \cup (U - H) \subseteq G$.*

Proof. Let G ba a nano π-open set and $Npint(H) \cup (U - H) \subseteq G$. Then $U - G \subseteq (U - Npint(H)) \cap H$, i.e., $(U - G) \subseteq Npcl(U - H) - (U - H)$. Since $U - H$ is nano πgp-closed, by Theorem 3.10, $U - G = \phi$ and hence $G = U$.

Conversely, let K be a nano π-open set of U and $K \subseteq H$. Since $Npint(H) \cup (U - H) = Nint(Ncl(H)) \cup (U - K)$ is nano π-open and $Npint(H) \cup (U - H) \subseteq Npint(H) \cup (U - K)$, by hypothesis, $Npint(H) \cup (U - K) = U$ and hence $K \subseteq Npint(H)$.

Theorem 3.24. *Let $H \subseteq V \subseteq U$ and V nano π-open and nano closed in U. If H is nano πgp-open in V, then H is nano πgp-open in U.*

Proof. Let K be any nano π-closed set and $K \subseteq H$. Since K is nano π-closed in V and H is nano πgp-open in V, $K \subseteq Npint_V(H)$ and then $K \subseteq Npint_U(H) \cap V$. Hence $K \subseteq Npint_U(H)$ and so H is nano πgp-open in U.

Theorem 3.25. *If H is nano πgp-open in U and $Npint(H) \subseteq P \subseteq H$, then P is nano πgp-open.*

Proof. Let $K \subseteq P$ and K be nano π-closed in U. Since H is nano πgp-open and $K \subseteq H$. $K \subseteq Npint(H)$ and then $K \subseteq Npint(P)$. Hence P is nano πgp-open.

Theorem 3.26. *A subset H of U is nano πgp-closed $\iff$ $Npcl(H) - H$ is nano πgp-open.*

Proof. Let $K \subseteq Npcl(H) - H$ and K be nano π-closed in U. Then by Theorem 3.10, $K = \phi$ and so $K \subseteq Npint(Npcl(H) - H)$. This shows that $Npcl(H) - H$ is nano πgp-open.

Conversely, let G be a nano π-open set of U and $H \subseteq G$. Then $Npcl(H) \cap (U - G) = Ncl(Nint(H)) \cap (U - G)$ is nano π-closed set contained in $Npcl(H) - H$. Since $Npcl(H) - H$ is nano πgp-open, by Theorem 3.22, $Npcl(H) \cap (U - G) \subseteq Npint(Npcl(H) - H = \phi)$ and hence $Npcl(H) \subseteq G$.

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