http://www.newtheory.org

Journal of New Theory

Received: 29.12.2014
Accepted: 05.01.2015

Year: **2015**, *Number*: **1** , *Pages*: **50-58**
Original Article[**]

On Soft Multi Continuous Functions

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Abstract – In this article, by using basic properties of soft multi topology and soft multi function, we define the notion of soft multi continuous function. We also introduce some basic definitions and theorems of the concept.

Keywords – *Soft multiset, Soft multi function, Soft multi topology, Soft multi continuous function.*

1 Introduction

Classical mathematical methods are not enough to solve the problems of daily life and also are not enough to meet the new requirements. Therefore, some theories such as Fuzzy set theory [23], Rough set theory [15], Soft set theory [12] and Multiset (or Bag) theory [22] have been developed to solve those problems.

Applications of these theories exists in many areas of mathematics. Shabir and Naz [17] defined the soft topological space and studied the concepts of soft open set, soft multi interior point, soft neighborhood of a point, soft separation axioms, and subspace of a soft topological space. Aygunoglu and Aygun [2] introduced the soft continuity of soft mapping, soft product topology and studied soft compactness and generalized Tychonoff theorem to the soft topological space. Min [11] gave some results on soft topological spaces. Zorlutuna et al. [24] also investigated soft interior point and soft neighborhood. There are some other studies on the structure of soft topological spaces ([3],[21]). Maji et al. [9] also initiated the more generalized concept of fuzzy soft sets which is a combination of fuzzy set and soft set. Tanay and Kandemir introduced topological structure of fuzzy soft set in [18] and gave a introductory theoretical base to carry further study on this concept. Following this study, some others ([20],[8],[1],[16]) studied on the concept of fuzzy soft topological spaces.

The concept of soft multisets which is combination of soft sets and multisets can be used to solve some real life problems. Also this concept can be used in many areas, such as data storage, computer science, information science, medicine, engineering, etc. The concept of soft multisets was introduced in [19]. Moreover, in [19],[13] and [14] soft multi topology and its some properties were given.

In this work we will recall soft multi function between two soft multiset. Then we will introduce soft multi continuous function on soft multi topological spaces and will give basic definitions and theorems of soft multi continuity.

[**]*Edited by* Serkan Karataş *(Area Editor) and* Naim Çağman *(Editor-in-Chief).*
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2 Preliminaries

2.1 Soft Sets, Multisets and Soft Multisets

In this section, we present the basic definitions of soft set, multiset and soft multiset which may be found in earlier studies [6, 12, 19].

Definition 2.1 (Soft set)**.** [12] Let U be an initial universe set and E be set of parameters. Let $P(U)$ denotes the power set of U and $A \subseteq U$. A pair (F,A) is called a soft set over U, where F is a mapping given by $F : A \to P(U)$.

Definition 2.2 (Multiset)**.** [6] An mset M drawn from the set X is represented by a function *Count* M or C_M defined as $C_M : X \to \mathbb{N}$. The word "multiset"is often shortened to "mset".

Let M be an mset from X with x appearing n times in M. It is denoted by $x \in^n M$. If $M = \{k_1/x_1, k_2/x_2, ..., k_n/x_n\}$, then, x_1 appearing k_1 times, x_2 appearing k_2 times and so on.

Definition 2.3. [6] Let M be an mset drawn from a set X. The support set of M denoted by M^* is a subset of X and $M^* = \{x \in X : C_M(x) > 0\}$. i.e., M^* is an ordinary set and it is also called root set.

The power set of an mset is the support set of the power mset and is denoted by $P^*(M)$.

Example 2.4. [5] Let $M = \{2/x, 3/y\}$ be an multiset. Then $M^* = \{x, y\}$ is the support set of M. The collection

$P(M) = \{3/\{2/x,1/y\}, 3/\{2/x,2/y\}, 6/\{1/x,1/y\}, 6/\{1/x,2/y\}, 2/\{1/x,3/y\}, 1/\{2/x\}, 2/\{1/x\}, 1/\{3/y\}, 3/\{2/y\}, 3/\{1/y\}, M, \emptyset\}$

is the power multiset of M. The collection

$P^*(M) = \{\{2/x,1/y\}, \{2/x,2/y\}, \{1/x,1/y\}, \{1/x,2/y\}, \{1/x,3/y\}, \{2/x\}, \{1/x\}, \{3/y\}, \{2/y\}, \{1/y\}, M, \emptyset\}$

is the support set of $P(M)$.

