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SOME CONTRIBUTION OF SOFT RUGULAR OPEN SETS TO SOFT SEPARATION AXIOMS IN SOFT QUAD TOPOLOGICAL SPACES

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Abstract-Our main interest in this study is to look for soft regular separations axioms in soft quad topological spaces. We talk over and focus our attention on soft regular separation axioms in soft quad topological spaces with respect to ordinary points and soft points. Moreover study the inherited characteristics at different angles with respect to ordinary points and soft points. Some of their central properties in soft quad topological spaces are also brought under examination.

Keywords- *Soft sets, soft topology, soft regular open set, soft regular closed set, soft quad topological space, soft R-qT_0 structure, R-soft qT_1structure, soft R-qT_2 structure, soft R-qT_3 structure and soft R-qT_4 structure.*

1 Introduction

In real life condition the complications in economics, engineering, social sciences, medical science etc. We cannot handsomely use the old-fashioned classical methods because of different types of uncertainties existing in these problems. To finish out these complications, some types of theories were put forwarded like theory of Fuzzy set, intuitionistic fuzzy set, rough set and bi polar fuzzy sets, inwhich we can safely use a mathematical methods for dealing with uncertainties. But, all these theories have their inherent worries. To overcome these difficulties in the year 1999, Russian scholar Molodtsov [4]introduced the idea of soft set as a new mathematical methods to deal with uncertainties. This is free from the above difficulties. Kelly [5] studied Bi topological spaces and discussed different results. Tapi et al. beautifully discussed separation axioms in quad topological spaces. Hameed and Abid discussed separation axioms in Tri-topological spaces.

Recently, in 2011, Shabir and Naz [7] initiated the idea of soft topological space and discussed different results with respect to ordinary points, they beautifully defined soft topology as a collection of τ of soft sets over X. they also defined the basic concept of soft topological spaces such as open set and closed soft sets, soft nbd of a point, soft separation axioms, soft regular and soft normal spaces and published their several performances. soft separation axioms are also discussed at detail. Aktas and Cagman [9] discussed soft sets and soft groups. Chen [10] discovered the parameterization reduction of soft sets and its applications. Feng et al. [11] studied soft semi rings and its applications. In the recent years, many interesting applications of soft sets theory and soft topology have been discussed at great depth [12,13,14,15,16,17,18,19,20,21,22] Kandil at al. [25] explained soft connectedness via soft ideal developed soft set theory. Kandil et al. [27] launched soft regular ity and normality based on semi open soft sets and soft ideals.

In [28,29,30,31,32,33,34,35,36] discussion is launched soft semi Hausdorff spaces via soft ideals, semi open and semi closed sets, separation axioms,decomposition of some type supra soft sets and soft continuity are discussed. Hussain and Ahmad [51] defined soft points, soft separation axioms in soft topological spaces with respect to soft points and used it in different results. Kandil et al. [52] studied soft semi separation axioms and some types of soft functions and their characteristics.

In this present paper, concept of soft regular separation axioms in soft quad topological spaces is broadcasted with respect to ordinary and soft points.

Many mathematicians made discussion over soft separation axioms in soft topological spaces at full length with respect to soft open set, soft b-open set, soft semi-open, soft α-open set and soft β-open set. They also worked over the hereditary properties of different soft topological structures in soft topology. In this present article h and is tried and work is encouraged over the gap that exists in soft quad-topology related to soft regular R- qT_0, soft regular R-qT_1 soft regular R-qT_2, soft regular R-qT_3 and soft regular-qT_4structures. Some propositions in soft quid topological spaces are discussed with respect to ordinary points and soft points. When we talk about distance between the points in soft topology then the concept of soft separation axioms will auto medically come in force. that is why these structures are catching our attentions. We hope that these results will be valuable for the future study on soft quad topological spaces to accomplish general framework for the practical applications and to solve the most complicated problems containing doubts in economics, engineering, medical, environment and in general mechanic systems of various varieties. In upcoming these beautiful soft topological structures may be extended in to soft n-topological spaces provided n is even.

2. Preliminaries

The following Definition s which are pre-requisites for present study

Definition 1 [4] Let X be an initial universe of discourse and E be a set of parameters. Let P (X) denotes the power set of X and A be a non-empty sub-set of E. A pair (F, A) is called a soft set over U, where F is a mapping given by $F\ A \rightarrow P\ (X)$

In other words, a set over X is a parameterized family of sub set of universe of discourse X . For $e \in A, F\ (e)$ may be considered as the set of e-approximate elements of the soft set (F, A) and if $e \notin A$ then $F(e) = \phi$ that is$F_{A=}\{F(e)\ e \in A \subseteq E, F\ A \rightarrow P(X)\}$ the family of all these soft sets over X denoted by $SS\ (X)\ _A$

Definition 2 [4] Let $F_{A,} G_B \in SS\ (X)\ _E$ then F_A, is a soft subset of G_B denoted by $F_A \sqsubseteq G_B$, if

1. $A \subseteq B$ and
2. $F(e) \subseteq G(e), \forall \in A$

In this case F_A is said to be a soft subset of G_B and G_B is said to be a soft super set $F_A, G_B \sqsupseteq F_A$

Definition 3 [6] Two soft subsets F_A and G_B over a common universe of discourse set X are said to be equal if F_A is a soft subset of G_B and G_B is a soft subset of F_A

Definition4 [6] The complement of soft subset (F, A) denoted by $(F, A)\ ^C$ is defined by $(F, A)\ ^C = (F^C, A)\ F^C \rightarrow P\ (X)$ is a mapping given by $F^C(e) = U - F\ (e)\ \forall e \in A$ and $F^C$is called the soft complement function of F . Clearly $(F^C)\ ^C$ is the same as F and $(\ (F, A)\ ^C)\ ^C = (F, A)$

Definition 5 [7] The difference between two soft subset (G, E) and (G, E) over common of universe discourse X denoted by $(F, E) \setminus (G, E)$ is defined as $F\ (e) \setminus G\ (e)$ for all $e \in E$

Definition 6 [7] Let (G, E) be a soft set over X and $x \in X$ We say that $x \in (F, E)$ and read as x belong to the soft set (F, E) whenever $x \in F\ (e)\ \forall e \in E$. The soft set (F, E) over X such that $F(e) = \{x\} \forall e \in E$ is called sing Let on soft point and denoted by $x, or\ (x, E)$

Definition 7 [6] A soft set (F, A) over X is said to be Null soft set denoted by $\overline{\emptyset}\ or\ \emptyset_A$ if $\forall e \in A, F(e) = \emptyset$

Definition 8 [6] A soft set (F, A) over X is said to be an absolute soft denoted by $\bar{A}\ or\ X_A$ if $\forall e \in A, F(e) = X$.

Clearly, we have $X_A^C = \emptyset_A\ and\ \emptyset_A^C = X_A$

Definition 9 [38] The soft set $(F, A) \in SS(X)A$ is called a soft point inX_A, denoted by e_F, if for the element $e \in A, F(e) \neq \{x\}$ and $F\ (e^/) = \phi$ if for all $e^/ \in A - \{e\}$.

A soft point is an element of a soft set F_A. The class of all soft sets over U is denoted by $S\ (U)$.

For Example $U = \{u_1, u_2, u_3\}, E = \{x_1, x_2, x_3\}, A = \{x_1, x_2\}$ and $F_A = \{ (x_1, \{u_1, u_2\}) \}, \{ (x_2, \{u_2, u_3\}) \}$. Then $F_{A_1} = \{(x_1, \{u_1\})\}, F_{A_2} = \{(x_1, \{u_2\})\}, F_{A_3} = \{(x_1, \{u_2, u_2\})\}$, $F_{A_4} = \{(x_2, \{u_2\})\}, F_{A_5} = \{(x_2, \{u_3\})\}, F_{A_6} = \{(x_2, \{u_1, u_3\})\}, F_{A_7} = \{(x_1, \{u_2\}, (x_{2,}\{u_2\})\}$, $F_{A_8} = \{(x_1, \{u_1\}, (x_2, \{u_3\})\}$,
$F_{A_9} = \{(x_1, \{u_1\})\}, (x_2, \{u_2, u_3\}) \}, F_{A_{10}} = \{(x_1, \{u_2\})\}, (x_2, \{u_2\}) \}, F_{A_{11}} = \{(x_1, \{u_2\}), (x_{2,}\{u_3\}) \}, F_{A_{12}} = \{(x_1, \{u_2\}), (x_{2,}\{u_2, u_3\}, F_{A_{13}} = \{(x_1, \{u_1, u_2\}), (x_2, \{u_2\}) \}, F_{A_{14}} = \{(x_1, \{u_1, u_2\}), (x_2, u_3\}, F_{A_{15}} = F_A, F_{A_{16}} = F_\emptyset$, are all soft sub sets of F_A.

Definition 10 [38] The soft point e_F is said to be in the soft set (G, A), denoted by $e_F \in (G, A)$ if for the element$e \in A, F(e) \subseteq G(e)$.

Definition 11 [6] The union of two soft sets (F, A) and (G, B) over the common universe of discourse X is the soft set (H, C), where, $C = AUB$ For all $e \in C$

$$H(e) = \begin{cases} F(e) & if\ e \in A - B \\ G(e) & if\ e \in (B - A) \\ F(e)UG(e), & if\ e \in A \cap B \end{cases}$$

Written as $(F, A) \cup (G, B) = (H, C)$

Definition 12 [6] The intersection $(H.C)$ of two soft sets (F, A) and (G, B) over common universe X, denoted $(F, A) \,\overline{\cap}\, (\mathrm{G}, \mathrm{B})$ is defined as $C = A \cap B$ and $H(e) = F(e) \cap G(e), \forall e \in C$

Definition 13 [41] Two soft sets $(G, A), (H, A)$ in $SS(X)_A$ are said to be soft disjoint, written $(G, A) \cap (H, A) = \emptyset_A$ if $G(e) \cap H(e) = \emptyset$ for all$e \in A$.

Definition 14 [38] The soft point e_G, e_H in X_A are disjoint, written $e_G \neq e_H$ if their corresponding soft sets (G, A) and (H, A) are disjoint.

Definition 15 [2] Let (F, E) be a soft set over X and Y be a non-empty sub set of X. Then the sub soft set of (F, E) over Y denoted by (Y_F, E), is defined as follow $Y_{F(\alpha)} = Y \cap F(\alpha), \forall \in E$ in other words

$$(Y_F, E) = Y \cap (F, E).$$

Definition 16 [3] Let τ be the collection of soft sets over X , then $\tau$is said to be a soft topology on X, if

1. $\emptyset, X$ belong to τ
2. The union of any number of soft sets in τ belongs to τ
3. The intersection of any two soft sets in τ belong to τ

The trip Let (X, F, E) is called a soft topological space.

Definition 17 [1] Let (X, F, E) be a soft topological space over X, then the member of τ are said to be soft open sets in X.

Definition 18 [1] Let (X, F, E) be a soft topological space over X. A soft set (F, A) over X is said to be a soft closed set in X if its relative complement $(F, E)^{C}$ belong to τ

Definition 19 [42] A soft set (A, E) in a soft topological space (X, τ, E) will be termed soft regular open set denoted as $S, R, O\ (X)$ if and only if there exists a soft open set $(F, E) = int\ (cl\ (F, E)$ and soft regular closed set if set $(F, E) = cl\ (in\ (F, E)$ denoted by as $S, R, C\ (X)$ in short h .

3. Soft Regular Separation Axioms of Soft Quad Topological Spaces

In this section we introduced soft regular Separation Axioms in soft Quad topological space with respect to ordinary points and discussed some results with respect to these points in detail.

Definition 20 Let $\left(\breve{X}, \overset{1}{\tau}, \breve{E}\right)$, $\left(\breve{X}, \overset{2}{\tau}, E\right)$, $\left(\breve{X}, \overset{3}{\tau}, \breve{E}\right)$ and $\left(\breve{X}, \overset{4}{\tau}, \breve{E}\right)$ be four different soft topologies on $\breve{X}$. Then $\left(\breve{X}, \overset{1}{\tau}, \overset{2}{\tau}, \overset{3}{\tau}, \overset{4}{\tau}, \breve{E}\right)$is called a *soft quad topological space.* The soft four topologies $\left(\breve{X}, \overset{1}{\tau}, \breve{E}\right)$, $\left(\breve{X}, \overset{2}{\tau}, \breve{E}\right)$, $\left(\breve{X}, \overset{3}{\tau}, E\right)$ and $\left(\breve{X}, \overset{4}{\tau}, \breve{E}\right)$ are independently satisfying the axioms of soft topology. The members of$\overset{1}{\tau}$ are called $\overset{1}{\tau}$ *soft open* set. and complement of$\overset{1}{\tau}$ soft open set is called $\overset{1}{\tau}$ *soft closed* set. Similarly, the member of $\overset{2}{\tau}$are called $\overset{2}{\tau}$ *soft open* sets and the complement of $\overset{2}{\tau}$ soft open sets are called $\overset{2}{\tau}$ *soft closed* set. The members of$\overset{3}{\tau}$ are called $\overset{3}{\tau}$ *soft open* set. and complement of$\overset{3}{\tau}$ soft open set is called $\overset{3}{\tau}$ *soft closed* set and the members of$\overset{4}{\tau}$ are called $\overset{4}{\tau}$ *soft open* set. and complement of$\overset{4}{\tau}$ soft open set is called $\overset{4}{\tau}$ *soft closed* set.

Definition 21 Let $\left(\breve{X}, \overset{1}{\tau}, \overset{2}{\tau}, \overset{3}{\tau}, \overset{4}{\tau}, \breve{E}\right)$ be a soft quad topological space over $\breve{X}$ and $\breve{Y}$be a non-empty subset of $\breve{X}$. Then$\overset{1}{\tau}_{\breve{Y}} = \{(Y_F, E)\ (F, \breve{E})\ \in \overset{1}{\tau})\}$,$\overset{2}{\tau}_{\breve{Y}} = \{(Y_G, E)\ (G, \breve{E})\ \in \overset{2}{\tau})\}$,$\overset{3}{\tau}_{\breve{Y}} = \{(\breve{Y}_H, \breve{E})\ (H, \breve{E})\ \in \overset{3}{\tau})\}$ and $\overset{4}{\tau}_{\breve{Y}} = \{(I_E, \breve{E})\ (I, \breve{E})\ \in \overset{4}{\tau})\}$are said to be the relative topological on Y. Then $\left(\breve{Y}, \overset{1}{\tau}, \overset{2}{\tau}, \overset{3}{\tau}, \overset{4}{\tau}, \breve{E}\right)$ is called relative soft quad-topological space $\left(X, \overset{1}{\tau}, \overset{2}{\tau}, \overset{3}{\tau}, \overset{4}{\tau}, \breve{E}\right)$.

Let $\left(\breve{X}, \overset{1}{\breve{\tau}}, \overset{2}{\breve{\tau}}, \overset{3}{\breve{\tau}}, \overset{4}{\breve{\tau}}, E\right)$ be a soft quad topological space over X, where $\left(\breve{X}, \overset{1}{\breve{\tau}}, \breve{E}\right)$, $\left(\breve{X}, \overset{2}{\breve{\tau}}, \breve{E}\right)$,$\left(\breve{X}, \overset{3}{\breve{\tau}}, \breve{E}\right)$ and $\left(\breve{X}, \overset{4}{\breve{\tau}}, \breve{E}\right)$ be four different soft topologies on $\breve{X}$. Then a sub set $(F, \breve{E})$ is said to be quad-open (in short h and q-open) if $(F, \breve{E}) \subseteq \overset{1}{\breve{\tau}} \cup \overset{2}{\breve{\tau}} \cup \overset{3}{\breve{\tau}} \cup \overset{4}{\breve{\tau}}$ and its complement is said to be soft q-closed.

