http://www.newtheory.org **ISSN: 2149-1402**

Journal of New Theory

Received: 27.11.2017
Published: 20.12.2017

Year: **2017**, *Number*: **19**, *Pages*: **63-81**
Original Article

CHARACTERIZATION OF SOFT α-SEPARATION AXIOMS AND SOFT β-SEPARATION AXIOMS IN SOFT SINGLE POINT SPACES AND IN SOFT ORDINARY SPACES

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Abstract - The main aim of this article is to introduce soft α and soft β separations axioms, soft α-separations axioms and soft β separations axioms in soft single point topology. We discuss soft $\alpha n - T_4$ separation axioms and soft $\beta n - T_4$ separation axioms in soft topological spaces with respect to ordinary points and soft points. Further study the hereditary properties at different angles with respect to ordinary points as well as with respect to soft points. Some of their fundamental properties in soft single point topological spaces are also studied.

Keywords - *Soft sets, soft points, soft α open set, soft α closed set, soft β open set, soft β closed in soft topological space, soft single point topology, soft α and soft β separation axioms.*

1. Introduction

In real life condition the problems in economics, engineering, social sciences, medical science etc. We cannot beautifully use the traditional classical methods because of different types of uncertainties presented in these problems. To overcome these difficulties, some kinds of theories were put forwarded like theory of Fuzzy set, intuitionistic fuzzy set, rough set and bi polar fuzzy sets, inwhich we can safely use a mathematical techniques for businessing with uncertainties. But, all these theories have their inherent difficulties. To overcome these difficulties in the year 1999, Russian scientist Molodtsov [4], initiated the notion of soft set as a new mathematical technique for uncertainties. Which is free from the above complications. In [4,5], Mololdtsov successfully applied the soft set theory in

different directions, such as smoothness of functions, game theory, operation research, Riemann integration, perron integration, probability ,theory of measurement and so on.
After presentation of the operations of soft sets [6], the properties and applications of the soft set theory have been studied increasingly [7,8,6]. Xiao et al. [9] and Pei and Maio [10] discussed the linkage between soft sets and information systems. They showed that soft sets are class of special information system. In the recent year, many interesting applications of soft sets theory have been extended by embedding the ideas of fuzzy sets [11,12,13,14,15,16,17,18,20,21,22] industrialized soft set theory, the operations of the soft sets are redefined and in indecision making method was constructed by using their new operations [23].

Recently, in 20011, Shabir and Naz [24] launched the study of soft Topological spaces, they beautiful defined soft Topology as a collection of τ of soft sets over X. They also defined the basic conception of soft topological spaces such as open set and closed soft sets, soft nbd of a point, soft separation axiom, soft regular and soft normal spaces and published their several behaviors. Min in [25] scrutinized some belongings of this soft separation axiom. In [26] Kandil et al. introduced some soft operations such as semi open soft, pre-open soft, α-open soft and β-open soft and examined their properties in detail. Kandil et al. [27] introduced the concept of soft semi–separation axioms, in particular soft semi-regular spaces. The concept of soft ideal was discussed for the first time by Kandil et al. [28]. They also introduced the concept of soft local function; these concepts are discussed with a view to find new soft topological from the original one, called soft topological spaces with soft ideal (X, τ, E, I).

Applications to different zone were further discussed by Kandil et al. [28,29,30,32, 33,34,35]. The notion of super soft topological spaces was initiated for the first time by El-Sheikh and Abd-e-Latif [36]. They also introduced new different types of sub-sets of supra soft topological spaces and study the dealings between them in great detail. Bin Chen [41] introduced the concept of semi open soft sets and studied their related properties, Hussain [42] discussed soft separation axioms. Mahanta [39] introduced semi open and semi closed soft sets. Arokialancy in [43] generalized soft g β closed and soft gs β closed sets in soft topology are exposed. Mukharjee [44] introduced some new bi topological notion with respect to ordinary points. Gocur and Kopuzlu [45] discussed some new properties on soft separation axioms in soft single point space over El-Sheikh and Abd-e-Latif [46] discussed Characterization of soft b-open sets in soft topological spaces and defined pre-open, semi-open, α-open and β-open soft sets in soft topological spaces with respect to ordinary points. Yumak and Kaymaker [47] discussed Soft β-open sets and their applications.

In this present paper the concept of soft α-T_i spaces (i=1, 2, 3) and soft β_i spaces (i=1, 2, 3) are introduced in soft single point space with respect to ordinary and soft points of a topological space. Soft $\alpha n - T_4$ space and Soft $\beta n - S_4$ are introduced in soft topological space with respect to ordinary and soft points. Many mathematicians discussed soft separation axioms in soft topological spaces at full length with respect to soft open set, soft b-open set, soft semi-open set. They also worked over the hereditary properties of different soft topological structures in soft topology. In this present work hand is tried and work is encouraged over the gap that exists in soft topology. Related to Soft spaces, some theorems in soft single topological spaces are discussed with respect to ordinary points as well as with respect to soft points. Focus is laid upon the characters of soft $\alpha n - T_4$ and soft $\beta n - S_4$ space and their sub spaces in soft topological structures. When we talk about the distances between the points in soft topology then the concept of soft separation axioms will automatically come in play. That is why these structures are catching our attentions. We hope that these results will be valuable for the future study on soft single point topological spaces to accomplish general framework for the practical applications and to solve the most intricate problems containing scruple in economics, engineering, medical, environment and in general mechanic systems of various kinds

2. Preliminaries

The following Definitions which are pre-requisites for present study.

Definition 1 [4]. Let X be an initial universe of discourse and E be a set of parameters. Let P(X) denotes the power set of X and A be a non-empty sub-set of E. A pair (F, A) is called a soft set over U, where F is a mapping given by $F: A \to P(X)$

In other words, a set over X is a parameterized family of sub set of universe of discourse X. For $e \in A, F(e)$ may be considered as the set of e-approximate elements of the soft set (F, A) and if $e \notin A$ then$F(e) = \phi$, that is $F_A = \{F(e): e \in A \subseteq E, F: A \to P(X)\}$ the family of all these soft sets over X denoted by$SS(X)_A$.

Definition 2 [4]. Let $F_A, G_B \in SS(X)_E$ then F_A is a soft subset of G_B denoted by $F_A \sqsubseteq G_B$, if

1. $A \subseteq B$ and
2. $F(e) \subseteq G(e), \forall \in A$

In this case F_A is said to be a soft subset of G_B and G_B is said to be a soft super setF_A, $G_B \sqsupseteq F_A$.

Definition 3 [6]. Two soft subsets F_A and G_B over a common universe of discourse set X are said to be equal if F_A is a soft subset of G_B and G_B is a soft subset of F_A.

Definition 4 [6]. The complement of soft subset(F, A) denoted by $(F, A)^C$ is defined by $(F, A)^C = (F^C, A) F^C \to P(X)$ is a mapping given by $F^C(e) = U - F(e) \forall e \in A$ and $F^C$is called the soft complement function of F. Clearly $(F^C)^C$ is the same as F and $((F, A)^C)^C = (F, A)$.

Definition 5 [7]. The difference between two soft subset (G, E) and (G, E) over common of universe discourse X denoted by $(F, E) - (G, E)$ is the soft set (H, E) where for all $e \in E. \overline{\emptyset} or\ \emptyset_A$ if $\forall e \in A, F(e) = \emptyset$.

