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ON NANO πgs -CLOSED SETS

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Abstract — In this paper, a new class of sets called πgs -closed sets in nano topological spaces is introduced and its properties are studied and studied of nano πgs -closed sets which is implied by that of nano gs -closed sets.

Keywords — Nano π -closed set, nano πg -closed set, nano πgp -closed sets and nano πgs -closed set

1 Introduction

Lellis Thivagar et al [5] introduced a nano topological space with respect to a subset X of an universe which is defined in terms of lower approximation and upper approximation and boundary region. The classical nano topological space is based on an equivalence relation on a set, but in some situation, equivalence relations are not suitable for coping with granularity, instead the classical nano topology is extended to general binary relation based covering nano topological space

Bhuvaneswari et.al [4] introduced and investigated nano g -closed sets in nano topological spaces. Recently, Rajasekaran et.al [8, 9] initiated the study nano πg -closed sets and new classes of sets called πgp -closed sets in nano topological spaces is introduced and its properties and studied of nano πgp -closed sets.

In this paper, a new class of sets called πgs -closed sets in nano topological spaces is introduced and its properties are studied and studied of nano πgs -closed sets which is implied by that of nano gs -closed sets.

2 Preliminaries

Throughout this paper $(U, \tau_R(X))$ (or X) represent nano topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset H of a

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space $(U, \tau_R(X))$, $Ncl(H)$ and $Nint(H)$ denote the nano closure of H and the nano interior of H respectively. We recall the following definitions which are useful in the sequel.

Definition 2.1. [7] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.

1. The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \subseteq X\}$, where $R(x)$ denotes the equivalence class determined by x .
2. The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \cap X \neq \phi\}$.
3. The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not - X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.

Property 2.2. [5] If (U, R) is an approximation space and $X, Y \subseteq U$; then

1. $L_R(X) \subseteq X \subseteq U_R(X)$;
2. $L_R(\phi) = U_R(\phi) = \phi$ and $L_R(U) = U_R(U) = U$;
3. $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$;
4. $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$;
5. $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$;
6. $L_R(X \cap Y) \subseteq L_R(X) \cap L_R(Y)$;
7. $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$;
8. $U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$;
9. $U_R U_R(X) = L_R U_R(X) = U_R(X)$;
10. $L_R L_R(X) = U_R L_R(X) = L_R(X)$.

Definition 2.3. [5] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then by the Property 2.2, $R(X)$ satisfies the following axioms:

1. U and $\phi \in \tau_R(X)$,
2. The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$,
3. The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ is a topology on U called the nano topology on U with respect to X . We call $(U, \tau_R(X))$ as the nano topological space. The elements of $\tau_R(X)$ are called as nano open sets and $[\tau_R(X)]^c$ is called as the dual nano topology of $[\tau_R(X)]$.

Remark 2.4. [5] If $[\tau_R(X)]$ is the nano topology on U with respect to X , then the set $B = \{U, \phi, L_R(X), B_R(X)\}$ is the basis for $\tau_R(X)$.

Definition 2.5. [5] If $(U, \tau_R(X))$ is a nano topological space with respect to X and if $H \subseteq U$, then the nano interior of H is defined as the union of all nano open subsets of H and it is denoted by $Nint(H)$.

That is, $Nint(H)$ is the largest nano open subset of H . The nano closure of H is defined as the intersection of all nano closed sets containing H and it is denoted by $Ncl(H)$.

That is, $Ncl(H)$ is the smallest nano closed set containing H .

Definition 2.6. A subset H of a nano topological space $(U, \tau_R(X))$ is called

1. nano semi open [5] if $H \subseteq Ncl(Nint(H))$.
2. nano regular-open [5] if $H = Nint(Ncl(H))$.
3. nano π -open [1] if the finite union of nano regular-open sets.
4. nano pre-open [5] if $H \subseteq Nint(Ncl(H))$.

The complements of the above mentioned sets is called their respective closed sets.

