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# FRAGMENTED CAESAR CIPHER 

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#### Abstract

In this study, we define a cipher method that is called fragmented Caesar cipher method that based on the basic logic of Caesar cipher. This new method has more possibility then the classical Caesar cipher because of the fragmented alphabet is used to cipher. We then construct a mathematical modeling and make a computer programs of the method.


Keywords - Caesar cipher, encryption, decryption, plaintext, ciphertext, fragmented Caesar cipher.

## 1 Introduction

One of the earliest known cryptographic systems was used by Julius Caesar. In the Caesar cipher, each letter in a plaintext is shifted by a letter a certain number of positions down the alphabet. The Caesar cipher can be decrypted an easy way with the brute-force attack. Since then a lots of technics of cipher have been developed to obtain an unbroken cipher technic. For examples, Omolara et al. [5], Patni [7, 8], Dey et al. [2] and Mishra [4]. More detailed explanations related to the Caesar cipher can be found in [9] and [6].

In this study, we define a cipher method that is called fragmented Caesar cipher method that is based on the Caesar cipher. In the fragmented Caesar cipher, the alphabet broken into small fragments and each letter in each fragment is replaced by a letter some arbitrary number of positions down. We then construct a mathematical

[^0]modeling and make a computer programs of the method. We finally give an example to show the cipher method is working successfully.

The present paper is a condensation of part of the dissertation [1].

## 2 Fragmented Caesar Cipher Method

In this section, we define a new cipher method which depends on the Caesar cipher method. In this method, we firstly divide the alphabet arbitrary fragments. And then each letter in each fragment is replaced by a letter some fixed number of positions down. Therefor, we call this method as fragmented Caesar cipher method or in sort FCC-method.

### 2.1 Mathematical Model of FCC-method

In this subsection, we first give a mathematical model of the FCC-method. We then write an algorithm of the FCC-method to make a computer program.

Throughout this paper, ASCII (American Standard Code for Information Interchange) is used, $I_{n}=\{1,2, \ldots, n\}$ for all $n \in \mathbb{N}$ is an index set and $U$ is a set of using characters which is ordered according to the ASCII.

Definition 1. Let $|U|=n$ and $X=\left\{x_{i}: i \in I_{n}\right\}$ be an ordered set according to the index set $I_{n}$. Then,

$$
\alpha: U \rightarrow X, \quad \alpha(i \text {-th element })=x_{i}, i \in I_{n},
$$

is called indexing function of $U$. Here, $x_{i}$ is called indexed element of $i$ th element of $U$ and the set $X$ is called indexed character set of $U$.

Example 2. Let $1,9, b, M,<$ and + be using characters. Then, the character set $U$ is written as $U=\{+, 1,9,<, M, b\}$ since

| $x$ | + | 1 | 9 | $<$ | $M$ | $b$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $A S C I I$ | 43 | 49 | 57 | 60 | 77 | 98 |

Therefore the indexed character set of $U$ is obtained as $X=\left\{x_{1}, x_{2}, x_{3}, x_{4}, x_{5}, x_{6}\right\}$ since

| $x$ | + | 1 | 9 | $<$ | $M$ | $b$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\alpha(x)$ | $x_{1}$ | $x_{2}$ | $x_{3}$ | $x_{4}$ | $x_{5}$ | $x_{6}$ |

Definition 3. Let $X$ be an indexed character set and $|X|=n$. Then, for $a_{p} \in \mathbb{N}$, $p \in I_{n}$, a fragmentation algorithm is set up as follows:

Algorithm of Fragmentation:
Step 1: Choose $a_{1}$ such that $2 \leq a_{1} \leq n_{1}=n-2$

Step 2: Let $n_{2}=n_{1}-a_{1}$. If $n_{2}>4$, choose $a_{2}$ such that $2 \leq a_{2} \leq n_{2}-2$, if not $a_{2}=n_{2}$ which means the process is terminated.
$\vdots$
Step p: Let $n_{p}=n_{p-1}-a_{p-1}$. If $n_{p}>4$, choose $a_{p}$ such that $2 \leq a_{p} \leq n_{p}-2$, if not $a_{p}=n_{p}$ which means the process is terminated.

Here, $p$ is called a fragment number, $a_{p}$ is number of characters in a fragment and $P=\left(a_{1}, a_{2}, \ldots, a_{p}\right)$ is called fragment key of $X$.