Definition 6 [7]. Let (G, E) be a soft set over$X$ and $x \in X$ We say that $x \in (F, E)$ and read as x belong to the soft set (F, E) whenever $x \in F(e) \forall e \in E$ The soft set (F, E) over X such that $F(e) = \{x\} \forall e \in E$ is called singleton soft point and denoted by $x_E, or\ (x, E)$.

Definition 7 [6]. A soft set (F, A) over X is said to be Null soft set denoted by $\overline{\emptyset}\ or\ \emptyset_A$ if $\forall e \in A, F(e) = \emptyset$.

Definition 8 [6]. A soft set (F, A) over X is said to be an absolute soft denoted by $\bar{A}$ *or* X_A if $\forall e \in A, F(e) = X$.

Clearly, we have. $X_A^c = \emptyset_A$ *and* $\emptyset_A^c = X_A$.

Definition 9 [7]. Let (G, E) be a soft set over X and $e_G \in X_A$, we say that $e_G \in (F, E)$ and read as e_G belong to the soft set (F, E) whenever $e_G \in F(e) \forall e \in E$. the soft set (F, E) over X such that $F(e) = \{e_G\}$, $\forall e \in E$ is called singleton soft point and denoted by e_G, *or* (e_G, E).

Definition 10 [42]. The soft set $(F, A) \in SSX_A$ is called a soft point in X_A, denoted by e_F, if for the element $e \in A, F(e) \neq \emptyset$ and $F(e^{/}) = \phi$ if for all $e^{/} \in A - \{e\}$

Definition 11 [42]. The soft point e_F is said to be in the soft set (G, A), denoted by $e_F \in (G, A)$ if for the element $e \in A, F(e) \subseteq G(e)$.

Definition 12 [42]. Two soft sets $(G, A), (H, A)$ in SSX_A are said to be soft disjoint, written

$$(G, A) \cap (H, A) = \emptyset_A \text{ If } G(e) \cap H(e) = \emptyset \forall\, e \in A.$$

Definition 13 [42]. The soft point $e_G, e_H \in X_A$ are disjoint, written $e_G \neq e_H$, if their corresponding soft sets (G, A) and (H, A) are disjoint.

Definition 14[6]. The union of two soft sets (F, A) and (G, B) over the common universe of discourse X is the soft set (H, C), where, $C = AUB \forall e \in C$

$$H(e) = \left\{ \begin{array}{ll} F(e) & if\ e \in A - B \\ G(e) & if\ e \in (B - A) \\ F(e), & if\ e \in A \cap B \end{array} \right\}$$

Written as $(F, A) \cup (G, B) = (H, C)$

Definition 15 [6]. The intersection $(H.C)$ of two soft sets (F, A) and (G, B) over common universe X, denoted $(F, A) \bar{\cap} (G, B)$ is defined as

$$C = A \cap B \text{ and } H(e) = F(e) \cap G(e), \forall e \in C.$$

Definition 16 [2]. Let (F, E) be a soft set over X and Y be a non-empty sub set of X. Then the sub soft set of (F, E) over Y denoted by (Y_F, E), is defined as follow $Y_{F(\alpha)} = Y \cap F(\alpha), \forall \in E$ in other words
$(Y_F, E) = Y \cap (F, E)$.

Definition 17 [2]. Let τ be the collection of soft sets over X , then $\tau$is said to be a soft topology on X, if

1. $\emptyset$, X belong to τ
2. The union of any number of soft sets in τ belongs to τ
3. The intersection of any two soft sets in τ belong to τ

The triplet (X, F, E) is called a soft topological space.

Definition 18 [1]. Let (X, τ, E) be a soft topological space over X then the member of τ are said to be soft open sets in X.

Definition 19 [1]. Let (X, τ, E) be a soft topological space over X. A soft set (F, A) over X is said to be a soft closed set in X if its relative complement $(F, E)^C$ belong to τ.

Definition 20 [46]. Let (X, τ, E) be a soft topological space and $(F, E) \subseteq SS(X)_E$ then (F, E) is said to be α-open soft set if $((F, E) \subseteq int(cl(int(F, E)))$.

The set of all α- open soft set is denoted $\alpha OS(X, \tau, E)$ or $BOS(X)$ and the set of all α-closed soft set is denoted by $\alpha CS(X, \tau, E)$ or $\alpha CS(X)$.

Definition 21 [46]. Let (X, τ, E) be a soft topological space and $(F, E) \subseteq SS(X)_E$ then (F, E) is called β open soft set $((F, E) \subseteq Cl(int(Cl(F, E))))$.

The set of all β *open soft* set is denoted by $S\beta O(X, \tau, E)$ or $S\beta O(X)$ and the set of all β *closed soft* set is denoted by $\beta CS(X, \tau, E)$ or $\beta CS(X)$.

Definition22[45]. Let X be an initial universe set, E be the set of parameters, x ∈ X and A be a subset of X. Let (A, E) be defined as A(e) = A, for all e ∈ E.

Then τ = {(A, E)|∀A ⊂ X} is a soft topology over X. In this case, τ is called soft Single point topology over X and (X, τ, E) is said to be a soft single point space over X.

Theorem*[48]. A sub space (Y, τ_Y, E) of a soft β T_1 space is soft βT_1.

3. Separation Axioms of Soft Topological Spaces With Respect to Ordinary Points as Well as Soft Points

Definition 23 [23]. Let (X, τ, A) be a soft Topological space over X and $x, y \in X$ such that $x \neq y$ if there exist at least one soft open set (F_1, A) OR (F_2, A) such that $x \in (F_1, A), y \notin (F_1, A)$ or $y \in (F_2, A), x \notin ((F_2, A)$ then (X, τ, A) is called a soft T_0 *space*.

Definition 24 [23]. Let (X, τ, A) be a soft Topological spaces over X and $x, y \in X$ such that $x \neq y$ if there exist soft open sets $(F_1, A)$and (F_2, A) such that $x \in (F_1, A), y \notin (F_1, A)$ and $y \in (F_2, A), x \notin ((F_2, A)$ then (X, τ, A) is called a soft T_1 *space*.

Definition 25 [23]. Let (X, τ, A) b e a soft Topological space over X and $x, y \in X$ such that $x \neq y$ if there exist soft open set (F_1, A) and (F_2, A) such that $x \in (F_1, A)$, and $y \in (F_2, A)$ and $F_1 \cap F_2 = \varphi$

Then (X, τ, A) is called soft T_2 spaces.

Definition 26 [42]. Let (X, τ, A) be a soft Topological space over X and $e_G, e_H \in X_A$ such that $e_G \neq e_H$ if we can search at least one soft open set (F_1, A) or (F_2, A) such that

$e_G \in (F_1, A), e_H \notin (F_1, A)$ or $e_H \in (F_2, A), e_G \notin ((F_2, A)$ then (X, τ, A) is called a soft T_0 space.

Definition 27 [42]. Let (X, τ, A) be a soft Topological spaces over X and $e_G, e_H \in X_A$ such that $e_G \neq e_H$ if we can search soft open sets $(F_1, A)$and (F_2, A) such that $e_G \in (F_1, A), e_H \notin (F_1, A)$ and $e_H \in (F_2, A), e_G \notin ((F_2, A)$ then (X, τ, A) is called a soft T_1 *space*.

Definition 28 [42]. Let (X, τ, A) b e a soft Topological space over X and $e_G, e_H \in X_A$ such that $e_G \neq e_H$ if we can search soft open set $(F_1, A)$and (F_2, A) such that $e_G \in (F_1, A)$, and $e_H \in (F_2, A)$

$(F_1, A) \cap (F_2, A) = \phi_A$ Then (X, τ, A) is called soft T_2 space.

