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GENERATING FUNCTIONS OF q-ANALOGUE OF I-FUNCTION SATISFYING TRUESDELL'S DESCENDING F_q-EQUATION

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Abstract – In this paper, the authors have obtained various forms of q-analogue of I-function satisfying Truesdell's descending F_q-equation. These forms have been employed to arrive at certain generating functions for q-analogue of I-function. Some particular cases of these results in terms of q-analogue of H-and G-functions which appear to be new have also been obtained.

Keywords – *F_q-equation, Generating function, q-analogue of I-Function, q-analogue of H-Function, q-analogue of G-Function.*

1 Introduction

Recent developments in the theory of Basic hypergeometric functions have gained much interest due to its introduction of certain new generalized forms of Basic hypergeometric functions. These functions are Mac-Roberts's E-Function, Meijer's G-Function, Fox's H-Function, Saxena's I-Function and their q-analogues. The q-analogue of I-Function have been introduced by Saxena et al. [1] in terms of Mellin-Barnes type basic contour integral as

$$I(z)=I^{m,n}_{A_i,B_i;r}\left[z;q\left|\begin{matrix}(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,A_i}\\(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,B_i}\end{matrix}\right.\right]=$$

$$\frac{1}{2\pi i}\int_L\frac{\prod_{j=1}^{m}G(q^{b_j-\beta_js})\prod_{j=1}^{n}G(q^{1-a_j+\alpha_js})}{\sum_{i=1}^{r}\left[\prod_{j=m+1}^{B_i}G(q^{1-b_{ji}+\beta_{ji}s})\prod_{j=n+1}^{A_i}G(q^{a_{ji}-\alpha_{ji}s})G(q^s)G(q^{1-s})\sin\pi s\right]}\pi z^s ds \qquad (1)$$

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where $\alpha_j, \beta_j, \alpha_{ji}, \beta_{ji}$ are real and positive, a_j, b_j, a_{ji}, b_{ji} are complex numbers and

$$G(q^{\alpha}) = \prod_{n=0}^{\infty} (1 - q^{\alpha+n})^{-1} = \frac{1}{(q^{\alpha};q)_{\infty}}$$

where L is contour of integration running from - i∞ to i∞ in such a manner so that all poles of $G(q^{b_j - \beta_j s}); 1 \le j \le m$ are to right of the path and those of $G(q^{1-a_j+\alpha_j s}); 1 \le j \le n$ are to left. The integral converges if *Re [s log (x) – log sin πs} < 0,* for large values of |s| on the contour L.

Setting $r = 1, A_i = A, B_i = B$ in equation (1) we get q-analogue of H-Function defined by Saxena et.al. [1] as follows:

$$H_{A,B}^{m,n}\left[z;q \middle| \begin{matrix} (a_j,\alpha_j)_{1,A} \\ (b_j,\beta_j)_{1,B} \end{matrix} \right] = \frac{1}{2\pi i} \int_L \frac{\prod_{j=1}^{m} G(q^{b_j-\beta_j s}) \prod_{j=1}^{n} G(q^{1-a_j+\alpha_j s})}{\prod_{j=m+1}^{B} G(q^{1-b_j+\beta_j s}) \prod_{j=n+1}^{A} G(q^{a_j-\alpha_j s}) G(q^s) G(q^{1-s}) \sin \pi s} \pi z^s ds \tag{2}$$

Further if we put $\alpha_j = \beta_j = 1$, equation (2) reduces to the basic analogue of Meijer's G-Function given by Saxena et. al. [1].

$$G_{A,B}^{m,n}\left[z;q \middle| \begin{matrix} a_1,a_2,...,a_A \\ b_1,b_2,...,b_B \end{matrix} \right] = \frac{1}{2\pi i} \int_L \frac{\prod_{j=1}^{m} G(q^{b_j-s}) \prod_{j=1}^{n} G(q^{1-a_j+s})}{\prod_{j=m+1}^{B} G(q^{1-b_j+s}) \prod_{j=n+1}^{A} G(q^{a_j-s}) G(q^s) G(q^{1-s}) \sin \pi s} \pi z^s ds \tag{3}$$