We can briefly choose values $a_{p}$ as follow, for $p \in I_{n}$ and $i \in I_{p}$,

$$
\begin{cases}2 \leq a_{1} \leq n_{1}, & \text { if } p=1, n_{1}=n-2 \\ 2 \leq a_{p} \leq n_{i}-2, & \text { if } p>1,4<n_{p}, n_{p}=n_{p-1}-a_{p-1} \\ n_{p}=a_{p}, & \text { if } p>1, n_{p}<4, n_{p}=n_{p-1}-a_{p-1}\end{cases}
$$

Example 4. Let $X$ be an indexed character set and $|X|=13$. If the fragmentation algorithm is working as follows,

Step 1: Choose $a_{1}=5$ such that $2 \leq a_{1} \leq n_{1}=13-2=11$,
Step 2: Choose $a_{2}=6$ such that $2 \leq a_{2} \leq 8-2$, because of $n_{2}=13-5=8$ and $8>4$,

Step 3: Choose $a_{3}=2$ because of $n_{3}=8-6=2$ and $2<4$.
Then, we obtain that $p=3$ and $P=(5,6,2)$.
Definition 5. Let $X=\left\{x_{1}, x_{2}, \ldots, x_{n}\right\}$ be an indexed character set. For all $i \in I_{n}$ and $k \in I_{n-i}$, the set $W=\left\{x_{i}, x_{i+1}, \ldots, x_{i+k}\right\}$ is called as a block subset of $X$ and denoted by $W \sqsubseteq X$.

Definition 6. Let $X$ be an indexed character set, $p$ be a number of fragment of $X$. If $X_{i} \sqsubseteq X$ has the following conditions, then family of set $\left\{X_{i}: i \in I_{p}\right\}$ is called an ordered fragmentation of $X$.

1. $\left|X_{i}\right|=a_{i}$,
2. $X_{i} \cap X_{j}=\emptyset$ for $i, j \in I_{p}, i \neq j$,
3. $X=\bigcup_{i \in I_{p}} X_{i}$,
4. $x_{\max \left(X_{i}\right)+1}=x_{\min \left(X_{i+1}\right)}$ for $i \in I_{p}$, where $x_{\min \left(X_{i}\right)}$ and $x_{\max \left(X_{i}\right)}$ be the first and the last element of $X_{i}$, respectively.

Here, the $X_{i}$ is called a fragment of $X$ for $i \in I_{p}$.
Example 7. Let us consider Example 4 where $X=\left\{x_{1}, x_{2}, \ldots, x_{13}\right\}$ and $P=(5,6,2)$. Then, for $a_{1}=5, a_{2}=6$ and $a_{3}=2$ the ordered fragmentation of $X$ are respectively as follow,

$$
\begin{aligned}
& X_{1}=\left\{x_{1}, x_{2}, x_{3}, x_{4}, x_{5}\right\} \\
& X_{2}=\left\{x_{6}, x_{7}, x_{8}, x_{9}, x_{10}, x_{11}\right\} \\
& X_{3}=\left\{x_{12}, x_{13}\right\}
\end{aligned}
$$

Therefor, the ordered fragmentation of $X$ is obtained as $\left\{X_{1}, X_{2}, X_{3}\right\}$.

Definition 8. Let $X_{i}$ be a fragment of $X$ and $a_{i}$ be the number of $X_{i}$ for all $i \in I_{p}$. If $0<r_{i}<a_{i}$, then $R=\left(r_{1}, r_{2}, \ldots, r_{p}\right)$ is called rotation key of $X$.

Here, the $r_{i}$ is called a number of rotation of $X_{i}$ for all $i \in I_{p}$.
Note that the key of this method has two part, one of them is a fragment key $P$ and the other is a rotation key $R$.

Example 9. Let us consider Example 4, if we choose number of rotations $r_{1}=3$, $r_{2}=4$ and $r_{3}=1$ for $X_{1}, X_{2}, X_{3}$, respectively. Then, the rotation key of $X$ would be $R=(3,4,1)$.
Definition 10. Let $X$ be an indexed character set, $\left\{X_{1}, X_{2}, \ldots, X_{p}\right\}$ be an ordered fragmentation of $X$ and $P=\left(a_{1}, a_{2}, \ldots, a_{p}\right)$ be a fragment key of $X$. Then, $m_{i}$ is defined by