Definition 29 [23]. Let (X, τ, E) be a soft topological space (G, E) be closed soft set in X and $e_G \in X_A$ such that $e_G \notin (G, E)$. If there occurs soft open sets (F_1, E) and (F_2, E) such that $e_G \in (F_1, E), (G, E) \subseteq (F_2, E)$ and $(F_1, E) \cap (F_2, E) = \varphi$. Then (X, τ, E) is called soft regular spaces. A soft regular T_1 *Space* is called soft T_3 space

Definition 30 [23]. Let (X, τ, E) be a soft topological space $(F_1, E), (G, E)$ be closed soft sets in X such that $(F, E) \cap (G, E) = \varphi$ if there exists open soft sets (F_1, E) and (F_2, E) such that $(F, E) \subseteq (F_1, E), (G, E) \subseteq (F_2, E)$ and $(F_1, E) \cap (F_2, E) = \varphi$ then (X, τ, E) is called soft normal space. A soft normal T_1 *Space* is called soft T_4 *Space*.

Definition 32 [45]. Let (X, τ, E) be a soft topological space X and $x, y \in X$ such that $x \neq y$. Let (F, E) and (G, E) be soft closed sets such that that $x \in (F, E)$ and $(F, E) \cap (G, E) = \phi$. If there exist soft open sets (F_1, E) and (F_2, E) such that $y \in (F_2, E)$, $(F, E) \subseteq (F_1, E)$, $(G, E) \subseteq (F_2, E)$ and $(F_1, E) \cap (F_2, E) = \phi$, then (X, τ, E) is called soft n-normal space.

Definition 33 [45]. Let (X, τ, E) be a soft topological space X. If (X, τ, E) is a soft n-normal space and T_1 space, then (X, τ, E) is a soft n-T_4 space.

4. Soft α Separation Axioms of Soft Single Point Topological Spaces

In this section we introduced the concept of soft αT_i spaces (i=1, 2, 3) in soft single point space with respect to ordinary and soft points of a soft single point topological space and some of its basic properties are studied and applied to different results in this section.

4.1 Soft α Separation Axioms of Soft Single Point Topological Spaces With Respect to Ordinary Points

In this section we introduced soft separation axioms in soft single point topological space with respect to ordinary points and discussed some results with respect to these points in detail.

Definition 34. Let (X, τ, E) be a soft topological space and $(x, E) \subseteq SS(X)_E$ then (x, E) is said to be α-open soft set if $((x, E) \subseteq int(cl(int(x, E)))$.

The set of all α- open soft set is denoted $\alpha OS(X,\tau,E)$ or $BOS(X)$ and the set of all α-closed soft set is denoted by $\alpha CS(X,\tau,E)$or $\alpha CS(X)$.

Definition 35. Let (X,τ,A) be a soft Topological space over X and $x,y \in X$ such that $x \neq y$ if there exist at least one soft α open set (F_1,A) OR (F_2,A) such that $x \in (F_1,A), y \notin (F_1,A)$ or $y \in (F_2,A), x \notin ((F_2,A)$then (X,τ,A) is called a soft $\alpha\ T_0\ space$.

Definition 36. Let (X,τ,A) be a soft Topological spaces over X and $x,y \in X$ such that $x \neq y$ if there exist soft α open sets (F_1,A) and (F_2,A) such that $x \in (F_1,A), y \notin (F_1,A)$ and $y \in (F_2,A), x \notin ((F_2,A)$ then (X,τ,A) is called a soft $\alpha\ T_1\ space$.

Definition 37. Let (X,τ,A) b e a soft topological space over X and $x,y \in X$ such that $x \neq y$ if there exist soft α open set $(F_1,A)$and (F_2,A) such that $y \notin (F_1,A)$, and $y \in (F_2,A)$ and $F_1 \cap F_2 = \varphi$

Then (X,τ,A) is called soft $\alpha\ T_2$ spaces.

Definition 38. Let (X,τ,E) be a soft topological space X and $x,y \in X$ such that$x \neq y$. Let (F,E) and (G,E) be soft α closed sets such that that $x \in (F,E)$ and $(F,E) \cap (G,E) = \phi$. If there exist soft α open sets (F_1,E) and (F_2,E) such that $y \in (F_2,E)$,$(F,E) \subseteq (F_1,E)$, $(G,E) \subseteq (F_2,E)$ and $(F_1,E) \cap (F_2,E) = \phi$, then (X,τ,E) is called soft α n-normal space.

Definition 39. Let (X,τ,E) be a soft topological space X. If (X,τ,E) is a soft α n-normal space and αT_1 space, then (X,τ,E) is a soft α n-T_4 space.

Theorem** A sub (Y,τ_Y,E) of a soft α T_1 space is soft αT_1.

Proof. Let $x,y \in Y$ such that$x \neq y$. Then $x,y \in X$ such that $x \neq y$. Hence there exists soft α open sets (F,E) and (G,E) such that $x \in (F,E), y \notin (F,E)$ and $y \in (G,E), x \notin ((G,E)$. Since$x \in Y$. Hence $x \in Y \cap (F,E) = (F_Y,E)$, (F,E) is soft α open set. Consider $,y \notin (F,A)$, This implies that, $y \notin F(e)\ for\ some\ e \in E.$ Therefore $y \notin Y \cap (F,E) = (F_Y,E)$. Similarly, if $y \in (G,E)$ and $x \notin ((G,A)$, Then $y \in (G_Y,E)$ and $x \notin (G_Y,E)$, Then $y \in (G_Y,E)$ and $x \notin (G_Y,E)$.Thus, (Y,τ_Y,E) of a soft α T_1 space is soft αT_1.

Theorem 1. Let X be an initial universe set, E be the set of parameters. If (X,τ,E) is a soft single point space, then each soft element of (X,τ,E) is both soft α open and soft α closed set.

Proof. Let X be an initial universe set, E be the set of parameters and(A,E) is a soft single point space. Let (A,E) be defined as$A(e) = A, \forall e \in E$. $\tau = \{(A,E): \forall A \subseteq X\}$ From Definition 22[45], since$\tau^{/} = \{(A,E)^{/}: \forall A^{/} \subseteq X\}, (A,E)^{/}$is a soft α open set$\forall A^{/} \subseteq X$. Thus (A,E) is soft α open and soft α closed set in $\forall A \subseteq X$.

Theorem 2. Let X be an initial universe set, E be the set of parameters. **If** (X,τ,E) is a soft single point space, then (X,τ_e) is a discrete space$\forall e \in E$.

Proof. Let X be an initial universe set, E be the set of parameters and (X,τ,E) is a soft single point space, (A,E) is defined as $A(e)=A, \forall\, e\in E$. Then $\tau=\{(A,E): \forall\, A\subseteq X\}$ is a soft topology over X from Definition 22[45]. Here A is soft α open set in $(X,\tau_e)\forall A\subseteq X \forall\, e\in E$. Thus (X,τ_e) is a discrete space for all $e\in E$.

Theorem 3. Let (X,τ,E) be a soft single point space X. Then (X,τ,E) is soft αT_1 space.