Farooq et. al. [2] defined the basic analogue of I-function in terms of Gamma function as follows

$$I_{q,A_i,B_i;R}^{m,n}\left[z;q \middle| \begin{matrix} (a_j,\alpha_j)_{1,n} \ (a_{ji},\alpha_{ji})_{n+1,A_i} \\ (b_j,\beta_j)_{1,m} \ (b_{ji},\beta_{ji})_{m+1,B_i} \end{matrix} \right] = \frac{1}{2\pi i} \int_L \frac{\prod_{j=1}^{m} \Gamma_q(b_j - \beta_j s) \prod_{j=1}^{n} \Gamma_q(1-a_j+\alpha_j s)}{\sum_{i=1}^{R} \left\{ \prod_{j=m+1}^{B_i} \Gamma_q(1-b_{ji}+\beta_{ji}s) \prod_{j=n+1}^{A_i} \Gamma_q(a_{ji}-\alpha_{ji}s) \right\} \Gamma_q(s)\Gamma_q(1-s)\sin \pi s} \pi z^s ds \tag{4}$$

In his effort towards achieving unification of special functions, Truesdell [3] has put forward a theory which yielded a number of results for special functions satisfying the so called Truesdell's F_q-equation. Agrawal B. M. [4] extended this theory further and derived results for descending F_q-equation. He obtained various properties like orthogonality, Rodrigue's and Schalafli's formulae for F_q-equation, which turn out to be special functions. Renu Jain et. al. [7] derived some generating functions of q-analogue of Mittag-Leffler function and Hermite polynomial satisfying Truesdell's ascending and descending Fq-equation. The function $F(z,\alpha)$ is said to satisfy the descending F-equation if

$$D_z^r F(z,\alpha) = F(z,\alpha - r) \tag{5}$$

For $F(z,\alpha)$ satisfying descending F-equation, Agrawal B. M. [6] has obtained following generating functions:

$$F(z+y,\alpha) = \sum_{n=0}^{\infty} y^n \frac{F(z,\alpha-n)}{n!} \tag{6}$$

The q-derivative of equation (5) can be written in the following manner:

$$D_{q,z}^r F(z,\alpha) = F(z,\alpha + r) \tag{7}$$

In order to obtain main result of this paper we will make use of the following results which we have obtained on multiplication formulae for q-analogue of Gamma functions:

$$\prod_{k=0}^{m-1} \Gamma_q\left(\frac{\alpha - r + k}{m}\right) = \frac{(1-q)^r q^{\frac{r}{m}(r-2\alpha+1)}}{(-1)^r (q^{1-\alpha};q)_r} \prod_{k=0}^{m-1} \Gamma_q\left(\frac{\alpha+k}{m}\right) \tag{8}$$

$$\prod_{k=0}^{m-1} \Gamma_q\left(1 - \frac{\alpha - r + k}{m}\right) = \frac{(q^{1-\alpha};q)_r}{(1-q)^r} \prod_{k=0}^{m-1} \Gamma_q\left(1 - \frac{\alpha+k}{m}\right) \tag{9}$$

In the results that follow, by $\Delta(\mu,\alpha)$ we shall mean the array of μ parameters

$$\frac{\alpha}{\mu}, \frac{\alpha+1}{\mu}, \ldots, \frac{\alpha+\mu-1}{\mu}; \qquad (\mu = 1,2,3,\ldots) \tag{10}$$

and $(\Delta(\mu,\alpha),\beta)$ stands for $\left(\frac{\alpha}{\mu},\beta\right), \left(\frac{\alpha+1}{\mu},\beta\right), \ldots, \left(\frac{\alpha+\mu-1}{\mu},\beta\right)$ (11)