Definition 2.7. A subset H of a nano topological space $(U, \tau_R(X))$ is called;

1. nano g -closed [3] if $Ncl(H) \subseteq G$, whenever $H \subseteq G$ and G is nano open.
2. nano gs -closed [2] if $Nscl(H) \subseteq G$, whenever $H \subseteq G$ and G is nano open.
3. nano πg -closed [8] if $Ncl(H) \subseteq G$, whenever $H \subseteq G$ and G is nano π -open.
4. nano gp -closed set [4] if $Npcl(H) \subseteq G$, whenever $H \subseteq G$ and G is nano open.
5. nano πgp -closed set [9] if $Npcl(H) \subseteq G$, whenever $H \subseteq G$ and G is nano π -open.

The complements of the above mentioned sets is called their respective open sets.

3 On Nano πgs -closed Sets

Definition 3.1. A subset H of a space $(U, \tau_R(X))$ is nano πgs -closed if $Nscl(H) \subseteq G$ whenever $H \subseteq G$ and G is nano π -open.

The complement of nano πgs -open if $H^c = U - H$ is nano πgs -closed.

Example 3.2. Let $U = \{1, 2, 3\}$ with $U/R = \{\{1\}, \{2, 3\}\}$ and $X = \{1\}$. Then the nano topology $\tau_R(X) = \{\phi, \{1\}, U\}$.

1. then $\{2, 3\}$ is nano πgs -closed set.

2. then $\{1\}$ is nano π gs-open set.

Definition 3.3. A subset H of a space $(U, \tau_R(X))$ is called a nano strong \mathcal{B}_Q -set if $Nint(Ncl(H)) = Ncl(Nint(H))$.

Example 3.4. In Example 3.2, then $\{1, 2\}$ is nano strong \mathcal{B}_Q -set.

Theorem 3.5. In a space $(U, \tau_R(X))$, the following properties are equivalent:

1. If H is nano gs-closed, then H is nano π gs-closed.
2. If H is nano π g-closed, then H is nano π gs-closed.

Proof. Obvious.

Remark 3.6. For a subset of a space $(U, \tau_R(X))$, we have the following implications:

$$\begin{array}{ccccc}
 & & \text{nano } \pi\text{-closed} & & \\
 & & \Downarrow & & \\
 \text{nano semi-closed} & \Leftarrow & \text{nano closed} & \Rightarrow & \text{nano pre-closed} \\
 \Downarrow & & \Downarrow & & \Downarrow \\
 \text{nano gs-closed} & \Leftarrow & \text{nano g-closed} & \Rightarrow & \text{nano gp-closed} \\
 \Downarrow & & \Downarrow & & \Downarrow \\
 \text{nano } \pi\text{gs-closed} & \Leftarrow & \text{nano } \pi\text{g-closed} & \Rightarrow & \text{nano } \pi\text{gp-closed}
 \end{array}$$

None of the above implications are reversible as shown by the following Examples.

Example 3.7. 1. Let $U = \{1, 2, 3, 4\}$ with $U/R = \{\{1, 2\}, \{3\}, \{4\}\}$ and $X = \{1, 4\}$. Then the nano topology $\tau_R(X) = \{\phi, \{4\}, \{1, 2\}, \{1, 2, 4\}, U\}$. Then $\{1\}$ is nano π gs-closed set but not nano π g-closed.

2. Let $U = \{1, 2, 3\}$ with $U/R = \{\{1, 3\}, \{2\}\}$ and $X = \{3\}$. Then the nano topology $\tau_R(X) = \{\phi, \{1, 3\}, U\}$. Clearly $\{1\}$ is nano π gs-closed set but not nano gs-closed.

Lemma 3.8. In a space $(U, \tau_R(X))$,

1. every nano open set is nano π gs-closed.
2. every nano closed set is nano π gs-closed.

Remark 3.9. The converses of statements in Lemma 3.8 are not necessarily true as seen from the following Examples.