Proof. Let (X,τ,E) be a soft single point space X and $x,y\in X$ such that $x\neq y$.Then there exist soft α open sets (x,E), (y,E) such that $x\in(x,E)\in\tau$, $y\notin(x,E)$ and $y\in(y,E)\in\tau$, $x\notin(y,E)$. Hence (X,τ,E) is soft αT_1 space.

Theorem 4. Let (X,τ,E) be a soft single point space over X then (X,τ,E) is soft αT_2 space.

Proof. Let (X,τ,E) be a soft single point space over X and $x,y\in X$ such that $x\neq y$. Then there exist soft α open sets (x,E) and (y,E) such that $x\in(x,E)\in\tau$, $y\in(y,E)\in\tau$ and $(x,E)\cap(y,E)=\phi$. Hence (X,τ,E) is soft αT_2 space.

Theorem 5. Let (X,τ,E) be a soft single point space over X. Then, (X,τ,E) is a soft αT_3 space.

Proof. Let (X,τ,E) be a soft single point space over X, (G,E) be a soft α closed set in X and $x\in X$ such that , $x\notin(G,E)$. From Theorem 1, there exists soft α open sets (x,E) and (G,E) such that $x\in(x,E)$, $(G,E)\subseteq(G,E)$and$(x,E)\cap(G,E)=\phi$. Also, from Theorem 3, (X,τ,E) is a soft αT_1 point space, so (X,τ,E) is soft αT_3 space.

Theorem 6. Let (X,τ,E) be a soft single point space over X then (X,τ,E) is a soft αT_4 space

Proof. Let (X,τ,E) be a soft single point space over $X$and let (F,E) and (G,E) be soft α closed sets in X such that $(F,E)\cap(G,E)=\phi$. From Theorem1, there exists soft α open sets (F,E) and (G,E) such that $F,E)\subseteq(F,E)$, $(G,E)\subseteq(G,E)$. Since $(F,E)\cap(G,E)=\phi$. (X,τ,E) Is called a soft α normal space. Also Theorem 3, (X,τ,E) is a soft αT_1 space, so (X,τ,E) is a soft αT_4.

Theorem 7. Let (X,τ,E) be a soft single point space over X and $x,y\in X$. Then (X,τ,E) is a soft α n- T_4 space.

Proof. Let (X,τ,E) be a soft single point space over X and$x,y\in X$, let (F,E) and (G,E) be soft α closed sets such that $x\in(F,E)$ and$(F,E)\cap(G,E)=\phi$. Then there exist soft α open sets (F,E) and (G,E) such that $y\in(G,E)$, $(F,E)\subseteq(F,E)$, $(G,E)\subseteq(G,E) and$ $(F,E)\cap(G,E)=\phi$ from Theorem 1, Thus (X,τ,E) is a soft α normal space. Also from Theorem 3, (X,τ,E) is soft αT_1 space so (X,τ,E) is a soft α n-T_4 space.

4.2 Soft α-Separation Axioms of Soft Single Point Topological Spaces With Respect to Soft Points

In this section we introduced soft α separation axiom in soft single point topological space with respect to soft points and discussed some results with respect to these points in detail.

Definition 40. Let (X,τ,E) be a soft topological space and $(e_G,E) \subseteq SS(X)_E$ then (e_G,E) is said to be α-open soft set if $((e_G,E) \subseteq int(cl(int(e_G,E)))$.

The set of all α- open soft set is denoted $\alpha OS(X,\tau,E)$ or $BOS(X)$ and the set of all α-closed soft set is denoted by $\alpha CS(X,\tau,E)$or $\alpha CS(X)$.

Definition 41. Let (X,τ,E) be a soft topological space X and $e_G,e_H \in X_E$ such that $e_G \neq e_H$. Let $(e_G,E)$and(e_H,E) be soft α closed sets such that that $e_G \in (e_G,E)$ and $(F,E) \cap (G,E) = \phi$. If there exist soft α open sets (F_1,E) and (F_2,E) such that $e_H \in (F_2,E)$,$(F,E) \subseteq (F_1,E)$, $(G,E) \subseteq (F_2,E)$ and $(F_1,E) \cap (F_2,E) = \phi$, then (X,τ,E) is called soft α n-normal space.

Definition 42. Let (X,τ,E) be a soft topological space X. If (X,τ,E) is a soft α n-normal space and α T_1 space, then (X,τ,E) is a soft α n-T_4 space.

Theorem 8. Let (X,τ,E) be a soft single point space$X$. Then (X,τ,E) is soft αT_1 space.

Proof. Let (X,τ,E) be a soft single point space X and $e_G,e_H \in X_E$such that $e_G \neq e_H$.Then there exist soft α open sets (e_G,E), (e_H,E) such that$e_G \in (e_G,E) \in \tau$,$e_H \notin (e_G,E)$ and $e_H \in (e_H,E) \in \tau$, $e_G \notin (e_H,E)$. Hence (X,τ,E) is soft αT_1 space.

Theorem 9. Let (X,τ,E) be a soft single point space over X then (X,τ,E) is soft αT_2 space

Proof. Let (X,τ,E) be a soft single point space over X_E and $e_G,e_H \in X_E$ such that $e_G \neq e_H$. Then there exist soft α open sets (e_G,E) and (e_H,E) such that $e_G \in (e_G,E) \in \tau$,$e_H \in (e_H,E) \in \tau$ and $(e_G,E) \cap (e_H,E) = \phi$. Hence (X,τ,E) is soft αT_2 space

Theorem 10. Let (X,τ,E) be a soft single point space over X .Then, (X,τ,E) is a soft αT_3 space.

Proof. Let (X,τ,E) be a soft single point space over X_E , (G,E) be a soft α closed set in X and $e_G \in X$ such that,$e_G \notin (G,E)$ From Theorem 1, there exists soft α open sets (e_G,E) and (G,E) such that$e_G \in (e_G,E)$, $(G,E) \subseteq (G,E)$and$(e_G,E) \cap (G,E) = \phi$. Also, from Theorem 8, (X,τ,E) is a soft αT_1 point space, so (X,τ,E) is soft αT_3 space. Theorem 11. Let (X,τ,E) be a soft single point space over X and $x,y \in X$.then (X,τ,E) is a soft α n- T_4 space.

Proof. Let (X,τ,E) be a soft single point space over X and $e_G,e_H \in X_E$, let (F,E) and (G,E) Be soft α closed sets such that $e_G \in (F,E)$and$(F,E) \cap (G,E) = \phi$. Then there exist soft α open sets (F,E) and (G,E) such that $e_H \in (G,E)$, $(F,E) \subseteq (F,E)$, $(G,E) \subseteq (G,E) and$ $(F,E) \cap (G,E) = \phi$ from Theorem 1, Thus (X,τ,E) is a soft α normal space. Also from Theorem 8, (X,τ,E) is soft αT_1 space so (X,τ,E) is a soft α n-T_4 space.

4.3 Soft β-Separation Axioms of Soft Single Point Space With Respect to Ordinary Points

In this section we introduced soft β-separation axioms in soft single point topological space with respect to ordinary points and discussed some results with respect to these points in detail.

Definition 43 [46]. Let (X,τ,E)be a soft topological space and $(x,E) \subseteq SS(X)_E$ then (x,E) is called β open soft set $((x,E) \subseteq Cl(int(Cl(x,E))))$.

The set of all β open soft set is denoted by $S\beta O(X,\tau,E)$ or $S\beta O(X)$ and the set of all β closed soft set is denoted by $\beta CS(X,\tau,E)$ or $\beta CS(X)$.