Definition 2.5 (Soft multiset)**.** [19] Let U be an multiset, E be set of parameters and $A \subseteq E$. Then a pair (F,A) is called a soft multiset where F is a mapping given by $F : A \to P^*(U)$. For $\forall e \in A$, multiset $F(e)$ represent by count function $C_{F(e)} : U^* \to \mathbb{N}$.

Example 2.6. [19] Let $U = \{1/x, 5/y, 3/z, 4/w\}$ and $E = \{p, q, r\}$. Define a mapping $F : E \to P^*(U)$ as follows:

$$F(p) = \{1/x, 2/y, 3/z\}, F(q) = \{4/w\} \text{ and } F(r) = \{3/y, 1/z, 2/w\}.$$

Then (F,A) is a soft multiset where for $\forall e \in A$, $F(e)$ multiset represent by count function $C_{F(e)} : U^* \to \mathbb{N}$, which are defined as follows:

$$\begin{array}{llll} C_{F(p)}(x) = 1, & C_{F(p)}(y) = 2, & C_{F(p)}(z) = 3, & C_{F(p)}(w) = 0, \\ C_{F(q)}(x) = 0, & C_{F(q)}(y) = 0, & C_{F(q)}(z) = 0, & C_{F(q)}(w) = 4, \\ C_{F(r)}(x) = 0, & C_{F(r)}(y) = 3, & C_{F(r)}(z) = 1, & C_{F(r)}(w) = 2. \end{array}$$

Then $(F,A) = \{F(p), F(q), F(r)\} = \{\{1/x, 2/y, 3/z\}, \{4/w\}, \{3/y, 1/z, 2/w\}\}$.

Definition 2.7. [19] For two soft multisets (F,A) and (G,B) over U, we say that (F,A) is a soft submultiset of (G,B) if

i. $A \subseteq B$

ii. $C_{F(e)}(x) \leq C_{G(e)}(x), \forall x \in U^*, \forall e \in A$

We write $(F,A)\tilde{\subset}(G,B)$.

In addition to (F,A) is a whole soft submultiset of (G,B) if $C_{F(e)}(x) = C_{G(e)}(x), \forall x \in U^*, \forall e \in A$.

Definition 2.8. [19] Let (F,A) and (G,B) be two soft multisets over U.

Equality $(F,A) = (G,B) \Leftrightarrow (F,A) \tilde{\subseteq} (G,B)$ and $(F,A) \tilde{\supseteq} (G,B)$.

Union $(H,C) = (F,A)\tilde{\cup}(G,B)$ where $C = A \cup B$ and $C_{H(e)}(x) = \max\{C_{F(e)}(x), C_{G(e)}(x)\}, \forall e \in A \cup B, \forall x \in U^*$.

Intersection $(H,C) = (F,A)\tilde{\cap}(G,B)$ where $C = A \cap B$ and $C_{H(e)}(x) = \min\{C_{F(e)}(x), C_{G(e)}(x)\}, \forall e \in A \cap B, \forall x \in U^*$.

Difference $(H,E) = (F,E)\backslash(G,E)$ where $C_{H(e)}(x) = \max\left\{C_{F(e)}(x) - C_{G(e)}(x), 0\right\}, \forall x \in U^*$.

Null A soft multiset (F,A) is said to be a NULL soft multiset denoted by Φ if for all $e \in A$, $F(e) = \emptyset$.

Complement The complement of a soft multiset (F,A) is denoted by $(F,A)^c$ and is defined by $(F,A)^c = (F^c, A)$ where $F^c : A \to P^*(U)$ is a mapping given by $F^c(e) = U \backslash F(e)$ for all $e \in A$ where $C_{F^c(e)}(x) = C_U(x) - C_{F(e)}(x), \forall x \in U^*$.

Definition 2.9. Let (F,E) be a soft set over U. The soft set (F,E) is called a soft multi point, denoted by x_e, if for the element $e \in E$, $F(e) = \{1/x\} = \{x\}$ and $F(\acute{e}) = \emptyset$, for all $\acute{e} \in E - \{e\}$.

Definition 2.10. Let (F,E) be a soft set over U and $e \in E$. $x_e \in (F,E)$ if for $C_{F(e)}(x) = n$, $n \geq 1$.

Example 2.11. Let $U = \{2/x, 1/y, 1/z, 3/w\}$, $E = \{e_1, e_2, e_3\}$ and $(F,A) = \{F(e_1), F(e_2)\} = \{\{1/x, 1/z, 2/w\}, \{1/y, 2/w\}\}$. Then $y_{e_2} \in (F,A)$ since $C_{F(e_2)}(y) = 1$. But $y_{e_1} \in (F,A)$ since $C_{F(e_1)}(y) = 0$.

