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INTERSECTIONAL SOFT SETS IN ORDERED GROUPOIDS

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Abstract – In this note, the notions of soft int-ordered groupoids and soft left (resp., right) ideals are introduced. The characterization of int-soft ordered groupoids in terms of characteristic and inclusive sets is discussed. The concepts of soft prime ideals and soft int-filters are also introduced, and the relation between them is investigated.

***Keywords* –** *Ordered groupoids, soft sets, int-soft filters, soft prime ideals.*

1. Introduction

The most successful theoretical approach to vagueness is undoubtedly fuzzy set theory introduced by Zadeh [14]. The theory is used commonly in different areas as engineering, medicine and economics, among others. The fuzzy set theory is based on the fuzzy membership function $\mu : X \to [0; 1]$. By the fuzzy membership function, we can determine the membership grade of an element with respect to a set. The fuzzy set theory has become very popular and has been used to solve problems in different areas. But there exists a difficulty: how to set the membership function in each particular case. The theory of soft sets is introduced by Molodtsov [8] as a new tool to discuss (vagueness) uncertainty. A soft set is a collection of approximate descriptions of an object. Each approximate description has two parts: a predicate and an approximate value set. In classical mathematics, we construct a mathematical model of an object and define the notion of the exact solution of this model. Usually the mathematical model is too complicated and we cannot find the exact solution. So, in the second step, we introduce the notion of approximate solution and calculate that solution. In the Soft Set Theory, we have the opposite approach to this problem. The initial description of the object has an approximate nature, and we do not need to introduce the notion of the exact solution. The absence of any restrictions on the approximate description in Soft Set Theory makes this theory very convenient and easily applicable in practice. Soft set theory has potential applications in many fields, including the smoothness of functions, game theory, operations research, Riemann integration, Perron integration, probability theory and measurement theory. Most

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of these applications have already been demonstrated in Molodtsov's paper [8]. Authors in [13] gave an application of soft sets to diagnose the prostate cancer risk. Cagman et al. [2] applied the soft set to the theory of groups. They studied the soft int-groups, which are different from the definition of soft groups in [1, 9]. This new approach is based on the inclusion relation and intersection of sets. It brings the soft set theory, set theory and the group theory together. Some supplementary properties of soft int-groups and normal soft int-groups, analogues to classical group theory and fuzzy group theory are introduced in [3, 10]. Recently, Ideal theory in semigroups based on soft int-semigroup is investigated in [11]. Authors in [12] discussed the applications of fuzzy soft sets to ordered semi group theory. Khan et al. [7], presented the concepts of a fuzzy soft left (right) ideal and fuzzy soft interior ideal over an ordered semigroup.

In this paper, the notions of soft int-ordered groupoids and soft left (resp., right) ideals are introduced. The characterization of soft int-ordered groupoids in terms of characteristic and inclusive sets is discussed. The concepts of soft prime ideals and soft int-filters are also introduced, and the relation between them is investigated.

2. Preliminaries

We denote by $(S,.,\leq)$ an ordered groupiod, that is, a groupiod $(S,.)$ with a simple order $\leq$ which satisfies the following condition:

$$\forall\, x,y,z \in S,\ x \leq y \text{ implies } xz \leq yz \text{ and } zx \leq zy\,.$$

Definition 2.1. [5] A non-empty subset A of S is called a left (resp. right) ideal of S if
1) $SA \subseteq A$ (resp. $\subseteq A$)
2) $a \in A, S \ni b \leq a$ implies $b \in A$.

Definition 2.2. [5] A (non-empty) set *A* is called an ideal of *S* if it is both a left and a right ideal of *S*.

Definition 2.3. [4] A subgroupoid F of S is called a filter of S if
1) $a, b \in S, ab \in F$ implies $b \in F$,
2) $a \in F, S \ni b \geq a$ implies $a \in F$.

For $A \subseteq S$, we define $(A] = \{t \in S : t \leq a \ for\ some\ a \in A\}$.

Let U be an initial universe set and let E be a set of parameters. Let $\mathrm{P(U)}$ denote the power set of U and $\mathrm{A, B, C, \ldots} \subseteq \mathrm{E}$.

