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A Study on Zagreb Indices of Vertex-Switching for Special Graph Classes

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Abstract — Many graph theorists have studied graph operations due to their applications and the advantages with heavy calculations. In a recent paper, the vertex-switching operation is analyzed, and some vertex-switched graphs are determined for some graph classes. This paper calculates the first Zagreb index and the second Zagreb index of vertex-switched star, complete bipartite, and tadpole graphs. It finally discusses the need for further research.

Keywords *Vertex switching, vertex switched graph, star graph, complete bipartite graph, tadpole graph*

Mathematics Subject Classification (2020) 05C07, 05C09

1. Introduction

A simple graph G consists of a set V of vertices (nodes) and a set E of edges (links) between some vertices so that no loops or multiple edges appear in G. For more information about graphs, see [1]. Many graph operations use vertices and edges for several purposes, such as vertex deletion and addition, edge deletion and addition, and edge contraction. Several such vertex and edge operations' applications can be found in [2–10].

In [11], Seidel has introduced a new type of vertex operation called vertex switching. For some fundamental properties and calculations, see [12–14]. For a graph $G = G(V, E)$ and a subset S of $V = V(G)$, the switching of G by S is the graph $G^S(V, E')$ obtained from G by removing all edges between S and $V - S$ and adding new edges between S and $V - S$ which are not in G. If the set S consists of a single vertex v, then, in particular, G^S, denoted briefly by G^v, is the vertex switching graph of G by v.

Some mathematical formulae called topological graph indices are used in obtaining some required properties of a given graph or a graph class. Some of these indices are useful in molecular chemistry and are alternatively called molecular descriptors. Two of these indices are the first and second Zagreb indices defined by Gutman and Trinajstic [15] as follows:

$$M_1(G) = \sum_{u \in V(G)} du^2 \quad \text{and} \quad M_2(G) = \sum_{uv \in E(G)} dudv$$

These two topological indices have been studied by many authors [16, 17]. In this study, we calculate

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the first and the second Zagreb indices of vertex switched star, complete bipartite, and tadpole graphs for the special case of $S = \{v\}$ is a single vertex set.

2. First Zagreb Index of Vertex Switched Graphs of Some Graph Classes

This section calculates the first Zagreb index of vertex-switched star, tadpole, and complete bipartite graphs. We start with complete bipartite graphs.

2.1. First Zagreb Index of Vertex Switched Graphs of Complete Bipartite Graphs

Firstly, we study the first Zagreb index of vertex switched complete bipartite graphs. Let two bipartite sets of the vertices in $K_{r,s}$ with $r \leq s$ be $A = \{u_1, u_2, \cdots, u_r\}$ and $B = \{v_1, v_2, \cdots, v_s\}$. By the definition of vertex switching operation, the case $r = 2$ shows a difference with the case $r \geq 3$ as we obtain a star graph in the former case, and in the latter case, a new graph class is obtained. We denote by $K_{r,s(t)}$ the graph obtained by joining t vertices to all of the s vertices in a bipartite graph $K_{r,s}$. For some examples of these graphs, see Figures 1 and 2 and more details, see [18].

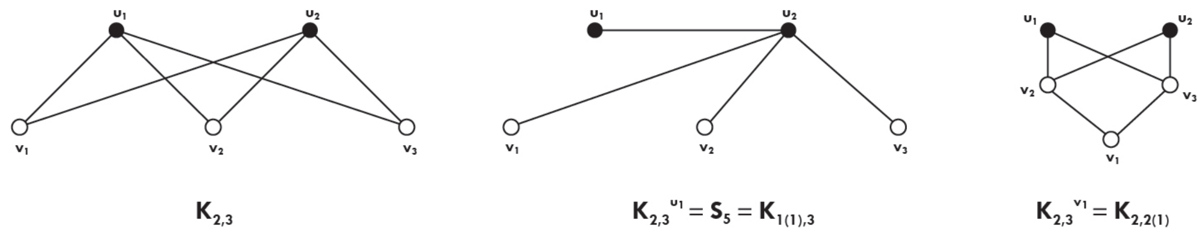

Figure 1. Vertex switching of $K_{2,3}$ where v belongs to different partition sets