3. 1 Soft Regular Separation Axioms of Soft Quad Topological Spaces with Respect to Ordinary Points

In this section we introduced soft semi separation axioms in soft quad topological space with respect to ordinary points and discussed some attractive results with respect to these points in detail.

Definition 22 Let $\left(\breve{X}, \overset{1}{\breve{\tau}}, \overset{2}{\breve{\tau}}, \overset{3}{\breve{\tau}}, \overset{4}{\breve{\tau}}, \breve{E}\right)$ be a soft quad topological space over $\breve{X}$ and $x, y \in \breve{X}$ such that $x \neq y$. if we can find soft q-open sets $(F, \breve{E})$ and (G, E) such that $x \in (F, \breve{E})$ and $y \notin (F, \breve{E})$ or $y \in (G, \breve{E})$ and $x \notin (G, \breve{E})$ *then* $\left(\breve{X}, \overset{1}{\breve{\tau}}, \overset{2}{\breve{\tau}}, \overset{3}{\breve{\tau}}, \overset{4}{\breve{\tau}}, \breve{E}\right)$ is called soft qT_0 space.

Definition 23 Let $\left(\breve{X}, \overset{1}{\breve{\tau}}, \overset{2}{\breve{\tau}}, \overset{3}{\breve{\tau}}, \overset{4}{\breve{\tau}}, \breve{E}\right)$ be a soft quad topological space over X and $x, y \in X$ such that $x \neq y$ if we can find two soft q-open sets $(F, \breve{E})$ and $(G, \breve{E})$ such that $x \in (F, \breve{E})$ and $y \notin (F, \breve{E})$ and $y \in (G, \breve{E})$ and $x \notin (G, \breve{E})$ then $\left(\breve{X}, \overset{1}{\breve{\tau}}, \overset{2}{\breve{\tau}}, \overset{3}{\breve{\tau}}, \overset{4}{\breve{\tau}}, \breve{E}\right)$ is called soft qT_1 space.

Definition 22 Let $\left(\breve{X}, \overset{1}{\breve{\tau}}, \overset{2}{\breve{\tau}}, \overset{3}{\breve{\tau}}, \overset{4}{\breve{\tau}}, \breve{E}\right)$be a soft quad topological space over X and $x, y \in X$ such that $x \neq y$. If we can find two q-open soft sets such that $x \in (F, \breve{E})$ and $y \in (G, \breve{E})$ moreover $(F, \breve{E}) \cap (G, \breve{E}) = \phi$. Then $(X, \tau_1, \tau_2, \tau_3, \tau_4, E)$ is called a soft qT_2 space.

Definition 25 Let $\left(\breve{X}, \overset{1}{\breve{\tau}}, \overset{2}{\breve{\tau}}, \overset{3}{\breve{\tau}}, \overset{4}{\breve{\tau}}, \breve{E}\right)$ be a soft topological space $(G, \breve{E})$ be q-closed soft set in X and $x \in X_A$ such that $x \notin (G, \breve{E})$. If there occurs soft q-open sets $(F_1, \breve{E})$ and $(F_2, \breve{E})$ such that $x \in (F_1, \breve{E})$, $(G, \breve{E}) \subseteq (F_2, \breve{E})$ and $(F_1, \breve{E}) \cap (F_2, \breve{E}) = \varphi$. Then $\left(\breve{X}, \overset{1}{\breve{\tau}}, \overset{2}{\breve{\tau}}, \overset{3}{\breve{\tau}}, \overset{4}{\breve{\tau}}, \breve{E}\right)$is called soft q-regular spaces. A soft q-regular qT_1 *Space* is called soft qT_3 space.

Then $\left(\breve{X},\overset{1}{\breve{\tau}},\overset{2}{\breve{\tau}},\overset{3}{\breve{\tau}},\overset{4}{\breve{\tau}},\breve{E}\right)$ is called a soft q-regular spaces. A soft q-regular $T_1\, Space.$ is called soft qT_3 space.

Definition 26 $\left(\breve{X},\overset{1}{\breve{\tau}},\overset{2}{\breve{\tau}},\overset{3}{\breve{\tau}},\overset{4}{\breve{\tau}},\breve{E}\right)$ be a *soft quad topological* space $(F_1,\breve{E})$, $(G,\breve{E})$ be closed soft sets in X such that $(F,\breve{E}) \cap (G,\breve{E}) = \varphi$ if there exists q-open soft sets $(F_1,\breve{E})$ and $(F_2,\breve{E})$ such that $(F,\breve{E}) \subseteq (F_1,\breve{E}),(G,\breve{E}) \subseteq (F_2,\breve{E})$ and $(F_1,\breve{E}) \cap (F_2,\breve{E}) = \varphi$. Then $\left(\breve{X},\overset{1}{\breve{\tau}},\overset{2}{\breve{\tau}},\overset{3}{\breve{\tau}},\overset{4}{\breve{\tau}},\breve{E}\right)$ is called a q-soft normal space. A soft q-normal $qT_1\, Space$ is called soft $qT_4\, Space$.

Definition 27 Let $\left(\breve{X},\overset{1}{\breve{\tau}},\overset{2}{\breve{\tau}},\overset{3}{\breve{\tau}},\overset{4}{\breve{\tau}},\breve{E}\right)$ be a soft topological space over X and $e_G, e_H \in X_A$ such that $e_G \neq e_H$ if there can happen at least one soft q-open set $(F_1,\breve{A})$ or $(F_2,\breve{A})$ such that $e_G \in (F_1,\breve{A}), e_H \notin (F_1,\breve{A})$ or $e_H \in (F_2,\breve{A}), e_G \notin ((F_2,\breve{A})$ then $\left(\breve{X},\overset{1}{\breve{\tau}},\overset{2}{\breve{\tau}},\overset{3}{\breve{\tau}},\overset{4}{\breve{\tau}},\breve{E}\right)$ is called a *soft* qT_0 *space.*

Definition 28 Let $\left(\breve{X},\overset{1}{\breve{\tau}},\overset{2}{\breve{\tau}},\overset{3}{\breve{\tau}},\overset{4}{\breve{\tau}},\breve{E}\right)$ be a soft topological spaces over X and $e_G, e_H \in X_{\breve{A}}$ such that $e_G \neq e_H$ if there can happen *soft q-open sets* $(F_1,\breve{A})$ and $(F_2,\breve{A})$ such that $e_G \in (F_1,\breve{A}), e_H \notin (F_1,\breve{A})$ and $e_H \in (F_2,\breve{A}), e_G \notin ((F_2,\breve{A})$ then $\left(\breve{X},\overset{1}{\breve{\tau}},\overset{2}{\breve{\tau}},\overset{3}{\breve{\tau}},\overset{4}{\breve{\tau}},\breve{E}\right)$ is called soft $qT_1\, space.$

Definition 29 Let $\left(\breve{X},\overset{1}{\breve{\tau}},\overset{2}{\breve{\tau}},\overset{3}{\breve{\tau}},\overset{4}{\breve{\tau}},\breve{E}\right)$ be a *soft topological* space over X and $e_G, e_H \in X_A$ such that $e_G \neq e_H$ if there can happen *soft q-open sets* $(F_1,\breve{A})$ and $(F_2,\breve{A})$ such that $e_G \in (F_1,\breve{A})$, and $e_H \in (F_2,\breve{A})$, $(F_1,\breve{A}) \cap (F_2,\breve{A}) = \phi_A$. Then $\left(\breve{X},\overset{1}{\breve{\tau}},\overset{2}{\breve{\tau}},\overset{3}{\breve{\tau}},\overset{4}{\breve{\tau}},\breve{E}\right)$ is called soft $qT_2\, space.$

Definition 30 Let $\left(\breve{X},\overset{1}{\breve{\tau}},\overset{2}{\breve{\tau}},\overset{3}{\breve{\tau}},\overset{4}{\breve{\tau}},\breve{E}\right)$ be a soft topological space $(G,\breve{E})$ be q-closed soft set in X and $e_G \in X_{\breve{A}}$ such that $e_G \notin (G,\breve{E})$. if there occurs soft q-open sets $(F_1,\breve{E})$ and $(F_2,\breve{E})$ such that $e_G \in (F_1,\breve{E})$, $(G,\breve{E}) \subseteq (F_2,\breve{E})$ and $(F_1,\breve{E}) \cap (F_2,E) = \breve{\emptyset}$. Then $\left(\breve{X},\overset{1}{\breve{\tau}},\overset{2}{\breve{\tau}},\overset{3}{\breve{\tau}},\overset{4}{\breve{\tau}},\breve{E}\right)$ is called soft q-regular spaces. A soft q-regular $qT_1\, Space$ is called soft qT_3 space.

Definition 31 In a *soft quad topological* space $\left(\breve{X},\overset{1}{\breve{\tau}},\overset{2}{\breve{\tau}},\overset{3}{\breve{\tau}},\overset{4}{\breve{\tau}},\breve{E}\right)$

1) $\overset{1}{\breve{\tau}}\cup\overset{2}{\breve{\tau}}$ is said to be *soft regular* T_0 space with respect to $\overset{3}{\breve{\tau}}\cup\overset{4}{\breve{\tau}}$ if for each pair of points $x,y\in\breve{X}$ such that $x\neq y$ there exists $\overset{1}{\breve{\tau}}\cup\overset{2}{\breve{\tau}}$ soft *regular* open set $(F,\breve{E})$ and a to $\overset{3}{\breve{\tau}}\cup\overset{4}{\breve{\tau}}$ soft *regular* open set $(G,\breve{E})$ such that $x\in(F,\breve{E})$ and $y\notin(G,\breve{E})$ or $y\in(G,\breve{E})$ and $x\notin(G,\breve{E})$ similarly, to $\overset{3}{\breve{\tau}}\cup\overset{4}{\breve{\tau}}$is said to be *soft regular* T_0 space with respect to $\overset{1}{\breve{\tau}}\cup\overset{2}{\breve{\tau}}$ if for each pair of points $x,y\in\breve{X}$such that $x\neq y$ there exists $\overset{3}{\breve{\tau}}\cup\overset{4}{\breve{\tau}}$ *regular open set* $(F,\breve{E})$ and $\overset{1}{\breve{\tau}}\cup\overset{2}{\breve{\tau}}$ soft *regular open* set $(G,\breve{E})$ such that $x\in(F,\breve{E})$ and $y\notin(F,\breve{E})$ or $y\in(G,\breve{E})$ and $x\notin(G,\breve{E})$. *soft quad topological spaces* $\left(\breve{X},\overset{1}{\breve{\tau}},\overset{2}{\breve{\tau}},\overset{3}{\breve{\tau}},\overset{4}{\breve{\tau}},\breve{E}\right)$is said to be *pair wise soft regular* T_0 space if $\overset{1}{\breve{\tau}}\cup\overset{2}{\breve{\tau}}$ is *soft regular* T_0 *space* with respect to $\overset{3}{\breve{\tau}}\cup\overset{4}{\breve{\tau}}$ and $\overset{3}{\breve{\tau}}\cup\overset{4}{\breve{\tau}}$ and is *soft regular* T_0 space with respect to $\overset{1}{\breve{\tau}}\cup\overset{2}{\breve{\tau}}$.

2) $\overset{1}{\breve{\tau}}\cup\overset{2}{\breve{\tau}}$ is said to be soft regular T_1 space with respect to $\overset{3}{\breve{\tau}}\cup\overset{4}{\breve{\tau}}$ if for each pair of points $x,y\in X$ such that $x\neq y$ there exists $\overset{1}{\breve{\tau}}\cup\overset{2}{\breve{\tau}}$ *soft regular open set* $(F,\breve{E})$ and to $\overset{3}{\breve{\tau}}\cup\overset{4}{\breve{\tau}}$ soft *regular open* set $(G,\breve{E})$ such that $x\in(F,\breve{E})$ and $y\notin(G,\breve{E})$ and $y\in(G,\breve{E})$ and $x\notin(G,\breve{E})$. Similarly, $\overset{3}{\breve{\tau}}\cup\overset{4}{\breve{\tau}}$is said to be soft regular T_1 space with respect to $\overset{1}{\breve{\tau}}\cup\overset{2}{\breve{\tau}}$ if for each pair of distinct points $x,y\in X$ such that $x\neq y$ there exists$\overset{3}{\breve{\tau}}\cup\overset{4}{\breve{\tau}}$ soft *regular open* set $(F,\breve{E})$ and a to $\overset{1}{\breve{\tau}}\cup\overset{2}{\breve{\tau}}$ soft *regular open* set $(G,\breve{E})$ such that $x\in(F,\breve{E})$ and $y\notin(F,\breve{E})$ and $y\in(G,\breve{E})$ and $x\notin(G,\breve{E})$. *soft quad topological spaces* ($\left(\breve{X},\overset{1}{\breve{\tau}},\overset{2}{\breve{\tau}},\overset{3}{\breve{\tau}},\overset{4}{\breve{\tau}},\breve{E}\right)$is said to be *pair wise soft regular* T_1 space if $\overset{1}{\breve{\tau}}\cup\overset{2}{\breve{\tau}}$is *soft regular* T_1 *space* with respect to $\overset{3}{\breve{\tau}}\cup\overset{4}{\breve{\tau}}$ and to $\overset{3}{\breve{\tau}}\cup\overset{4}{\breve{\tau}}$is *soft regular* T_1 space with respect to $\overset{3}{\breve{\tau}}\cup\overset{4}{\breve{\tau}}$.