Definition 2.4. [8] A soft set (α, A) over U is defined to be the set of ordered pairs

$$(\alpha, A) = \{(x, \alpha(x)) : x \in E;\ \alpha(x) \in P(U)\};$$

where $\alpha: E \to P(U)$ such that $\alpha(x) = \emptyset$ if $x \notin A$.

Definition 2.5. [11] Let (α, A) and (β, A) be two soft sets. Then, (α, A) is a soft subset of (β, A), denoted by $(\alpha, A) \sqsubseteq (\beta, A)$ if $\alpha(x) \subseteq \beta(x)$ for all $x \in A$ and (α, A); (β, A) are called soft equal, denoted by $(\alpha, A) = (\beta, A)$ if and only if $\alpha(x) = \beta(x)$ for all $x \in A$.

Definition 2.6. [11] Let (α, A) and (β, A) be two soft sets. Then, union $(\alpha, A) \sqcup (\beta, A)$ and intersection $(\alpha, A) \sqcap (\beta, A)$ are defined by

$$(\alpha \sqcup \beta)(x) = \alpha(x) \cup \beta(x),$$
$$(\alpha \sqcap \beta)(x) = \alpha(x) \cap \beta(x),$$

respectively.

Definition 2.7. A soft set (α, S) in a groupoid S is called a soft int- subgroupoid of S if

$$\alpha(xy) \supseteq \alpha(x) \cap \alpha(y) \text{ for all } x, y \in S.$$

3. Soft Left and Soft Right Ideals in Ordered Groupoids

In what follows, we take $E = S$, as a set of parameters , which is a groupoid unless otherwise stated. For a nonempty subset A of S, define a map $\chi_A : S \to P(U)$ as follows:

$$\chi_A(x) = \begin{cases} U & \text{if } x \in A, \\ \emptyset & \text{otherwise.} \end{cases}$$

Then (χ_A, S) is a soft set over U, which is called the characteristic soft set (see [11]).

Lemma 3.1. If $(S, \cdot, \leq)$ is an ordered groupoid and $\emptyset \neq A \subseteq S$, the characteristic Soft set $(\chi_{(A]}, S)$ is satisfying the condition:

$$\forall\, x, y \in S, x \leq y \text{ implies } \chi_{(A]}(x) \supseteq \chi_{(A]}(y).$$

Proof. By definition, $\chi_{(A]}$ is a mapping from S into $\{U, \emptyset\} \subset P(U)$. Suppose $x, y \in S$, $x \leq y$ .If $y \notin (A]$, then $\chi_{(A]}(y) = \emptyset$ and $\chi_{(A]}(x) \supseteq \chi_{(A]}(y)$. Consider the case $y \in (A]$, then $\chi_{(A]}(y) = U$. Because $y \in (A]$, there exists $z \in A$ such that $y \leq z$ and consequently $x \leq z$. Thus $x \in (A]$. Therefore, $\chi_{(A]}(x) = U$, $\chi_{(A]}(x) \supseteq \chi_{(A]}(y)$. ∎

Proposition 3.2. Let $(S, \cdot, \leq)$ be an ordered groupoid and $\emptyset \neq A \subseteq S$. Then $A = (A]$ if and only if (χ_A, S) satifies

$$\forall\, x, y \in S, x \leq y \text{ implies } \chi_A(x) \supseteq \chi_A(y).$$

Proof. Assume that $A = (A]$, the desired result comes directly from lemma 3.1. Conversely, suppose that for $x, y \in S$, $x \leq y$ implies $\chi_A(x) \supseteq \chi_A(y)$. Let $x \in (A]$. There exists $y \in A$ so that $x \leq y$. By the hypothesis, we have $\chi_A(x) \supseteq \chi_A(y)$. Since $y \in A$, we have $\chi_A(y) = U$. Thus $\chi_A(x) = U$ and $x \in A$. Therefore $A = (A]$. ∎

Here, we introduce the concepts of soft (left, right) ideals in ordered groupoids and characterize them in terms of soft sets.

Definition 3.3. Let $(S, \cdot, \leq)$ be an ordered groupoid. A soft set (α, S) over U is called a soft left ideal over U if

1) $\alpha(xy) \supseteq \alpha(y)$ for all $x, y \in S$,
2) $x \leq y$ implies $\alpha(x) \supseteq \alpha(y)$.