Example 3.10. In Example 3.2,

1. then $\{2, 3\}$ is nano π gs-closed set but not nano open.
2. then $\{1, 2\}$ is nano π gs-closed set but not nano closed.

Theorem 3.11. In a space $(U, \tau_R(X))$, the following properties are equivalent:

1. H is nano π -open and nano π gs-closed.

2. H is nano regular-open.

Proof. (1) \Rightarrow (2). By (1) $Nscl(H) \subseteq H$, since H is nano π -open and nano π gs-closed. Thus $Nint(Ncl(H)) \subseteq H$. Since H is nano open, then H is clearly nano pre-open and thus $H \subseteq Nint(Ncl(H))$. Therefore $Nint(Ncl(H)) \subseteq H \subseteq Nint(Ncl(H))$ or equivalently $H = Nint(Ncl(H))$, which shows that H is nano regular-open.

(2) \Rightarrow (1). Every nano regular-open set is nano π -open. For the second claim note that nano regular-open sets are even nano semi-closed.

Corollary 3.12. *If H is nano π -open and nano π gs-closed, then H is nano semi-closed and hence nano gs-closed.*

Proof. By assumption and Theorem 3.11, H is nano regular-open. Thus H is nano semi-closed. Since every nano semi-closed set is nano gs-closed, H is nano gs-closed.

Remark 3.13. *In a space $(U, \tau_R(X))$, $Nscl(U - H) = U - sint(H)$, for any subset H of a space U .*

Theorem 3.14. *In a space $(U, \tau_R(X))$, $H \subseteq U$ is nano π gs-open $\iff F \subseteq Nsint(H)$ whenever K is nano π -closed and $K \subseteq H$.*

Proof. Necessity. Let H be nano π gs-open. Let K be nano π -closed and $K \subseteq H$. Then $U - H \subseteq U - K$ where $U - K$ is nano π -open. nano π gs-closedness of $U - H$ implies $Nscl(U - H) \subseteq U - K$. By Remark 3.13, $Nscl(U - H) = U - Nsint(H)$. So $K \subseteq Nsint(H)$.

Sufficiency. Suppose K is nano π -closed and $K \subseteq H$ imply $K \subseteq Nsint(H)$. Let $U - H \subseteq G$ where G is nano π -open. Then $U - G \subseteq H$ where $U - G$ is nano π -closed. By hypothesis $U - G \subseteq Nsint(H)$. That is $U - Nsint(H) \subseteq G$. By Remark 3.13, $Nscl(U - H) \subseteq G$. So, $U - H$ is nano π gs-closed and H is nano π gs-open.

Theorem 3.15. *In a space $(U, \tau_R(X))$, the following properties are equivalent:*

1. H is nano π -clopen.
2. H is nano π -open, a nano strong \mathcal{B}_Q -set and nano π gs-closed.

Proof. (1) \Rightarrow (2) is Obvious.

(1) \Rightarrow (2). By Theorem 3.11, H is nano regular-open. Since H is a nano strong \mathcal{B}_Q -set, $H = Nint(Ncl(H)) = Ncl(Nint(H))$. So H is nano regular-closed. This shows that H is nano π -closed and hence H is nano π -clopen.

Theorem 3.16. *In a space $(U, \tau_R(X))$, the union of two nano π gs-closed sets is nano π gs-closed.*

Proof. Let $H \cup Q \subseteq G$, then $H \subseteq G$ and $Q \subseteq G$ where G is nano π -open. As H and Q are π gs-closed, $Ncl(H) \subseteq G$ and $Ncl(Q) \subseteq G$. Hence $Ncl(H \cup Q) = Ncl(H) \cup Ncl(Q) \subseteq G$.

Example 3.17. *In Example 3.2, then $H = \{2\}$ and $Q = \{3\}$ is nano π gs-closed sets. Clearly $H \cup Q = \{2, 3\}$ is nano π gs-closed.*

Theorem 3.18. *In a space $(U, \tau_R(X))$, the intersection of two nano π gs-open sets are nano π gs-open.*

Proof. Obvious by Theorem 3.16.