Theorem 12. Let X be an initial universe set, E be the set of parameters. If(X,τ,E) is a soft single point space, then each soft element of (X,τ,E) is both soft β open and soft β closed set.

Proof. Let X be an initial universe set, E be the set of parameters and(A,E) is a soft single point space. Let (A,E) be defined as$A(e) = A, \forall e \in E$. $\tau = \{(A,E) : \forall A \subseteq X\}$ From Definition 22[45], since$\tau^{/} = \{(A,E)^{/} : \forall A^{/} \subseteq X\}, (A,E)^{/}$is a soft β open set$\forall A^{/} \subseteq X$. Thus (A,E) is soft β open and soft β closed set in $\forall A \subseteq X$.

Theorem 13. Let X be an initial universe set, E be the set of parameters. If (X,τ,E) is a soft single point space, then (X,τ_e) is a discrete space $\forall e \in E$.

Proof. Let X be an initial universe set, E be the set of parameters and(X,τ,E) is a soft single point space, (A,E) is defined as $A(e) = A, \forall e \in E$. Then $\tau = \{(A,E) : \forall A \subseteq X\}$ is a soft topology over X Definition 22[45]. Here A is soft β open set in $(X,\tau_e) \forall A \subseteq X \forall e \in E$. Thus (X,τ_e) is a discrete space for all $e \in E$.

Theorem 14. Let (X,τ,E) be a soft single point space$X$. Then (X,τ,E) is soft βT_1 space.

Proof. Let (X,τ,E) be a soft single point space $X$and $x,y \in X$ such that $x \neq y$.Then there exist soft β open sets (x,E), (y,E) such that$x \in (x,E) \in \tau$, $y \notin (x,E)$ and $y \in (y,E) \in \tau$, $x \notin (y,E)$. Hence (X,τ,E) is soft βT_1 space.

Theorem 15. Let (X,τ,E) be a soft single point space over X then (X,τ,E) is soft βT_2 space

Proof. Let (X,τ,E) be a soft single point space over $X$and$x,y \in X$ such that $x \neq y$. Then there exist soft β open sets (x,E) and (y,E) such that $x \in (x,E) \in \tau$, $y \in (y,E) \in \tau$ and $(x,E) \cap (y,E) = \phi$. Hence (X,τ,E) is soft βT_2 space

Theorem 16. Let (X,τ,E) be a soft single point space over X . Then, (X,τ,E) is a soft βT_3 space.

Proof. Let (X, τ, E) be a soft single point space over , (G, E) be a soft β closed set in X and $x \in X$ such that , $x \notin (G, E)$. From Theorem 12, there exists soft β open sets (x, E) and (G, E) such that $x \in (x, E)$, $(G, E) \subseteq (G, E)$and$(x, E) \cap (G, E) = \phi$. Also, from Theorem 12, there exists soft β open sets (F, E) and (G, E) such that $F, E) \subseteq (F, E)$, $(G, E) \subseteq (G, E)$. Since $(F, E) \cap (G, E) = \phi$.$(X, \tau, E)$ Is called a soft β normal space. Also Theorem 14, (X, τ, E) is a soft βT_1 space, so (X, τ, E) is a soft βT_4. (X, τ, E) is a soft βT_1 point space, so (X, τ, E) is soft βT_3 space.

Theorem 17. Let (X, τ, E) be a soft single point space over X then (X, τ, E) is a soft βT_4 space

Proof. Let (X, τ, E) be a soft single point space over $X$and let (F, E) and (G, E) be soft β closed sets in X such that$(F, E) \subseteq (F, E)$, $(G, E) \subseteq (G, E)$. Since $(F, E) \cap (G, E) = \phi$. From Theorem 12. (X, τ, E) is called a soft β normal space. Also Theorem 14, (X, τ, E) is a soft βT_1 space, so (X, τ, E) is a soft βT_4.

Theorem 18. Let (X, τ, E) be a soft single point space over X and $x, y \in X$.then (X, τ, E) is a soft β n- T_4 space.

Proof. Let (X, τ, E) be a soft single point space over X and, $y \in Xx, y \in X$, let (F, E) and (G, E) Be soft β closed sets such that $x \in (F, E)$ and $(F, E) \cap (G, E) = \phi$. Then there exist soft β open sets (F, E) and (G, E) such that $y \in (G, E)$, $(F, E) \subseteq (F, E)$, $(G, E) \subseteq (G, E) and$ $(F, E) \cap (G, E) = \phi$ from Theorem 12. Thus (X, τ, E) is a soft β normal space. Also from Theorem14, (X, τ, E) is soft βT_1 space so (X, τ, E) is a softβ n-T_4 space.

4.4 Soft β-Separation Axioms of Soft Single Point Topological Spaces With Respect to Soft Points

In this section we introduced soft β-separation axioms in soft single point topological space with respect to soft points and discussed some results with respect to these points in detail.

Theorem 19. Let (X, τ, E) be a soft single point space X. Then (X, τ, E) is soft βT_1 space.

Proof. Let (X, τ, E) be a soft single point space X and $e_G, e_H \in X_E$ such that $e_G \neq e_H$. Then there exist soft β open sets (e_G, E), (e_H, E) such that$e_G \in (e_G, E) \in \tau$, $e_H \notin (e_G, E)$ and $e_H \in (e_H, E) \in \tau$, $e_G \notin (e_H, E)$. Hence (X, τ, E) is soft βT_1 space.

Theorem 20. Let (X, τ, E) be a soft single point space over X then (X, τ, E) is soft βT_2 space

Proof. Let (X, τ, E) be a soft single point space over X_E and $e_G, e_H \in X_E$ such that $e_G \neq e_H$. Then there exist soft β open sets (e_G, E) and (e_H, E) such that $e_G \in (e_G, E) \in \tau$, $e_H \in (e_H, E) \in \tau$ and$(e_G, E) \cap (e_H, E) = \phi$. Hence (X, τ, E) is soft βT_2 space

Theorem 21. Let (X, τ, E) be a soft single point space over X . Then, (X, τ, E) is a soft βT_3 space.

Proof. Let (X,τ,E) be a soft single point space over X_E , (G,E) be a soft β closed set in X and $e_G \in X$ such that, $e_G \notin (G,E)$ From Theorem 1, there exists soft β open sets (e_G,E) and (G,E) such that $e_G \in (e_G,E)$, $(G,E) \subseteq (G,E)$ and $(e_G,E) \cap (G,E) = \phi$. Also, from Theorem 8, (X,τ,E) is a soft βT_1 point space, so (X,τ,E) is soft βT_3 space.

Proof. Let (X,τ,E) be a soft single point space over X_E , (G,E) be a soft β closed set in X and $e_G \in X$ such that,$e_G \notin (G,E)$ From Theorem 12, there exists soft β open sets (e_G,E) and (G,E) such that$e_G \in (e_G,E)$, $(G,E) \subseteq (G,E)$ and $(e_G,E) \cap (G,E) = \phi$. Also, from Theorem 19, (X,τ,E) is a soft βT_1 point space, so (X,τ,E) is soft βT_3 space.

Theorem 22. Let (X,τ,E) be a soft single point space over X and $x,y \in X$.then (X,τ,E) is a soft β n- T_4 space.