3) $\overset{1}{\breve{\tau}}\cup\overset{2}{\breve{\tau}}$is said to be soft regular T_2 space with respect to $\overset{1}{\breve{\tau}}\cup\overset{2}{\breve{\tau}}$ if for each pair of points $x,y\in\breve{X}$ such that $x\neq y$ there exists a $\overset{1}{\breve{\tau}}\cup\overset{2}{\breve{\tau}}$ soft *regular* open set $(F,\breve{E})$ and a $\overset{3}{\breve{\tau}}\cup\overset{4}{\breve{\tau}}$ soft *regular open* set $(G,\breve{E})$ such that $x\in(F,\breve{E})$ and $y\in(G,\breve{E})$, $(F,\breve{E})\cap(G,\breve{E})=\phi$. Similarly, $\overset{3}{\breve{\tau}}\cup\overset{4}{\breve{\tau}}$is said to be *soft regular* T_2 space with respect to $\overset{1}{\breve{\tau}}\cup\overset{2}{\breve{\tau}}$ if for each pair of points $x,y\in\breve{X}$ such that $x\neq y$there exists a $\overset{3}{\breve{\tau}}\cup\overset{4}{\breve{\tau}}$ *soft regular open* set (F,E) and a $\overset{1}{\breve{\tau}}\cup\overset{2}{\breve{\tau}}$ *soft regular open* set $(G,\breve{E})$ such that $x\in(F,\breve{E})$, $y\in(G,\breve{E})$

and $(F,\breve{E}) \cap (G,\breve{E}) = \phi$. The soft quad totopological space $\left(\breve{X},\overset{1}{\breve{\tau}},\overset{2}{\breve{\tau}},\overset{3}{\breve{\tau}},\overset{4}{\breve{\tau}},\breve{E}\right)$is said to be pair wise soft regular T_2 space if $\overset{1}{\breve{\tau}} \cup \overset{2}{\breve{\tau}}$is soft regular T_2 space with respect to $\overset{3}{\breve{\tau}} \cup \overset{4}{\breve{\tau}}$ and $\overset{3}{\breve{\tau}} \cup \overset{4}{\breve{\tau}}$ is soft regular T_2 space with respect to $\overset{1}{\breve{\tau}} \cup \overset{2}{\breve{\tau}}$

Definition 32 In a soft quad topological space $\left(\breve{X},\overset{1}{\breve{\tau}},\overset{2}{\breve{\tau}},\overset{3}{\breve{\tau}},\overset{4}{\breve{\tau}},\breve{E}\right)$

1) $\overset{1}{\breve{\tau}} \cup \overset{2}{\breve{\tau}}$ is said to be soft *regular* qT_3 space with respect to a $\tau_3 \cup \tau_4$ if $\overset{1}{\breve{\tau}} \cup \overset{2}{\breve{\tau}}$is soft *regular* T_1 space with respect to $\overset{3}{\breve{\tau}} \cup \overset{4}{\breve{\tau}}$ and for each pair of points $x, y \in \breve{X}$such that $x \neq y$ there exists $\overset{1}{\breve{\tau}} \cup \overset{2}{\breve{\tau}}$. soft *regular* closed set $(G,\breve{E})$ such that $x \notin (G,\breve{E})$, a$\overset{1}{\breve{\tau}} \cup \overset{2}{\breve{\tau}}$ soft *regular open* set $(F_1,\breve{E})$ and $\overset{3}{\breve{\tau}} \cup \overset{4}{\breve{\tau}}$ soft *regular* open set $(F_2,\breve{E})$ such that $x \in (F_1,\breve{E}), (G,\breve{E}) \subseteq (F_2,\breve{E})$ and $(F_1,\breve{E}) \cap (F_2,\breve{E}) = \emptyset$. Similarly,$\overset{3}{\breve{\tau}} \cup \overset{4}{\breve{\tau}}$is said to be soft regular T_3 space with respect to $\overset{1}{\breve{\tau}} \cup \overset{2}{\breve{\tau}}$ if $\overset{3}{\breve{\tau}} \cup \overset{4}{\breve{\tau}}$ is soft regular T_1 space with respect to $\tau_1 \cup \tau_2$ and for each pair of points $x, y \in \breve{X}$such that $x \neq y$ there exists a $\tau_3 \cup \tau_4$ soft *regular* closed set $(G,\breve{E})$ such that $x \notin (G,\breve{E})$, $\tau_3 \cup \tau_4$ soft regular open set $(F_1,\breve{E})$ and $\overset{1}{\breve{\tau}} \cup \overset{2}{\breve{\tau}}$ soft *regular open* set $(F_2,\breve{E})$ such that $x \in (F_1,\breve{E}), (G,\breve{E}) \subseteq (F_2,\breve{E})$ and $(F_1,\breve{E}) \cap (F_2,\breve{E}) = \phi$. $\left(\breve{X},\overset{1}{\breve{\tau}},\overset{2}{\breve{\tau}},\overset{3}{\breve{\tau}},\overset{4}{\breve{\tau}},\breve{E}\right)$is said to be pair wise soft regular T_3 space if $\overset{1}{\breve{\tau}} \cup \overset{2}{\breve{\tau}}$ is soft regular T_3 space with respect to $\overset{3}{\breve{\tau}} \cup \overset{4}{\breve{\tau}}$ and $\overset{3}{\breve{\tau}} \cup \overset{4}{\breve{\tau}}$is soft s *regular* T_3 space with respect to $\overset{1}{\breve{\tau}} \cup \overset{2}{\breve{\tau}}$.

2) $\overset{1}{\breve{\tau}} \cup \overset{2}{\breve{\tau}}$is said to be soft regular T_4 space with respect to $\overset{3}{\breve{\tau}} \cup \overset{4}{\breve{\tau}}$ if $\overset{1}{\breve{\tau}} \cup \overset{2}{\breve{\tau}}$is soft regular T_1 space with respect to $\overset{3}{\breve{\tau}} \cup \overset{4}{\breve{\tau}}$, there exists a $\overset{1}{\breve{\tau}} \cup \overset{2}{\breve{\tau}}$ soft *regular* closed set $(F_1,\breve{E})$ and $\overset{3}{\breve{\tau}} \cup \overset{4}{\breve{\tau}}$ soft *regular* closed set $(F_2,\breve{E})$ such that $(F_1,\breve{E}) \cap (F_2,\breve{E}) = \emptyset$. Also there exists $(F_3,\breve{E})$ and $(G_1,\breve{E})$ such that $(F_3,\breve{E})$ is soft $\overset{1}{\breve{\tau}} \cup \overset{2}{\breve{\tau}}$ *regular open* set, $(G_1,\breve{E})$ is soft $\overset{3}{\breve{\tau}} \cup \overset{4}{\breve{\tau}}$ *regular open* set such that $(F_1,\breve{E}) \subseteq (F_3,\breve{E})$, $(F_2,\breve{E}) \subseteq (G_1,\breve{E})$. Similarly,$\overset{3}{\breve{\tau}} \cup \overset{4}{\breve{\tau}}$is said to be soft regular T_4 space with respect to $\overset{1}{\breve{\tau}} \cup \overset{2}{\breve{\tau}}$ if $\overset{3}{\breve{\tau}} \cup \overset{4}{\breve{\tau}}$is soft regular T_1 space with respect to $\overset{1}{\breve{\tau}} \cup \overset{2}{\breve{\tau}}$, there exists $\overset{3}{\breve{\tau}} \cup \overset{4}{\breve{\tau}}$ soft *regular* closed set $(F_1,\breve{E})$ and $\overset{1}{\breve{\tau}} \cup \overset{2}{\breve{\tau}}$ soft *regular* closed set $(F_2,\breve{E})$ such that $(F_1,\breve{E}) \cap (F_2,\breve{E}) = \phi$. Also there exist $(F_3,\breve{E})$ and $(G_1,\breve{E})$ such that $(F_3,\breve{E})$ is soft $\overset{3}{\breve{\tau}} \cup \overset{4}{\breve{\tau}}$ *regular* i open set, $(G_1,\breve{E})$ is soft $\overset{1}{\breve{\tau}} \cup \overset{2}{\breve{\tau}}$ *regular open* set such that $(F_1,\breve{E}) \subseteq$

$(F_3,\breve{E}),(F_2,\breve{E}) \subseteq (G_1,\breve{E})$ and $(F_3,\breve{E}) \cap (G_1,\breve{E}) = \phi$. Thus, $(\breve{X},\overset{1}{\tilde{\tau}},\overset{2}{\tilde{\tau}},\breve{E})$ is said to be pair wise soft regular T_4 space if $\overset{1}{\tilde{\tau}} \cup \overset{2}{\tilde{\tau}}$ is soft regular T_4 space with respect to $\overset{3}{\tilde{\tau}} \cup \overset{4}{\tilde{\tau}}$ and $\tau_3 \cup \tau_4$ is soft regular T_4 space with respect to $\overset{1}{\tilde{\tau}} \cup \overset{2}{\tilde{\tau}}$.

Proposition 1. Let $(\breve{X},\tau,\breve{E})$ be a soft topological space over X. if $(\breve{X},\tau,\breve{E})$ is soft-R_3-space , then for all $x \in \breve{X}$, $x_E = (x,\breve{E})$ is regular-closed soft set.

Proof. We want to prove that $x_{\breve{E}}$is regular-closed soft set, which is sufficient to prove that $x_{\breve{E}}^c$ is regular soft-open set for all $y \in \{x\}^c$. Since $(X,\tau,\breve{E})$ is soft R_3-space , then there exists soft regular set sets $(F,\breve{E})_Y$ and (G,E) such that $y_{\breve{E}} \subseteq (F,\breve{E})_Y$ and $x_{\breve{E}} \cap (F,\breve{E})_Y = \phi$ and $x_{\breve{E}} \subseteq (G,\breve{E})$ and $y_{\breve{E}} \cap (G,\breve{E}) = \phi$. It follows that , $\cup_{y\in(x)^c}(F,\breve{E})_y \subseteq x_{\breve{E}}^c$. Now, we want to prove that $x_{\breve{E}}^c \subseteq \cup_{y\in(x)^c} (F,\breve{E})_y$. Let $\cup_{y\in(x)^c} (F,E)_y = (H,E)$. where $H(e) = \cup_{y\in(x)^c}(F(e))_y$ for all $e \in \breve{E}$. Since, $x_{\breve{E}}^c(e) = \{x\}^c$ for all $e \in \breve{E}$from Definition 6, so, for all $y \in \{x\}^c$ and $e \in \breve{E} x_{\breve{E}}^c(e) = \{x\}^c = \cup_{y\in(x)^c}\{y\} = \cup_{y\in(x)^c}F(e)_y = H(e)$. Thus, $x_{\breve{E}}^c \subseteq \cup_{y\in(x)^c} (F,\breve{E})_y$ from Definition 2, and so$x_{\breve{E}}^c = \cup_{y\in(x)^c} (F,\breve{E})_y$. This means that , $x_{\breve{E}}^c$ is soft regular-open set for all$y \in \{x\}^c$. Hence $x_{\breve{E}}$ is soft regular-closed set.

Proposition 2. Let $(\breve{Y},\overset{Y}{\tilde{\tau}},\breve{E})$ be a soft sub space of a soft topological space $(\breve{X},\tau,\breve{E})$ and $(F,\breve{E}) \in SS(\breve{X})$ then,

1. if $(F,\breve{E})$ is soft regular open *soft* set in $\breve{Y}$ and $\breve{Y} \in \tau$, then $(F,\breve{E}) \in \tau$.

2. $(F,\breve{E})$ is soft regular *open soft* set in $\breve{Y}$ if and only if $(F,\breve{E}) = Y \cap (G,\breve{E})$ for some $(G,\breve{E}) \in \tau$.

3. $(F,\breve{E})$ is soft regular *closed soft* set in$\breve{Y}$ if and only if $(F,\breve{E}) = \breve{Y} \cap (H,\breve{E})$ for some $(H,\breve{E})$ *is* τ *soft* regular *close* set.

Proof. 1) Let $(F,\breve{E})$ be a soft regular open set in $\breve{Y}$, then there does exists a soft regular open set (G,E) in $\breve{X}$ such that $(F,\breve{E}) = \breve{Y} \cap (G,\breve{E})$. Now, if $\breve{Y} \in \tau$ then $\breve{Y} \cap (G,\breve{E}) \in \tau$ by the third condition of the definition of a soft topological space and hence $(F,\breve{E}) \in \tau$.

2) Fallows from the definition of a soft subspace.

3) if $(F,\breve{E})$ is soft regular closed in Y then we have $(F,\breve{E}) = \breve{Y} \backslash (G,\breve{E})$, for some $(G,\breve{E}) \in \tau_{\breve{Y}}$. Now,$(G,\breve{E}) = \breve{Y} \cap (H,\breve{E})$ for some soft regular open set $(H,\breve{E}) \in \tau$. for any$\alpha \in \breve{E}$. $F(\alpha) = \breve{Y}(\alpha) \backslash G(\alpha) = \breve{Y} \backslash G(\alpha) = Y \backslash (Y(\alpha) \cap H(\alpha)) = \breve{Y} \backslash (\breve{Y} \cap H(\alpha)) = Y \backslash H(\alpha) = \breve{Y} \cap (X \backslash H(\alpha)) = \breve{Y} \cap (H(\alpha))^C = \breve{Y}(\alpha) \cap (H(\alpha))^C$. Thus $(F,\breve{E}) = \breve{Y} \cap (H,\breve{E})'$is soft regular closed in $\breve{X}$ as$(H,\breve{E}) \in \tau$. Conversely, suppose that $(F,\breve{E}) = \breve{Y} \cap$

$(G,\breve{E})$ for some soft regular closed set (G,E) in $\breve{X}$. This qualifies us to say that $(G,\breve{E})^{/} \in \tau$. Now, if $(G,\breve{E}) = (X,\breve{E}) \setminus (H,\breve{E})$ where $(H,\breve{E})$ is soft regular open.

Proposition 3. Let $\left(\breve{X},\overset{1}{\breve{\tau}},\overset{2}{\breve{\tau}},\overset{3}{\breve{\tau}},\overset{4}{\breve{\tau}},\breve{E}\right)$ be a soft quad topological space over $\breve{X}$. Then, if $\left(\breve{X},\overset{1}{\breve{\tau}},\overset{2}{\breve{\tau}},\breve{E}\right)$ and $\left(\breve{X},\overset{3}{\breve{\tau}},\overset{4}{\breve{\tau}},\breve{E}\right)$ are soft regular T_3 space then $\left(\breve{X},\overset{1}{\breve{\tau}},\overset{2}{\breve{\tau}},\overset{3}{\breve{\tau}},\overset{4}{\breve{\tau}},\breve{E}\right)$ is a pair wise soft regular T_2 space.

Proof. Suppose $\left(\breve{X},\overset{1}{\breve{\tau}},\overset{2}{\breve{\tau}},\breve{E}\right)$ is a soft regular T_3 space with respect to $\left(\breve{X},\overset{3}{\breve{\tau}},\overset{4}{\breve{\tau}},\breve{E}\right)$ then according to definition for $x,y \in \breve{X}$, which distinct, by using Proposition 1, $(\breve{Y},\breve{E})$ is soft *regular* closed set in $\overset{3}{\breve{\tau}} \cup \overset{4}{\breve{\tau}}$ and $x \notin (\breve{Y},\breve{E})$ there exists a$\overset{1}{\breve{\tau}} \cup \overset{2}{\breve{\tau}}$ soft *regular* open set $(F,\breve{E})$ and a $\overset{3}{\breve{\tau}} \cup \overset{4}{\breve{\tau}}$ soft *regular* open set $(G,\breve{E})$ such that $x \in (F,\breve{E}), y \in (Y,\breve{E}) \subseteq (G,\breve{E})$ and $(F_1,\breve{E}) \cap (F_2,\breve{E}) = \phi$. Hence $\overset{1}{\breve{\tau}} \cup \overset{2}{\breve{\tau}}$is soft regular T_2 space with respect to $\overset{3}{\breve{\tau}} \cup \overset{4}{\breve{\tau}}$. Similarly, if $\left(\breve{X},\overset{3}{\breve{\tau}},\overset{4}{\breve{\tau}},\breve{E}\right)$ is a soft regular T_3 space with respect to $\left(\breve{X},\overset{1}{\breve{\tau}},\overset{2}{\breve{\tau}},\breve{E}\right)$ then according to definition for $x,y \in \breve{X}, x \neq y$, by using Theorem 2, (x, $\breve{E}$) is *regular* closed soft set in $\overset{1}{\breve{\tau}} \cup \overset{2}{\breve{\tau}}$ and $y \notin (x,E)$ there exists a $\overset{3}{\breve{\tau}} \cup \overset{4}{\breve{\tau}}$ soft *regular open* set $(F,\breve{E})$ and a $\overset{1}{\breve{\tau}} \cup \overset{2}{\breve{\tau}}$ soft *regular open* set $(G,\breve{E})$ such that $y \in (F,\breve{E})$, $x \in (x,\breve{E}) \subseteq (G,\breve{E})$ and $(F_1,\breve{E}) \cap (F_2,\breve{E}) = \phi$. Hence $\overset{3}{\breve{\tau}} \cup \overset{4}{\breve{\tau}}$is soft regular T_2 space. This implies that $\left(\breve{X},\overset{1}{\breve{\tau}},\overset{2}{\breve{\tau}},\overset{3}{\breve{\tau}},\overset{4}{\breve{\tau}},\breve{E}\right)$is a pair wise soft regular T_2 space.