Definition 3.4. Let $(S, ., \leq)$ be an ordered groupoid. A soft set (α, S) over U is called a soft right ideal over U if

1) $\alpha(xy) \supseteq \alpha(x)$ for all $x, y \in S$,
2) $x \leq y$ implies $\alpha(x) \supseteq \alpha(y)$.

A soft set (α, S) over U is called a soft ideal over U if it is both a soft left and a soft right ideal over U.

Theorem 3.5.[11] For any nonempty subset A of a groupoid S, the following are equivalent.

1) A is a left (resp., right) ideal of S.
2) The characteristic soft set (χ_A, S) is a soft left (resp., right) ideal over U.

Theorem 3.6. Let $(S, ., \leq)$ be an ordered groupoid, and $\emptyset \neq A \subseteq S$. Then A is a left (right) ideal of S if and only if (χ_A, S) is a soft left (right) ideal over U .

Proof. Assume that A is a left ideal of S. For any $x, y \in S$, $x \leq y$. If $y \notin A$ then $\chi_A(y) = \emptyset$ and $\chi_A(x) \supseteq \chi_A(y)$. It is clear that $\chi_A(xy) \supseteq \emptyset = \chi_A(y)$. If $y \in A$, then $x.y \in A$ and $\chi_A(y) = U$. Since $x \leq y$ and A a left ideal of S, we have $y \in A$ and so $\chi_A(x) = U$. Thus again $\chi_A(x) \geq \chi_A(y)$ and $\chi_A(xy) = U = \chi_A(y)$. Therefore, (χ_A, S) is a soft left ideal over U. Similarly, (χ_A, S) is a soft right ideal over U when A is a left ideal of S. Conversely, suppose that (χ_A, S) is a soft left ideal over U. Let $x \in S$ and $y \in A$ such that $x \leq y$. Then $\chi_A(y) = U$, and so $\chi_A(xy) \supseteq \chi_A(y) = U$. Since (χ_A, S) is a soft left ideal over U and $\leq y$, we have $\chi_A(x) \supseteq \chi_A(y)$. Since $y \in A$, $\chi_A(y) = U$. Then $\chi_A(x) = U$, and $x \in A$. The rest of the proof is a consequence of theorem 3.5. Similarly, we can show that if (χ_A, S) is a soft right ideal over U, then A is a right ideal of S. ■

Definition 3.7.[10] For a soft set (α, A) over U and a non-empty subset V of U, the V-inclusive set of (α, A), denoted by α^V, is defined to be the set

$$\alpha^V = \{x \in A: V \subseteq \alpha(x)\}.$$

As a generalization of Theorem 3.6, we have the following result.

Theorem 3.8. Let $(S, \cdot, \leq)$ be an ordered groupoid and (α, S) a soft set over U. Then (α, S) is a soft ideal over U if and only if α^V is an ideal of S provided $\alpha^V = \emptyset$.

Proof. Assume that (α, S) is a soft ideal over U. Let $x \in \alpha^V$, then $\alpha(x) \supseteq V$. Since (α, S) is a soft ideal, we have $\alpha(xy) \supseteq \alpha(x) \supseteq V$ and $\alpha(yx) \supseteq \alpha(x) \supseteq V$ for all $y \in S$. Thus $xy \in \alpha^V$ and $yx \in \alpha^V$. Furthermore, let $x \in \alpha^V$ and $y \in S$ such that $y \leq x$. Then $y \in \alpha^V$. Indeed, since $x \in \alpha^V$, $\alpha(x) \supseteq V$, and (α, S) is a soft ideal over U, we have $\alpha(y) \supseteq \alpha(x) \supseteq V$, so $y \in \alpha^V$. Therefore, α^V is an ideal of S. Conversely, let α^V be an ideal of S for every non-empty subset $V \subseteq U$. For any $\in S$, take $V = \alpha(x)$. Then $x \in \alpha^V$. Since α^V is an ideal of S, we have $xy \in \alpha^V$ and so $\alpha(xy) \supseteq V = \alpha(x)$, for all $y \in S$.