2 Generating Functions for q-analogue of I-Function

(A): Renu Jain et. al.[5] obtained various forms of I-Function which satisfy Truesdell's descending F-equation and hence in this connection, we established in this section the different forms of q-analogue of I-Function which satisfy Truesdell's descending F_q-equation:

$$(I) \; \left(q^{\frac{\alpha}{2}\left[\frac{1-\rho}{\rho}\right]} z\right)^{\alpha-1} I_{p_i,q_i,l}^{m,n}\left[\frac{q^{\alpha h(\lambda+1)}}{z^{h\lambda}};q \left| \begin{matrix} \{\Delta(\lambda,\alpha),h\},\{(a_j,\alpha_j)_{\lambda+1,n}\},\{(a_{ji},\alpha_{ji})_{n+1,p_i-\rho}\},\{\Delta(\rho,\alpha),h\} \\ \{(b_j,\beta_j)_{1,m}\},\{(b_{ji},\beta_{ji})_{m+1,q_i-\rho}\},\{\Delta(\rho,\alpha),h\} \end{matrix}\right.\right] \tag{12}$$

$$(II)\ \frac{\left(q^{\frac{-\alpha}{2}\left[\frac{1+\lambda}{\lambda}\right]}z\right)^{\alpha-1}}{(1-q)^{\alpha}} I^{m,n}_{p_i,q_i,l}\left[\frac{q^{\alpha h(\lambda-1)}}{z^{h\lambda}};q\left|\begin{matrix}\{\Delta(2\lambda,2\alpha),h\},\{(a_j,\alpha_j)_{2\lambda+1,n}\},\{(a_{ji},\alpha_{ji})_{n+1,p_i}\}\\ \left\{\Delta(\lambda,\alpha+\frac{1}{2}),h\right\},\{(b_j,\beta_j)_{\lambda+1,m}\},\{(b_{ji},\beta_{ji})_{m+1,q_i}\}\end{matrix}\right.\right] \tag{13}$$

$$(III)\ \frac{\left(q^{\frac{-\alpha}{2(\alpha-1)}\left[\frac{3\alpha+1}{3\lambda}+\alpha-1\right]}z\right)^{\alpha-1}}{(1-q)^{\alpha}} I^{m,n}_{p_i,q_i,l}\left[\frac{q^{\alpha h(\lambda+1)}}{z^{h\lambda}};q\left|\begin{matrix}\{\Delta(3\lambda,3\alpha),h\},\{(a_j,\alpha_j)_{3\lambda+1,n}\},\{(a_{ji},\alpha_{ji})_{n+1,p_i}\}\\ \left\{\Delta(\lambda,\alpha+\frac{2}{3}),h\right\},\{(b_j,\beta_j)_{\lambda+1,m}\},\{(b_{ji},\beta_{ji})_{m+1,q_i-\lambda}\},\left\{\Delta(\lambda,\alpha+\frac{1}{3}),h\right\}\end{matrix}\right.\right] \tag{14}$$

$$(IV)\ \left(q^{\frac{-\alpha}{2}}z\right)^{\alpha-1} e^{\pi i\alpha} I^{m,n}_{p_i,q_i,l}\left[\frac{q^{\alpha h\lambda}}{z^{h\lambda}};q\left|\begin{matrix}\{\Delta(\lambda,\alpha),h\},\{(a_j,\alpha_j)_{\lambda+1,n}\},\{(a_{ji},\alpha_{ji})_{n+1,p_i}\}\\ \{(b_j,\beta_j)_{1,m}\},\{(b_{ji},\beta_{ji})_{m+1,q_i}\}\end{matrix}\right.\right] \tag{15}$$

$$(V)\ \left(q^{\frac{\alpha}{2}(\frac{1-\lambda}{\lambda})}z\right)^{\alpha-1} I^{m,n}_{p_i,q_i,l}\left[\frac{q^{h\alpha(\lambda-1)}}{z^{h\lambda}};q\left|\begin{matrix}\{(a_j,\alpha_j)_{1,n}\},\{(a_{ji},\alpha_{ji})_{n+1,p_i-\lambda}\},\{\Delta(\lambda,\alpha),h\}\\ \{(b_j,\beta_j)_{1,m}\},\{(b_{ji},\beta_{ji})_{m+1,q_i}\}\end{matrix}\right.\right] \tag{16}$$