We consider the situation where the switched vertex belongs to A:

Theorem 2.1. Let $v \in A$. Then,

$$M_1(K^v_{2,s}) = s^2 + 3s + 2$$

Proof. We need to delete all the edges $vv_1, vv_2, \cdots, vv_s$ incident to v and add a new edge between v and each vertex w in G which are not adjacent to v in G. Thus, v becomes a pendant vertex in $K^v_{2,s}$. There is only one vertex w in G as $r = 2$. Hence, w will be connected to all the other vertices in G when we form $K^v_{2,s}$, and all these vertices are adjacent only to w, causing the graph to be a star graph. The total number of those vertices is $2 + s - 1 = s + 1$. Adding w, we obtain that this star graph will be S_{2+s}. For more details, see [18]. The vertex partition of $K^v_{2,s}$ is provided in Table 1:

Table 1. The vertex partition of $K^v_{2,s}$, for $v \in A$

d_u	# u
1	$s+1$
$s+1$	1

stands for "the number of".

Hence, the first Zagreb index of $K^v_{2,s}$ is

$$M_1(K^v_{2,s}) = M_1(S_{2+s}) = 1^2(s+1) + 1(s+1)^2 = s^2 + 3s + 2$$

□

We consider the situation where the switched vertex belongs to B:

Theorem 2.2. Let $v \in B$. Then,

$$M_1(K^v_{2,s}) = 3s^2 + 3s - 6$$

PROOF. Let $v \in B$. We delete both edges vu_1 and vu_2 and add new $s-1$ edges vv_2, vv_3, $\cdots$, vv_s. Moreover, u_1 and u_2 are adjacent to all the vertices in $B-\{v\}$ which is the graph $K^1_{2,s-1}$ that is the graph obtained by joining v to $s-2$ vertices in $B-\{v\}$ in $K_{2,s-2}$, giving a complete bipartite graph $K_{2,s-1}$. Thus, the vertex partition of $K^v_{2,s}$ is provided in Table 2:

Table 2. The vertex partition of $K^v_{2,s}$, for $v \in B$

d_u	$\#\ u$
$s-1$	3
3	$s-1$

stands for "the number of".

Hence,

$$M_1(K^v_{2,s}) = M_1(K^1_{2,s-1}) = 3(s-1)^2 + 3^2(s-1) = 3s^2 + 3s - 6$$

□

We consider the first Zagreb index of a vertex switched graph $K_{r,s}$ with $3 \leq r \leq s$. For an illustration, see Figure 2, where $r=3$ and $s=4$.

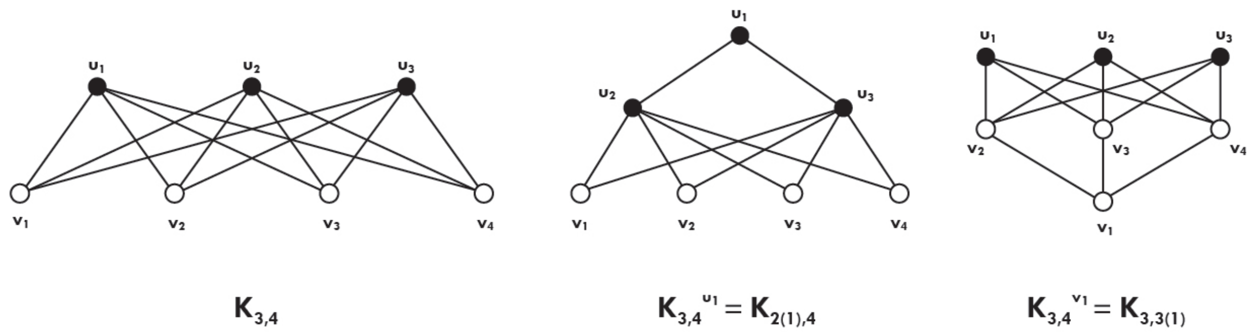

Figure 2. Two possible vertex switchings of the complete bipartite graph $K_{3,4}$

Theorem 2.3. Let $K_{r,s}$ be a complete bipartite graph with $3 \leq r \leq s$. If $v \in A$, then

$$M_1(K^v_{r,s}) = s^2r + r^2s + r^2 - s^2 - r - s$$

and if $v \in B$, then

$$M_1(K^v_{r,s}) = s^2r + r^2s + s^2 - r^2 - r - s$$

PROOF. For $v \in A$, $M_1(K^v_{r,s}) = M_1(K^1_{s,r-1})$. Therefore, the vertex partition of $K^v_{r,s}$ is provided in Table 3:

Table 3. The vertex partition of $K^v_{r,s}$, for $v \in A$

d_u	$\#\ u$
$s+1$	$r-1$
$r-1$	$s+1$

stands for "the number of".