Moreover, if $x \leq y$ then $\alpha(x) \supseteq \alpha(y)$. Indeed: Let $\alpha(y) = W$. Then $y \in \alpha^W$. Since α^W is an ideal of S, we have $x \in \alpha^W$. Then $(x) \supseteq W = \alpha(y)$. Therefore, (α, S) is a soft right ideal over U. In a similar way, we can show that (α, S) is also a soft left ideal over U, and so (α, S) is a soft ideal over U. ■

For an ordered groupoid S, let (θ, S) be the soft set over U defined by $\theta(x) = U$ for all $x \in S$. Let $a \in S$, define $A_a = \{(x,y) \in S \times S : a \leq xy\}$. For two soft sets (α, S) and (β, S) , we define The soft product of (α, S) and (β, S) as the soft set $(\alpha \circ \beta, S)$ over U defined by

$$(\alpha o \beta)(a) = \begin{cases} \bigcup_{(x,y)\in A_a} \{\alpha(x) \cap \beta(y)\} & if\ A_a \neq \emptyset \\ \emptyset & otherwise \end{cases}$$

Here, we give equivalent definitions of soft right (resp. left) ideals and soft ideals.

Theorem 3.9.. Let $(S,.,\leq)$ be an ordered groupoid. A soft set (α, S) over U is called a soft left ideal over U if and only if

1) $(\theta \circ \alpha, S) \sqsubseteq (\alpha, S)$,
2) $x \leq y$ implies $\alpha(x) \supseteq \alpha(y)$.

Proof. Assume that (α, S) is a soft left ideal over U. Let $a \in S$. Then $(\theta \circ \alpha)(a) \subseteq \alpha(a)$. Indeed: If $A_a = \emptyset$, then $(\theta \circ \alpha)(a) = \emptyset \subseteq \alpha(a)$. Let $A_a \neq \emptyset$. Then

$$(\theta \circ \alpha)(a) = \bigcup_{(x,y)\in A_a} \{\theta(x) \cap \alpha(y)\}$$

$$= \bigcup_{(x,y)\in A_a} \{\alpha(y)\}$$

Now, we show that $\alpha(y)) \subseteq \alpha(a)$ for every $(x, y) \in A_a$. In fact: If $(x, y) \in A_a$, then $a \leq xy$. Since (α, S) is a soft left ideal, we have $\alpha(a) \supseteq \alpha(xy) \supseteq \alpha(y)$. Therefore we have

$$(\theta \circ \alpha)(a) = \bigcup_{(x,y)\in A_a} \{\theta(x) \cap \alpha(y)\}$$

$$= \bigcup_{(x,y)\in A_a} \{\alpha(y)\} \subseteq \alpha(a)$$

Conversely, let $x, y \in S$. By hypothesis, we have $(\theta \circ \alpha)(xy) \subseteq \alpha(xy)$. Since $(x, y) \in A_{xy}$, we have

$$(\theta \circ \alpha)(xy) = \bigcup_{(s,t)\in A_{xy}} \{\theta(s) \cap \alpha(t)\}$$

$$\supseteq \theta(x) \cap \alpha(y) = \alpha(y)$$

Hence we obtain $\alpha(xy) \supseteq \alpha(y)$, that is, (α, S) is a soft left ideal over U . ■

In a similar argument we prove the following result**.**

Theorem 3.10. Let $(S,.,\leq)$ be an ordered groupoid. A soft set (α,S) over U is called a soft ideal ideal over U if and only if

1) $(\alpha \circ \theta, S) \sqsubseteq (\alpha, S)$,
2) $x \leq y$ implies $\alpha(x) \supseteq \alpha(y)$.

Theorem 3.11. Let $(S,.,\leq)$ be an ordered semigroup. S is regular if and only if for every soft set (α,S) over U we have $(\alpha,S) \sqsubseteq (\alpha \circ \theta \circ \alpha, S)$.

Proof. Assume that S is regular and that (α, S) is a soft set over U. For $a \in S$ there exists $x \in S$ such that $a \leq axa$. Since $(ax, a) \in A_a$, we have

$$(\alpha \circ \theta \circ \alpha)(a) = \bigcup_{(s,t)\in A_a} \{(\alpha \circ \theta)(s) \cap \alpha(t)\}$$

$$\supseteq (\alpha \circ \theta)(ax) \cap \alpha(a).$$

Since $(a, x) \in A_{ax}$, we have

$$(\alpha \circ \theta)(ax) = \bigcup_{(u,v)\in A_{ax}} \{(\alpha)(u) \cap \theta(v)\}$$

$$\supseteq \alpha(a) \cap \theta(x) = \alpha(a).$$

Hence we have $(\alpha \circ \theta \circ \alpha)(a) \supseteq \alpha(a)$. Therefore, $(\alpha, S) \sqsubseteq (\alpha \circ \theta \circ \alpha, S)$. Conversely, Let $a \in S$. By hypothesis, we have

$$\chi_{\{a\}}(a) = U \subseteq (\chi_{\{a\}} \circ \theta \circ \chi_{\{a\}})(a).$$