Definition 2.12. [19] Let V be a non-empty submultiset of U, then $\tilde{V}$ denotes the soft multiset (V,E) over U for which $V(e) = V$, for all $e \in E$.

In particular, (U,E) will be denoted by $\tilde{U}$.

2.2 Soft Multi Function

In this section, we recall soft multi function which was given in [13].

Definition 2.13. [13] Let X be multiset and E be set of parameters. Then the collection of all soft multisets over X with parameters from E is called a soft multi class and is denoted as X_E.

Definition 2.14. [13] Let X_E and Y_K be two soft multi class. Let $\varphi : X^* \to Y^*$ and $\psi : E \to K$ be two function. Then the pair (φ, ψ) is called a soft multi function and denoted by $f = (\varphi, \psi) : X_E \to Y_K$ is defined as follows:
Let (F,E) be a soft multiset in X_E. Then the image of (F,E) under soft multi function f is soft multiset in Y_K defined by $f(F,E)$, where for $k \in \psi(E) \subseteq K$ and $y \in Y^*$,

$$C_{f(F,E)(k)}(y) = \begin{cases} \sup\limits_{e \in \psi^{-1}(k) \cap E, x \in \varphi^{-1}(y)} C_{F(e)}(x), & \text{if } \psi^{-1}(k) \neq \emptyset, \varphi^{-1}(y) \neq \emptyset; \\ 0, & \text{otherwise.} \end{cases}$$

Let (G,K) be a soft multiset in Y_K. Then the inverse image of (G,K) under soft multi function f is soft multiset in X_E defined by $f^{-1}(G,K)$, where for $e \in \psi^{-1}(K) \subseteq E$ and $x \in X^*$,

$$C_{f^{-1}(G,K)(e)}(x) = C_{G(\psi(e))}(\varphi(x)).$$

f is said to be injective (onto or surjective) if both $\varphi : X^* \to Y^*$ and $\psi : E \to K$ are injective (onto or surjective) mappings. If f is both injective as well as surjective, then f is said to be a soft multi bijective function.

Example 2.15. [13] Let $X = \{2/a, 3/b, 4/c, 5/d\}$, $Y = \{5/x, 4/y, 3/z, 2/w\}$, $E = \{e_1, e_2, e_3, e_4\}$, $K = \{k_1, k_2, k_3\}$ and X_E, Y_K, classes of soft multisets. Let $\varphi : X^* \to Y^*$ and $\psi : E \to K$ be two function defined as

$$\varphi(a) = z, \quad \varphi(b) = y, \quad \varphi(c) = y, \quad \varphi(d) = x,$$
$$\psi(e_1) = k_1, \quad \psi(e_2) = k_3, \quad \psi(e_3) = k_2, \quad \psi(e_4) = k_1.$$

Choose two soft multisets in X_E and Y_K, respectively, as

$$(F,A) = \{e_1 = \{1/a, 2/b, 1/d\}, e_3 = \{3/b, 2/c, 1/d\}, e_4 = \{2/a, 5/d\}\},$$
$$(G,B) = \{k_1 = \{4/x, 2/w\}, k_2 = \{1/x, 1/y, 2/z, 2/w\}\}$$