$$
m_{i}= \begin{cases}0, & i=0 \\ m_{i-1}+a_{i}, & i \in I_{p}\end{cases}
$$

and called a module of $X_{i}$ for all $i \in I_{p}$.
It is clear to see that $x_{m_{i}}=x_{\max \left(X_{i}\right)}$ for $i \in I_{p}$.
Definition 11. Let $X$ be an indexed character set and $\left\{X_{1}, X_{2}, \ldots, X_{p}\right\}$ be an ordered fragmentation of $X$. If $m_{i}$ is a module of $X_{i}$ for all $i \in I_{p}$ and $R=\left(r_{1}, r_{2}, \ldots, r_{p}\right)$ is a rotation key of $X$, then $X_{i}$-rotation function, denoted by $\beta_{i}$, is defined by

$$
\beta_{i}: X_{i} \rightarrow X_{i}, \beta_{i}\left(x_{t}\right)= \begin{cases}x_{t+r_{i}}, & t+r_{i} \leq m_{i} \\ x_{\left(t+r_{i}\right)\left(\bmod m_{i}\right)+m_{i-1}}, & t+r_{i}>m_{i}\end{cases}
$$

where $t \in I_{a_{i}}$.
Definition 12. Let $X$ be an indexed character set and $\left\{X_{1}, X_{2}, \ldots, X_{p}\right\}$ be an ordered fragmentation of $X$. If $\beta_{i}$ is an $X_{i}$-rotation function for all $i \in I_{p}$, then the following function

$$
\beta: X \rightarrow X, \beta(x)=\left\{\begin{array}{cc}
\beta_{1}(x), & x \in X_{1} \\
\beta_{2}(x), & x \in X_{2} \\
\vdots & \\
\beta_{p}(x), & x \in X_{p}
\end{array}\right.
$$

is called a rotation function of $X$.
Definition 13. Let $\alpha: U \rightarrow X$ be an indexing function. Then for all $t \in I_{n}$, inverse of $\alpha$ is called a characterization function and defined by

$$
\alpha^{-1}: X \rightarrow U, \quad \alpha^{-1}\left(x_{t}\right)=" t \text {-th element of } \mathrm{U} "
$$

Definition 14. If $\alpha: U \rightarrow X, \alpha^{-1}: X \rightarrow U$ and $\beta: X \rightarrow X$ be indexing, characterization and rotation functions, respectively, then an encryption function on $U$ is defined by

$$
\gamma: U \rightarrow U, \quad \gamma(x)=\alpha^{-1}(\beta(\alpha(x)))
$$

Definition 15. Let $X$ be an indexed character set and $\left\{X_{1}, X_{2}, \ldots, X_{p}\right\}$ be an ordered fragmentation of $X$. If $R=\left(r_{1}, r_{2}, \ldots, r_{p}\right)$ is a rotation key of $X, \beta_{i}: X_{i} \rightarrow X_{i}$ is an $X_{i}$-rotation function and $m_{i}$ is a module of $X_{i}$ for all $i \in I_{p}$, then inverse of rotation function of $X_{i}$, denoted by $\beta_{i}^{-1}$, is defined by

$$
\begin{aligned}
& \beta_{i}^{-1}: X_{i} \rightarrow X_{i}, \\
& \beta_{i}^{-1}\left(x_{t}\right)=\left\{\begin{array}{ll}
x_{t+m_{i}-\left(r_{i}+m_{i-1}\right)}, & t+m_{i}-\left(r_{i}+m_{i-1}\right) \leq m_{i} \\
x_{\left(t+m_{i}-\left(r_{i}+m_{i-1}\right)\right)\left(\text { modm m }_{i}\right)+m_{i-1}}, & t+m_{i}-\left(r_{i}+m_{i-1}\right)>m_{i}
\end{array} \text { where } t \in\right.
\end{aligned}
$$ $I_{a_{i}}$.

Definition 16. Let $X$ be an indexed character set and $\left\{X_{1}, X_{2}, \ldots, X_{p}\right\}$ be an ordered fragmentation of $X$. If $\beta_{i}^{-1}$ is an inverse of rotation function of $X_{i}$ for all $i \in I_{p}$, then the following function

$$
\beta^{-1}: X \rightarrow X, \beta^{-1}(x)=\left\{\begin{array}{cc}
\beta_{1}^{-1}(x), & x \in X_{1} \\
\beta_{2}^{-1}(x), & x \in X_{2} \\
\vdots & \\
\beta_{p}^{-1}(x), & x \in X_{p}
\end{array}\right.
$$

is called a inverse of rotation function of $X$.
Definition 17. If $\alpha: U \rightarrow X, \alpha^{-1}: X \rightarrow U, \beta^{-1}: X \rightarrow X$ and $\gamma: U \rightarrow U$ be indexing, characterization, inverse of rotation and encryption functions, respectively, then a decryption function on $U$ is defined by

$$
\gamma^{-1}: U \rightarrow U, \quad \gamma^{-1}(x)=\alpha^{-1}\left(\beta^{-1}(\alpha(x))\right)
$$