Thus

$$(\chi_{\{a\}} \circ \theta \circ \chi_{\{a\}})(a) = U.$$

If $A_a = \emptyset$, then $(\chi_{\{a\}} \circ \theta \circ \chi_{\{a\}})(a) = \emptyset$, a contradiction. So $A_a \neq \emptyset$, then

$$(\chi_{\{a\}} \circ \theta \circ \chi_{\{a\}})(a) = \bigcup_{(x,y)\in A_a} \{(\chi_{\{a\}} \circ \theta)(x) \cap \chi_{\{a\}}(y)\}.$$

Claim: There exists $(x, y) \in A_a$ such that $(\chi_{\{a\}} \circ \theta)(x) \neq \emptyset$ and $\chi_{\{a\}}(y) \neq \emptyset$.
Proof. Suppose that $(\chi_{\{a\}} \circ \theta)(x) = \emptyset$ or $\chi_{\{a\}}(y) = \emptyset$ for every $(x, y) \in A_a$, then $(\chi_{\{a\}} \circ \theta)(x) \cap \chi_{\{a\}}(y) = \emptyset$ for every $(x, y) \in A_a$. This implies that

$$(\chi_{\{a\}} \circ \theta \circ \chi_{\{a\}})(a) = \bigcup_{(x,y)\in A_a} \{(\chi_{\{a\}} \circ \theta)(u) \cap \chi_{\{a\}}(v)\} = \emptyset.$$

But this contradicts

$$(\chi_{\{a\}} \circ \theta \circ \chi_{\{a\}})(a) = U.$$

If $y \neq a$, then $\chi_{\{a\}}(y) = \emptyset$. By the claim, we have $y = a$, $(x, a) \in A_a$ and $a \leq xa$. Let A_a be empty, then $(\chi_{\{a\}} \circ \theta)(x) = \emptyset$. By the claim, we have $A_a \neq \emptyset$. Then

$$(\chi_{\{a\}} \circ \theta)(x) = \bigcup_{(s,t)\in A_x} \{\chi_{\{a\}}(s) \cap \theta(t)\} = \bigcup_{(s,t)\in A_x} \{\chi_{\{a\}}(s)\}$$

If $s \neq a$ for every $(s,t) \in A_x$, then $\chi_{\{a\}}(s) = \emptyset$. Hence $(\chi_{\{a\}} \circ \theta)(x) = \emptyset$. By the claim, there exists $(s,t) \in A_x$ such that $s = a$. Then $(a,t) \in A_x$ and $x \leq at$. Thus we have $a \leq xa \leq ata$. Therefore, S is regular. ■

4. Soft Filters in Ordered Groupoids

In this section, we introduce the concept of int-soft filters in ordered groupoids, and we characterize filters of ordered groupoids in terms of int-soft filters.

Definition 4.1. Let $(S, \cdot, \leq)$ be an ordered groupoid. A soft set (α, S) over U is called a soft int-filter over U if

1) $x \leq y \Rightarrow \alpha(x) \leq \alpha(y)$.
2) $\alpha(xy) = \alpha(x) \cap \alpha(y)\ \forall\, x, y \in S$.

It is well known that a subset A of a groupoid S is a subgroupoid iff the soft set (χ_A, S) is a soft int-groupoid over U [11].

Proposition 4.2. Let $(S, \cdot, \leq)$ be an ordered groupoid and $\emptyset \neq F \subseteq S$. Then F is a filter of S if and only if the soft set (χ_F, S) is a soft int-filter over U.