Then soft multiset image of (F,A) under $f : X_E \to Y_K$ is obtained as

$$C_{f(F,A)(k_1)}(x) = \begin{cases} \sup\limits_{e\in\psi^{-1}(k_1)\cap A, a\in\varphi^{-1}(x)} C_{F(e)}(a), & \text{if } \psi^{-1}(k_1)\neq\emptyset, \varphi^{-1}(x)\neq\emptyset; \\ 0, & \text{otherwise.}\end{cases}$$
$$= \begin{cases} \sup\limits_{e\in\{e_1,e_4\}, a\in\{d\}} C_{F(e)}(a), & \text{if } \psi^{-1}(k_1)\neq\emptyset, \varphi^{-1}(x)\neq\emptyset; \\ 0, & \text{otherwise.}\end{cases}$$
$$= \sup\left\{C_{F(e_1)}(d), C_{F(e_4)}(d)\right\}$$
$$= 5,$$
$$C_{f(F,A)(k_1)}(y) = \sup\left\{C_{F(e_1)}(b), C_{F(e_4)}(b), C_{F(e_1)}(c), C_{F(e_4)}(c)\right\} = 2,$$
$$C_{f(F,A)(k_1)}(z) = \sup\left\{C_{F(e_1)}(a), C_{F(e_4)}(a)\right\} = 2,$$
$$C_{f(F,A)(k_1)}(w) = 0 \text{ (since } \varphi^{-1}(w)=\emptyset\text{)},$$
$$C_{f(F,A)(k_2)}(x) = \begin{cases} \sup\limits_{e\in\psi^{-1}(k_2)\cap A, a\in\varphi^{-1}(x)} C_{F(e)}(a), & \text{if } \psi^{-1}(k_2)\neq\emptyset, \varphi^{-1}(x)\neq\emptyset; \\ 0, & \text{otherwise.}\end{cases}$$
$$= \begin{cases} \sup\limits_{e\in\{e_3\}, a\in\{d\}} C_{F(e)}(a), & \text{if } \psi^{-1}(k_2)\neq\emptyset, \varphi^{-1}(x)\neq\emptyset; \\ 0, & \text{otherwise.}\end{cases}$$
$$= \sup\left\{C_{F(e_3)}(d)\right\}$$
$$= 1,$$
$$C_{f(F,A)(k_2)}(y) = \sup\left\{C_{F(e_3)}(b), C_{F(e_3)}(c)\right\} = 3,$$
$$C_{f(F,A)(k_2)}(z) = \sup\left\{C_{F(e_3)}(a)\right\} = 0,$$
$$C_{f(F,A)(k_2)}(w) = 0 \text{ (since } \varphi^{-1}(w)=\emptyset\text{)}.$$

Consequently, we have
$$(f(F,A),B) = \{k_1 = \{5/x, 2/y, 2/z\}, k_2 = \{1/x, 3/y\}\}.$$

Soft multiset inverse image of (G,B) under $f : X_E \to Y_K$ is obtained as
$$C_{f^{-1}(G,B)(e_3)}(a) = C_{G(\psi(e_3))}(\varphi(a)) = C_{G(k_2)}(z) = 2,$$
$$C_{f^{-1}(G,B)(e_3)}(b) = C_{G(\psi(e_3))}(\varphi(b)) = C_{G(k_2)}(y) = 1,$$
$$C_{f^{-1}(G,B)(e_3)}(c) = C_{G(\psi(e_3))}(\varphi(c)) = C_{G(k_2)}(y) = 1,$$
$$C_{f^{-1}(G,B)(e_3)}(d) = C_{G(\psi(e_3))}(\varphi(d)) = C_{G(k_2)}(x) = 1,$$
$$C_{f^{-1}(G,B)(e_4)}(a) = C_{G(\psi(e_4))}(\varphi(a)) = C_{G(k_1)}(z) = 0,$$
$$C_{f^{-1}(G,B)(e_4)}(b) = C_{G(\psi(e_4))}(\varphi(b)) = C_{G(k_1)}(y) = 0,$$
$$C_{f^{-1}(G,B)(e_4)}(c) = C_{G(\psi(e_4))}(\varphi(c)) = C_{G(k_1)}(y) = 0,$$
$$C_{f^{-1}(G,B)(e_4)}(d) = C_{G(\psi(e_4))}(\varphi(d)) = C_{G(k_1)}(x) = 4.$$
Consequently, we have
$$(f^{-1}(G,B),D) = \{e_3 = \{2/a, 1/b, 1/c, 1/d\}, e_4 = \{4/d\}\}.$$

Theorem 2.16. [13] Let $f : X_E \to Y_K$ be a soft multi function, $(F,A), (F_i,A)$ soft multisets in X_E and $(G,B), (G_i,B)$ soft multisets in Y_K.

(1) $f(\Phi) = \Phi$, $f(\tilde{X}) \tilde{\subseteq} \tilde{Y}$,

(2) $f^{-1}(\Phi) = \Phi$, $f^{-1}(\tilde{Y}) = \tilde{X}$,

(3) $f((F_1,A_1)\tilde{\cup}(F_2,A_2)) = f(F_1,A_1)\tilde{\cup} f(F_2,A_2)$.
In general, $f(\tilde{\cup}_{i\in I}(F_i,A_i)) = \tilde{\cup}_{i\in I}\ f(F_i,A_i)$,

(4) $f^{-1}((G_1,B)\tilde{\cup}(G_2,B)) = f^{-1}(G_1,B)\tilde{\cup} f^{-1}(G_2,B)$.
In general, $f^{-1}(\tilde{\cup}_{i\in I}(G_i,B)) = \tilde{\cup}_{i\in I}\ f^{-1}(G_i,B)$,