Proof. Let (X,τ,E) be a soft single point space over X and $e_G,e_H \in X_E$, let (F,E) and (G,E) be soft β closed sets such that $e_G \in (F,E)$ and $(F,E) \cap (G,E) = \phi$. Then there exist soft β open sets (F,E) and (G,E) such that $e_H \in (G,E)$, $(F,E) \subseteq (F,E)$, $(G,E) \subseteq (G,E) and$ $(F,E) \cap (G,E) = \phi$ from Theorem 12, Thus (X,τ,E) is a soft β normal space. Also from Theorem 19, (X,τ,E) is soft βT_1 space so (X,τ,E) is a soft β n- T_4 space.

4.5 Soft α-Separation Axioms of Soft Topological Spaces With Respect to Ordinary Points

In this section we introduced soft α separation axioms in soft topological space with respect to ordinary points and discussed some results with respect to these points in detail. Soft αT_4 may not be a soft αT_3 space and soft αT_2 space. But breaking news is that we launched a new soft α separation axioms which is both soft αT_3 space and soft αT_2 space. It enjoys all the properties of both the soft αT_3 space and soft αT_2 space.

Theorem 23. Let (Y,τ_Y,E) be a soft sub space of a soft topological space (X,τ,E) and $(F,E) \in SS(X)$ then
1) If (F,E) is α *open soft* set in Y and $Y \in \tau$, then $(F,E) \in \tau$.
2) (F,E) is α *open soft* set in Y if and only if $(F,E) = Y \cap (G,E)$ for some $(G,E) \in \tau$.
3) (F,E) is α *closed soft* set in Y if and only if $(F,E) = Y \cap (H,E)$ for some $(H,E)\ is\ \tau$ *α close soft* set.

Proof. 1) Let (F,E) be a soft α set in Y then there does exists a soft α open set (G,E) in X such that $(F,E) = Y \cap (G,E)$. Now, if $Y \in \tau$ then $Y \cap (G,E) \in \tau$ by the third condition of the definition of a soft topological space and hence$(F,E) \in \tau$.

2) Fallows from the definition of a soft subspace.

3) If (F,E) is soft α closed in Y then we have $(F,E) = Y(G,E)$, for some $(G,E) \in \tau_Y$. Now, $(G,E) = Y \cap (H,E)$ for some soft α open set $(H,E) \in \tau$. for any $\gamma \in E$. $F(\gamma) = Y(\gamma) \setminus G(\gamma)$ $=Y\setminus G(\gamma)$ $=$ $Y(Y(\gamma) \cap H(\gamma))$ $=Y\setminus(Y \cap H(\gamma)) = Y\setminus H(\gamma) = Y \cap (X\setminus H(\gamma))$ $=$ $Y \cap (H(\gamma))^c = Y(\gamma) \cap (H(\gamma))^c$. Thus $(F,E) = Y \cap (H,E)$ is soft α closed in X as$(H,E) \in \tau$. Conversely, suppose that $(F,E) = Y \cap (G,E)$ for some soft α close set (G,E) in X. This qualifies us to say

that$(G,E)^{-} \in \tau$. Now, if $(G,E) = (X,E)(H,E)$ where (H,E) is soft α open set τ then for any $\gamma \in EF(\gamma) = Y(\gamma) \cap G(\gamma) = Y \cap G(\gamma)$ $=Y\cap (X(\gamma)\backslash H(\gamma))$ $=Y \cap (X(\gamma)\backslash H(\gamma))$ $=Y \cap (X\backslash H(\gamma))Y\backslash H(\gamma)$ $=Y\backslash(Y \cap H(\gamma))$ $=Y(\gamma)\backslash(Y(\gamma) \cap H(\gamma))$. Thus $(F,E) = Y\backslash\big(Y \cap (H,E)\big)$. Since $(H,E) \in \tau$, So$(Y \cap (H,E) \in \tau$. So $(Y \cap (H,E) \in \tau_Y$ and hence (F,E) is soft α closed in Y.

Theorem 24. Let (X,τ,E) be a soft topological space over$X$. And let (Y,E) be a soft α closed set in X and let $(G,E) \subseteq (Y,E)$. Then, (G,E) is a soft α closed set in sub space Y iff, (G,E) Is a soft α closed set in X.

Proof. This implies since (G,E) is a soft α closed set in soft sub space Y, there exists a soft α closed set (F,E) in X such that $(G,E) = Y \cap (F,E)$ from Theorem 23, Because (Y,E) and (F,E) are soft α closed set in X. Is implied by Since (G,E) is a soft α closed set in X and $(G,E) = Y \cap (G,E)$, (G,E) is a soft α closed set in sub space Y from Theorem 23.

Theorem 25. Let (X,τ,E) be a soft topological space over X and Y be a non-empty soft set of X. If (X,τ,E) is a soft $\alpha n = T_4$ space and Y be a soft α closed set, (Y,τ_Y,E) is a soft $\alpha n = T_4$ space.

Proof. Let (X,τ,E) is a soft $\alpha n = T_4$ space and Y be a soft α closed set in X. Because (X,τ,E) is soft αT_1 space, (Y,τ_Y,E) is soft αT_1 from ***Theorem*****. Let,(F,E) and (G,E) be soft α closed set in Y such that that$x \in (F,E)$. Then (F,E) and (G,E) are soft α closed sets in X from Theorem 1, Because (X,τ,E) is a soft $\alpha n = T_4$ space, $(F,E) \cap (G,E) = \phi$. Since (X,τ,E) is a soft α n-normal space, there exists soft α open sets and (F_1,E) and (F_2,E) such that and$y \in (F_2,E)$, $(F,E) \subseteq (F_1,E)$, $(G,E) \subseteq (F_2,E)$ from **Theorem 1**, In this case,$y \in Y \cap (F_2,E)$, $(F,E) \subseteq Y \cap (F_1,E)$, $(G,E) \subseteq Y \cap (F_2,E)$ and $\big(Y \cap (F_1,E)\big) \cap \big(Y \cap (F_2,E)\big) = \phi$. Hence (Y,τ_Y,E) is a soft α n-normal space, so (Y,τ_Y,E) is a soft $\alpha n = T_4$ space.

Theorem 26. Soft $\alpha n = T_4$ space is soft αT_3 space.

Proof. Let (X,τ,E) be a soft $\boldsymbol{\alpha n = T_4}$ space over$X$ and let $x \in X$. And let (F,E) and let (G,E) be soft α closed sets such that let $x \in (F,E)$ and $(F,E) \cap (G,E) = \phi$. Then there exists soft α open sets (F_1,E) and (F_2,E) such that $y \in (F_2,E)$,$(F,E) \subseteq (F_1,E)$, $(G,E) \subseteq (F_2,E)$ and $(F_1,E) \cap (F_2,E) = \phi$. Because of $x \in (F,E)$ and $(F,E) \cap (G,E) = \phi$, $x \notin (G,E)$ (for all α∈E, $x \notin (G(\alpha)$. Then $(G,E) \subseteq (F_2,E) \in \tau$, $x \in (F,E)(F_1,E) \in \tau$ and $(F_1,E) \cap (F_2,E) = \phi$. And then, (X,τ,E) is soft α regular space. Also (X,τ,E) is soft αT_1 space, so (X,τ,E) is soft αT_3 space.

4.6 Soft α-Separation Axioms of Soft Topological Spaces With Respect to Soft Points

In this section we introduced soft **α** separation axioms in soft topological space with respect to soft points and discussed some results with respect to these points in detail. Soft $\boldsymbol{\alpha T_4}$ may not be a soft $\boldsymbol{\alpha T_3}$ space soft α T_2 space

But breaking news is that we launched a new soft α separation axiom which is both soft αT_3 space and soft αT_2 space. It enjoys all the properties of both the soft αT_3 space and soft αT_2 space.