Hence,

$$M_1(K^v_{r,s}) = (s-1)^2(r+1) + (r+1)^2(s-1) = s^2r + r^2s + r^2 - s^2 - r - s$$

For $v \in B$, $M_1(K^v_{r,s}) = M_1(K^1_{r,s-1})$. Therefore, the vertex partition of $K^v_{r,s}$ is provided in Table 4:

Table 4. The vertex partition of $K^v_{r,s}$, for $v \in B$

d_u	$\# u$
$s-1$	$r+1$
$r+1$	$s-1$

\# stands for "the number of".

Hence,

$$M_1(K^v_{r,s}) = M_1(K^1_{r,s-1}) = (s-1)^2(r+1) + (r+1)^2(s-1) = s^2r + r^2s + s^2 - r^2 - r - s$$

□

2.2. First Zagreb Index of Vertex Switched Graphs of Star Graphs

This section looks for the vertex-switched graphs of the star graphs. As there are two different types of vertices in a star graph, we have the following sub-cases:

Theorem 2.4. Let S_n be a star graph, v_1 be the central vertex, and all the remaining vertices be $v_2, v_3, \cdots, v_n$. Then,

$$M_1(S_n^{v_1}) = 0$$

and

$$M_1(S_n^{v_i}) = 2n^2 - 4n$$

for $i \in \{2, 3, \cdots, n\}$, where $K_{1,1,n-2}$ is the complete tripartite graph.

PROOF. First, determine $S_n^{v_1}$. As v_1 is connected to all the other vertices and there are no other edges in S_n, the remaining graph, when we delete all the incident $n-1$ edges to v_1, will have no edges, that is, the resulting graph will be N_n, see the graph in the middle in Figure 3, and hence $M_1(S_n^{v_1}) = 0$.

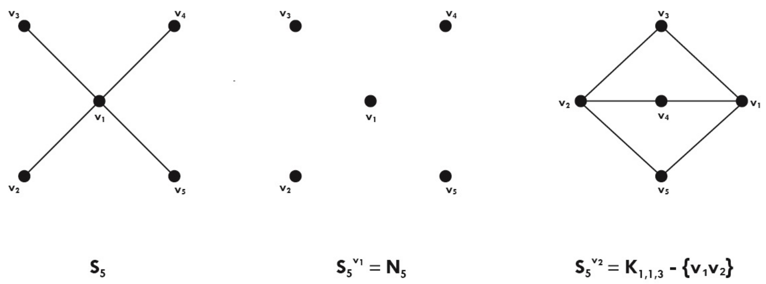

Figure 3. Two possible vertex switchings of a star graph

Secondly, determine $S_n^{v_i}$ where v_i is a vertex different then v_1. When we switch the vertex v_i, all vertices $v_2, v_3, \cdots, v_{i-1}, v_{i+1}, \cdots, v_n$ will be adjacent to only v_1 and v_i, each. This graph is an example of the triangular book graph without a spine; see the graph on the right in Figure 3, for $n = 5$, and Figure 4, for $n > 5$. That is, $S_n^{v_i} = K_{1,1,n-2} - \{v_1v_i\}$.