(5) $f((F_1,A)\tilde{\cap}(F_2,A)) \tilde{\subseteq} f(F_1,A)\tilde{\cap} f(F_2,A)$.
In general, $f(\tilde{\cap}_{i\in I}(F_i,A)) \tilde{\subseteq} \tilde{\cap}_{i\in I}\ f(F_i,A)$,

(6) $f^{-1}((G_1,B)\tilde{\cap}(G_2,B)) = f^{-1}(G_1,B)\tilde{\cap} f^{-1}(G_2,B)$.
In general, $f^{-1}(\tilde{\cap}_{i\in I}(G_i,B)) = \tilde{\cap}_{i\in I}\ f^{-1}(G_i,B)$,

(7) If $(F_1,A)\tilde{\subseteq}(F_2,A)$, then $f(F_1,A)\tilde{\subseteq} f(F_2,A)$,

(8) If $(G_1,B)\tilde{\subseteq}(G_2,B)$, then $f^{-1}(G_1,B)\tilde{\subseteq} f^{-1}(G_2,B)$.

2.3 Soft Multi Topology

In this section, we recall soft multi topology which was given in [19].

Definition 2.17. [19] Let $\tau \subseteq X_E$, then τ is said to be a soft multi topology on X if the following conditions hold.

i. Φ,$\tilde{X}$ belong to τ.

ii. The union of any number of soft multisets in τ belongs to τ.

iii. The intersection of any two soft multisets in τ belongs to τ.

τ is called a soft multi topology over X and the binary (X_E, τ) is called a soft multi topological space over X.

The members of τ are said to be soft multi open sets in X.

A soft multiset (F, E) over X is said to be a soft multi closed set in X, if its complement $(F, E)^c$ belongs to τ .

Example 2.18. [19] Let $X = \{2/x, 3/y, 4/z, 5/w\}$, $E = \{p, q\}$ and $\tau = \{\Phi, \tilde{X}, (F_1, E), (F_2, E), (F_3, E)\}$ where $(F_1, E), (F_2, E), (F_3, E)$ are soft multisets over X, defined as follows
$F_1\,(p) = \{1/x, 2/y, 3/z\}\,,$ $F_1\,(q) = \{4/w\}$
$F_2\,(p) = X,$ $F_2\,(q) = \{1/x, 3/y, 4/z, 5/w\}$
$F_3\,(p) = \{2/x, 3/y, 3/z, 1/w\}\,,$ $F_3\,(q) = \{1/x, 4/w\}\,.$

Then τ defines a soft multi topology on X and hence (X_E, τ) is a soft multi topological space over X.

Remark 2.19. In the example above, without allowing the repetitions for the elements of X, we obtain soft topology on X. Thus the concept of soft multi topology is more general than that of soft topology.

Definition 2.20. [19] Let X be multiset, E be the set of parameters.

- let τ^1 be the collection of all soft multisets which can be defined over X. Then τ^1 is called the soft multi discrete topology on X and (X_E, τ^1) is said to be a soft multi discrete space over X.
- $\tau^0 = \{\Phi, \tilde{X}\}$ is called the soft multi indiscrete topology on X and (X_E, τ^0) is said to be a soft indiscrete space over X.

3 Soft Multi Continuous Functions

In this section, we define soft multi continuous functions and examine their properties.

Definition 3.1. Let (X_E, τ) and (Y_K, σ) be two soft multi topological spaces, $f : (X_E, \tau) \to (Y_K, \sigma)$ be a soft multi function. For each soft multi open neighbourhood (G, B) of $f(x)_k$, if there exists a soft multi open neighbourhood (F, A) of x_e such that $f((F, A))\tilde{\subset}(G, B)$ then f is said to be soft multi continuous function on x_e. If f is soft multi continuous function for all x_e, then f is called soft multi continuous function.

Theorem 3.2. Let (X_E, τ) and (Y_K, σ) be two soft multi topological spaces, $f : (X_E, \tau) \to (Y_K, \sigma)$ be a soft multi function. Then f is soft multi continuous function if and only if $f^{-1}((G, B))$ is a soft multi open set in X_E, for each soft multi open set (G, B) in Y_K.