Proposition 4. Let $\left(\breve{X},\overset{1}{\breve{\tau}},\overset{2}{\breve{\tau}},\overset{3}{\breve{\tau}},\overset{4}{\breve{\tau}},\breve{E}\right)$be a soft quad topological space over $\breve{X}$. $\left(\breve{X},\overset{1}{\breve{\tau}},\overset{2}{\breve{\tau}},\breve{E}\right)$ and $\left(\breve{X},\overset{3}{\breve{\tau}},\overset{4}{\breve{\tau}},\breve{E}\right)$ are soft regular T_3 space then $\left(\breve{X},\overset{1}{\breve{\tau}},\overset{2}{\breve{\tau}},\overset{3}{\breve{\tau}},\overset{4}{\breve{\tau}},\breve{E}\right)$is a pair wise soft regular T_3 space.

Proof. Suppose $\left(\breve{X},\overset{1}{\breve{\tau}},\overset{2}{\breve{\tau}},\breve{E}\right)$ is a soft regular T_3 space with respect to $\left(\breve{X},\overset{1}{\breve{\tau}},\overset{2}{\breve{\tau}},\breve{E}\right)$ then according to definition for $x,y \in X, x \neq y$ there exists a $\left(\breve{X},\overset{1}{\breve{\tau}},\overset{2}{\breve{\tau}},\breve{E}\right)$ soft *regular open* set $(F,\breve{E})$ and a $\left(\breve{X},\overset{3}{\breve{\tau}},\overset{4}{\breve{\tau}},\breve{E}\right)$ soft *regular open* set $(G,\breve{E})$ such that $x \in (F,\breve{E})$ and $y \notin (F,\breve{E})$ or $y \in (G,\breve{E})$ and $x \notin (G,\breve{E})$ and for each point $x \in X$ and each $\left(\breve{X},\overset{1}{\breve{\tau}},\overset{2}{\breve{\tau}},\breve{E}\right)$ *regular* closed soft set $(G_1,\breve{E})$ such that $x \notin (G_1,\breve{E})$ there exists

$\left(\breve{X},\overset{1}{\breve{\tau}},\overset{2}{\breve{\tau}},\breve{E}\right)$ soft *regular* open set $(F_1,\breve{E})$ and $\left(\breve{X},\overset{3}{\breve{\tau}},\overset{4}{\breve{\tau}},\breve{E}\right)$ soft *regular* open set (F_2,E) such that $x\in\left(F_1,\breve{E}\right),(G_1,\breve{E})\subseteq(F_2,\breve{E})$ and $\left(F_1,\breve{E}\right)\cap\left(F_2,\breve{E}\right)=\phi$. Similarly, to $\left(\breve{X},\overset{3}{\breve{\tau}},\overset{4}{\breve{\tau}},\breve{E}\right)$ is a soft regular T_3 space with respect to $\left(\breve{X},\overset{1}{\breve{\tau}},\overset{2}{\breve{\tau}},\breve{E}\right)$ So according to definition for $x,y\in X,x\neq y$ there exists a $\left(\breve{X},\overset{3}{\breve{\tau}},\overset{4}{\breve{\tau}},\breve{E}\right)$ soft *regular* open set $(F,\breve{E})$ and a $\left(\breve{X},\overset{1}{\breve{\tau}},\overset{2}{\breve{\tau}},\breve{E}\right)$ soft *regular open* set (G,E) such that $x\in(F,\breve{E})$ and $y\notin(F,\breve{E})$ or $y\in(G,\breve{E})$ and $x\notin(G,\breve{E})$ and for each point $x\in\breve{X}$ and each $\left(\breve{X},\overset{3}{\breve{\tau}},\overset{4}{\breve{\tau}},\breve{E}\right)$ *regular* closed soft set $(G_1,\breve{E})$ such that $x\notin(G_1,\breve{E})$ there exists $\left(\breve{X},\overset{3}{\breve{\tau}},\overset{4}{\breve{\tau}},\breve{E}\right)$ soft *regular open* set $\left(F_1,\breve{E}\right)$ and a $\left(\breve{X},\overset{1}{\breve{\tau}},\overset{2}{\breve{\tau}},\breve{E}\right)$ soft semi open set $\left(F_2,\breve{E}\right)$ such that $x\in\left(F_1,\breve{E}\right),(G_1,\breve{E})\subseteq(F_2,\breve{E})$ and $\left(F_1,\breve{E}\right)\cap\left(F_2,\breve{E}\right)=\phi$. Hence $\left(\breve{X},\overset{1}{\breve{\tau}},\overset{2}{\breve{\tau}},\overset{3}{\breve{\tau}},\overset{4}{\breve{\tau}},\breve{E}\right)$is pair wise soft regular T_3 space.

Proposition 5. If $\left(\breve{X},\overset{1}{\breve{\tau}},\overset{2}{\breve{\tau}},\overset{3}{\breve{\tau}},\overset{4}{\breve{\tau}},\breve{E}\right)$ be a soft quad topological space over X. $\left(\breve{X},\overset{1}{\breve{\tau}},\overset{2}{\breve{\tau}},\breve{E}\right)$ and $\left(\breve{X},\overset{3}{\breve{\tau}},\overset{4}{\breve{\tau}},\breve{E}\right)$ are soft regular T_4 space then $\left(\breve{X},\overset{1}{\breve{\tau}},\overset{2}{\breve{\tau}},\overset{3}{\breve{\tau}},\overset{4}{\breve{\tau}},\breve{E}\right)$ is pair wise soft regular T_4 space.

Proof. Suppose $\left(\breve{X},\overset{1}{\breve{\tau}},\overset{2}{\breve{\tau}},\breve{E}\right)$ is soft regular T_4 space with respect to $\left(\breve{X},\overset{3}{\breve{\tau}},\overset{4}{\breve{\tau}},\breve{E}\right)$ So according to definition for $x,y\in\breve{X},x\neq y$ there exist a $\left(\breve{X},\overset{1}{\breve{\tau}},\overset{2}{\breve{\tau}},\breve{E}\right)$ soft *regular open* set $(F,\breve{E})$ and a $\left(\breve{X},\overset{3}{\breve{\tau}},\overset{4}{\breve{\tau}},\breve{E}\right)$ soft *regular* open set $(G,\breve{E})$ such that $x\in(F,\breve{E})$ and $y\notin(F,\breve{E})$ or $y\in(G,\breve{E})$ and $x\notin(G,\breve{E})$ each $\left(\breve{X},\overset{1}{\breve{\tau}},\overset{2}{\breve{\tau}},\breve{E}\right)$ soft *regular* closed set $(F_1,\breve{E})$ and a $\left(\breve{X},\overset{3}{\breve{\tau}},\overset{4}{\breve{\tau}},\breve{E}\right)$ soft *regular* closed set $(F_2,\breve{E})$ such that $\left(F_1,\breve{E}\right)\cap\left(F_2,\breve{E}\right)=\phi$. There exist (F_3,E) and $\left(G_1,\breve{E}\right)$ such that $\left(F_3,\breve{E}\right)$ is soft $\left(\breve{X},\overset{3}{\breve{\tau}},\overset{4}{\breve{\tau}},\breve{E}\right)$ and soft regular open set $(G_1,\breve{E})$ is soft $\left(\breve{X},\overset{1}{\breve{\tau}},\overset{2}{\breve{\tau}},\breve{E}\right)$ *regular open* set $\left(F_1,\breve{E}\right)\subseteq\left(F_3,\breve{E}\right),(F_2,\breve{E})\subseteq(G_1,\breve{E})$ and $\left(F_3,\breve{E}\right)\cap\left(G_1,\breve{E}\right)=\phi$. Similarly, $\left(\breve{X},\overset{3}{\breve{\tau}},\overset{4}{\breve{\tau}},\breve{E}\right)$ is soft regular T_4 space with respect to τ_1 so according to definition for $x,y\in\breve{X},x\neq y$ there exists a $\left(X,\tau_3,\tau_4,\breve{E}\right)$ soft *regular* open set $(F,\breve{E})$ and a $\left(\breve{X},\overset{1}{\tilde{\tau}},\overset{2}{\tilde{\tau}},\breve{E}\right)$ soft *regular open* set $(G,\breve{E})$ such that $x\in(F,\breve{E})$ and $y\notin(F,\breve{E})$ or $y\in(G,\breve{E})$ and $x\notin(G,\breve{E})$ and for each $\left(\breve{X},\overset{3}{\tilde{\tau}},\overset{4}{\tilde{\tau}},\breve{E}\right)$ soft *regular* closed set (F_1,E) and $\left(\breve{X},\overset{1}{\tilde{\tau}},\overset{2}{\tilde{\tau}},\breve{E}\right)$ soft *regular* closed set $(F_2,\breve{E})$ such that $(F_1,E)\cap(F_2,E)=\phi$. there exists soft *regular open* sets$\left(F_3,\breve{E}\right)$ and $(G_1,\breve{E})$ such that $\left(F_3,\breve{E}\right)$ is soft $\left(\breve{X},\overset{1}{\tilde{\tau}},\overset{2}{\tilde{\tau}},\breve{E}\right)$ *regular* open set $(G_1,\breve{E})$ is soft $\left(\breve{X},\overset{3}{\tilde{\tau}},\overset{4}{\tilde{\tau}},\breve{E}\right)$ *regular open* set such that $\left(F_1,\breve{E}\right)\subseteq\left(F_3,\breve{E}\right),(F_2,\breve{E})\subseteq(G_1,\breve{E})$ and $\left(F_3,\breve{E}\right)\cap\left(G_1,\breve{E}\right)=\phi$. Hence $\left(\breve{X},\overset{1}{\tilde{\tau}},\overset{2}{\tilde{\tau}},\overset{3}{\tilde{\tau}},\overset{4}{\tilde{\tau}},\breve{E}\right)$ is pair wise soft regular T_4 space.

Proposition 6. Let $\left(\breve{X},\overset{1}{\tilde{\tau}},\overset{2}{\tilde{\tau}},\overset{3}{\tilde{\tau}},\overset{4}{\tilde{\tau}},\breve{E}\right)$ be a soft quad topological space over X and Y be a non-empty subset of X. if $(X,\overset{1^Y}{\tilde{\tau}}\ \overset{2^Y}{\tilde{\tau}},\overset{3^Y}{\tilde{\tau}},\overset{4^Y}{\tilde{\tau}},\breve{E})$ is pair wise soft regular T_3 space. Then $(\breve{Y},\overset{1^Y}{\tilde{\tau}}\ \overset{2^Y}{\tilde{\tau}},\overset{3^Y}{\tilde{\tau}},\overset{4^Y}{\tilde{\tau}},\breve{E})$ is pair wise soft regular T_3 space. `

Proof. First we prove that $(\breve{Y},\overset{1^Y}{\tilde{\tau}},\overset{2^Y}{\tilde{\tau}},\overset{3^Y}{\tilde{\tau}},\overset{4^Y}{\tilde{\tau}},\breve{E})$ is pair wise soft regular T_1 space.

Let $x,y\in X,x\neq y$ if $\left(\breve{X},\overset{1}{\tilde{\tau}},\overset{2}{\tilde{\tau}},\overset{3}{\tilde{\tau}},\overset{4}{\tilde{\tau}},\breve{E}\right)$ is pair wise space then this implies that $\left(\breve{X},\overset{1}{\tilde{\tau}},\overset{2}{\tilde{\tau}},\overset{3}{\tilde{\tau}},\overset{4}{\tilde{\tau}},\breve{E}\right)$ is pair wise soft space. So there exists $\left(\breve{X},\overset{1}{\tilde{\tau}},\overset{2}{\tilde{\tau}},\breve{E}\right)$ soft *regular open* $(F,\breve{E})$ and $\left(\breve{X},\overset{3}{\tilde{\tau}},\overset{4}{\tilde{\tau}},\breve{E}\right)$ soft *regular* open set $(G,\breve{E})$ such that $x\in(F,\breve{E})$ and $y\notin(F,\breve{E})$ or $y\in(G,\breve{E})$ and $x\notin(G,\breve{E})$ now $x\in\breve{Y}\ and\ x\notin(G,\breve{E})$. Hence $x\in\breve{Y}\cap\left(F,\breve{E}\right)=(Y_F,E)$ then $y\notin Y\cap(\alpha)$ for some $\alpha\in\breve{E}$. this means that $\alpha\in\breve{E}$ then $y\notin Y\cap F(\alpha)$ for some $\alpha\in\breve{E}$.

Therefore, $y\notin Y\cap\left(F,\breve{E}\right)=\left(Y_F,\breve{E}\right)$. Now $y\in\breve{Y}$ and $y\in\left(G,\breve{E}\right)$. Hence $y\in Y\cap\left(G,\breve{E}\right)=\left(G_Y,\breve{E}\right)$ where $\left(G,\breve{E}\right)\in(X,\tau_3,\tau_4,\breve{E})$. Consider $x\notin(G,E)$ this means that $\alpha\in\breve{E}$ then $x\notin Y\cap G(\alpha)$ for some α$\in\breve{E}$. Therefore $x\notin\breve{Y}\cap\left(G,\breve{E}\right)=(G_Y,\breve{E})$ thus $(\breve{Y},\tau_{1\breve{Y}},\tau_{2\breve{Y}},\tau_{3\breve{Y}},\tau_{4\breve{Y}},\breve{E})$ is pair wise soft regular T_1 space.

Now we prove that $(\breve{Y},\overset{1^Y}{\tilde{\tau}}\ \overset{2^Y}{\tilde{\tau}},\overset{3^Y}{\tilde{\tau}},\overset{4^Y}{\tilde{\tau}},\breve{E})$ is pair wise soft regular T_3 space then $(\breve{Y},\overset{1^Y}{\tilde{\tau}}\ \overset{2^Y}{\tilde{\tau}},\overset{3^Y}{\tilde{\tau}},\overset{4^Y}{\tilde{\tau}},\breve{E})$ is pair wise soft R-regular space.