Theorem 27. Let (X,τ,E) be a soft topological space over$X$ and Y be a non-empty soft set of X . If (X,τ,E) is a soft $\alpha n - T_4$ space and Y be a soft α closed set, (Y,τ_Y,E) is a soft $\alpha n - T_4$ space.

Proof. Let (X,τ,E) is a soft $\alpha n = T_4$ space and Y be a soft α closed set in$X$. Because (X,τ,E) is soft αT_1 space, (Y,τ_Y,E) is soft αT_1 from ***Theorem*****. Let, (F,E) and (G,E) be soft α closed set in Y such that that $e_G \in (F,E)$. Then (F,E) and (G,E) are soft α closed sets in X from Theorem 24. Because (X,τ,E) is a soft $\alpha n - T_4$ space $(F,E) \cap (\mathrm{G,E}) = \phi$. Since (X,τ,E) is a soft α n-normal space, there exists soft α open sets and $(F_1,E)$and (F_2,E) such that and $e_H \in (F_2,E)$, $(F,E) \subseteq (F_1,E)$, $(\mathrm{G,E}) \subseteq (F_2,E)$ and $(F_1,E) \cap (F_2,E) = \phi$. $Y \cap (F_1,E)$ and $Y \cap (F_2,E)$ are soft α open sets in Y.

Theorem 24. In this case,$e_H \in Y \cap (F_2,E)$, $(F,E) \subseteq Y \cap (F_1,E)$, $(G,E) \subseteq Y \cap (F_2,E)$ and $\left(Y \cap (F_1,E)\right) \cap \left(Y \cap (F_2,E)\right) = \phi$. Hence (Y,τ_Y,E) is a soft α n-normal space, so (Y,τ_Y,E) is a soft $\alpha n - T_4$ space.

Theorem 28. Soft $\alpha n - T_4$ space is soft αT_3 space.

Proof. Let(X,τ,E) be a soft $\alpha n - T_4$ space over$X$ and let $x \in X$. And let (F,E) and let (G,E) be soft α closed sets such that let $e_G \in (F,E)$ and $(F,E) \cap (\mathrm{G,E}) = \phi$. Then there exists soft α open sets (F_1,E) and (F_2,E) such that $e_H \in (F_2,E)$,$(F,E) \subseteq (F_1,E)$, $(G,E) \subseteq (F_2,E)$ and $(F_1,E) \cap (F_2,\mathrm{E}) = \phi$. Because of $e_G \in (F,E)$ and $(F,E) \cap (\mathrm{G,E}) = \phi$, $e_G \notin (G,E)$ (for all α∈E, $e_G \notin (G(\alpha)$. Then $(\mathrm{G,E}) \subseteq (F_2,E) \in \tau$, $e_G \in (F,E)(F_1,E) \in \tau$ and $(F_1,E) \cap (F_2,E) = \phi$. And then, (X,τ,E) is soft α regular space. Also (X,τ,E) is soft αT_1 space, so (X,τ,E) is soft $\alpha n - T_4$ space.

4.7 Soft β-Separation Axioms of Soft Topological Spaces With Respect to Ordinary Points

In this section we introduced soft β-separation axioms in soft topological space with respect to ordinary points and discussed some results with respect to these points in detail. Soft βT_4 may not be a soft βT_3 space and soft βT_2 space. But breaking news is that we launched a new soft β-separation axiom which is both soft βT_3 space and soft βT_2 space. It enjoys all the properties of both the soft βT_3 space and soft βT_2 space.

Theorem 29. Let (Y,τ_Y,E) be a soft sub space of a soft topological space (X,τ,E) and $(F,E) \in SS(X)$ then

1) If(F,E) is β *open soft* set in Y and$Y \in \tau$, then$(F,E) \in \tau$.
2) (F,E) is β *open soft* set in Y if and only if $(F,E) = Y \cap (G,E)$ for some $(G,E) \in \tau$.
3) (F,E) is β *closed soft* set in Y if and only if $(F,E) = Y \cap (H,E)$ for some $(H,E)\ is\ \tau\beta$ *close soft* set.

Proof. 1) Let (F,E) be a soft β open set in Y then there does exists a soft β open set (G,E) in X such that $(F,E) = Y \cap (G,E)$. Now, if $Y \in \tau$ then $Y \cap (G,E) \in \tau$ by the third condition of the definition of a soft topological space and hence$(F,E) \in \tau$.

2) Fallows from the definition of a soft subspace.

3) If (F,E) is soft β closed in Y then we have $(F,E) = Y(G,E)$, for some $(G,E) \in \tau_Y$. Now, $(G,E) = Y \cap (H,E)$ for some soft β open set $(H,E) \in \tau$. for any $\gamma \in E$. $F(\gamma) = Y(\gamma) \setminus G(\gamma)$ $= Y \setminus G(\gamma)$ $=$ $Y \setminus (Y(\gamma) \cap H(\gamma))$ $=$ $Y \setminus (Y \cap H(\gamma)) = Y \setminus H(\gamma) = Y \cap (X \setminus H(\gamma))$ $=$ $Y \cap (H(\gamma))^C = Y(\gamma) \cap (H(\gamma))^C$. Thus $(F,E) = Y \cap (H,E)'$ is soft β closed in X as $(H,E) \in \tau$. Conversely, suppose that $(F,E) = Y \cap (G,E)$ for some soft β close set (G,E) in X. This qualifies us to say that $(G,E)' \in \tau$.Now, if $(G,E) = (X,E) \setminus (H,E)$ where (H,E) is soft βopen set τ then for any $\gamma \in E F(\gamma) = Y(\gamma) \cap G(\gamma) = Y \cap G(\gamma)$ $= Y \cap (X(\gamma) \setminus H(\gamma)$ $= Y \cap (X(\gamma) \setminus H(\gamma))$ $= Y \cap (X \setminus H(\gamma)) Y \setminus H(\gamma)$ $= Y \setminus (Y \cap H(\gamma))$ $= Y(\gamma) \setminus (Y(\gamma) \cap H(\gamma))$. Thus $(F,E) = Y \setminus (Y \cap (H,E))$. Since $(H,E) \in \tau$, So $(Y \cap (H,E) \in \tau$. So $(Y \cap (H,E) \in \tau_Y$ and hence (F,E) is soft β closed in Y.

Theorem 30. Let (X,τ,E) be a soft topological space over X. And let (Y,E) be a soft β closed set in X and let $(G,E) \subseteq (Y,E)$. Then, (G,E) is a soft β closed set in sub space Y iff, (G,E) is a soft β closed set in X.

Proof. This implies since (G,E) is a soft β closed set in soft sub space Y, there exists a soft β closed set (F,E) in X such that $(G,E) = Y \cap (F,E)$ from Theorem 29. Because (Y,E) and (F,E) are soft β closed set in X.Is implied by Since (G,E) is a soft β closed set in X and, $(G,E) = Y \cap (G,E)$, (G,E) is a soft β closed set in sub space Y from Theorem 29.

Theorem 31. Let (X,τ,E) be a soft topological space over X and Y be a non-empty soft set of X. If (X,τ,E) is a soft β$n - T_4$ space and Y be a soft β closed set, (Y,τ_Y,E) is a soft β$n - T_4$ space.

