

EVALUATION OF THE SUITABILITY OF GLOBAL GRADIENT ALGORITHM AND INVERSE MATRIX METHOD FOR STEADY-STATE ANALYSIS OF WATER DISTRIBUTION NETWORKS

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ABSTRACT

Two methods for steady-state analysis of water distribution networks (WDNs) were evaluated in this paper: Global Gradient Algorithm (GGA) and inverse matrix method (IMM). Both methodologies are based on the solution of nonlinear system of equations formed by mass and energy balances within a WDN. A number of WDNs were solved using both methods. Results showed that GGA solves the WDNs in around 70 to 75% less number of iterations than IMM. Besides, single GGA solution of a WDN takes less time compared to IMM solution (about 57% faster for a network of 31 elements). In contrast, single iteration takes 127 to 552 μ s with IMM while it takes 235 to 773 μ s with GGA depending on the number of network elements. Evaluations are based on the speed, the accuracy, and ease of handling the iteration procedures and it was concluded that, although GGA provides faster solutions, both methods are fast enough and IMM, with its simpler iteration procedure, could be beneficial if employed for teaching WDNs in engineering education.

Keywords: Water distribution networks, systems of nonlinear equations, global gradient algorithm, inverse matrix method

İÇME SUYU ŞEBEKELERİNİN KARARLI DURUM ANALİZİ İÇİN KÜRESEL GRADYEN ALGORİTMASI VE TERS MATRİS YÖNTEMİNİN UYGUNLUĞUNUN DEĞERLENDİRİLMESİ

ÖZ

Bu çalışmada içme suyu şebekelerinin kararlı akım analizi için kullanılabilen iki yöntem değerlendirilmiştir: Küresel Gradyen Algoritması (KGA) ve ters matris yöntemi (TMY). Her iki yöntem de şebeke için yazılan kütle ve enerji korunum denklemlerinden oluşan, doğrusal olmayan denklem sisteminin çözümüne dayanmaktadır. Bu çalışmada her iki yöntem de kullanılarak bir dizi şebeke çözümü yapılmıştır. Sonuçlar, KGA'nın TMY'ye göre yaklaşık %70 ile %75 oranında daha az iterasyonla sonuca ulaştığını göstermiştir. Ayrıca, KGA ile tek şebeke çözümü, TMY'ye göre daha az zaman almaktadır (31 elemanlı bir şebeke için yaklaşık %57 oranında daha hızlı). Buna karşın, şebekedeki eleman sayısına bağlı olarak TMY'de tek iterasyon 127 ile 552 μ s'de tamamlanırken KGA'da 235 ile 773 μ s sürmektedir. Değerlendirmeler metodların hızı, doğruluğu ve iterasyon prosedürlerinin kolaylığına göre yapılmış olup, KGA daha hızlı çözümler üretmesine karşın her iki yöntemin de yeteri kadar hızlı olduğunu söylemek mümkündür ve iterasyon prosedürleri çok daha kolay olduğu için TMY'nin mühendislik eğitiminde içme suyu şebekelerini öğretmek için kullanılması KGA'ya göre daha avantajlı olabilir.

Anahtar Kelimeler: İçme suyu şebekeleri, doğrusal olmayan denklem sistemleri, küresel gradyen algoritması, ters matris yöntemi

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1. INTRODUCTION

A water distribution network (WDN) is an engineered system of pipe networks for the purpose of supplying safe and clean water to cities. A WDN is usually a complicated system of pipes laid out in closed- or open-loops, water tanks (also called service tanks), reservoirs, main transmission lines, pumps, valves, surge tanks, etc. Design and operation of WDNs, therefore, involves somewhat complicated procedures involving hydraulic analysis of the systems or parts of them.

Along with economic and population growth, growth of cities in expanding mode and in concentrating mode, which involves construction of taller buildings in city centers, is inevitable. In Turkey, this process has accelerated in recent years with the implementation of regulations in association with the 2012 Turkish law pertaining to urban transformation of cities. In the process, the risky buildings are demolished and new, modern and durable buildings are constructed instead. Besides, new residential zones away from earthquake risks are being opened up. Further, number of homes in new buildings are greater than older buildings (taller buildings and smaller home meters) due usually to economic reasons which brings about new variables to be considered in the design procedure such as the problem of water demand prediction. Thus, the problem of designing, upgrading/improving, expanding, and operating of WDNs have become even more complicated and have led to a number of subsequent problems associated with not only water demand predictions, but also water allocation, water supply safety as well as WDN operation and sustainable management [1]. In this aspect, a thorough understanding of the design and operation of a WDN becomes a priority for especially civil and environmental engineers. Besides, contents and objectives of courses related with water distribution network design and operation in engineering curricula need to be revised to include good engineering practice for expanding/improving WDNs.