Example 3.19. In Example 3.2, then $H = \{1, 3\}$ and $Q = \{1, 2\}$ is nano π gs-open. Clearly $H \cap Q = \{1\}$ is nano π gs-open.

Remark 3.20. In a space $(U, \tau_R(X))$, the union of two nano π gs-closed sets but not nano π gs-closed.

Example 3.21. In Example 3.7(1), then $H = \{1\}$ and $Q = \{4\}$ is nano π gs-closed sets. Clearly $H \cup Q = \{1, 4\}$ is but not nano π gs-closed.

Remark 3.22. In a space $(U, \tau_R(X))$, the intersection of two nano π gs-open sets but not nano π gs-open.

Example 3.23. In Example 3.7(1), then $H = \{3, 4\}$ and $Q = \{1, 2, 3\}$ is nano π gs-open sets. Clearly $H \cap Q = \{3\}$ is but not nano π gs-open.

Theorem 3.24. Let H be nano π gs-closed. Then $Nscl(H) - H$ does not contain any non-empty nano π -closed set.

Proof. Let K be a nano π -closed set such that $K \subseteq Nscl(H) - H$. Then $K \subseteq U - H$ implies $H \subseteq U - K$. Therefore $Nscl(H) \subseteq U - K$. That is $K \subseteq U - Nscl(H)$. Hence $K \subseteq Nscl(H) \cap (U - Nscl(H)) = \phi$. This shows $K = \phi$.

Theorem 3.25. If H is nano π gs-closed and $H \subseteq P \subseteq Nscl(H)$, then P is nano π gs-closed.

Proof. Let H be nano π gs-closed and $P \subseteq G$, where G is nano π -open. Then $H \subseteq P$ implies $H \subseteq G$. Since H is nano π gs-closed, $Nscl(H) \subseteq G$. $P \subseteq Nscl(H)$ implies $Nscl(P) \subseteq Nscl(H)$. Therefore $Nscl(P) \subseteq G$ and hence P is nano π gs-closed.

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References

- [1] Adinatha C. Upadhya, *On quasi nano p-normal spaces*, International Journal of Recent Scientific Research, 8/6 (2017) 17748-17751.
- [2] K. Bhuvaneshwari and K. Ezhilarasi, *On Nano semi generalized and Nano generalized semi closed sets*, IJMCAR, 4/3 (2014) 117-124.
- [3] K. Bhuvaneshwari and K. Mythili Gnanapriya, *Nano Generalized closed sets*, International Journal of Scientific and Research Publications, 4/5 (2014) 1-3.
- [4] K. Bhuvaneshwari and K. Mythili Gnanapriya, *On Nano Generalised Pre Closed Sets and Nano Pre Generalised Closed Sets in Nano Topological Spaces*, International Journal of Innovative Research in Science, Engineering and Technology, 3/10 (2014) 16825-16829.

- [5] M. Lellis Thivagar and Carmel Richard, *On Nano forms of weakly open sets*, International Journal of Mathematics and Statistics Invention, 1/1 (2013) 31-37.
- [6] C. R. Parvathy and , S. Praveena, *On Nano Generalized Pre Regular Closed Sets in Nano Topological Spaces*, IOSR Journal of Mathematics (IOSR-JM), 13/2 (2017) 56-60.
- [7] Z. Pawlak, *Rough sets*, International journal of computer and Information Sciences, 11/5 (1982) 341-356.
- [8] I. Rajasekaran and O. Nethaji, *On some new subsets of nano topological spaces*, Journal of New Theory, 16 (2017) 52-58.
- [9] I. Rajasekaran, T. Siva Subramania Raja and O. Nethaji, *On nano πgp -closed sets*, Journal of New Theory, 19 (2017) 20-26.