Let $y \in \breve{Y}$ and $(G, \breve{E})$ be a soft *regular* closed set in Y such that $y \notin (G, E)$ where $(G, E) \in \left(\breve{X}, \overset{1}{\breve{\tau}}, \overset{2}{\breve{\tau}}, \overset{3}{\breve{\tau}}, \overset{4}{\breve{\tau}}, \breve{E}\right)$ then $(G, \breve{E}) = (Y, \breve{E}) \cap (F, \breve{E})$ for some soft *regular* closed set in $(\breve{X}, \overset{1}{\breve{\tau}}^{Y} \overset{2}{\breve{\tau}}^{Y}, \overset{3}{\breve{\tau}}^{Y}, \overset{4}{\breve{\tau}}^{Y}, \breve{E})$ Hence $y \notin (Y, \breve{E}) \cap (F, \breve{E})$ but $y \in (Y, \breve{E})$, so $y \notin (F, \breve{E})$ since $\left(\breve{X}, \overset{1}{\breve{\tau}}, \overset{2}{\breve{\tau}}, \overset{3}{\breve{\tau}}, \overset{4}{\breve{\tau}}, \breve{E}\right)$is soft regular T_3 space $(X, \tau_1, \tau_2, \tau_3, \tau_4, \breve{E})$ is soft *regular* space so there exists $\left(\breve{X}, \overset{1}{\breve{\tau}}, \overset{2}{\breve{\tau}}, \breve{E}\right)$ soft *regular open* set $(F_1, \breve{E})$ and $\left(\breve{X}, \overset{3}{\breve{\tau}}, \overset{4}{\breve{\tau}}, \breve{E}\right)$ soft *regular* open set $(F_2, \breve{E})$ such that

$$y \in (F_1, \breve{E}), (G, E) \subseteq (F_2, \breve{E})$$
$$(F_1, E)(F_2, E) = \phi$$

Take $(G_1, \breve{E}) = (Y, \breve{E}) \cap (F_2, \breve{E})$ then $(G_1, \breve{E}), (G_2, \breve{E})$ are soft *regular* open set in $\breve{Y}$ such that

$$y \in (G_1, \breve{E}), (G, \breve{E}) \subseteq (Y, \breve{E}) \cap (F_2, \breve{E}) = (G_2, \breve{E})$$
$$(G_1, \breve{E}) \cap (G_2, \breve{E}) \subseteq (F_1, \breve{E}) \cap (F_2, \breve{E}) = \phi$$
$$(G_1, \breve{E}) \cap (G_2, \breve{E}) = \phi$$

There fore$\overset{1}{\breve{\tau}}^{Y} \cup \overset{2}{\breve{\tau}}^{Y}$ is soft R-regular space with respect to $\overset{3}{\breve{\tau}}^{Y} \cup \overset{4}{\breve{\tau}}^{Y}$. Similarly, Let $y \in \breve{Y}$ and $(G, \breve{E})$ be a soft *regular* closed sub set in $\breve{Y}$ such that $y \notin (G, \breve{E})$, where $(G, \breve{E}) \in$ ($\left(\breve{X}, \overset{3}{\breve{\tau}}, \overset{4}{\breve{\tau}}, \breve{E}\right)$ then $(G, \breve{E}) = (Y, \breve{E}) \cap (F, \breve{E})$ where $(F, \breve{E})$ is some soft *regular* closed set in $\left(\breve{X}, \overset{1}{\breve{\tau}}, \overset{2}{\breve{\tau}}, \breve{E}\right)$ $y \notin (Y, \breve{E}) \cap (F, \breve{E})$ But $y \in (Y, \breve{E})$ so $y \notin (F, \breve{E})$ since $\left(\breve{X}, \overset{1}{\breve{\tau}}, \overset{2}{\breve{\tau}}, \breve{E}\right)$ is soft R-regular space so there exists $\left(\breve{X}, \overset{3}{\breve{\tau}}, \overset{4}{\breve{\tau}}, \breve{E}\right)$ soft *regular* open set $(F_1, \breve{E})$ and $\left(\breve{X}, \overset{1}{\breve{\tau}}, \overset{2}{\breve{\tau}}, \breve{E}\right)$ soft *regular* open set (F_2, E). Such that

$$y \in (F_1, \breve{E}), (G, \breve{E}) \subseteq (F_2, \breve{E})$$
$$(F_1, \breve{E}) \cap (F_2, \breve{E}) = \phi$$

Take

$$(G_1, \breve{E}) = (Y, \breve{E}) \cap (F_1, \breve{E})$$
$$(G_1, \breve{E}) = (Y, \breve{E}) \cap (F_1, \breve{E})$$

Then (G_1, E) and $(G_2, \breve{E})$ are soft *regular* open set in Y such that

$$y \in (G_1, \breve{E}), (G, \breve{E}) \subseteq (Y, \breve{E}) \cap (F_2, \breve{E}) = (G_2, \breve{E})$$
$$(G_1, \breve{E}) \cap (G_2, \breve{E}) \subseteq (F_1, \breve{E}) \cap (F_2, \breve{E}) = \phi$$

Therefore $\overset{3}{\tau}^{Y} \cup \overset{4}{\tau}^{Y}$ is soft *regular* space with respect $\overset{1}{\tau}^{Y} \cup \overset{2}{\tau}^{Y} \Rightarrow (\breve{Y}, \overset{1}{\tau}^{Y} \overset{2}{\tau}^{Y}, \overset{3}{\tau}^{Y}, \overset{4}{\tau}^{Y}, \breve{E})$ is pair wise soft regular T_3 space.

Proposition 7. Let $\left(\breve{X}, \overset{1}{\tau}, \overset{2}{\tau}, \overset{3}{\tau}, \overset{4}{\tau}, \breve{E}\right)$ be a soft quad topological space over $\breve{X}$ and $\breve{Y}$ be a soft *regular* closed sub space of X. If $\left(\breve{X}, \overset{1}{\tau}, \overset{2}{\tau}, \overset{3}{\tau}, \overset{4}{\tau}, \breve{E}\right)$ is pair wise soft regular T_4 space then $(\breve{Y}, \overset{1}{\tau}^{Y} \overset{2}{\tau}^{Y}, \overset{3}{\tau}^{Y}, \overset{4}{\tau}^{Y}, \breve{E})$ is pair wise soft regular T_4 space.

Proof. Since $\left(\breve{X}, \overset{1}{\tau}, \overset{2}{\tau}, \overset{3}{\tau}, \overset{4}{\tau}, \breve{E}\right)$ is pair wise soft regular T_4 space. So this implies that $\left(\breve{X}, \overset{1}{\tau}, \overset{2}{\tau}, \overset{3}{\tau}, \overset{4}{\tau}, \breve{E}\right)$ is pair wise soft regular T_1 space as proved above.

We prove $\left(\breve{X}, \overset{1}{\tau}, \overset{2}{\tau}, \overset{3}{\tau}, \overset{4}{\tau}, \breve{E}\right)$ is pair wise soft *regular* normal space. Let $(G_1, \breve{E}), (G_2, \breve{E})$ be soft *regular* closed sets in $\breve{Y}$ such that

$$(G_1, \breve{E}) \cap (G_2, \breve{E}) = \phi$$

Then

$$(G_1, \breve{E}) = (Y, \breve{E}) \cap (F_1, \breve{E})$$

and

$$(G_2, \breve{E}) = (\breve{Y}, \breve{E}) \cap (F_2, \breve{E})$$

For some soft *regular* closed sets such that $(F_1, \breve{E})$ is soft *regular* closed set in $\overset{1}{\tau} \cup \overset{2}{\tau}$ soft *regular* closed set $(F_2, \breve{E})$ in $\overset{3}{\tau}^{Y} \cup \overset{4}{\tau}^{Y}$ and $(F_1, \breve{E}) \cap (F_2, \breve{E}) = \phi$ From Proposition 2**.** Since, $\breve{Y}$ is soft *regular* closed sub set of X then $(G_1, \breve{E}), (G_2, \breve{E})$ are soft *regular* closed sets in $\breve{X}$such that

$$(G_1, \breve{E}) \cap (G_2, \breve{E}) = \phi$$

Since $\left(\breve{X}, \overset{1}{\tau}, \overset{2}{\tau}, \overset{3}{\tau}, \overset{4}{\tau}, \breve{E}\right)$ is pair wise softs *regular* normal space. So there exists soft *regular open* sets $(H_1, \breve{E})$ and (H_2, E) such that $(H_1, \breve{E})$ is soft *regular open* set in $\tau_1 \cup \tau_2$ and $(H_2, \breve{E})$ is soft *regular* open set in $\overset{3}{\tau}^{Y} \cup \overset{4}{\tau}^{Y}$ such that

$$(G_1, \breve{E}) \subseteq (H_1, \breve{E})$$
$$(G_2, \breve{E}) \subseteq (H_2, \breve{E})$$
$$(H_1, \breve{E}) \cap (H_2, \breve{E}) = \phi$$

Since

$$\left(G_1,\breve{E}\right),\left(G_2,\breve{E}\right)\subseteq\ (Y,\breve{E})$$

Then

$$\left(G_1,\breve{E}\right)\subseteq\left(Y,\breve{E}\right)\cap\ (H_1,\breve{E})$$
$$\left(G_2,\breve{E}\right)\subseteq\left(Y,\breve{E}\right)\cap\ (H_2,\breve{E})$$

and

$$\left[\left(Y,\breve{E}\right)\cap\left(H_1,\breve{E}\right)\right]\cap\left[\left(Y,\breve{E}\right)\cap\left(H_2,\breve{E}\right)\right]=\phi$$

Where $\left(Y,\breve{E}\right)\cap\left(H_1,\breve{E}\right)$ and $\left(Y,\breve{E}\right)\cap\left(H_2,\breve{E}\right)$ are soft *regular* open sets in Y there fore $\overset{1}{\ddot{\tau}}{}^{Y}\cup\overset{2}{\ddot{\tau}}{}^{Y}$ is soft *regular* normal space with respect to $\overset{3}{\ddot{\tau}}{}^{Y}\cup\overset{4}{\ddot{\tau}}{}^{Y}$. Similarly, Let $\left(G_1,\breve{E}\right),\left(G_2,\breve{E}\right)$ be soft *regular* closed sub set in Y such that

$$\left(G_1,\breve{E}\right)\cap\left(G_2,\breve{E}\right)=\phi$$

Then

$$\left(G_1,\breve{E}\right)=\ (Y,\breve{E})\ \cap\left(F_1,\breve{E}\right)$$

and

$$\left(G_2,\breve{E}\right)=\ (Y,\breve{E})\ \cap\left(F_2,\breve{E}\right)$$

For some soft *regular* closed sets such that $(F_1,\breve{E})$ is soft *regular* closed set in $\overset{3}{\ddot{\tau}}\cup\overset{4}{\ddot{\tau}}$ *and* $(F_2,\breve{E})$ soft *regular* closed set in $\overset{1}{\ddot{\tau}}{}^{Y}\cup\overset{2}{\ddot{\tau}}{}^{Y}$ and $\left(F_1,\breve{E}\right)\left(F_2,\breve{E}\right)=\phi$ from Proposition 2. Since, $\breve{Y}$ is soft *regular* closed sub set in $\breve{X}$ then $\left(G_1,\breve{E}\right),\left(G_2,\breve{E}\right)$ are soft *regular* closed sets in $\breve{X}$ such that

$$\left(G_1,\breve{E}\right)\cap\left(G_2,\breve{E}\right)=\phi$$

Since $\left(\breve{X},\overset{1}{\ddot{\tau}},\overset{2}{\ddot{\tau}},\overset{3}{\ddot{\tau}},\overset{4}{\ddot{\tau}},\breve{E}\right)$ is pair wise soft *regular* normal space so there exists soft *regular* open sets $\left(H_1,\breve{E}\right)$ and $\left(H_2,\breve{E}\right)$. Such that $\left(H_1,\breve{E}\right)$ is soft *regular open* set is $\overset{3}{\ddot{\tau}}\cup\overset{4}{\ddot{\tau}}$ and (H_2,E) is soft *regular open* set in $\overset{1}{\ddot{\tau}}{}^{Y}\cup\overset{2}{\ddot{\tau}}{}^{Y}$ such that

$$\left(G_1,\breve{E}\right)\subseteq\left(H_1,\breve{E}\right)$$
$$\left(G_2,\breve{E}\right)\subseteq\left(H_2,\breve{E}\right)$$
$$\left(H_1,\breve{E}\right)\cap\left(H_2,\breve{E}\right)=\phi$$

Since

$$\left(G_1,\breve{E}\right),\left(G_2,\breve{E}\right)\subseteq\ (Y,\breve{E})$$

Then

$$\left(G_1,\breve{E}\right)\subseteq\left(Y,\breve{E}\right)\cap\ (H_1,\breve{E})$$
$$\left(G_2,\breve{E}\right)\subseteq\left(Y,\breve{E}\right)\cap\ (H_2,\breve{E})$$

and

$$\left[\left(Y,\breve{E}\right)\cap\left(H_1,\breve{E}\right)\right]\cap\left[\left(Y,\breve{E}\right)\cap\left(H_2,\breve{E}\right)\right]=\phi$$

Where $(Y,\breve{E}) \cap (H_1,\breve{E})$ and $(Y,\breve{E}) \cap (H_2,\breve{E})$ are soft *regular open* sets in $\breve{Y}$ there fore $\overset{3}{\breve{\tau}}{}^{Y} \cup \overset{4}{\breve{\tau}}{}^{Y}$ is soft *regular* normal space with respect to $\overset{1}{\breve{\tau}}{}^{Y} \cup \overset{2}{\breve{\tau}}{}^{Y} \Rightarrow (\breve{Y}, \overset{1}{\breve{\tau}}{}^{Y} \overset{2}{\breve{\tau}}{}^{Y}, \overset{3}{\breve{\tau}}{}^{Y}, \overset{4}{\breve{\tau}}{}^{Y}, \breve{E})$ is pair wise soft regular T_4 space.

3. 2 Soft Regular Separation Axioms in Soft Quad Topological Spaces with Respect to Soft Points

In this section, we introduced soft topological structures known as soft regular separation axioms in soft quad topology with respect to soft points. With the applications of these soft *regular* separation axioms different result are brought under examination.