Over the years, a number of algorithms for steady-state hydraulic analysis of WDNs have been proposed and successfully applied including Hardy-Cross method [2], linear theory method [3, 4], Newton-Raphson method [5], global gradient algorithm [6], and co-tree flows method [7]. In Turkey, on the other hand, most of design engineers employ dead-end approach or Hardy-Cross method. In dead-end approach, the design engineer need to define paths and dead-ends while Hardy-Cross method is based on defining closed loops with branching pipes excluded from the analysis. Research on WDN design and operation is still ongoing with main focus especially on pressure-driven approach (PDA), rather than demand-driven approach (DDA), to solving nonlinear system of mass and energy balance equations [8-16]. In general, DDA to hydraulic analysis of WDNs in the design stage is considered to be safer since the design of WDNs is based on the assumption that the network will be capable of delivering desired operating pressures even under worst-case conditions.

In recent years, the global gradient algorithm (GGA) has gained great attraction for steady-state analysis of a WDN because of its robustness and speed. GGA is able to handle pipe flow direction without user interference during the analysis, is capable of handling looped- and branched pipe networks, and most importantly initial estimations of pipe flows do not necessarily need to satisfy mass balance. For its overwhelming features over several methods, GGA is preferred in most of today's software packages like EPANET2 and WaterGEMs [6, 17].

In view of engineering education, usually Hardy-Cross method is employed in undergraduate projects [2, 18]. However, branching pipes are excluded from calculations in this method; also, the student is required to define loops. Besides, a set of initial estimates of pipe flows that satisfies mass balance is necessary. After this, the calculations are complicated and time-consuming and initial estimation of pipe flows need to be repeated at each time step if extended period simulation is performed. Thus, students are usually lost in calculations and cannot spare time for understanding better how a hydraulic equilibrium is established within a WDN and how a WDN is operated. In order to overcome this, popular approach in recent years for teaching WDNs involves computerized methods, i.e. some instructors teach the computational algorithms using a commercial or open-source software. However, using software packages for this purpose requires licensing of commercial software packages (usually students need to buy a licensed copy of the software) or teaching a scripting language in case of an open-source software is used [6]. In any case, learning the operating aspects of WDNs could be possible while mathematical background of WDN design problems is overlooked [6]. In the last few years, the instructors realized the importance of mathematical aspects of the design problem and started to develop their own methods of teaching the mathematical background of WDN design [19-21]. The motivation for this study comes from the need of easily teaching the mathematical background of WDN design problems. For this purpose, the solutions of mass and energy balance equations for a given WDN by GGA and inverse matrix method (IMM) were considered in a technical standpoint. Using GGA could improve students' understanding, however, GGA is not meant for hand calculations and it is very difficult even for the most brilliant students to implement GGA that involves such a wide range of matrix operations. Instead, IMM could be employed for steady-state hydraulic analysis of a WDN once mass and energy balance equations are established. Considering that the system of nonlinear equations for nodal heads and pipe flows can easily be solved on an MS Excel sheet even without implementing a computer

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program, the students could comprehend the basics of hydraulic analysis and could spare time for better understanding of how a WDN operates.

The purpose of this study is to compare global gradient algorithm and inverse matrix method for steady-state hydraulic analysis of water distribution networks in regard to undergraduate engineering education. For this purpose, two Visual Basic for Applications (VBA) subroutines were implemented in MS Excel 2010 and the subroutines were run a number of times to assess their speed and stability. Two methods were evaluated for their possible use in undergraduate engineering education.

2. MATERIAL AND METHODS

2.1. Hydraulics

The well-known Darcy-Weisbach formula was used for calculating the headloss in a pipe:

$$h_f = \frac{8fL}{g\pi^2 D^5} Q^2 \tag{1}$$

where h_f is the friction headloss (m H₂O), f is friction factor (dimensionless), L is pipe length (m), D is pipe diameter (m), and Q is flowrate (m³/s). A conditional function was used for estimating friction factor. For laminar flow, i.e. $Re < 2000$, well-known Hagen-Poiseuille formula (Eqn. 2a) was used. Under turbulent conditions, i.e. $Re \geq 4000$, explicit Swamee-Jain formula (Eqn 2b) was employed [21].

$$f = \frac{64}{Re} \tag{2a}$$

$$f = \frac{1}{\left[-2 \log\left(\frac{RR}{3.7} + \frac{5.74}{Re^{0.9}}\right)\right]^2} \tag{2b}$$

For transient conditions, the friction factor is calculated by cubic interpolation from Moody’s diagram [17]. In Eqn. 2b, RR is the relative roughness of the pipe (dimensionless), that’s the roughness height of the pipe divided by its diameter while Reynolds number was calculated using

$$Re = \frac{4\rho Q}{\pi\mu D} \tag{3}$$

where ρ is the density of water (kg/m³) and μ is the dynamic viscosity (kg/m·s). For calculating minor headloss, equivalent-pipe-length method was used:

$$h_m = \frac{8Q^2}{g\pi^2 D^4} \sum_i K_i \tag{4}$$

where h_m is the minor headloss (m H₂O) and K_i is the minor loss coefficient of the i^{th} element on the pipe (dimensionless).