Proof. $\Rightarrow$: Let (G, B) be a soft multi open set in Y_K and $x_e \in f^{-1}((G, B))$ be an arbitrary soft multi point. Then $f(x)_k = f(x_e) \in f(f^{-1}((G, B)))\tilde{\subseteq}(G, B)$. Since f is soft multi continuous function, there exists soft multi open set $x_e \in (F, A)$ such that $f((F, A))\tilde{\subset}(G, B)$. This implies that $x_e \in (F, A)\tilde{\subseteq} f^{-1}(f((F, A)))\tilde{\subseteq}(f^{-1}((G, B))$ is a soft multi open set in X_E. □

$\Leftarrow$:Let x_e be a soft multi point and $f(x)_k \in (G, B)$ be an arbitrary soft multi open neighbourhood. Then $(F, A) = f^{-1}((G, B))$ is a soft multi open set in X_E, $x_e \in (F, A)$ and $f((F, A))\tilde{\subset}(G, B)$.

Theorem 3.3. Let (X_E, τ) and (Y_K, σ) be two soft multi topological spaces, $f : (X_E, \tau) \to (Y_K, \sigma)$ be a soft multi function. Then f is soft multi continuous function if and only if $f^{-1}((G, B))$ is a soft multi closed set in X_E, for each soft multi closed set (G, B) in Y_K.

Proof. This proof is similar to Theorem 3.2. □

Example 3.4. Let (X_E, τ) and (Y_K, σ) be two soft multi topological spaces. If $\tau = \tau^1$, then every function $f : (X_E, \tau) \to (Y_K, \sigma)$ is soft multi continuous. Also if $\sigma = \tau^0$, then every function $f : (X_E, \tau) \to (Y_K, \sigma)$ is soft multi continuous.

Example 3.5. Let $X = \{6/a, 7/b, 8/c\}$, $Y = \{8/x, 9/y, 7/z\}$, $E = \{e_1, e_2, e_3, e_4\}$ and $K = \{k_1, k_2, k_3\}$. Let $\tau = \{\Phi, \tilde{X}, (F_1, A_1), (F_2, A_2)\}$ and $\sigma = \{\Phi, \tilde{Y}, (G_1, B_1), (G_2, B_2)\}$ where soft multisets defined as follows

$(F_1, A_1) = \{F_1(e_1), F_1(e_2)\} = \{\{1/a, 2/b, 2/c\}, \{1/a, 2/b, 2/c\}\}$,
$(F_2, A_2) = \{F_2(e_1), F_2(e_2), F_2(e_2)\} = \{\{6/a, 7/b, 7/c\}, \{6/a, 7/b, 7/c\}, \{6/a, 4/b, 4/c\}\}$,
$(G_1, B_1) = \{G_1(k_1), G_1(k_2)\} = \{\{3/x, 2/y, 3/z\}, \{5/x, 9/y, 6/z\}\}$,
$(G_2, B_2) = \{G_2(k_1), G_2(k_2), G_2(k_3)\} = \{\{3/x, 7/y, 6/z\}, \{8/x, 9/y, 7/z\}, \{2/x, 4/y, 6/z\}\}$.

Then (X_E, τ) and (Y_K, σ) are soft multi topological spaces over X and Y, respectively. Let $\varphi : X^* \to Y^*$ and $\psi : E \to K$ be two function defined as

$\varphi(a) = z, \quad \varphi(b) = y, \quad \varphi(c) = y,$
$\psi(e_1) = k_1, \quad \psi(e_2) = k_1, \quad \psi(e_3) = k_3, \quad \psi(e_4) = k_2.$

Since $f^{-1}(\Phi) = \Phi$, $f^{-1}(\tilde{Y}) = \tilde{X}$, $f^{-1}((G_1, B_1)) = (F_1, A_1)$ and $f^{-1}((G_2, B_2)) = (F_2, A_2)$, then function $f : (X_E, \tau) \to (Y_K, \sigma)$ is soft multi continuous.

Theorem 3.6. Let (X_E, τ) and (Y_K, σ) be two soft multi topological spaces, $f : (X_E, \tau) \to (Y_K, \sigma)$ be a soft multi function. Then the following statements are equivalent:

i. f is soft multi continuous function,

ii. For each soft multi set (F, A) in X_E, $f(\overline{(F, A)}) \tilde{\subseteq} \overline{f((F, A))}$,

iii. For each soft multi set (G, B) in Y_K, $\overline{f^{-1}((G, B))} \tilde{\subseteq} f^{-1}(\overline{(G, B)})$.

iv. For each soft multi set (G, B) in Y_K, $f^{-1}((G, B)^\circ) \tilde{\subseteq} f^{-1}((G, B))^\circ$