Definition 33 In a *soft quad topological* space $\left(\breve{X}, \overset{1}{\breve{\tau}}, \overset{2}{\breve{\tau}}, \overset{3}{\breve{\tau}}, \overset{4}{\breve{\tau}}, \breve{E}\right)$

1) $\overset{1}{\breve{\tau}} \cup \overset{2}{\breve{\tau}}$ said to be *soft regular* T_0 space with respect to $\overset{3}{\breve{\tau}} \cup \overset{4}{\breve{\tau}}$ if for each pair of distinct points $e_G, e_H \in X_A$there happens $\overset{1}{\breve{\tau}} \cup \overset{2}{\breve{\tau}}$ soft *regular* open set $(F,\breve{E})$ and a $\overset{3}{\breve{\tau}} \cup \overset{4}{\breve{\tau}}$ soft *regular* open set $(G,\breve{E})$ such that $e_G \in (F,\breve{E})$ and $e_H \notin (G,\breve{E})$, Similarly, $\overset{3}{\breve{\tau}} \cup \overset{4}{\breve{\tau}}$is said to be *soft regular* T_0 space with respect to $\overset{1}{\breve{\tau}} \cup \overset{2}{\breve{\tau}}$ if for each pair of distinct points $e_G, e_H \in X_A$there happens $\overset{3}{\breve{\tau}} \cup \overset{4}{\breve{\tau}}$ *soft regular open set* (F,E) and a $\overset{1}{\breve{\tau}} \cup \overset{2}{\breve{\tau}}$ *regular* soft open set (G,E) such that $e_G \in (F,\breve{E})$ and $e_H \notin (F,\breve{E})$ or $e_H \in (G,\breve{E})$ and $e_G \notin (G,\breve{E})$. *Soft quad topological spaces* $\left(\breve{X}, \overset{1}{\breve{\tau}}, \overset{2}{\breve{\tau}}, \overset{3}{\breve{\tau}}, \overset{4}{\breve{\tau}}, \breve{E}\right)$ is said to be *pair wise soft regular* T_0 space if $\overset{1}{\breve{\tau}} \cup \overset{2}{\breve{\tau}}$ is *soft regular* T_0 *space* with respect to $\overset{3}{\breve{\tau}} \cup \overset{4}{\breve{\tau}}$ and $\overset{3}{\breve{\tau}} \cup \overset{4}{\breve{\tau}}$ is soft regular T_0 spaces with respect to $\overset{1}{\breve{\tau}} \cup \overset{2}{\breve{\tau}}$

2) $\overset{1}{\breve{\tau}} \cup \overset{2}{\breve{\tau}}$is said to be soft *regular* T_1 space with respect to $\overset{3}{\breve{\tau}} \cup \overset{4}{\breve{\tau}}$ if for each pair of distinct points $e_G, e_H \in X_A$ there happens $\overset{1}{\breve{\tau}} \cup \overset{2}{\breve{\tau}}$ *soft regular open set* $(F,\breve{E})$ and $\overset{3}{\breve{\tau}} \cup \overset{4}{\breve{\tau}}$ soft *regular open* set $(G,\breve{E})$ such that $e_G \in (F,\breve{E})$ and $e_H \notin (G,\breve{E})$ and $e_H \in (G,\breve{E})$ and $e_G \notin (G,E)$. Similarly, $\overset{3}{\breve{\tau}} \cup \overset{4}{\breve{\tau}}$is said to be soft regular T_1 space with respect to $\overset{1}{\breve{\tau}} \cup \overset{2}{\breve{\tau}}$ if for each pair of distinct points $e_G, e_H \in X_A$there exist $\overset{3}{\breve{\tau}} \cup \overset{4}{\breve{\tau}}$ soft *regular open* set $(F,\breve{E})$ and a $\overset{1}{\breve{\tau}} \cup \overset{2}{\breve{\tau}}$ soft *regular* open set $(G,\breve{E})$ such that $e_G \in (F,\breve{E})$ and $e_H \notin (G,\breve{E})$ and $e_H \in (G,\breve{E})$ and $e_G \notin (G,\breve{E})$. *Soft quad topological space* $\left(\breve{X}, \overset{1}{\breve{\tau}}, \overset{2}{\breve{\tau}}, \overset{3}{\breve{\tau}}, \overset{4}{\breve{\tau}}, \breve{E}\right)$ is said to be *pair wise soft regular* T_1 space if $\overset{1}{\breve{\tau}} \cup \overset{2}{\breve{\tau}}$ is *soft regular* T_1 *space* with respect to $\overset{3}{\breve{\tau}} \cup \overset{3}{\breve{\tau}}$ and $\overset{3}{\breve{\tau}} \cup \overset{4}{\breve{\tau}}$is soft regular T_1 spaces with respect to $\overset{1}{\breve{\tau}} \cup \overset{2}{\breve{\tau}}$.

3) $\overset{1}{\breve{\tau}} \cup \overset{2}{\breve{\tau}}$ is said to be soft regular T_2 space with respect to $\overset{3}{\breve{\tau}} \cup \overset{4}{\breve{\tau}}$, if for each pair of distinct points $e_G, e_H \in X_A$ there happens a $\overset{1}{\breve{\tau}} \cup \overset{2}{\breve{\tau}}$ soft *regular* open set $(F, \breve{E})$ and a $\overset{3}{\breve{\tau}} \cup \overset{4}{\breve{\tau}}$ soft *regular* open set $(G, \breve{E})$ such that $e_G \in (F, \breve{E})$ and $e_H \notin (G, \breve{E})$ and $e_H \in (G, \breve{E})$ and $e_G \notin (G, \breve{E})$ and $(F, \breve{E}) \cap (G, \breve{E}) = \phi$. Similarly, $\tau_3 \cup \tau_4$ is aid to be *soft regular* T_2 space with respect to $\overset{1}{\breve{\tau}} \cup \overset{2}{\breve{\tau}}$ if for each pair of distinct points $e_G, e_G \in X_A$ there happens $\overset{3}{\breve{\tau}} \cup \overset{4}{\breve{\tau}}$ *soft regular open* set (F, E) and $\overset{1}{\breve{\tau}} \cup \overset{2}{\breve{\tau}}$ *soft regular open* set $(G, \breve{E})$ such that $e_G \in (F, \breve{E})$ and $e_G \in (G, \breve{E})$ and $(F, \breve{E}) \cap (G, \breve{E}) = \phi$. The soft quad topological space $\left(\breve{X}, \overset{1}{\breve{\tau}}, \overset{2}{\breve{\tau}}, \overset{3}{\breve{\tau}}, \overset{4}{\breve{\tau}}, \breve{E}\right)$ is said to be pair wise soft regular T_2 space if $\overset{1}{\breve{\tau}} \cup \overset{2}{\breve{\tau}}$ is soft regular T_2 space with respect to $\overset{3}{\breve{\tau}} \cup \overset{4}{\breve{\tau}}$ and $\overset{3}{\breve{\tau}} \cup \overset{4}{\breve{\tau}}$ is soft regular T_2 space with respect to $\overset{1}{\breve{\tau}} \cup \overset{2}{\breve{\tau}}$.

Definition 34 In a soft quad topological space $\left(\breve{X}, \overset{1}{\breve{\tau}}, \overset{2}{\breve{\tau}}, \overset{3}{\breve{\tau}}, \overset{4}{\breve{\tau}}, \breve{E}\right)$

1) $\overset{1}{\breve{\tau}} \cup \overset{2}{\breve{\tau}}$ is said to be soft regular T_3 space with respect to $\overset{3}{\breve{\tau}} \cup \overset{4}{\breve{\tau}}$ if $\overset{1}{\breve{\tau}} \cup \overset{2}{\breve{\tau}}$ is soft regular T_1 space with respect to $\tau_3 \cup \tau_4$ and for each pair of distinct points $e_G, e_H \in X_A$, there exists $\overset{1}{\breve{\tau}} \cup \overset{2}{\breve{\tau}}$.

regular closed soft set $(G, \breve{E})$ such that $e_G \notin (G, \breve{E})$, $\overset{1}{\breve{\tau}} \cup \overset{2}{\breve{\tau}}$ soft *regular open* set $(F_1, \breve{E})$ and $\overset{3}{\breve{\tau}} \cup \overset{4}{\breve{\tau}}$ soft *regular open* set $(F_2, \breve{E})$ such that $e_G \in (F_1, \breve{E}), (G, \breve{E}) \subseteq (F_2, \breve{E})$ and $(F_1, \breve{E}) \cap (F_2, \breve{E}) = \emptyset$. Similarly, $\tau_3 \cup \tau_4$is said to be soft regular T_3 space with respect to $\overset{1}{\breve{\tau}} \cup \overset{2}{\breve{\tau}}$ if $\overset{3}{\breve{\tau}} \cup \overset{4}{\breve{\tau}}$ is soft regular T_1 space with respect to $\overset{1}{\breve{\tau}} \cup \overset{2}{\breve{\tau}}$ and for each pair of distinct points $e_G, e_H \in X_A$there exists a $\overset{3}{\breve{\tau}} \cup \overset{4}{\breve{\tau}}$ soft *regular* closed set (G, E) such that $e_G \notin (G, E)$, $\overset{3}{\breve{\tau}} \cup \overset{4}{\breve{\tau}}$ soft *regular* open set (F_1, E) amd $\overset{1}{\breve{\tau}} \cup \overset{2}{\breve{\tau}}$ soft *regular* open set (F_2, E) such that $e_G \in (F_1, E), (G, E) \subseteq (F_2, \breve{E}$ and $(F_1, \breve{E}) \cap (F_2, \breve{E}) = \phi$. $\left(\breve{X}, \overset{1}{\breve{\tau}}, \overset{2}{\breve{\tau}}, \overset{3}{\breve{\tau}}, \overset{4}{\breve{\tau}}, \breve{E}\right)$ is said to be pair wise soft regular T_3 space if $\overset{1}{\breve{\tau}} \cup \overset{2}{\breve{\tau}}$is soft regular T_3 space with respect to $\overset{3}{\breve{\tau}} \cup \overset{4}{\breve{\tau}}$ and $\overset{3}{\breve{\tau}} \cup \overset{4}{\breve{\tau}}$is soft regular T_3 space with respect to $\overset{1}{\breve{\tau}} \cup \overset{2}{\breve{\tau}}$.

2) $\overset{1}{\breve{\tau}} \cup \overset{2}{\breve{\tau}}$is said to be soft regular T_4 space with respect to $\overset{3}{\breve{\tau}} \cup \overset{4}{\breve{\tau}}$ if $\overset{1}{\breve{\tau}} \cup \overset{2}{\breve{\tau}}$is soft regular T_1 space with respect to $\overset{3}{\breve{\tau}} \cup \overset{4}{\breve{\tau}}$, there exists a $\overset{1}{\breve{\tau}} \cup \overset{2}{\breve{\tau}}$ soft *regular* closed set (F_1, E) and $\overset{3}{\breve{\tau}} \cup \overset{4}{\breve{\tau}}$ soft *regular* closed set $(F_2, \breve{E})$ such that $(F_1, \breve{E}) \cap (F_2, \breve{E}) = \emptyset$,also,open there exists $(F_3, \breve{E})$ and $(G_1, \breve{E})$ such that $(F_3, \breve{E})$ is soft $\overset{1}{\breve{\tau}} \cup \overset{2}{\breve{\tau}}$ *regular*

open set, (G_1, E) is soft $\overset{3}{\breve{\tau}} \cup \overset{4}{\breve{\tau}}$ *regular* set such that $(F_1, \breve{E}) \subseteq (F_3, \breve{E})$, $(F_2, \breve{E}) \subseteq (G_1, \breve{E})$. Similarly, $\overset{3}{\breve{\tau}} \cup \overset{4}{\breve{\tau}}$is said to be soft regular T_4 space with respect to $\overset{1}{\breve{\tau}} \cup \overset{2}{\breve{\tau}}$ if $\overset{3}{\breve{\tau}} \cup \overset{4}{\breve{\tau}}$is soft regular T_1 space with respect to $\overset{1}{\breve{\tau}} \cup \overset{2}{\breve{\tau}}$there exists $\overset{3}{\breve{\tau}} \cup \overset{4}{\breve{\tau}}$ soft *regular* closed set $(F_1, \breve{E})$ and $\overset{1}{\breve{\tau}} \cup \overset{2}{\breve{\tau}}$ soft *regular* closed set $(F_2, \breve{E})$ such that $(F_1, \breve{E}) \cap (F_2, \breve{E}) = \phi$. Also there exists $(F_3, \breve{E})$ and $(G_1, \breve{E})$ such that $(F_3, \breve{E})$ is soft $\overset{3}{\breve{\tau}} \cup \overset{4}{\breve{\tau}}$ *regular* open set, $(G_1, \breve{E})$ is soft $\overset{1}{\breve{\tau}} \cup \overset{2}{\breve{\tau}}$ *regular* soft set such that $(F_1, E) \subseteq (F_3, E), (F_2, E) \subseteq (G_1, E)$ and $(F_3, E) \cap (G_1, E) = \phi$. Thus, $(\left(\breve{X}, \overset{1}{\breve{\tau}}, \overset{2}{\breve{\tau}}, \overset{3}{\breve{\tau}}, \overset{4}{\breve{\tau}}, \breve{E}\right)$is said to be pair wise soft regular T_4 space if $\overset{1}{\breve{\tau}} \cup \overset{2}{\breve{\tau}}$is soft regular T_4 space with respect to $\overset{3}{\breve{\tau}} \cup \overset{4}{\breve{\tau}}$ and to $\overset{3}{\breve{\tau}} \cup \overset{4}{\breve{\tau}}$is soft regular T_4 space with respect to $\overset{1}{\breve{\tau}} \cup \overset{2}{\breve{\tau}}$.

Proposition 8. Let $\left(\breve{X}, \overset{1}{\breve{\tau}}, \overset{2}{\breve{\tau}}, \overset{3}{\breve{\tau}}, \overset{4}{\breve{\tau}}, \breve{E}\right)$ be a soft topological space over $X.\left(\breve{X}, \overset{1}{\breve{\tau}}, \overset{2}{\breve{\tau}}, \overset{3}{\breve{\tau}}, \overset{4}{\breve{\tau}}, \breve{E}\right)$ is soft *regular* T_3 space , then for all $e_G \in X_E e_G = (e_G, \breve{E})$ is soft *regular*-closed set.

Proof. We want to prove that $e_G$is *regular* closed soft set, which is sufficient to prove that e_G^c is *regular open* soft set for all $e_H \in \{e_G\}^c$. Since $(X, \tau_1, \tau_2, \tau_3, \tau_4, \breve{E})$ is soft regular T_3 space , then there exists soft *regular* sets $(F, E)_{e_H}$ and (G, E) such that $e_{H_E} \subseteq (F, \breve{E})_{e_H}$ and $e_{G_E} \cap (F, \breve{E})_{e_H} = \phi$ and $e_{G_E} \subseteq (G, \breve{E})$ and $e_{H_E} \cap (G, \breve{E}) = \phi$. It follows that, $\cup_{e_H \in (e_G)^c} (F, \breve{E})_{e_H} \subseteq e_{G_E}^c$ Now, we want to prove that $e_{G_{\breve{E}}}^c \subseteq \cup_{e_H \in (e_G)^c} (F, \breve{E})_{e_H}$. Let $\cup_{e_H \in (e_G)^c} (F, \breve{E})_{e_H} = (H, \breve{E})$. Where $H(e) = \cup_{e_H \in (e_G)^c} (F(e)_{e_H}$ for all $e \in \breve{E}$. Since $e_{G_E}^c(e) = (e_G)^c$for all $e \in \breve{E}$ from Definition 9, so, for all $e_H \in \{e_G\}^c$ and $e \in \breve{E} e_{G_E}^c(e) = \{e_G\}^c = \cup_{e_H \in (e_G)^c} \{e_H\} = \cup_{e_H \in (e_G)^c} F(e)_{e_H} = H(e)$. Thus, $e_{G_E}^c \subseteq \cup_{e_H \in (e_G)^c} (F, E)_{e_H}$ from Definition 2, and so, $e_{G_E}^c = \cup_{e_H \in (e_G)^c} (F, E)_{e_H}$. This means that , $e_{G_E}^c$ is soft *regular open* set for all $e_H \in \{e_G\}^c$. Therefore, $e_{G_E}$is *regular* closed soft set.

Proposition 9. Let $\left(\breve{X}, \overset{1}{\breve{\tau}}, \overset{2}{\breve{\tau}}, \overset{3}{\breve{\tau}}, \overset{4}{\breve{\tau}}, \breve{E}\right)$be a soft quad topological space over X. Then, if $\left(\breve{X}, \overset{1}{\breve{\tau}}, \overset{2}{\breve{\tau}}, \breve{E}\right)$ and $\left(\breve{X}, \overset{3}{\breve{\tau}}, \overset{4}{\breve{\tau}}, \breve{E}\right)$are soft regular qT_3 space , then $\left(\breve{X}, \overset{1}{\breve{\tau}}, \overset{2}{\breve{\tau}}, \overset{3}{\breve{\tau}}, \overset{4}{\breve{\tau}}, \breve{E}\right)$is a pair wise soft regular T_2 space.