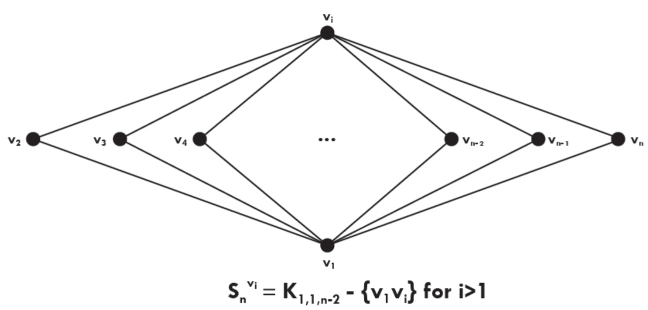

Figure 4. Vertex switching at a pendant vertex of a star graph giving triangular book graph without a spine

Therefore, the vertex partition of $S_n^{v_i}$ is provided in Table 5:

Table 5. Vertex partition of $S_n^{v_i}$, for $v_2, v_3, \cdots, v_{i-1}, v_{i+1}, \cdots, v_n$

d_u	$\#\ u$
2	$n-2$
$n-2$	2

stands for "the number of".

Hence,

$$M_1(S_n^{v_i}) = M_1(K_{1,1,n-2} - \{v_1v_i\}) = 2^2(n-2) + (n-2)^2 2 = 2n^2 - 4n$$

□

2.3. First Zagreb Index of Vertex Switched Graphs of Tadpole Graphs

This section calculates the first Zagreb index of vertex-switched tadpole graphs $T_{r,s}$. This case is much more complicated than the previously considered graph classes, as many different types of vertices exist. Let the vertex at which the path and cycle parts of the tadpole graph intersect be v_1, the remaining vertices on the cycle in clockwise order be $v_2, v_3, \cdots, v_r$, and the remaining vertices starting from the neighbor vertex v_{r+1} be $v_{r+1}, v_{r+2}, \cdots, v_{r+s}$ (see Figure 5).

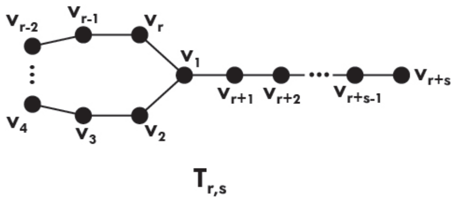

Figure 5. Tadpole graph $T_{r,s}$

Using the symmetry in $T_{r,s}$, we study the first Zagreb index of the vertex switched graphs $T_{r,s}^v$ for v is v_1, v_2, v_3, v_{r+1}, v_{r+2}, v_{r+s-1}, and v_{r+s} as $T_{r,s}^{v_r} = T_{r,s}^{v_2}$, $T_{r,s}^{v_{r-1}} = T_{r,s}^{v_3}$, etc. In Figure 6, we illustrated all the possible vertex-switched tadpole graphs:

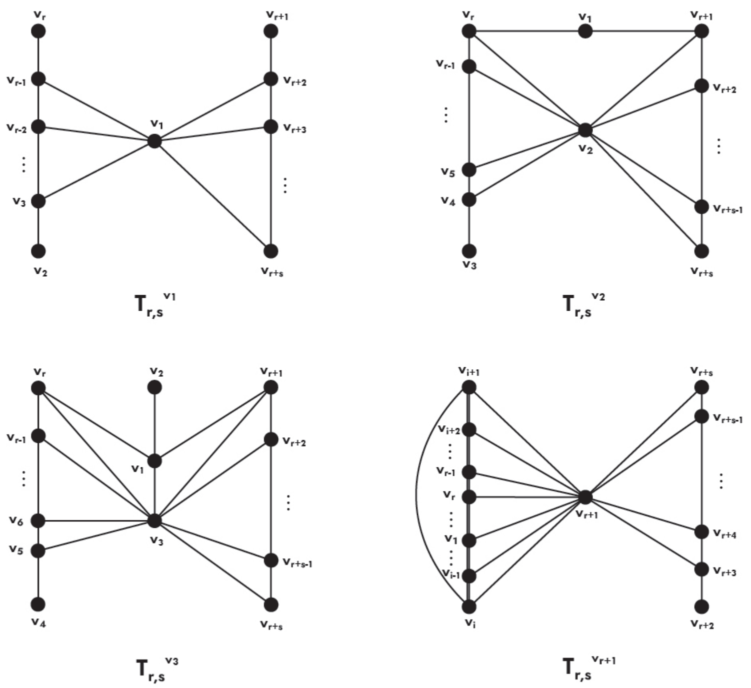

Figure 6. All the possible vertex switched tadpole graphs $T_{r,s}$

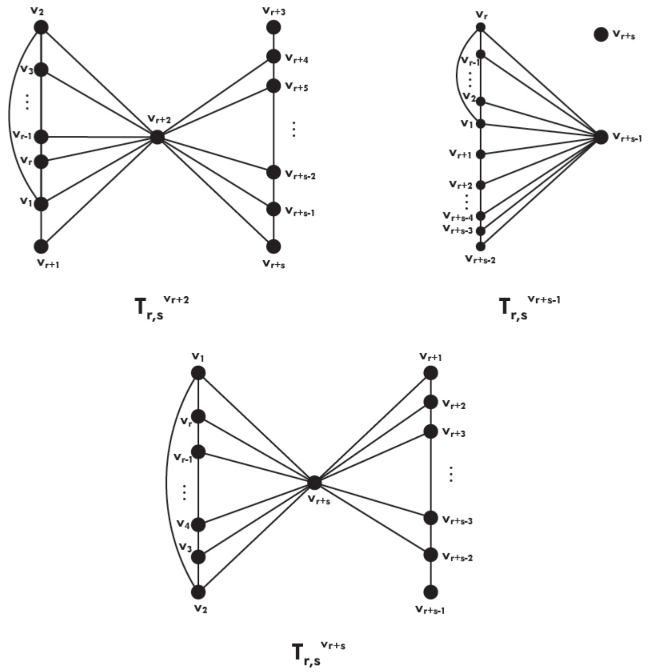

Figure 6. (Continued) All the possible vertex switched tadpole graphs $T_{r,s}$

Theorem 2.5. Let $T_{r,s}$ be a tadpole graph, v_1 be the central vertex, and all the remaining vertices be $v_2, v_3, \cdots, v_n$. Then,

$$M_1(T_{r,s}^{v_1}) = r^2 + s^2 + 2rs + r + s - 22$$

$$M_1(T_{r,s}^{v_2}) = r^2 + s^2 + 2rs + 3r + 3s - 18$$

$$M_1(T_{r,s}^{v_3}) = r^2 + s^2 + 2rs + 3r + 3s - 14$$

$$M_1(T_{r,s}^{v_{r+1}}) = r^2 + s^2 + 2rs + 5r + 5s - 18$$

$$M_1(T_{r,s}^{v_{r+2}}) = r^2 + s^2 + 2rs + 5r + 5s - 20$$

$$M_1(T_{r,s}^{v_{r+s-1}}) = r^2 + s^2 + 2rs + 5r + 5s - 3$$

and

$$M_1(T_{r,s}^{v_{r+s}}) = r^2 + s^2 + 2rs + 5r + 5s - 18$$

PROOF. The vertex partition for $T_{r,s}^{v_1}$ is provided in Table 6:

Table 6. The vertex partition of $T_{r,s}^{v_1}$

d_u	$\#\ u$
1	3
2	1
3	$r+s-5$
$r+s-4$	1

\# stands for "the number of".

Hence, its first Zagreb index is as follows:

$$M_1(T_{r,s}^{v_1}) = 1^2 \cdot 3 + 2^2 \cdot 1 + 3^2 \cdot (r+s-5) + (r+s-4)^2 \cdot 1 = r^2 + s^2 + 2rs + r + s - 22$$

Moreover, the vertex partition for $T_{r,s}^{v_2}$ is provided in Table 7:

Table 7. The vertex partition of $T_{r,s}^{v_2}$

d_u	$\#\ u$
1	1
2	2
3	$r+s-4$
$r+s-3$	1

stands for "the number of".

Thus,

$$M_1(T_{r,s}^{v_2}) = 1^2 \cdot 1 + 2^2 \cdot 2 + 3^2 \cdot (r+s-4) + (r+s-3)^2 \cdot 1 = r^2 + s^2 + 2rs + 3r + 3s - 18$$

Further, the vertex partition for $T_{r,s}^{v_3}$ is provided in Table 8:

Table 8. The vertex partition of $T_{r,s}^{v_3}$

d_u	$\#\ u$
1	2
2	1
3	$r+s-5$
4	1
$r+s-3$	1

stands for "the number of".