Proof. Assume that F is a filter of S and that $x, y \in S, x \leq y$. If $x \notin F$, then $\chi_F(x) = \emptyset$. Hence $\chi_F(x) \subseteq \chi_F(y)$. If $x \in F$, then $\chi_F(x) = U$. Since $y \geq x \in F$, we have $y \in F$. Then $\chi_F(y) = U$, and again $\chi_F(x) \subseteq \chi_F(y)$. In order to show that $\chi_F(xy) = \chi_F(x) \cap \chi_F(y)$ for all $x, y \in S$, let $x, y \in S$ such that $x.y \notin F$. Then $\chi_F(xy) = \emptyset$. Moreover $x.y \notin F$ implies $x \notin F$ or $y \notin F$. Then $\chi_F(x) = \emptyset$. or $\chi_F(y) = \emptyset$. So $\chi_F(x) \cap \chi_F(y) = \emptyset$, and $\chi_F(xy) = \chi_F(x) \cap \chi_F(y)$ for all $x, y \in S$. Now, consider $x.y \in F$. Then $\chi_F(xy) = U$. Since $x.y \in F$, we have $x \in F$ and $y \in F$. Then $\chi_F(x) = \chi_F(y) = U$, whence $\chi_F(x) \cap \chi_F(y) = U$ and $\chi_F(xy) = U = \chi_F(x) \cap \chi_F(y)$.
Conversely, let (χ_F, S) be a soft int-filter over U. By the condition 2 of definition 4.1, F is a sub-groupoid of S. Let $x, y \in S$, $xy \in F$. Since (χ_F, S) is a soft int-filter over U, then $\chi_F(xy) = \chi_F(x) \cap \chi_F(y)$. Since $y \in F$, we have $\chi_F(xy) = U$. Then $\chi_F(x) \cap \chi_F(y) = U$, $\chi_F(x) = \chi_F(y) = U$, and $x, y \in F$. Let $x \in F$, $x \leq y$. Then we have $\chi_F(x) = U$. Since $x \leq y$, we have $\chi_F(x) \leq \chi_F(y)$. Therefore, $\chi_F(y) = U$, and $y \in F$. This completes the proof. ■

In the rest of this section, we give the relation between int-soft filters and soft prime ideals of ordered groupoids. In an ordered groupoid, a non-empty subset F is a filter if and only if $S \setminus F = \emptyset$ or $S \setminus F$ is a prime ideal of S [6]. An analogous result holds for soft sets, as well.

Definition 4.3. Let S be an ordered groupoid and (α, S) a soft set over . The complement of (α, S) is the soft set (α^c, S) defined by

$$\alpha^c: S \to P(U)$$

where $\alpha^c(x) = U \setminus \alpha(x)$.

Lemma 4.4. Let S be a groupoid and (α, S) a soft set over U. The following are equivalent:
1) $\alpha(xy) = \alpha(x) \cap \alpha(y), \forall x, y \in S$,
2) $\alpha^c(xy) = \alpha^c(x) \cup \alpha^c(y) \; \forall x, y \in S$.

Proof. Straightforward. ■

Definition 4.5. For a groupoid S, a soft set (α, S) over U is called a soft prime ideal over U if

$$\alpha(xy) \subseteq \alpha(x) \cup \alpha(y), \forall x, y \in S .$$

Theorem 4.6. Let $(S, \cdot, \leq)$ be an ordered groupoid and (α, S) a soft set over U. Then (α, S) is a soft int-filter over U if and only if (α^c, S) is an soft prime ideal over U .

Proof. Suppose (α, S) is a soft int-filter over U. Let $x, y \in S$, $x \leq y$. Then, we have $\alpha(x) \subseteq \alpha(y)$. Then $\alpha^c(x) = U \setminus \alpha(x) \supseteq U \setminus \alpha(y) = \alpha^c(y)$. Now, for any $x, y \in S$, we have $\alpha(xy) = \alpha(x) \cap \alpha(y)$. Then by lemma 4.4, $\alpha^c(xy) = \alpha^c(x) \cup \alpha^c(x)$. Therefore, (α^c, S) is an soft prime ideal over U. Conversely, Let $x, y \in S$, $x \leq y$. Since (α^c, S) is a soft ideal, we have $\alpha^c(x) \supseteq \alpha^c(y)$, and consequently $\alpha(x) \subseteq \alpha(y)$. Since (α^c, S) is an soft prime ideal, we have $\alpha^c(xy) = \alpha^c(x) \cup \alpha^c(x). \forall x, y \in S$. Then, by lemma 4.4, $\alpha(xy) = \alpha(x) \cap \alpha(y)$. Therefore, (α, S) is a soft int-filter over U. ■

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