Proof. Suppose if $\left(\breve{X}, \overset{1}{\breve{\tau}}, \overset{2}{\breve{\tau}}, \breve{E}\right)$ is a soft regular T_3 space with respect to $\left(\breve{X}, \overset{3}{\breve{\tau}}, \overset{4}{\breve{\tau}}, \breve{E}\right)$, then according to definition for, $e_G \neq e_{H,} e_G, e_H \in X_A$, by using Theorem 8, $(e_H, \breve{E})$ is soft *regular* closed set in $\left(\breve{X},\overset{3}{\tau},\overset{4}{\tau},\breve{E}\right)$ and $e_G \notin (e_H,\breve{E})$ there exist a $\left(\breve{X},\overset{1}{\tau},\overset{2}{\tau},\breve{E}\right)$ soft *regular* open set (F,E) and a $\left(\breve{X},\overset{3}{\tau},\overset{4}{\tau},\breve{E}\right)$ soft *regular* open set (G,E) such that $e_G \in (F,\breve{E})$, $e_H \in (y,\breve{E}) \subseteq (G,\breve{E})$ and $(F_1,\breve{E}) \cap (F_2,\breve{E}) = \phi$. Hence $\left(\breve{X},\overset{1}{\tau},\overset{2}{\tau},\breve{E}\right)$ is soft regular T_2 space with respect to $\left(\breve{X},\overset{3}{\tau},\overset{4}{\tau},\breve{E}\right)$ Similarly, if $\left(\breve{X},\overset{3}{\tau},\overset{4}{\tau},\breve{E}\right)$ is a soft regular T_3 space with respect to $\left(\breve{X},\overset{1}{\tau},\overset{2}{\tau},\breve{E}\right)$, then according to definition for, $e_G \neq e_H, e_G, e_H \in X_A$, by using Theorem 8, $(e_G,\breve{E})$ is *regular* closed soft set in $\left(\breve{X},\overset{1}{\tau},\overset{2}{\tau},\breve{E}\right)$ is and $y \notin (x,\breve{E})$ there exists a $\left(\breve{X},\overset{3}{\tau},\overset{4}{\tau},\breve{E}\right)$ soft *regular open* set $(F,\breve{E})$ and a $\left(\breve{X},\overset{1}{\tau},\overset{2}{\tau},\breve{E}\right)$ soft *regular* open set $(G,\breve{E})$ such that $e_H \in (F,\breve{E})$, $e_G \in (x,\breve{E}) \subseteq (G,\breve{E})$ and $(F_1,\breve{E}) \cap (F_2,\breve{E}) = \phi$. Hence $\left(\breve{X},\overset{3}{\tau},\overset{4}{\tau},\breve{E}\right)$ is a soft regular T_2 space. Thus $\left(\breve{X},\overset{1}{\tau},\overset{2}{\tau},\overset{3}{\tau},\overset{4}{\tau},\breve{E}\right)$ is a pair wise soft s *regular* T_2 space.

Proposition 10. Let $\left(\breve{X},\overset{1}{\tau},\overset{2}{\tau},\overset{3}{\tau},\overset{4}{\tau},\breve{E}\right)$ be a soft quad topological space over $\breve{X}$. if $\left(\breve{X},\overset{1}{\tau},\overset{2}{\tau},\breve{E}\right)$ and $\left(\breve{X},\overset{3}{\tau},\overset{4}{\tau},\breve{E}\right)$ are soft regular T_3 space then $\left(\breve{X},\overset{1}{\tau},\overset{2}{\tau},\overset{3}{\tau},\overset{4}{\tau},\breve{E}\right)$ is a pair wise soft regular T_3 space.

Proof. Suppose $\left(\breve{X},\overset{1}{\tau},\overset{2}{\tau},\breve{E}\right)$ is a soft regular T_3 space with respect to $\left(\breve{X},\overset{3}{\tau},\overset{4}{\tau},\breve{E}\right)$ then according to definition for $e_G, e_H \in X_A e_G \neq e_H$ there happens $\tau_1 \cup \tau_2$ soft *regular open* set (F,E) and a $\overset{3}{\tau} \cup \overset{4}{\tau}$ soft *regular* open set (G,E) such that $e_G \in (F,\breve{E})$ and $e_H \notin (F,\breve{E})$ or $e_H \in (G,\breve{E})$ and $e_G \notin (G,\breve{E})$ and for each point $e_G \in X_A$ and each $\overset{1}{\tau} \cup \overset{2}{\tau}$ *regular* closed soft set $(G_1,\breve{E})$ such that $e_G \notin (G_1,\breve{E})$ there happens a $\overset{1}{\tau} \cup \overset{2}{\tau}$ soft *regular* open set $(F_1,\breve{E})$ and $\overset{3}{\tau} \cup \overset{4}{\tau}$ soft *regular open* set $(F_2,\breve{E})$ such that $e_G \in (F_1,\breve{E}), (G_1,\breve{E}) \subseteq (F_2,\breve{E})$ and $(F_1,\breve{E}) \cap (F_2,\breve{E}) = \phi$. Similarly $\left(\breve{X},\overset{3}{\tau},\overset{4}{\tau},\breve{E}\right)$ is a soft regular T_3 space with respect to $\left(\breve{X},\overset{1}{\tau},\overset{2}{\tau},\breve{E}\right)$ So according to definition for $e_G, e_H \in X_A$, $e_G \neq e_H$ there exists a $\overset{3}{\tau} \cup \overset{4}{\tau}$ soft *regular open* set $(F,\breve{E})$ and $\overset{1}{\tau} \cup \overset{2}{\tau}$ soft *regular open* set $(G,\breve{E})$ such that $e_H \in (F,\breve{E})$ and $e_H \notin (F,\breve{E})$ or $e_H \in (G,\breve{E})$ and

$e_G \notin (G,\breve{E})$ and for each point $e_G \in X_A$ and each $\overset{3}{\breve{\tau}} \cup \overset{4}{\breve{\tau}}$s *regular* closed soft set $(G_1,\breve{E})$ such that $e_G \notin (G_1,\breve{E})$ there exists $\overset{3}{\breve{\tau}} \cup \overset{4}{\breve{\tau}}$ soft *regular* open set $(F_1,\breve{E})$ and $\overset{1}{\breve{\tau}} \cup \overset{2}{\breve{\tau}}$ soft *regular open* set $(F_2,\breve{E})$ such that $e_G \in (F_1,\breve{E}), (G_1,\breve{E}) \subseteq (F_2,\breve{E})$ and $(F_1,\breve{E}) \cap (F_2,\breve{E}) = \phi$. Hence $\left(\breve{X}, \overset{1}{\breve{\tau}}, \overset{2}{\breve{\tau}}, \overset{3}{\breve{\tau}}, \overset{4}{\breve{\tau}}, \breve{E}\right)$ is pair wise soft *regular* T_3 space.

Proposition 11. if $\left(\breve{X}, \overset{1}{\breve{\tau}}, \overset{2}{\breve{\tau}}, \overset{3}{\breve{\tau}}, \overset{4}{\breve{\tau}}, \breve{E}\right)$ be a soft quad topological space over $\breve{X}$. $\left(\breve{X}, \overset{1}{\breve{\tau}}, \overset{2}{\breve{\tau}}, \breve{E}\right)$ and $\left(\breve{X}, \overset{3}{\breve{\tau}}, \overset{4}{\breve{\tau}}, \breve{E}\right)$ are soft regular T_4 space then $\left(\breve{X}, \overset{1}{\breve{\tau}}, \overset{2}{\breve{\tau}}, \overset{3}{\breve{\tau}}, \overset{4}{\breve{\tau}}, \breve{E}\right)$ is pair wise soft regular T_4 space.

Proof. Suppose $\left(\breve{X}, \overset{1}{\breve{\tau}}, \overset{2}{\breve{\tau}}, \breve{E}\right)$ is soft regular T_4 space with respect to $\left(\breve{X}, \overset{3}{\breve{\tau}}, \overset{4}{\breve{\tau}}, \breve{E}\right)$ So according to definition for $e_G, e_H \in \breve{X}$, $e_G \neq e_H$ there happens a $\overset{1}{\breve{\tau}} \cup \overset{2}{\breve{\tau}}$ soft *regular* open set $(F,\breve{E})$ and a $\overset{3}{\breve{\tau}} \cup \overset{4}{\breve{\tau}}$ soft *regular* open set $(G,\breve{E})$ such that $e_G \in (F,\breve{E})$ and $e_H \notin (F,\breve{E})$ or $e_H \in (G,E)$ and $e_G \notin (G,\breve{E})$ each $\overset{1}{\breve{\tau}} \cup \overset{2}{\breve{\tau}}$ soft *regular* closed set (F_1,E) and a $\overset{3}{\breve{\tau}} \cup \overset{4}{\breve{\tau}}$ soft *regular* closed set (F_2,E) such that $(F_1,E) \cap (F_2,E) = \phi$. There occurs (F_3,E) and (G_1,E) such that (F_3,E) is soft $\overset{3}{\breve{\tau}} \cup \overset{4}{\breve{\tau}}$ *regular open* set (G_1,E) is soft a $\overset{1}{\breve{\tau}} \cup \overset{2}{\breve{\tau}}$ *regular* open set $(F_1,E) \subseteq (F_3,E), (F_2,E) \subseteq (G_1,E)$ and $(F_3,E) \cap (G_1,E) = \phi$. Similarly, $\overset{3}{\breve{\tau}} \cup \overset{4}{\breve{\tau}}$ is soft regular T_4 space with respect to $\overset{1}{\breve{\tau}} \cup \overset{2}{\breve{\tau}}$ so according to definition for $e_G, e_H \in X_A, e_G \neq e_H$ there happens a $\overset{3}{\breve{\tau}} \cup \overset{4}{\breve{\tau}}$ soft *regular* open set $(F,\breve{E})$ and a $\overset{1}{\breve{\tau}} \cup \overset{2}{\breve{\tau}}$ soft *regular open* set $(G,\breve{E})$ such that $e_G \in (F,\breve{E})$ and $e_H \notin (F,\breve{E})$ or $e_H \in (G,\breve{E})$ and $e_G \notin (G,\breve{E})$ and for each $\overset{3}{\breve{\tau}} \cup \overset{4}{\breve{\tau}}$ soft *regular* closed set $(F_1,\breve{E})$ and $\overset{1}{\breve{\tau}} \cup \overset{2}{\breve{\tau}}$ soft *regular* closed set $(F_2,\breve{E})$ such that $(F_1,\breve{E}) \cap (F_2,\breve{E}) = \phi$. there occurs $(F_3,\breve{E})$ and $(G_1,\breve{E})$ such that $(F_3,\breve{E})$ is soft $\overset{3}{\breve{\tau}} \cup \overset{4}{\breve{\tau}}$ *regular open* set $(G_1,\breve{E})$ is soft $\overset{1}{\breve{\tau}} \cup \overset{2}{\breve{\tau}}\, regular$semi open set such that $(F_1,\breve{E}) \subseteq (F_3,\breve{E}), (F_2,\breve{E}) \subseteq (G_1,\breve{E})$ and $(F_3,\breve{E}) \cap (G_1,\breve{E}) = \phi$ hence $\left(\breve{X}, \overset{1}{\breve{\tau}}, \overset{2}{\breve{\tau}}, \overset{3}{\breve{\tau}}, \overset{4}{\breve{\tau}}, \breve{E}\right)$ is pair wise soft regular T_4 space.

Proposition 12. Let $\left(\breve{X},\overset{1}{\tilde{\tau}},\overset{2}{\tilde{\tau}},\overset{3}{\tilde{\tau}},\overset{4}{\tilde{\tau}},\breve{E}\right)$ be a soft quad topological space over $\breve{X}$ and $\breve{Y}$ be a non-empty subset of $\breve{X}$. if $(\breve{Y},\overset{1}{\tilde{\tau}}^{Y}\ \overset{2}{\tilde{\tau}}^{Y},\overset{3}{\tilde{\tau}}^{Y},\overset{4}{\tilde{\tau}}^{Y},\breve{E})$ is pair wise soft regular T_3 space. Then $(\breve{Y},\overset{1}{\tilde{\tau}}^{Y}\ \overset{2}{\tilde{\tau}}^{Y},\overset{3}{\tilde{\tau}}^{Y},\overset{4}{\tilde{\tau}}^{Y},\breve{E})$ is pair wise soft regular T_3 space.

Proof. First we prove that $(\breve{Y},\overset{1}{\tilde{\tau}}^{Y}\ \overset{2}{\tilde{\tau}}^{Y},\overset{3}{\tilde{\tau}}^{Y},\overset{4}{\tilde{\tau}}^{Y},\breve{E})$ is pair wise soft *regular* T_1 space.

Let $e_G, e_H \in X_A, e_G \neq e_H$ if $\left(\breve{X},\overset{1}{\tilde{\tau}},\overset{2}{\tilde{\tau}},\overset{3}{\tilde{\tau}},\overset{4}{\tilde{\tau}},\breve{E}\right)$ is pair wise soft space then this implies that $\left(\breve{X},\overset{1}{\tilde{\tau}},\overset{2}{\tilde{\tau}},\overset{3}{\tilde{\tau}},\overset{4}{\tilde{\tau}},\breve{E}\right)$ is pair wise soft $\overset{1}{\tilde{\tau}} \cup \overset{2}{\tilde{\tau}}$ space. So there exists $\overset{1}{\tilde{\tau}} \cup \overset{2}{\tilde{\tau}}$ soft *regular* open set $(G,\breve{E})$ such that $e_G \in (F,\breve{E})$ and $e_H \notin (F,\breve{E})$ or $e_H \in (G,E)$ and $e_G \notin (G,E)$ now$e_G \in Y$ *and* $e_G \notin (G,\breve{E})$. Hence $e_G \in Y \cap (F,\breve{E}) = (Y_F,\breve{E})$ then $e_H \notin Y \cap F\ (\alpha)$ for some $\alpha \in \breve{E}$, this means that $\alpha \in \breve{E}$ then $e_H \notin \breve{Y} \cap F\ (\alpha)$ for some $\alpha \in \breve{E}$.

There fore, $e_H \notin Y \cap (F,\breve{E}) = (Y_F,\breve{E})$. Now $e_H \in \breve{Y}$ and $e_H \in (G,\breve{E})$. Hence, $e_H \in \breve{Y} \cap (G,\breve{E}) = (G_Y,\breve{E})$ where$(G,\breve{E}) \in \overset{3}{\tilde{\tau}} \cup \overset{4}{\tilde{\tau}}$. Consider $x \notin (G,\breve{E})$. this means that $\alpha \in E$ then $x \notin \breve{Y} \cap G\ (\alpha)$ for some$\alpha \in \breve{E}$. There fore $e_G \notin \breve{Y} \cap (G,\breve{E}) = (G_Y,E)$ thus $((\breve{Y},\overset{1}{\tilde{\tau}}^{Y}\ \overset{2}{\tilde{\tau}}^{Y},\overset{3}{\tilde{\tau}}^{Y},\overset{4}{\tilde{\tau}}^{Y},\breve{E})$ is pair wise soft regular T_1 space.