Proof. **i.** $\Rightarrow$ **ii.** Let f be soft multi continuous function and (F, A) be soft multi set in X_E. Since $f((F, A)) \tilde{\subseteq} \overline{f((F, A))}$, then $(F, A) \tilde{\subseteq} f^{-1}(f((F, A))) \tilde{\subseteq} f^{-1}(\overline{f((F, A))})$. Using this statement and continuity of f , we have soft multi closed set $f^{-1}(\overline{f((F, A))})$ in Y_K and $\overline{f^{-1}(\overline{f((F, A))})} = f^{-1}(\overline{f((F, A))})$. Then $\overline{(F, A)} \tilde{\subseteq} f^{-1}(\overline{f((F, A))})$ and so $f(\overline{(F, A)}) \tilde{\subseteq} \overline{f((F, A))}$.

ii. $\Rightarrow$ **iii.** Let (G, B) be a soft multi set in Y_K and $f^{-1}((G, B)) = (F, A)$. By part ii., we have $f(\overline{(F, A)}) = f(\overline{f^{-1}((G, B))}) \tilde{\subseteq} \overline{f(f^{-1}((G, B)))} \tilde{\subseteq} \overline{(G, B)}$. Then $\overline{f^{-1}((G, B))} = \overline{(F, A)} \tilde{\subseteq} f^{-1}(f(\overline{(F, A)})) \tilde{\subseteq} f^{-1}(\overline{(G, B)})$.

iii. $\Rightarrow$ **iv.** Let (G, B) be a soft multi set in Y_K. Using part iii., we have $\overline{f^{-1}((G, B)^c)} \tilde{\subseteq} f^{-1}(\overline{(G, B)^c})$. Since $(G, B)^\circ = (\overline{(G, B)^c})^c$, then $f^{-1}((G, B)^\circ) = f^{-1}((\overline{(G, B)^c})^c) = (f^{-1}(\overline{(G, B)^c}))^c \tilde{\subseteq} (\overline{f^{-1}((G, B)^c)})^c = f^{-1}((G, B))^\circ$.

iv. $\Rightarrow$ **i.** Let (G, B) be a soft multi open set in Y_K . Since $f^{-1}((G, B)) = f^{-1}((G, B)^\circ) \tilde{\subseteq} f^{-1}((G, B))^\circ$, we have $f^{-1}((G, B))^\circ = f^{-1}((G, B))$. Then $f^{-1}((G, B))$ is soft multi open set in X_E and so f is soft multi continuous function.

□

Definition 3.7. Let (X_E, τ) and (Y_K, σ) be two soft multi topological spaces, $f : (X_E, \tau) \to (Y_K, \sigma)$ be a soft multi function.

– [13] Soft multi function f is called soft multi open if $f((F, A))$ is a soft multi open set in Y_K, for each soft multi open set (F, A) in X_E.

– Soft multi function f is called soft multi closed if $f((F,A))$ is a soft multi closed set in Y_K, for each soft multi closed set (F,A) in X_E.

Theorem 3.8. Let (X_E,τ) and (Y_K,σ) be two soft multi topological spaces, $f:(X_E,\tau)\to(Y_K,\sigma)$ be a soft multi function.

i. f is a soft multi open function if and only if for each soft multi set (F,A) in X_E, $f((F,A)^\circ)\tilde{\subseteq}(f((F,A)))^\circ$ is satisfied.

ii. f is a soft multi closed function if and only if for each soft multi set (F,A) in X_E, $\overline{f((F,A))}\tilde{\subseteq}f(\overline{(F,A)})$ is satisfied.

Proof. **i.** Let f be a soft multi open function and (F,A) be a soft multiset in X_E. Since $(F,A)^\circ\tilde{\subseteq}(F,A)$ and f is soft multi open function, then $f((F,A)^\circ)\tilde{\subseteq}f((F,A))$ and $f((F,A)^\circ)$ is a soft multi open set in Y_K. Thus $f((F,A)^\circ)\tilde{\subseteq}(f((F,A)))^\circ$.

Let (F,A) be any soft open multiset in X_E. Using $(F,A)=(F,A)^\circ$, we have $f((F,A))=f((F,A)^\circ)\tilde{\subseteq}(f((F,A)))^\circ$. Then $f((F,A))=(f((F,A)))^\circ$ and so f is a soft multi open function.

ii. Let f be a soft multi closed function and (F,A) be a soft multiset in X_E. Since $(F,A)\tilde{\subseteq}\overline{(F,A)}$ and f is soft multi closed function, then $f((F,A))\tilde{\subseteq}f(\overline{(F,A)})$ and $f(\overline{(F,A)})$ is a soft multi closed set in Y_K. Thus $\overline{f((F,A))}\tilde{\subseteq}f(\overline{(F,A)})$.