Thereby,

$$M_1(T_{r,s}^{v_3}) = 1^2 \cdot 2 + 2^2 \cdot 1 + 3^2 \cdot (r+s-5) + 4^2 \cdot 1 + (r+s-3)^2 \cdot 1 = r^2 + s^2 + 2rs + 3r + 3s - 14$$

Besides, the vertex partition for $T_{r,s}^{v_{r+1}}$ is provided in Table 9:

Table 9. The vertex partition of $T_{r,s}^{v_{r+1}}$

d_u	$\#\ u$
1	1
2	1
3	$r+s-3$
$r+s-2$	1

stands for "the number of".

Therefore,

$$M_1(T_{r,s}^{v_{r+1}}) = 1^2 \cdot 1 + 2^2 \cdot 1 + 3^2 \cdot (r+s-3) + (r+s-2)^2 \cdot 1 = r^2 + s^2 + 2rs + 5r + 5s - 18$$

In addition, the vertex partition for $T_{r,s}^{v_{r+2}}$ is provided in Table 10:

Table 10. The vertex partition of $T_{r,s}^{v_{r+2}}$

d_u	$\#\ u$
1	1
2	2
3	$r+s-5$
4	1
$r+s-2$	1

stands for "the number of".

Hence,

$$M_1(T_{r,s}^{v_{r+2}}) = 1^2 \cdot 1 + 2^2 \cdot 2 + 3^2 \cdot (r+s-5) + (r+s-2)^2 \cdot 1 + 4^2 \cdot 1 = r^2 + s^2 + 2rs + 5r + 5s - 20$$

Moreover, the vertex partition for $T_{r,s}^{v_{r+s-1}}$ is provided in Table 11:

Table 11. The vertex partition of $T_{r,s}^{v_{r+s-1}}$

d_u	# u
2	1
3	$r+s-3$
4	1
$r+s-2$	1

stands for "the number of".

Thus,

$$M_1(T_{r,s}^{v_{r+s-1}}) = r^2 + s^2 + 2rs + 5r + 5s - 3$$

The vertex partition for $T_{r,s}^{v_{r+s}}$ is provided in Table 12:

Table 12. The vertex partition of $T_{r,s}^{v_{r+s}}$

d_u	# u
1	1
2	1
3	$r+s-3$
$r+s-2$	1

stands for "the number of".

Therefore,

$$M_1(T_{r,s}^{v_{r+s}}) = 1^2 \cdot 1 + 2^2 \cdot 1 + 3^2 \cdot (r+s-3) + (r+s-2)^2 \cdot 1 = r^2 + s^2 + 2rs + 5r + 5s - 18$$

□

3. Second Zagreb index of switched graphs of some graph classes

This section calculates the second Zagreb index of a switched star, tadpole, and complete bipartite graphs. Due to the difference in definitions of the first and second Zagreb indices, we use another method. This method uses edge partition, which depends on determining all the pairs of vertex degrees where these vertex pairs form an edge in the graph.

Theorem 3.1. If $v \in A$, then

$$M_2(K_{2,s}^v) = M_2(S_{2+s}) = (s+1)^2$$

and if $v \in B$, then

$$M_2(K_{2,s}^v) = M_2(K_{2,s-1}^1) = 9(s-1)^2$$

Proof. If $v \in A$, the edge partition of $K_{2,s}^v$ is provided in Table 13:

Table 13. The vertex partition of $K_{2,s}^v$ for $v \in A$

(d_u, d_v)	# uv
$(s+1, 1)$	$s+1$

stands for "the number of".

Then, the second Zagreb index of $K_{2,s}^v$ is as follows:

$$M_2(K_{2,s}^v) = M_2(S_{2+s}) = (s+1)^2$$

If $v \in B$, then the edge partition of $K_{2,s}^v$ is provided in Table 14:

Table 14. The vertex partition of $K_{2,s}^v$ for $v \in B$

(d_u, d_v)	$\#\ uv$
$(3, s-1)$	$3(s-1)$

stands for "the number of".