Proof. Let (X,τ,E) is a soft β$n = T_4$ space and Y be a β soft closed set in X. Because (X,τ,E) is soft βT_1 space, (Y,τ_Y,E) is soft βT_1 from ***Theorem**** [48]. Let, (F,E) and (G,E) be soft β closed set in Y such that that $x \in (F,E)$. Then (F,E) and (G,E) are soft β closed sets in X from Theorem 30. Because (X,τ,E) is a soft β$n - T_4$ space, $(F,E) \cap (G,E) = \phi$. Since (X,τ,E) is a soft β n-normal space, there exists soft β open sets and (F_1,E) and (F_2,E) such that and $y \in (F_2,E)$, $(F,E) \subseteq (F_1,E)$, $(G,E) \subseteq (F_2,E)$ and $(F_1,E) \cap (F_2,E) = \phi$. $Y \cap (F_1,E)$ and $Y \cap (F_2,E)$ are soft β open sets in Y from Theorem 29. In this case, $y \in Y \cap (F_2,E)$, $(F,E) \subseteq Y \cap (F_1,E)$, $(G,E) \subseteq Y \cap (F_2,E)$ and $(Y \cap (F_1,E)) \cap (Y \cap (F_2,E)) = \phi$. Hence $(Y,\tau_Y S,E)$ is a soft β n-normal space, so (Y,τ_Y,E) is a soft β$n - T_4$ space.

Theorem 32. Soft β$n - T_4$ space is soft βT_3 space.

Proof. Let (X,τ,E) be a soft **β**$n - T_4$ space over X and let $x \in X$. And let (F,E) and let (G,E) be soft β closed sets such that let $x \in (F,E)$ and $(F,E) \cap (G,E) = \phi$. Then there exists soft β open sets (F_1,E) and (F_2,E) such that $y \in (F_2,E)$, $(F,E) \subseteq (F_1,E)$, $(G,E) \subseteq (F_2,E)$ and $(F_1,E) \cap (F_2,E) = \phi$. Because of $x \in (F,E)$ and $(F,E) \cap (G,E) = \phi$, $x \notin (G,E)$ (for all γ∈ E, $x \notin (G(\gamma)$. Then $(G,E) \subseteq (F_2,E) \in \tau$, $x \in (F,E)(F_1,E) \in \tau$ and $(F_1,E) \cap (F_2,E) = \phi$. And then, (X,τ,E) is soft β regular space. Also (X,τ,E) is soft βT_1 space, so (X,τ,E) is soft βT_3 space.

4.8 Soft β-Separation Axioms of Soft Topological Spaces With Respect to Soft Points

In this section we introduced soft β-separation axioms in soft topological space with respect to soft points and discussed some results with respect to these points in detail. Soft βT_4 may not be a soft βT_3 space

But breaking news is that we launched a new soft β-separation axiom which is both soft βT_3 space and soft βT_2 space. It enjoys all the properties of both the soft βT_3 space and soft βT_2 space.

Theorem 33. Let (X,τ,E) be a soft topological space over$X$ and Y be a non-empty soft set of X .If (X,τ,E) is a soft β$n-T_4$ space and Y be a soft β closed set, (Y,τ_Y,E) is a soft β$n=T_4$ space.

Proof. Let (X,τ,E) is a soft β$n-T_4$ space and Y be a β soft closed set in X. Because (X,τ,E) is soft βT_1 space, (Y,τ_Y,E) is soft βT_1 from ***Theorem**** [48]. Let, (F,E) and (G,E) be soft β closed set in Y such that that$e_G \in (F,E)$. Then (F,E) and (G,E) are soft β closed sets in X from Theorem 30. . Because (X,τ,E) is a soft β$n=T_4$ space$(F,E)\cap(\text{G},\text{E})=\phi$. Since $(X,\tau,E)$is a soft β n-normal space, there exists soft β open setsand (F_1,E) and (F_2,E)such that and$e_H \in (F_2,E)$, $(F,E)\subseteq(F_1,E)$, $(\text{G},\text{E})\subseteq(F_2,E)$ and $(F_1,E)\cap(F_2,E)=\phi$. $Y\cap(F_1,E)$ and $Y\cap(F_2,E)$ are soft β open sets in Y from Theorem 29. In this case,$e_H \in Y\cap(F_2,E)$, $(F,E)\subseteq Y\cap(F_1,E)$, $(G,E)\subseteq Y\cap(F_2,E)$ and $\left(Y\cap(F_1,E)\right)\cap\left(Y\cap(F_2,E)\right)=\phi$. Hence (Y,τ_Y,E) is a soft β n-normal space, so (Y,τ_Y,E) is a soft β$n-T_4$ space.

Theorem 34. Soft β$n-T_4$ space is soft βT_3 space.

Proof. Let(X,τ,E) be a soft β$n-T_4$ space over$X$ and let $x\in X$. And let (F,E) and let (G,E) be soft β closed sets such that let $e_G \in (F,E)$ and $(F,E)\cap(\text{G},\text{E})=\phi$. Then there exists soft β open sets (F_1,E) and (F_2,E) such that $e_H \in (F_2,E)$,$(F,E)\subseteq(F_1,E)$, $(G,E)\subseteq(F_2,E)$ and $(F_1,E)\cap(F_2,E)=\phi$. Because of$e_G \in (F,E)$ and $(F,E)\cap(\text{G},\text{E})=\phi$, $e_G \notin (G,E)$ (for all $\gamma \in$E, $e_G \notin (G(\gamma)$.Then $(\text{G},\text{E})\subseteq(F_2,E)\in\tau$, $e_G \in (F,E)(F_1,E)\in\tau$ and $(F_1,E)\cap(F_2,E)=\phi$. And then, (X,τ,E) is soft β regular space. Also (X,τ,E) is soft βT_1 space, so (X,τ,E) is soft βT_3 space.

5. Conclusion

Topology is the most important branch of mathematics which deals with mathematical structures. Recently, many researchers have studied the soft set theory which is initiated by Molodtsov [4] and safely applied to many problems which contain uncertainties in our social life. Shabir and Naz in [23] introduced and deeply studied the concept of soft topological spaces. They also studied topological structures and exhibited their several properties with respect to ordinary points. In this present paper the concept of soft αT_i spaces (i=1, 2, 3) and soft β_i spaces (i=1, 2, 3) are introduced in soft single point space with respect to ordinary and soft points of a topological space. Soft α$n-T_4$ space and Soft β$n-S_4$ are introduced in soft topological space with respect to ordinary and soft points.

Many mathematicians discussed soft separation axioms in soft topological spaces at full length with respect to soft open set, soft b-open set, soft semi-open set and soft set. They also worked over the hereditary properties of different soft topological structures in soft topology. In this present work hand is tried and work is encouraged over the gap that exists in soft topology. Related to Soft spaces, some theorems in soft single topological spaces are discussed with respect to ordinary points as well as with respect to soft points. Focus is laid upon the characters of soft $\alpha n - T_4$ and soft $\beta n - S_4$ space and their sub spaces in soft topological structures. We also beautifully discussed some soft transmissible properties with respect to ordinary as well as soft points. We hope that these results in this paper will help the researchers for strengthening the toolbox of soft topology. In the next study, we extend the concept of semi open, Pre-open and b^{**}open soft sets in soft bi topological spaces with respect to ordinary as well as soft points. We also extended these axioms to different results. These soft separation axioms would be useful for the growth of the theory of soft topology to solve complex problems, comprising doubts in economics, engineering, medical etc.

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