Finally, the total headloss through a pipe can be calculated as the sum of friction and minor losses as follows:

$$h_L = \alpha_1 Q^2 + \alpha_2 Q^2 \tag{5}$$

where h_L is the total headloss through the pipe (m H₂O), α_1 and α_2 are the headloss coefficients calculated by Eqn. 1 and Eqn. 4, respectively.

2.2. Mass and Energy Balances

In a water distribution network of n unknown-head nodes (simply called nodes), k known-head nodes (tanks or reservoirs), and p links with unknown flowrates that connects all nodes (unknown- and known-head nodes) within the network, the mass balance equation in matrix form can be written as:

$$q = AF \tag{6}$$

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where q is the vector of nodal demands (n by 1), A is a matrix (n by p) that defines the system topology for unknown-head nodes, and F is the vector of link flowrates (p by 1). Elements of matrix A are 1 if the i^{th} pipe enters the j^{th} node, -1 if the i^{th} pipe leaves the j^{th} node, and 0 if the i^{th} pipe is not connected to the j^{th} node ($i = 1, 2, \dots, p$ and $j = 1, 2, \dots, n$).

For a given network defined by the topology matrix A , the conservation of energy can be expressed in matrix form by

$$h_L = RF^2 \quad (7)$$

where h_L is the vector of total headlosses in links (p by 1) and R is the diagonal pipe resistance matrix (p by p). Considering that the headloss through a pipe is the difference of the heads between the starting and ending nodes of the pipe, and that some of the pipes within the network may be connected to known-head nodes, the energy equation takes the form of

$$RF + A^T H = -BH_f \quad (8)$$

where H is the vector of unknown nodal heads (n by 1), B is another topology matrix (p by k) that is similar to A^T and that defines connections of k known-head nodes, and H_f is the vector of nodal heads of k known-head nodes (k by 1). The exponent of the pipe flow vector in Eqn. 8 is shown as equal to unity. The reason for this is the need for linearization of the equation. In this way, considering the flow direction is from the starting node to the ending node of the pipe, the elements of the resistance matrix are defined so that

$$r_{ij} = \begin{cases} 0 & \Leftrightarrow i \neq j \\ \alpha_{1,i}|F_i| + \alpha_{2,i}|F_i| - \frac{a_i - b_i|F_i|^{c_i}}{|F_i|} & \Leftrightarrow i = j \end{cases} \quad (9)$$

where r_{ij} is the element of the resistance matrix in i^{th} row and j^{th} column, $\alpha_{1,i}$ and $\alpha_{2,i}$ are the headloss coefficients in the i^{th} pipe, a_i , b_i , and c_i are the pump curve coefficients in the i^{th} pipe.

Eqn. 6 and 8 form a linear system of equations as follows:

$$\begin{bmatrix} 0 & A \\ A^T & R \end{bmatrix} \begin{bmatrix} H \\ F \end{bmatrix} = \begin{bmatrix} q \\ -BH_f \end{bmatrix} \quad (10)$$

2.3. Global Gradient Algorithm

Once the matrices A , q , B , and H_f are created, global gradient algorithm follows an iterative procedure to calculate nodal heads and pipe flows. The procedure is as follows [6]:

Step 1. Initialize pipe flows (F vector) with an arbitrary set of values that do not necessarily satisfy mass balance.

Step 2. Calculate pipe resistance matrix (R).

Step 3. Calculate nodal head estimations using

$$H^{i+1} = -[AM^{-1}R^{-1}A^T]^{-1}\{AM^{-1}[F^i + R^{-1}BH_f] + [q - AF^i]\} \quad (11a)$$

where i is the iteration counter starting from zero and M is a diagonal matrix (p by p). The diagonal elements of M are equal to 2 (for Darcy-Weisbach).

Step 4. Calculate pipe flow estimations using

$$F^{i+1} = [I - M^{-1}]F^i - M^{-1}R^{-1}[A^T H^{i+1} + BH_f] \quad (11b)$$

where I is the identity matrix (p by p).

Step 5. Check convergence. If converged, leave the iterations and report H^{i+1} and F^{i+1} . If not, return to **Step 2** with the set of H^{i+1} and F^{i+1} calculated in the last iteration.

The most important advantage of GGA is that the initial flowrates do not necessarily need to satisfy mass balance. This feature of the method makes it very useful especially in extended period simulations in which a great number of steady-state solutions of the network are required. Further, GGA is a robust method in that the solution converges very fast (usually less than 10 iterations regardless of the initial flowrates selected) and the

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results are stable. One major drawback of the method is related with educational issues. The method is very difficult for undergraduate students to understand and to implement in a computer program, which reduces its importance in engineering curricula.