Then, the second Zagreb index of $K_{2,s}^v$ is as follows:

$$M_2(K_{2,s}^v) = M_2(K_{2,s-1}^1) = 9(s-1)^2$$

□

Theorem 3.2. Let $K_{r,s}$ be a complete bipartite graph with $3 \leq r \leq s$. If $v \in A$, then

$$M_2(K_{r,s}^v) = M_2(K_{s,r-1}^1) = (s+1)^2(r-1)^2$$

and if $v \in B$, then

$$M_2(K_{r,s}^v) = M_2(K_{r,s-1}^1) = (s-1)^2(r+1)^2$$

Proof. If $v \in A$, then the edge partition of $K_{r,s}^v$ is provided in Table 15:

Table 15. The vertex partition of $K_{r,s}^v$ for $v \in A$

(d_u, d_v)	$\#\ uv$
$(s+1, r-1)$	$(s+1)(r-1)$

stands for "the number of".

Therefore,

$$M_2(K_{r,s}^v) = M_2(K_{s,r-1}^1) = (s+1)^2(r-1)^2$$

If $v \in B$, then the edge partition of $K_{r,s}^v$ is provided in Table 16:

Table 16. The vertex partition of $K_{r,s}^v$ for $v \in B$

(d_u, d_v)	$\#\ uv$
$(r+1, s-1)$	$(r+1)(s-1)$

stands for "the number of".

Hence,

$$M_2(K_{r,s}^v) = M_2(K_{s,r-1}^1) = (s-1)^2(r+1)^2$$

□

Theorem 3.3. Let S_n be a star graph, v_1 be the central vertex, and all the remaining vertices be $v_2, v_3, \cdots, v_n$. Then,

$$M_2(S_n^{v_1}) = 0$$

and

$$M_2(S_n^{v_i}) = M_2(K_{1,1,n-2} - \{v_1 v_i\}) = 4(n-2)^2$$

for $i \in \{2, 3, \cdots, n\}$, where $K_{1,1,n-2}$ is the tripartite graph.

PROOF. Because $M_2(S_n^{v_1}) = N_n$, it is obvious that

$$M_2(S_n^{v_1}) = 0$$

For $i \in \{2, 3, \cdots, n\}$, the edge partition for $S_n^{v_i}$ is provided in Table 17:

Table 17. The edge partition of $S_n^{v_i}$ for $i \in \{2, 3, \cdots, n\}$

(d_u,d_v)	# uv
$(2,n-2)$	$2(n-2)$

stands for "the number of".

Hence,

$$M_2(S_n^{v_i}) = M_2(K_{1,1,n-2} - \{v_1v_i\}) = 4(n-2)^2$$

□

Theorem 3.4. Let $T_{r,s}$ be a tadpole graph, v_1 be the central vertex, and all the remaining vertices be $v_2, v_3, \cdots, v_n$. Then,

$$M_2(T_{r,s}^{v_1}) = 3r^2 + 3s^2 + 6rs - 16r - 16s + 4$$

$$M_2(T_{r,s}^{v_2}) = 3r^2 + 3s^2 + 6rs - 10r - 10s - 3$$

$$M_2(T_{r,s}^{v_3}) = 3r^2 + 3s^2 + 6rs - 9r - 9s + 1$$

$$M_2(T_{r,s}^{v_{r+1}}) = 3r^2 + 3s^2 + 6rs - 4r - 4s - 13$$

$$M_2(T_{r,s}^{v_{r+2}}) = 3r^2 + 3s^2 + 6rs - 4r - 4s - 8$$

$$M_2(T_{r,s}^{v_{r+s-1}}) = 3r^2 + 3s^2 + 6rs - 3r - 3s$$

and

$$M_2(T_{r,s}^{v_{r+s}}) = 3r^2 + 3s^2 + 6rs - 4r - 4s - 13$$

PROOF. The edge partition for $T_{r,s}^{v_1}$ is provided in Table 18:

Table 18. The edge partition of $T_{r,s}^{v_1}$

(d_u,d_v)	# uv
(1,3)	3
(2,3)	1
$(2,r+s-4)$	1
$(3,r+s-4)$	$r+s-5$
(3,3)	$r+s-7$

stands for "the number of".