Now, we prove that $(\breve{Y},\overset{1}{\tilde{\tau}}^{Y}\ \overset{2}{\tilde{\tau}}^{Y},\overset{3}{\tilde{\tau}}^{Y},\overset{4}{\tilde{\tau}}^{Y},\breve{E})$ is pair wise soft regular T_3 space.

Let $e_H \in \breve{Y}$ and $(G,\breve{E})$ be soft *regular* closed set in $\breve{Y}$ such that $e_H \notin (G,\breve{E})$ where $(G,\breve{E}) \in \overset{1}{\tilde{\tau}} \cup \overset{2}{\tilde{\tau}}$then $(G,\breve{E}) = (\breve{Y},\breve{E}) \cap (F,\breve{E})$ for some soft *regular* closed set in $\overset{1}{\tilde{\tau}} \cup \overset{2}{\tilde{\tau}}$hence$e_H \notin (Y,\breve{E}) \cap (F,\breve{E})$ but $e_H \in (Y,\breve{E})$, so $e_H \notin (F,\breve{E})$ since $\left(\breve{X},\overset{1}{\tilde{\tau}},\overset{2}{\tilde{\tau}},\overset{3}{\tilde{\tau}},\overset{4}{\tilde{\tau}},\breve{E}\right)$ is soft regular T_3 space $\left(\breve{X},\overset{1}{\tilde{\tau}},\overset{2}{\tilde{\tau}},\overset{3}{\tilde{\tau}},\overset{4}{\tilde{\tau}},\breve{E}\right)$is soft R-regular space so there happens $\tau_1 \cup \tau_2$ soft *regular open* set $(F_1,\breve{E})$ and $\overset{3}{\tilde{\tau}} \cup \overset{4}{\tilde{\tau}}$ soft *regular* open set $(F_2,\breve{E})$ such that

$$e_H \in (F_1,\breve{E}), (G,\breve{E}) \subseteq (F_2,\breve{E})$$
$$(F_1,\breve{E})(F_2,\breve{E}) = \breve{\emptyset}$$

Take $(G_1,\breve{E}) = (Y,\breve{E}) \cap (F_2,\breve{E})$ then $(G_1,\breve{E}),(G_2,\breve{E})$ are soft *regular open* sets in $\breve{Y}$ such that

$$e_H \in (G_1,\breve{E}), (G,\breve{E}) \subseteq (Y,\breve{E}) \cap (F_2,\breve{E}) = (G_2,\breve{E})$$

$$\left(G_1,\breve{E}\right) \cap \left(G_2,\breve{E}\right) \subseteq \left(F_1,\breve{E}\right) \cap \left(F_2,\breve{E}\right) = \breve{\emptyset}$$
$$\left(G_1,\breve{E}\right) \cap \left(G_2,\breve{E}\right) = \breve{\emptyset}$$

Therefore, $\left(\breve{X},\overset{1}{\breve{\tau}},\overset{2}{\breve{\tau}},\breve{E}\right)$ is soft R-regular space with respect to $\overset{3}{\breve{\tau}}^{Y} \cup \overset{4}{\breve{\tau}}^{Y}$ Similarly, Let $e_H \in \breve{Y}$ and $\left(G,\breve{E}\right)$ be a soft *regular* closed sub set in $\breve{Y}$ such that $e_H \notin \left(G,\breve{E}\right)$, Where $(G,E) \in \overset{3}{\breve{\tau}} \cup \overset{4}{\breve{\tau}}$ then $\left(G,\breve{E}\right) = (Y,\breve{E}) \cap (F,\breve{E})$ where $\left(F,\breve{E}\right)$ is some soft *regular* closed set in $\overset{3}{\breve{\tau}} \cup \overset{4}{\breve{\tau}}$ $e_H \notin (Y,E) \cap (F,E)$ but $e_H \in (Y,E)$ so $e_H \notin (F,E)$ since $\left(\breve{X},\overset{1}{\breve{\tau}},\overset{2}{\breve{\tau}},\overset{3}{\breve{\tau}},\overset{4}{\breve{\tau}},\breve{E}\right)$ is soft *R*-regular space so there happens $\overset{3}{\breve{\tau}} \cup \overset{4}{\breve{\tau}}$ soft *regular* open set $(F_1,\breve{E})$ and $\overset{1}{\breve{\tau}} \cup \overset{2}{\breve{\tau}}$ soft *regular open* set $(F_{2,},\breve{E})$. Such that

$$e_H \in \left(F_1,\breve{E}\right), \left(G,\breve{E}\right) \subseteq \left(F_2,\breve{E}\right)$$
$$\left(F_1,\breve{E}\right) \cap \left(F_{2,},\breve{E}\right) = \phi$$

Take

$$\left(G_1,\breve{E}\right) = \left(Y_,,\breve{E}\right) \cap \left(F_1,\breve{E}\right)$$
$$\left(G_1,\breve{E}\right) = \left(Y_,,\breve{E}\right) \cap \left(F_1,\breve{E}\right)$$

Then $\left(G_{1,},\breve{E}\right)$ and $\left(G_{2,},\breve{E}\right)$ are soft *regular open* set in,$\breve{Y}$ such that

$$e_H \in \left(G_1,\breve{E}\right), \left(G,\breve{E}\right) \subseteq \left(Y,\breve{E}\right) \cap \left(F_2,\breve{E}\right) = \left(G_2,\breve{E}\right)$$
$$\left(G_1,\breve{E}\right) \cap \left(G_2,\breve{E}\right) \subseteq \left(F_1,\breve{E}\right) \cap \left(F_2,\breve{E}\right) = \breve{\emptyset}.$$

Therefore $\overset{3}{\breve{\tau}}^{Y} \cup \overset{4}{\breve{\tau}}^{Y}$ is soft R-regular space.

4 Conclusions

A soft set with single specific topological structure is unable to shoulder up the responsibility to construct the whole theory. So to make the theory healthy, some additional structures on soft set has to be introduced. It makes, it more springy to develop the soft topological spaces with its infinite applications. In this regards we introduced soft topological structure known as soft quad topological structure with respect to soft regular open sets. In this article, topology is the supreme branch of pure mathematics which deals with mathematical structures. Freshly, many scholars have studied the soft set theory which is coined by Molodtsov [4] and carefully applied to many difficulties which contain uncertainties in our social life. Shabir and Naz [7] familiarized and deeply studied the origin of soft topological spaces. They also studied topological structures and exhibited their several properties with respect to ordinary points.

In the present work, we constantly study the behavior of soft regular separation axioms in soft quad topological spaces with respect to soft points as well as ordinary points of a soft topological space. We introduce soft regular qT_0structure, soft regular qT_1 structure, soft regular qT_2 structure, soft regular qT_3 and soft regular qT_4structure with respect to soft and ordinary points. In future we will plant these structures in different results. More over defined soft regular T_0structure w. r. t. soft regular T_1structure and vice versa, soft regular T_1structure w. r. t soft regular T_2structure and vice versa and soft regular T_3 space w. r. t soft regular T_4 and vice versa with respect to ordinary and soft points in soft quad topological spaces and studied their activities in different results with respect to ordinary and soft points. We also planted these axioms to different results. These soft semi separation axioms in quad structure would be valuable for the development of the theory of soft topology to solve complicated problems, comprising doubts in economics, engineering, medical etc. We also attractively discussed some soft transmissible properties with respect to ordinary as well as soft points. I have fastidiously studied numerous homes on the behalf of soft topology, and lastly I determined that soft topology is totally linked or in other sense we can correctly say that soft topology (Separation Axioms) are connected with structure. Provided if it is related with structures then it gives the idea of non-linearity beautifully. In other ways we can rightly say soft topology is somewhat directly proportional to non-linearity. Although we use non-linearity in Applied Math. So it is not wrong to say that soft topology is applied Math in itself. It means that soft topology has the taste of both of pure and applied math. In future I will discuss Separation Axioms in soft topology With respect to soft points. We expect that these results in this article will do help the researchers for strengthening the tool box of soft topological structures. In the forthcoming, we spread the idea of soft α-open, and soft b^{**}open sets in soft quad topological structure with respect to ordinary and soft points.

References

[1] B. M. Ittanagi, 2014, soft bi-topological spaces, International Journal of Computer Application (0975 8887), v107-No. 7.
[2] S. A El-Sheikh, Rodyna A. Hosny and A. M. Abd-e-Latif, 2015, Characterization of b soft separation Axioms in soft topological spaces in Inf. Sci. Let t. 4 (3) 125-133.
[3] M. Naz, M. Shabir and M. I. Ali, 2015, Separation axiom in bi soft topological spaces, Arxiv. 1850 of . 00866 vl [Math, GN]
[4] D. A Molodtsov, 1999, soft set theory first results, Comp. Math. Appl., 37, 19-31.
[5] J. C. Kelly, 1963, Bitopological spaces proc London math-soc, 13, 71-81.
[6] P. K. Maji, R, Biswas and A. R. Roy, 2003, soft set theory, Comp. Math. Appl., 45, 555-562.
[7] M. Shabir and M. Naz, on soft topological space , 2011, Comp. Math. Appl., 61, 1786-1799.
[8] H. Aktas and N. Cagman, 2007, soft sets and soft groups, Inform. Sci 1 (77), 2726-2735.
[9] Chen, 2005, the paramatrization reduction of soft sets and application,Comp. Math, Appl., 49,757-763.
[10] F. Feng, Y. B. Jun, X. Z. Zhao, 2008, soft semi rings, Comp. Math. Appl., 56, 2621-2628
[11] Z. Kong, L. Gao, L. Wang, S. Li, 2008, The normal parameter reduction of soft set and its algorithm. Comp., Math. Appl., 56, 3029-3027.

[12] Y. B. Jun, C. H. Park, 2008, Application of soft sets and ideal theory of BCK/BCI-algebra, Inf, Sci 178, 2466-2475.
[13] Pie and D. Miao, 2005, Form soft sets to information system,in X. Hu, Q. Liu,A. Skowron, T. Y . Lin, R. R. Yager, B. Zhang (Eds), Proceedings of Granular Computing,Vol. 2,IEEE,pp. 617-621.
[14] Majumdar and S. K Samanta, 2010, on soft mapping, Comp. Math. Appl., 60, 2666-2672.
[15] B. Jun, 2008, soft BSK/BCI-algehra, Comp. Math. Appl., 56, 1408-1413.
[16] Shahir and M. I. Ali, 2009) soft ideals and generalized fuzzy ideals in semi-groups, New Math. Wat. Comp. 5,599-615
[17] Y. Zou and Z. Xiao, 2008, Data analysis approach of soft sets under incomp Let e information,
Knowledge-Based System, 21, 941-945.
[18] A. Aygunoglu and H. Aygun, Some notes on soft topological space , Neural Comp and Applic DoI 10. 1007/s. 00521-011-0722-3.
[19] Husain, B Ahmad, 2011, Some properties of soft topological spaces, Comp. Math. App., 62, 4058-4067.
[20] D. N Geogiou, A. C Megart is, V. I. 2013, petropounlos On soft topological spaces, Appl, Math. Inf, Sci 7 (2), 1889-190
[21] Hen B, 2013, soft semi open sets and related properties in soft topological spaces. Appl, math, Inf, sci 7 (1), 287-294.
[22] A. Kandil, O. A . E. Tantawy, S. A,EI Sheikh and A. M, Abd-e-Lat if, 2015, γ operation and decomposition of some form of soft continuity of soft topological spaces via soft ideal, Ann Fuzzy Math, inf, 9 (3),385-402.

[23] A. Kandil, O. A . E. Tantawy, S. A, EI Sheikh and A. M, Abd-e-Latif, 2014, soft connectedness via soft ideal, Journal of New result in science, 4,90-108.
[24] A. Kandil, O. A . E. Tantawy, S. A, EI Sheikh and A. M, Abd-e-Latif, 2014, soft ideal theory, soft local function and generalize soft topological spaces, Appl. Math. Inf. Sci, 8 (4),1595-1603.
[25] A. Kandil. O. A . E. Tantawy, S. A,EI Sheikh and A. M, Abd-e-Latif, 2015, soft regular ity and normality based on semi open soft sets and soft ideals,App. Math. Inf. Sci. Let t, 3,47-55.
[26] A. Kandil, O. A . E. Tantawy, S. A,EI Sheikh and A. M, Abd-e-Latif, 2014) soft semi compactness, via soft ideals, Appl. Math . Inf. Sci, 8 (5),2297-2306
[27] A. Kandil, O. A . E. Tantawy, S. A,EI Sheikh and A. M, Abd-e-Latif, 2014) soft semi (Quasi) husdorff spaces via soft ideals, South Asian, J. Math, 4 (6),265-284.

[28] I. Zorlutana,N. Akdog,W. K. Min, S. Atmaca, 2012, Remarks on soft topological spaces, Annals, of Fuzzy Mathematics and informatics 3 (2),171-185
[29] J. Mahanta. P. K. Das, 2012, On soft topological spaces Via semi open and semi closed soft sets, ar XIV 1203. 1433VI [math, G. N]
[30] A. Kandil'O. A. E. Tantawy, S. A El-Sheikh and A. M Abd-Latif, 2014, Sopra generalized closed soft sets with respect to an soft ideal in supra soft topological spaces Appl. Math . Inf. Sci, (4), 1731-1740.
[31] S. A El-Sheikh and A. M Abd-e-Latif, 2014, Decomposition of some type of supra soft sets and soft continuity, international journal of mathematics trends and technology, 9 (1), 37-56.
[32] L. A. Zadeh, 1965, Fuzzy sets, information and control 8,338-353.

[33] Q. M. Sun, Z. L Zhang, J. Liu soft sets and soft modules, proceeding of rough sets and Knowledge technology, third international conference, RSKT 2008,17-19 May, Chengdu, Chaina, PP 403-409, 2008.
[34] C. H. Park, Y. B. Jun, M. A. Ozturk, 2008, soft WS Algebras, Communications of the Korean Mathematical Society 23 (3), 313-324.
[35] I. Arockiarani and A. A. Lancy, 2013, Generalized soft gβ-closed sets and soft gsβ-closed sets in soft topological spaces, IJMRS, 4 (2), 1-7.
[36] B. Chen, 2013, soft semi-open sets and related properties in soft topological spaces. Appl. Math. Inf. Sci, 7 (1), 287-294.
[37] A. S. Mashhour, M. E. Abd El-Monsef and S. N. El-Deeb, 1982, A note on semi-continuity and pre-continuity. Indian J. Pure Appl. Math. 13 (10), 1119-1123.
[38] S. Hussain, B. Ahmad, 2015, soft separation axioms in soft topological space submitted to elsvier.
[39] A. Kandil, O. A . E. Tantawy, S. A,EI Sheikh and A. M, Abd-e-Latif, 2014, soft semi separation axioms and some
[40] U. D. Tapi,Ranu Sharma,Bhagyashri A. Deole, 2014, Separation Axioms in Quad topological spaces, Bul Let in of Mathematical Science and Applications İS SN 2278-9634, Vol. 10, pp 16-18.
[41] N. F. Hameed and Yahya Abid, 2011,Certain types of separation axioms in Tri-topological spaces,Iraqi Journal of science,Vol 52,No. 2,PP212-217.
[42] Y. Yumak and A. K. Kaymaci, 2015, some β-Open sets and their Applications, journal of New Theory, 4 (2015) 80-89.