2.4. Inverse Matrix Method

Once the matrices A , q , B , and H_f are created, the nodal heads (H) and pipe flows (F) can be calculated in one step by inverse matrix method. However, the procedure is iterative since the diagonal elements of pipe resistance matrix are so defined that they are functions of pipe flows (Eqn. 9). Before starting the iterative procedure, two matrices need to be created as C ($n+p$ by $n+p$) and Y ($n+p$ by 1) as follows:

$$C_{(n+p) \times (n+p)} = \begin{bmatrix} 0_{n \times n} & A_{n \times p} \\ A_{p \times n}^T & R_{p \times p} \end{bmatrix} \tag{12a}$$

$$Y_{(n+p) \times 1} = \begin{bmatrix} q_{n \times 1} \\ (-BH_f)_{p \times 1} \end{bmatrix} \tag{12b}$$

All elements of C and Y are constant during the iterations except the diagonal elements of the C matrix $c_{n+1, n+1}$ through $c_{n+p, n+p}$. The values of these diagonal elements are updated during iterations.

The iterative procedure is as follows:

Step 1. Initialize pipe flows (F vector) with an arbitrary set of values that do not necessarily satisfy mass balance.

Step 2. Calculate pipe resistances and update $c_{n+1, n+1}$ through $c_{n+p, n+p}$ with each.

Step 3. Calculate the inverse of C as C^{-1} .

Step 4. Calculate all nodal heads and pipe flows using

$$U = C^{-1}Y \tag{13}$$

The resulting vector (U) has a dimension of $n+p$ by 1. The first n elements of the vector are nodal heads and the rest are the pipe flows.

Step 5. Check convergence. If converged, leave the iterations and report H and F . If not, return to **Step 2** with pipe flows (F vector) calculated in the last iteration.

Although inverse matrix method is not usually preferred for solving systems of linear equations due to stability problems, the iterations are easier. Besides, the time required to complete one iteration with IMM is shorter than that with GGA. In the view of engineering education, IMM could be easily understood by students and offers a great advantage for teaching WDN design in engineering curricula. The iterations of IMM are simpler and one can perform iterations even on an MS Excel sheet without implementing a VBA code. If desired, implementation of IMM in a computer program is much simpler than implementing GGA.

2.5. Implementation

Both GGA and IMM were implemented separately as subroutines using MS Excel Visual Basic for Applications (VBA7.0). For both algorithms, a main subroutine was also implemented. The matrices A , q , B , and H_f were created in the main subroutine. Since the elements of pipe resistance matrix are functions of pipe flows, its elements were updated in related subroutines during iterations. In the main subroutine, the time is recorded and the matrices A , q , B , and H_f were passed to the related GGA and IMM subroutines. The subroutines were called from the main subroutine 1000 times (to minimize uncertainties in runtimes) and after the calculations are complete, the calculation times were recorded for both methods. A simplified block diagram of the implemented subroutines is shown in Fig 1. For matrix multiplication, inverse, and transpose operations, MS Excel worksheet functions were used. In calculations, water temperature is assumed as 5°C. The program codes were run using a computer operating on a Windows 7 Pro SP1 (x64) with Intel Core i5-2500K CPU at 3.30 GHz and 16 GB RAM at 2133 MHz.

A Visual Basic for Applications code of the IMM iterations is given in Fig. 2. In the code, N and P are the number of unknown-head nodes and the number of pipes, respectively; the U_{Old} and U are old and new values of calculated unknowns (nodal heads and pipe flows) in two successive iterations; Ls , Ds , es , and Ks are arrays of the lengths, diameters, roughness heights, and total minor loss coefficients of the pipes. Iterations start after initializing unknowns with arbitrary numbers. A maximum number of iterations were defined by $MAXITR$. In the

code, external functions were shown in red. The *Minverse* and *MMult* are MS Excel built-in functions and *RESISTANCECOEFFICIENT* is a user-defined VBA function specifically implemented to calculate the resistance coefficient of a pipe (Eqn. 9) given with its length, diameter, roughness height, total minor loss coefficient and flowrate. All elements of the matrix *C* and *Y* except the pipe resistance coefficients in *C* (the diagonal elements $c_{n+1, n+1}$ through $c_{n+p, n+p}$) are constant during iterations. In the iterations, only a number of elements equal to the number of pipes within the WDN are updated. Obviously, the iterations are simple to perform on MS Excel sheet and implementing a computer program for an undergraduate student with a fundamental knowledge on engineering programming is easily achievable.

3. RESULTS AND DISCUSSION

3.1. Simulations

Both GGA and IMM aim to solve Eqn. 10 by different approaches. Since both methods are based on the same mass and energy balance equations, the calculated nodal heads and pipe flows by both methods will always be the same as long as the convergence criteria are the same. Therefore, the comparison of these two methods is based on the number of iterations and time of solution.