By the definition of the second Zagreb index,

$$M_2(T_{r,s}^{v_1}) = 3r^2 + 3s^2 + 6rs - 16r - 16s + 4$$

The edge partition for $T_{r,s}^{v_2}$ is provided in Table 19:

Table 19. The edge partition of $T_{r,s}^{v_2}$

(d_u,d_v)	# uv
(1,3)	1
(2,3)	3
$(2,r+s-3)$	1
$(3,r+s-3)$	$r+s-4$
(3,3)	$r+s-6$

stands for "the number of".

By the definition of second Zagreb index,

$$M_2(T_{r,s}^{v_2}) = 3r^2 + 3s^2 + 6rs - 10r - 10s - 3$$

The edge partition for $T_{r,s}^{v_3}$ is provided in Table 20:

Table 20. The edge partition of $T_{r,s}^{v_3}$

(d_u,d_v)	# uv
$(1,3)$	1
$(1,4)$	1
$(2,3)$	1
$(3,r+s-3)$	$r+s-5$
$(2,r+s-3)$	1
$(4,r+s-3)$	1
$(3,3)$	$r+s-7$

stands for "the number of".

Hence,

$$M_2(T_{r,s}^{v_3}) = 3r^2 + 3s^2 + 6rs - 9r - 9s + 1$$

The edge partition for $T_{r,s}^{v_{r+1}}$ is provided in Table 21:

Table 21. The edge partition of $T_{r,s}^{v_{r+1}}$

(d_u,d_v)	# uv
$(1,3)$	1
$(2,3)$	1
$(3,r+s-2)$	$r+s-3$
$(2,r+s-2)$	1
$(3,3)$	$r+s-4$

stands for "the number of".

Therefore,

$$M_2(T_{r,s}^{v_{r+1}}) = 3r^2 + 3s^2 + 6rs - 4r - 4s - 13$$

The edge partition for $T_{r,s}^{v_{r+2}}$ is provided in Table 22:

Table 22. The edge partition of $T_{r,s}^{v_{r+2}}$

(d_u,d_v)	# uv
$(1,3)$	1
$(2,3)$	1
$(2,4)$	1
$(3,3)$	$r+s-7$
$(3,r+s-2)$	$r+s-5$
$(4,r+s-2)$	1
$(2,r+s-2)$	2
$(3,4)$	2

stands for "the number of".

Thus,

$$M_2(T_{r,s}^{v_{r+2}}) = 3r^2 + 3s^2 + 6rs - 4r - 4s - 8$$

The edge partition for $T_{r,s}^{v_{r+s-1}}$ is provided in Table 23:

Table 23. The edge partition of $T_{r,s}^{v_{r+s-1}}$

(d_u,d_v)	$\#\ uv$
$(2,3)$	1
$(3,3)$	$r+s-6$
$(3,r+s-2)$	$r+s-4$
$(2,r+s-2)$	1
$(3,4)$	3
$(4,r+s-2)$	1

stands for "the number of".

Thereby,

$$M_2(T_{r,s}^{v_{r+s-1}}) = 3r^2 + 3s^2 + 6rs - 3r - 3s$$

□

4. Conclusion

Several vertex and edge operations are frequently used in graph theory to obtain several properties of given graph types. A recently introduced operation is vertex switching. Several preliminary results have just been published on this operation. In this paper, we considered two important degree-based topological indices, the first and second Zagreb indices, for the vertex-switched graphs. The other topological indices can be calculated for the vertex-switched graphs of different type graph classes. As there are over 3000 such indices, a large research area is related to this problem. Moreover, graph operations, derived graphs, and other graph parameters can be studied for the vertex-switched graphs. Such studies might have chemical applications, and we can obtain physicochemical properties of many molecular structures, including nanocones, nanomaterials, dendrimers, and chains. This will increase the importance of the studied areas and may cause the occurrence of new study areas. As there are many diverse definitions of Zagreb-type graph indices, one can also study the exponential Zagreb indices defined in [19]. Similarly, an analysis of Zagreb indices over zero divisor graphs can be given in detail using the results in [17].

Author Contributions

The author read and approved the final version of the paper.

Conflicts of Interest

The author declares no conflict of interest.

Ethical Review and Approval

No approval from the Board of Ethics is required.

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