$$(VI)\ \left(q^{\frac{\alpha}{2}\left(\frac{1-\lambda}{\lambda}-\frac{1}{\rho}\right)}z\right)^{\alpha-1} e^{\pi i\alpha} I^{m,n}_{p_i,q_i,l}\left[\frac{q^{\alpha h\lambda}}{z^{h\lambda}};q\left|\begin{matrix}\{\Delta(\rho,\alpha),h\}\{(a_j,\alpha_j)_{\rho+1,n}\},\{(a_{ji},\alpha_{ji})_{n+1,p_i-\lambda}\},\{\Delta(\lambda,\alpha),h\}\\ \{\Delta(\rho,\alpha),h\}\{(b_j,\beta_j)_{\rho+1,m}\},\{(b_{ji},\beta_{ji})_{m+1,q_i}\}\end{matrix}\right.\right] \tag{17}$$

Assuming that form (12) is $A(z,\alpha)$, replacing q-analogue of I-Function by its definition (4) and then interchanging order of integration and differentiation, which is justified under the conditions of convergence [2], we observe

$$D^r_{q,z}A(z,\alpha)$$
$$=\frac{1}{2\pi i}\int_L \frac{\prod_{j=1}^{m}\Gamma_q(b_j-\beta_j s)\prod_{k=0}^{\lambda-1}\Gamma_q\left(1-\frac{\alpha+k}{\lambda}-hs\right)\prod_{j=\lambda+1}^{n}\Gamma_q(1-a_j+\alpha_j s)\, q^{\frac{\alpha(\alpha-1)(1-\rho)}{2\rho}} q^{\alpha h(\lambda+1)s} D^r_{q,z}\left\{z^{\alpha-1-h\lambda s}\right\}\pi ds}{\sum_{i=1}^{l}\left\{\prod_{j=m+1}^{q_i-\rho}\Gamma_q(1-b_{ji}+\beta_{ji}s)\prod_{k=0}^{\rho-1}\Gamma_q\left(1-\frac{\alpha+k}{\rho}+hs\right)\prod_{j=n+1}^{p_i-\rho}\Gamma_q(a_{ji}-\alpha_{ji}s)\prod_{k=0}^{\rho-1}\Gamma_q\left(\frac{\alpha+k}{\rho}-hs\right)\right\}\Gamma_q(s)\Gamma_q(1-s)\sin\pi s} \tag{18}$$

Now results (8) and (9) lead to two very important identities

$$\prod_{k=0}^{\lambda-1}\Gamma_q\left(\frac{\alpha+k}{\lambda}-hs\right)=\frac{(-1)^r(q^{1-(\alpha-h\lambda s)};q)_r}{(1-q)^r q^{\frac{r}{2m}(r-2(\alpha-h\lambda s)+1)}}\prod_{k=0}^{\lambda-1}\Gamma_q\left(\frac{\alpha-r+k}{\lambda}-hs\right) \tag{19}$$

$$\prod_{k=0}^{\lambda-1}\Gamma_q\left(1-\frac{\alpha+k}{\lambda}+hs\right)=\frac{(1-q)^r}{(q^{1-(\alpha-h\lambda s)};q)_r}\prod_{k=0}^{\lambda-1}\Gamma_q\left(1-\frac{\alpha-r+k}{\lambda}+hs\right) \tag{20}$$

Using these identities (19) and (20) we see that (18) takes the form

$$D_{q,z}^{r}A(z,\alpha)=\left[A(z,\alpha-r)\right] \tag{21}$$

This is the Truesdell's form of descending F-equation.
Similarly forms (13) to (17) can be shown to satisfy Truesdell's descending F-equation.

(B): In this section we employ forms (12) to (17), to establish the following generating functions for q-analogue of I-Functions using Truesdell's descending F-equation technique:

(I)

$$(1+q^{\alpha})^{\alpha-1}I_{p_i,q_i,l}^{m,n}\left[(1+q^{\alpha})^{-h\lambda}x;q\left|\begin{matrix}\{\Delta(\lambda,\alpha),h\},\{(a_j,\alpha_j)_{\lambda+1,n}\},\{(a_{ji},\alpha_{ji})_{n+1,p_i-\rho}\},\{\Delta(\rho,\alpha),h\}\\ \{(b_j,\beta_j)_{1,m}\},\{(b_{ji},\beta_{ji})_{m+1,q_i-\rho}\},\{\Delta(\rho,\alpha),h\}\end{matrix}\right.\right]$$
$$=\sum_{r=0}^{\infty}\frac{\left(q^{(-\alpha+r/2+1/2)(\frac{1-\rho}{\rho})+\alpha}\right)^r}{r!}I_{p_i,q_i,l}^{m,n}\left[q^{-rh(\lambda+1)}x;q\left|\begin{matrix}\{\Delta(\lambda,\alpha-r),h\},\{(a_j,\alpha_j)_{\lambda+1,n}\},\{(a_{ji},\alpha_{ji})_{n+1,p_i-\rho}\},\{\Delta(\rho,\alpha-r),h\}\\ \{(b_j,\beta_j)_{1,m}\},\{(b_{ji},\beta_{ji})_{m+1,q_i-\rho}\},\{\Delta(\rho,\alpha-r),h\}\end{matrix}\right.\right] \tag{22}$$

(II)

$$(1+q^{\alpha})^{\alpha-1}I_{p_i,q_i,l}^{m,n}\left[(1+q^{\alpha})^{-h\lambda}x;q\left|\begin{matrix}\{\Delta(2\lambda,2\alpha),h\},\{(a_j,\alpha_j)_{2\lambda+1,n}\},\{(a_{ji},\alpha_{ji})_{n+1,p_i}\}\\ \left\{\Delta(\lambda,\alpha+\frac{1}{2}),h\right\},\{(b_j,\beta_j)_{\lambda+1,m}\},\{(b_{ji},\beta_{ji})_{m+1,q_i}\}\end{matrix}\right.\right]$$
$$=\sum_{r=0}^{\infty}\frac{\left(q^{\left(\alpha-\frac{1}{2}-\frac{r}{2}\right)\left(\frac{1+\lambda}{\lambda}\right)+\alpha}(1-q)\right)^r}{r!}I_{p_i,q_i,l}^{m,n}\left[q^{rh(1-\lambda)}x;q\left|\begin{matrix}\{\Delta(2\lambda,2\alpha-2r),h\},\{(a_j,\alpha_j)_{2\lambda+1,n}\},\{(a_{ji},\alpha_{ji})_{n+1,p_i}\}\\ \left\{\Delta(\lambda,\alpha-r+\frac{1}{2}),h\right\},\{(b_j,\beta_j)_{\lambda+1,m}\},\{(b_{ji},\beta_{ji})_{m+1,q_i}\}\end{matrix}\right.\right] \tag{23}$$

(III)

$$(1+q^{\alpha})^{\alpha}I_{p_i,q_i,l}^{m,n}\left[(1+q^{\alpha})^{-h\lambda}x;q\left|\begin{matrix}\{\Delta(3\lambda,3\alpha),h\},\{(a_j,\alpha_j)_{3\lambda+1,n}\},\{(a_{ji},\alpha_{ji})_{n+1,p_i}\}\\ \left\{\Delta(\lambda,\alpha+\frac{2}{3}),h\right\},\{(b_j,\beta_j)_{\lambda+1,m}\},\{(b_{ji},\beta_{ji})_{m+1,q_i-\lambda}\},\left\{\Delta(\lambda,\alpha+\frac{1}{3}),h\right\}\end{matrix}\right.\right]$$
$$=\sum_{r=0}^{\infty}\frac{\left(q^{\left(2\alpha+\frac{\alpha}{\lambda}-\frac{r}{2\lambda}-\frac{r}{2}+\frac{1}{6\lambda}+\frac{1}{2}\right)}(1-q)\right)^r}{r!}I_{p_i,q_i,l}^{m,n}\left[q^{-rh(1+\lambda)}x;q\left|\begin{matrix}\{\Delta(3\lambda,3\alpha-3r),h\},\{(a_j,\alpha_j)_{3\lambda+1,n}\},\{(a_{ji},\alpha_{ji})_{n+1,p_i}\}\\ \left\{\Delta(\lambda,\alpha-r+\frac{2}{3}),h\right\},\{(b_j,\beta_j)_{\lambda+1,m}\},\{(b_{ji},\beta_{ji})_{m+1,q_i-\lambda}\},\left\{\Delta(\lambda,\alpha-r+\frac{1}{3}),h\right\}\end{matrix}\right.\right] \tag{24}$$