Let (F,A) be any soft closed multiset in X_E. Using $(F,A)=\overline{(F,A)}$, we have $\overline{f((F,A))}\tilde{\subseteq}f(\overline{(F,A)})=f((F,A))$. Then $f((F,A))=\overline{f((F,A))}$ and so f is a soft multi closed function. □

Theorem 3.9. Let (X_E,τ) and (Y_K,σ) be two soft multi topological spaces, $f:(X_E,\tau)\to(Y_K,\sigma)$ be a soft multi bijection function. f is a soft multi open function if and only if f is a soft multi closed function.

Proof. For any soft multiset (F,A) in X_E, $f((F,A)^c)=(f((F,A)))^c$.

Let (F,A) be any soft closed multiset in X_E. Then $(F,A)^c$ is soft open multiset in X_E. Since f is a soft multi open function and $f((F,A)^c)=(f((F,A)))^c$, then $(f((F,A)))^c$ is soft open multiset in Y_K. Thus $f((F,A))$ is soft closed multiset in Y_K and so f is a soft multi closed function.

Let (F,A) be any soft open multiset in X_E. Then $(F,A)^c$ is soft closed multiset in X_E. Since f is a soft multi closed function and $f((F,A)^c)=(f((F,A)))^c$, then $(f((F,A)))^c$ is soft closed multiset in Y_K. Thus $f((F,A))$ is soft open multiset in Y_K and so f is a soft multi open function. □

Theorem 3.10. Let (X_E,τ) and (Y_K,σ) be two soft multi topological spaces, $f:(X_E,\tau)\to(Y_K,\sigma)$ be a soft multi injective function. f is a soft multi continuous and a soft multi open function if and only if for each soft multi set (F,A) in X_E, $(f((F,A)))^\circ=f((F,A)^\circ)$ is satisfied.

Proof. It can be proved easily using Theorem 3.6 and Theorem 3.8. □

Theorem 3.11. Let (X_E,τ) and (Y_K,σ) be two soft multi topological spaces, $f:(X_E,\tau)\to(Y_K,\sigma)$ be a soft multi function. f is a soft multi continuous and a soft multi closed function if and only if for each soft multi set (F,A) in X_E, $\overline{f((F,A))}=f(\overline{(F,A)})$ is satisfied.

Proof. It can be proved easily using Theorem 3.6 and Theorem 3.8. □

Theorem 3.12. Let (X_E,τ) and (Y_K,σ) be two soft multi topological spaces, $f:(X_E,\tau)\to(Y_K,\sigma)$ be a soft multi bijection function. f is a soft multi continuous if and only if f is a soft multi open (closed) function.

Proof. Let (G,B) be a soft multi open (closed) set in Y_K. Since f is soft multi continuous, then $f^{-1}((G,B))$ is soft multi open (closed) set in X_E. Thus it clear that f^{-1} is soft multi open (closed) function.

Let (G,B) be a soft multi open (closed) set in Y_K. Since f^{-1} is soft multi open (closed) function, then $f^{-1}((G,B))$ is soft multi open (closed) set in X_E. Thus it clear that f^{-1} is soft multi continuous function. □

Definition 3.13. Let (X_E, τ) and (Y_K, σ) be two soft multi topological spaces, $f : (X_E, \tau) \to (Y_K, \sigma)$ be a soft multi function. If f is a soft multi bijection, soft multi continuous and f^{-1} is a soft multi continuous function, then f is said to be soft multi homeomorphism from X to Y . When a homeomorphism f exists between X and Y , we say that X is soft multi homeomorphic to Y.

Theorem 3.14. Let (X_E, τ) and (Y_K, σ) be two soft multi topological spaces, $f : (X_E, \tau) \to (Y_K, \sigma)$ be a soft multi function. Then the following statements are equivalent:

i. f is a soft multi homeomorphism,

ii. f is a soft multi continuous and soft multi open function,

iii. f is a soft multi continuous and soft multi closed function,

iv. For each soft multi set (F, A) in X_E, $\overline{f((F, A))} = f(\overline{(F, A)})$.

Proof. **i.**⇔ **ii.** İt is clear from Theorem 3.12.

ii.⇔ **iii.** İt is clear from Theorem 3.9.

iii.⇔ **iv.** İt is clear from Theorem 3.11.

□

4 Conclusion

In this work, we introduced soft multi continuous function , soft multi open function, soft multi closed function and soft multi homeomorphism. Also we gave some basic properties of these concepts.

Acknowledgement

We are grateful to the refree for his valueable comments and suggestions.

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