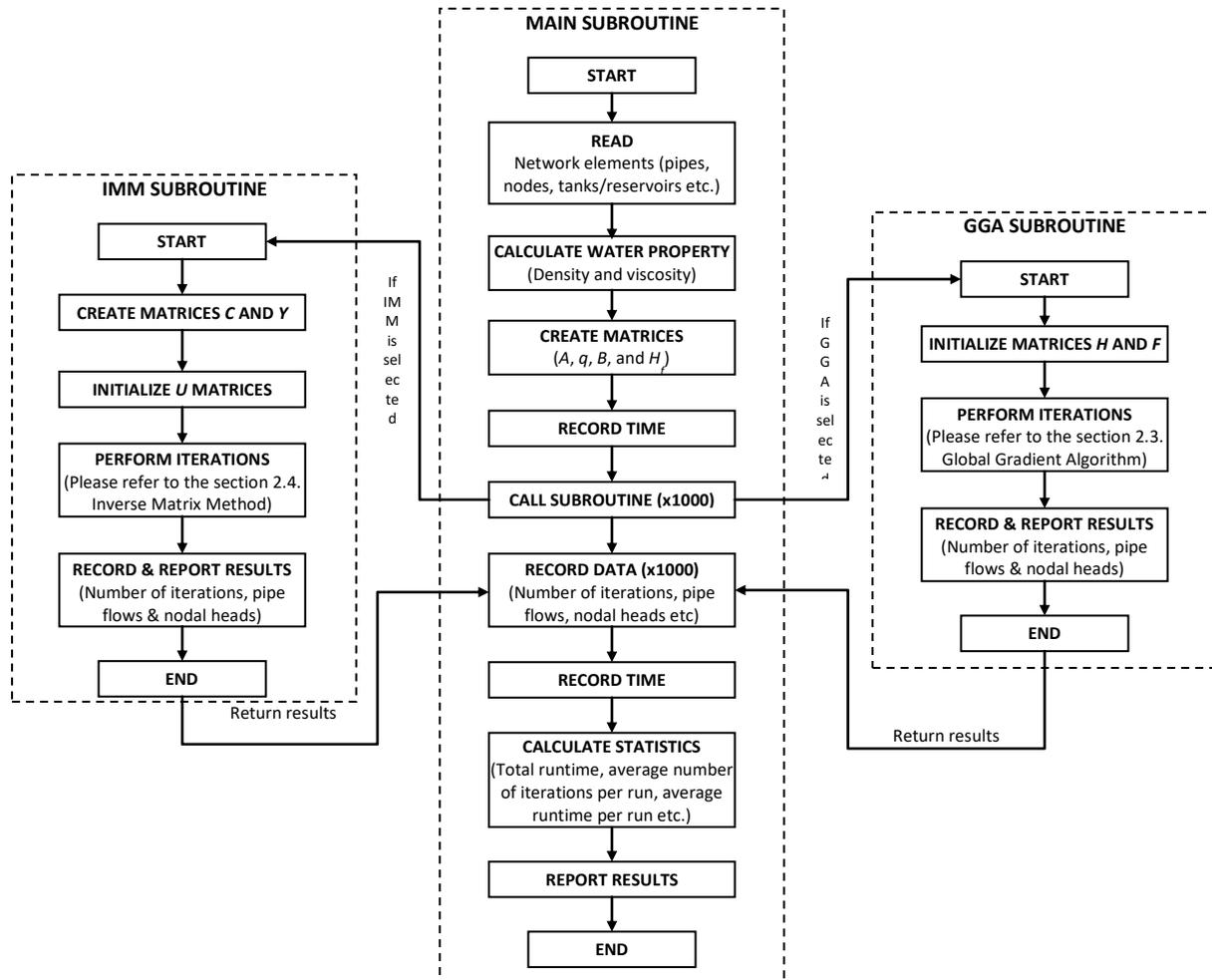


Figure 1. Simplified block diagram of implemented computer program

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'CREATE THE MATRICES C AND Y

'INITIALIZE UNKNOWNNS, THAT'S U_OLD
ReDim U_Old(1 To N + P, 1 To 1)
For I = 1 To N + P
    U_Old(I, 1) = Rnd
Next I
'PERFORM ITERATIONS
For ITERATOR = 1 To MAXITR
    'Calculate pipe resistances
    For I = 1 To P
        C(I + N, I + N) = RESISTANCECOEFFICIENT(Ls(I), Ds(I), es(I), Ks(I), U_Old(I + N, 1))
    Next I
    C_INV = Application.WorksheetFunction.MInverse(C)
    U = Application.WorksheetFunction.MMult(C_INV, Y)
    'Check convergence
    Convergent = True
    For I = 1 To N + P
        rel_err = Abs((U(I, 1) - U_Old(I, 1)) / U(I, 1)) * 100
        If rel_err > MAXERR Then
            Convergent = False
            U_Old = U
            Exit For
        End If
    Next I
    If Convergent Then Exit For
Next ITERATOR
'REPORT RESULTS, THAT'S U AND Convergent
    