(IV)

$$(1+q^{\alpha})^{\alpha-1}I_{p_i,q_i,l}^{m,n}\left[(1+q^{\alpha})^{-h\lambda}x;q\left|\begin{matrix}\{\Delta(\lambda,\alpha),h\},\{(a_j,\alpha_j)_{\lambda+1,n}\},\{(a_{ji},\alpha_{ji})_{n+1,p_i}\}\\ \{(b_j,\beta_j)_{1,m}\},\{(b_{ji},\beta_{ji})_{m+1,q_i}\}\end{matrix}\right.\right]$$
$$=\sum_{r=0}^{\infty}\frac{(-1)^r\left(q^{2\alpha+\frac{r}{2}-\frac{1}{2}}\right)^r}{r!}I_{p_i,q_i,l}^{m,n}\left[q^{-rh\lambda}x;q\left|\begin{matrix}\{\Delta(\lambda,\alpha-r),h\},\{(a_j,\alpha_j)_{\lambda+1,n}\},\{(a_{ji},\alpha_{ji})_{n+1,p_i}\}\\ \{(b_j,\beta_j)_{1,m}\},\{(b_{ji},\beta_{ji})_{m+1,q_i}\}\end{matrix}\right.\right] \tag{25}$$

(V)

$$(1+q^{\alpha})^{\alpha-1} I^{m,n}_{p_i,q_i,l}\left[(1+q^{\alpha})^{-h\lambda}x;q\left|\begin{matrix}\{(a_j,\alpha_j)_{1,n}\},\{(a_{ji},\alpha_{ji})_{n+1,p_i-\lambda}\},\{\Delta(\lambda,\alpha),h\}\\ \{(b_j,\beta_j)_{1,m}\},\{(b_{ji},\beta_{ji})_{m+1,q_i}\}\end{matrix}\right.\right]$$

$$=\sum_{r=0}^{\infty}\frac{\left(q^{\left(\frac{1-\lambda}{\lambda}\right)\left[-\alpha+\frac{r}{2}+\frac{1}{2}\right]+\alpha}\right)^r}{r!} I^{m,n}_{p_i,q_i,l}\left[q^{rh(1-\lambda)}x;q\left|\begin{matrix}\{(a_j,\alpha_j)_{1,n}\},\{(a_{ji},\alpha_{ji})_{n+1,p_i-\lambda}\},\{\Delta(\lambda,\alpha-r),h\}\\ \{(b_j,\beta_j)_{1,m}\},\{(b_{ji},\beta_{ji})_{m+1,q_i}\}\end{matrix}\right.\right] \quad (26)$$

(VI)

$$(1+q^{\alpha})^{\alpha-1} I^{m,n}_{p_i,q_i,l}\left[(1+q^{\alpha})^{-h\lambda}x;q\left|\begin{matrix}\{\Delta(\rho,\alpha),h\}\{(a_j,\alpha_j)_{\rho+1,n}\},\{(a_{ji},\alpha_{ji})_{n+1,p_i-\lambda}\},\{\Delta(\lambda,\alpha),h\}\\ \{\Delta(\rho,\alpha),h\}\{(b_j,\beta_j)_{\rho+1,m}\},\{(b_{ji},\beta_{ji})_{m+1,q_i}\}\end{matrix}\right.\right]$$

$$=\sum_{r=0}^{\infty}\frac{(-1)^r\left(q^{\left(\frac{1}{\lambda}-\frac{1}{\rho}-1\right)\left[-\alpha+\frac{r}{2}+\frac{1}{2}\right]+\alpha}\right)^r}{r!} I^{m,n}_{p_i,q_i,l}\left[q^{-rh\lambda}x;q\left|\begin{matrix}\{\Delta(\rho,\alpha-r),h\}\{(a_j,\alpha_j)_{\rho+1,n}\},\{(a_{ji},\alpha_{ji})_{n+1,p_i-\lambda}\},\{\Delta(\lambda,\alpha-r),h\}\\ \{\Delta(\rho,\alpha-r),h\}\{(b_j,\beta_j)_{\rho+1,m}\},\{(b_{ji},\beta_{ji})_{m+1,q_i}\}\end{matrix}\right.\right] \quad (27)$$