```

Figure 2. Core of IMM iterations in VBA

Four sample networks were created without regard to what ranges of operating pressures and flow velocities apply since the main purpose of the study is to evaluate feasibility of the use of global gradient algorithm and inverse matrix method for undergraduate education. The number of elements, that’s, the total number of pipes, known-head nodes, and unknown-head nodes, were 15, 20, 23, and 31, respectively in the networks Network #1, Network #2, Network #3, and Network #4. In the sample networks, all pipe lengths were between 10 and 150 m, all the pipe diameters were between 100 and 300 mm, total water demands were between 100 and 150 L/s, roughness heights of all pipes were 0.0015 mm, and total minor loss coefficients of all pipes were zero. Pipes were assumed to be of high-density polyethylene material.

In all of the sample networks, one known-head node (a service tank) is provided to find a unique solution of the networks. The hydraulic grade at the service tank was assumed as 70 m H₂O for all simulations. For all networks, the elevations of the unknown-head nodes are assumed as 0 meters, which means that the hydraulic grade calculated at each node is also equal to the operating pressure at that node. For all runs and both methods, the maximum change criterion for convergence test was defined as 0.001%. Schematic view of one of the sample networks (Network #4) is shown in Fig. 3.

Hydraulic simulation of all sample networks were performed 1000 times using both global gradient algorithm and inverse matrix method. Total runtimes of the subroutines and number of iterations till convergence changed due to the fact that randomly selected initial values to pipe flows (between 0 and 1 m³/s) were assigned. All runtimes and number of iterations were recorded by a VBA algorithm (creating *A*, *q*, *B*, and *H_f* matrices as well as reporting results were excluded from runtimes). Runtimes and number of iterations till convergence are shown in Table 1.

Obviously GGA converges in considerably less iterations compared to IMM. Minimum numbers of iterations with IMM until convergence is achieved were between 33.1 and 39.3 while, with GGA, they were 8.7 and 12.1. Average number of iterations for IMM ranged from 46.1 to 49.4 while GGA converged in 12.6 to 13.3 iterations. The percent discrepancies between average number of iterations by IMM and GGA ranged from 69.27% to 73.95%. A similar pattern was observed for maximum number of iterations. Finally, IMM converges to a unique solution of the network in around 33 to 53 iterations depending on the number of elements within the network as well as the distance of the initial values vector to the exact solution. A similar pattern was observed for GGA. GGA converged to unique solutions of the networks in 9 to 16 iterations depending on the number of elements within the network as well as the distance of the initial values vector to the exact solution.

With respect to the calculated nodal heads and pipe flows both by GGA and IMM, no significant differences were observed. Percent discrepancies between calculated nodal heads and pipe flows were always less than 0.001%.

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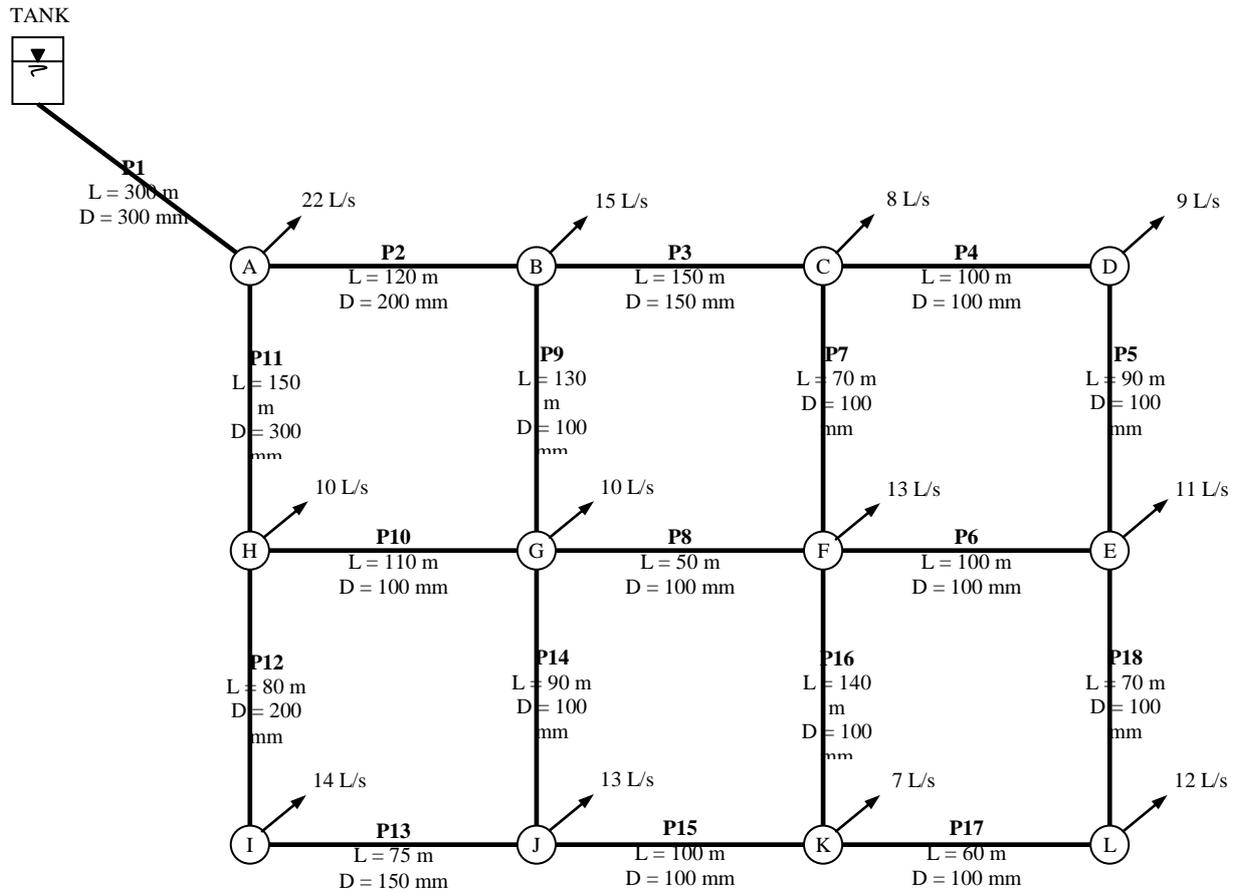


Figure 3. Schematic view of one of the sample networks (Network #4) and nodal demands

Average times required by both methods to complete a single solution of a given network were also summarized in Table 1. Average times per run by IMM (runtime until a unique solution to a given network is obtained) were between 6.29 ± 0.04 to 25.8 ± 0.06 ms, while GGA took 3.03 ± 0.03 to 11.1 ± 0.06 ms to complete single run. Obviously, GGA converges to the unique solution of a given network in considerably less time. For sample networks, GGA converged by 51.80% to 56.94% faster than IMM. Further, time per single run for both methods increases with the number of elements within the network. Fig. 4 shows the change of time required for single run of a given network by both methods.

All of the performance indicators up to here (number of iterations, accuracy, and average calculation time) show that global gradient algorithm is better when compared to inverse matrix method. On the other hand, time required to complete one iteration with IMM is much less than that with GGA (Table 1). This is especially the case for smaller networks, i.e. less number of elements within the network (Fig. 4). An IMM iteration for sample networks took 127 to 552 μ s while a GGA iteration took 235 to 773 μ s to complete. This clearly shows that, for sample networks in this study, a GGA iteration is up to 85% slower than an IMM iteration. The main reason for this is that an IMM iteration requires only two steps after updating resistance coefficients, namely calculating the inverse of coefficients matrix (C^{-1}) and multiplying Y vector with it ($C^{-1}Y$). In this regard, an IMM iteration has the advantage over GGA iteration since it is very easy to handle the IMM iteration.

In educational view, a water distribution network must be considered on a different scale than any real engineering project in that student projects are always smaller, i.e. up to 100 or 150 elements at most. Based on Fig. 4, a water distribution network with 100 elements would take up to 0.5 seconds by IMM and 0.2 seconds by GGA to solve, while it would take 1.2 seconds by IMM and 0.5 seconds by GGA to solve a 150-element-network. Thus, for a water distribution network of student-project-scale, the calculation times by both methods do not cause a significant difference. In light of this, 10 selected senior environmental engineering students were instructed to solve a given pipe network by both methods and several questions were asked at the end of their work. Questions were as follows:

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Table 1. Number of iterations and runtimes obtained by GGA and IMM

Performance measures	Networks ⁴			
	Network #1	Network #2	Network #3	Network #4
Minimum number of iterations ¹				
IMM	33.1±2.64	34.7±2.16	37.2±1.14	39.3±0.48
GGA	8.70±0.48	9.90±0.32	10.0±0.47	12.1±0.57
Discrepancy ²	73.72%	71.47%	73.12%	69.21%
Average number of iterations ¹				
IMM	49.4±0.09	46.4±0.03	46.1±0.04	46.7±0.06
GGA	12.9±0.03	13.3±0.02	12.6±0.03	14.4±0.02
Discrepancy ²	73.95%	71.40%	72.6%	69.27%
Maximum number of iterations ¹				
IMM	53.2±0.42	51.1±0.32	51.9±0.32	52.1±0.32
GGA	14.9±0.32	15.2±0.42	14.0±0.00	16.0±0.00
Discrepancy ²	71.99%	70.25%	73.03%	69.29%
Time per single run (ms) ¹				
IMM	6.29±0.04	9.86±0.03	13.0±0.03	25.8±0.06
GGA	3.03±0.03	4.68±0.02	5.64±0.03	11.1±0.06
Discrepancy ²	51.80%	52.51%	56.58%	56.94%
Estimated time per iteration (µs) ³				
IMM	127	212	282	552
GGA	235	352	446	773
Discrepancy ²	85.04%	66.04%	58.16%	40.04%
Discrepancy between calculated heads	<0.001%	<0.001%	<0.001%	<0.001%
Discrepancy between calculated flows	<0.001%	<0.001%	<0.001%	<0.001%

¹Based on 1000 runs for each network.

²Based on average values.

³Calculated as the ratio of time per single run to average number of iterations, and expressed in microseconds.

⁴Uncertainties are given as standard deviations of 10 values measured over 1000 runs.

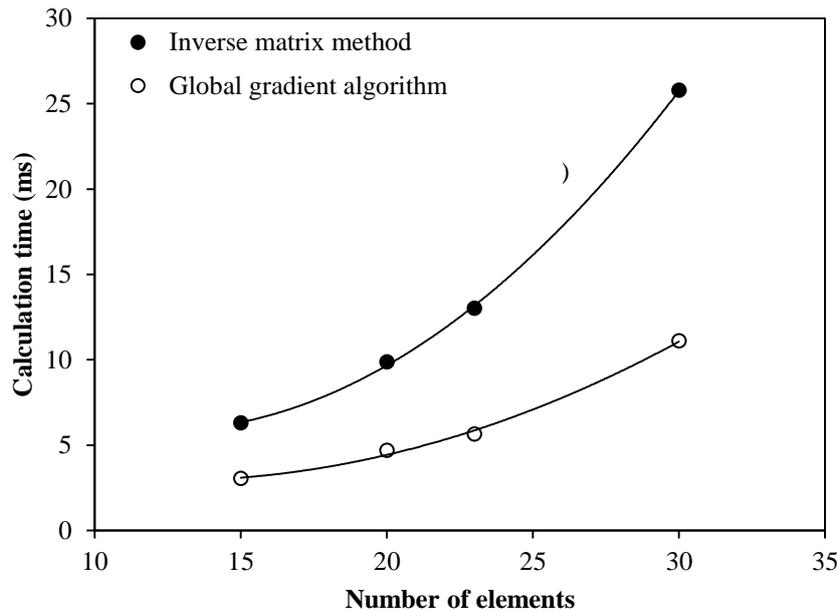


Figure 4. Change of runtime by inverse matrix method and global gradient algorithm with respect to number of elements within the network

S. DEMİR

1. Which method did you use to solve the given network? (Answers: a. GGA; b. IMM)