Proof: To establish (22) we substitute the form (12) in Truesdell's descending F-equation (6) and replace z by $\frac{y}{q^{\alpha}}$ and $\frac{q^{\alpha h+2\alpha h\lambda}}{y^{h\lambda}}$ by x in succession to get the required result. Similarly, result (24) can be proved by substituting the form (14) in Truesdell's descending F-equation (6) and using same replacement.
To establish (23) we substitute the form (13) in Truesdell's descending F-equation (6) and replace z by $\frac{y}{q^{\alpha}}$ and $\frac{q^{2\alpha h\lambda-\alpha h}}{y^{h\lambda}}$ by x in succession to get the required result. Similarly, result (26) can be proved by substituting the form (16) in Truesdell's descending F-equation (6) and using same replacement.
To establish (25) we substitute the form (15) in Truesdell's descending F-equation (6) and replace z by $\frac{y}{q^{\alpha}}$ and $\frac{q^{2\alpha h\lambda}}{y^{h\lambda}}$ by x in succession to get the required result. Similarly, result (27) can be proved by substituting the form (17) in Truesdell's descending F-equation (6) and using same replacement.

3 Special Cases

These results yield as special cases of certain generating function for q-analogue of Fox's H-Function [1] and q-analogue of Meijer's G-Function [1].

(i): If we take $l=1$ then the series (22) reduces to generating function of q-analogue of Fox's H-Function.

$$(1+q^{\alpha})^{\alpha-1} H_{P,Q}^{m,n}\left[(1+q^{\alpha})^{-h\lambda} x; q \middle| \begin{matrix} \{\Delta(\lambda,\alpha),h\},(a_j,\alpha_j),\{\Delta(\rho,\alpha),h\} \\ (b_j,\beta_j),\{\Delta(\rho,\alpha),h\} \end{matrix}\right]$$

$$= \sum_{r=0}^{\infty} \frac{\left(q^{(-\alpha+r/2+1/2)(\frac{1-\rho}{\rho})+\alpha}\right)^r}{r!} H_{P,Q}^{m,n}\left[q^{-rh(\lambda+1)} x; q \middle| \begin{matrix} \{\Delta(\lambda,\alpha-r),h\},(a_j,\alpha_j),\{\Delta(\rho,\alpha-r),h\} \\ (b_j,\beta_j),\{\Delta(\rho,\alpha-r),h\} \end{matrix}\right] \quad (28)$$

Again taking $\alpha_j = \beta_j = 1 \, and \; h = 1$ in (28), it gives Meijer's G-Function as:

$$(1+q^{\alpha})^{\alpha-1} H_{P,Q}^{m,n}\left[(1+q^{\alpha})^{-\lambda} x; q \middle| \begin{matrix} \Delta(\lambda,\alpha), a_j, \Delta(\rho,\alpha) \\ b_j, \Delta(\rho,\alpha) \end{matrix}\right]$$

$$= \sum_{r=0}^{\infty} \frac{\left(q^{(-\alpha+r/2+1/2)(\frac{1-\rho}{\rho})+\alpha}\right)^r}{r!} H_{P,Q}^{m,n}\left[q^{-r(\lambda+1)} x; q \middle| \begin{matrix} \Delta(\lambda,\alpha-r), a_j, \Delta(\rho,\alpha-r) \\ b_j, \Delta(\rho,\alpha-r) \end{matrix}\right] \quad (29)$$

Similarly, (23) to (27) can be employed to yield apparently new and interesting results for q-analogue of Fox's H-Function and Meijer's G-Function [1].

4 Conclusions

The results proved in this paper give some contributions to the theory of Truesdell's F_q-equation and are believed to be new to the theory of q-calculus and are likely to find certain applications in the theory of q-calculus.

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