2. Why did you prefer this method? (Answers: a. It is easier to employ this method; b. Friends used this method; c. The other was difficult)
3. On what platform did you implement the solution? (Answers: a. Spreadsheet (MS Excel etc.); b. Computer program (VB, VBA, Matlab etc.))

All of the students returned their solutions within a week. The solutions were checked for their correctness first, and then the students are asked to answer the questionnaire. All of the students preferred IMM for solving the given network (Q. 1). The student answers to Q. 2 showed that most of the students find IMM is easier to apply. Six students picked the first answer (a. It is easier to employ this method), two picked the second (b. Friends used this method) and the other two picked the third (c. The other method was difficult). For Q. 3, eight students prepared an MS Excel spreadsheet table for solving the example system, while one implemented a simple VBA code to perform each iteration manually (without an outer loop for iterations, each iteration was performed by a simple code and the calculated values are reported back on the spreadsheet). The other student did not answer the third question. The results clearly showed that the students prefer IMM over GGA. Although GGA provides a faster solution to the problem, the students seem more interested in easiness of the calculation procedures.

4. CONCLUSIONS

Global gradient algorithm (GGA) and inverse matrix method (IMM) for steady-state hydraulic analyses of water distribution networks were evaluated in this study. Evaluations were performed based on number of iterations required to converge a unique solution of a given network, calculation speed, accuracy, and most importantly the ease of method's iterations, and implementation of the methods in a computer program or on an MS Excel worksheet. Sample networks for assessing the methods' performances were created and solved using MS Excel VBA subroutines specifically implemented to measure runtimes of the methods.

Because of the fact that, in older methods like dead-end and Hardy-Cross, the initial estimates of pipe flows need to be distributed in such a way that the direction and magnitudes of flowrates must satisfy the mass balance equations, both GGA and IMM methods without the need for sophisticated estimates of initial pipe flows are certainly more preferable over dead-end and Hardy-Cross methods. In this aspect, the benefits of using IMM or GGA for teaching purposes could be understood better when extended period simulations are required. To decide between IMM and GGA, one should consider the fact that IMM iterations include simpler matrix operations, i.e. only calculating the inverse of a matrix and multiplying it with another, compared to longer and more complicated iterative procedure of GGA, which involves about 20 matrix multiplications and about 10 matrix inversions. Further, IMM iterations are also very easy to perform on MS Excel worksheets. A worksheet template could also be prepared by the instructor to help students, which could even reduce calculation times. Although GGA provides faster solutions, the time required for calculations does not differ much for small WDNs; and employing the IMM algorithm, rather than GGA, for teaching WDNs in engineering education could be beneficial. A student survey also supported the conclusions withdrawn here. As a result, the students would be presented by the chance of completing computational design in short periods of time, and can spare time for developing a deeper understanding of how hydraulic equilibrium in a water distribution network is established and how a water distribution network is designed and operated efficiently.

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