It is clear to see that $\gamma^{-1}(x)=\alpha^{-1}\left(\beta^{-1}\left(\left(\alpha^{-1}\right)^{-1}(x)\right)\right)=\alpha^{-1}\left(\beta^{-1}(\alpha(x))\right)$.
Definition 18. Let $U$ be a character set, $P$ be a fragment key, $R$ be a rotation key and $\gamma$ be an encryption function. The four tuple ( $U, P, R, \gamma$ ) is called an FCCencryption on $U$. The four tuple ( $U, P, R, \gamma^{-1}$ ) is called an FCC- decryption on $U$.

### 2.2 FCC-Encryption Algorithm

Assume that $U$ is a character set and $X$ is an indexed character set. Then, an algorithm of the FCC-encryption is set up as follows:

## Algorithm of FCC-Encryption:

Step 1: Find the fragment number $p$ and the $P=\left(a_{1}, a_{2}, \ldots, a_{p}\right)$,
Step 2: Choose the $R=\left(r_{1}, r_{2}, \ldots, r_{p}\right)$ according to the $P$,
Step 3: Find the $\left\{X_{i}: i \in I_{p}\right\}$ and the module $m_{i}$ for each $X_{i}$,
Step 4: Find the $\beta_{i}\left(x_{t}\right)$ for $x_{t} \in X_{i} t \in I_{a_{i}}$ and $i \in I_{p}$,
Step 5: Find the $\alpha^{-1}\left(x_{t}\right)$ for $x_{t} \in X_{i}, t \in I_{a_{i}}$ and $i \in I_{p}$,
Step 6: Find the $\gamma(x)$ for $x \in U$.

Example 19. Let

$$
U=\{c, \mathrm{~d}, \mathrm{e}, \mathrm{f}, \mathrm{~g}, \mathrm{~g}, \mathrm{~h}, \mathrm{l}, \mathrm{a}, \mathrm{~b}, \mathrm{c}, \mathrm{~m}, \mathrm{n}, \mathrm{i}, \mathrm{j}, \mathrm{k}, \mathrm{l}, \mathrm{u}, \ddot{\mathrm{u}}, \mathrm{o}, \ddot{\mathrm{o}}, \mathrm{p}, \mathrm{r}, \mathrm{~s}, \mathrm{~s}, \mathrm{t}, \mathrm{z}, \mathrm{v}, \mathrm{y}\}
$$

and

$$
X=\left\{x_{1}, x_{2}, x_{3}, x_{4}, x_{5}, x_{6}, x_{7}, x_{8}, x_{9}, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}, x_{17}, \ldots, x_{29}\right\}
$$

Then,
Step 1: By using the algorithm of fragmentation, we can obtain the fragment number $p=4$ and the $P=(11,6,9,3)$ where $a_{1}=11, a_{2}=6, a_{3}=9$ and $a_{4}=3$.

Step 2: For $a_{1}=11, a_{2}=6, a_{3}=9$ and $a_{4}=3$ the rotation key is obtained as $R=(3,4,7,2)$ since $0<r_{1}=3<a_{1}=11, \quad 0<r_{2}=4<a_{2}=6, \quad 0<r_{3}=7<$ $a_{3}=9, \quad 0<r_{1}=2<a_{4}=3$.

Step 3: For $a_{i}(i=1,2,3,4)$ the fragments of $X, X_{i}$, are obtained as,

$$
\begin{array}{ll}
\text { for } a_{1}=11, & X_{1}=\left\{x_{1}, x_{2}, x_{3}, x_{4}, x_{5}, x_{6}, x_{7}, x_{8}, x_{9}, x_{10}, x_{11}\right\} \\
\text { for } a_{2}=6, & X_{2}=\left\{x_{12}, x_{13}, x_{14}, x_{15}, x_{16}, x_{17}\right\} \\
\text { for } a_{3}=9, & X_{3}=\left\{x_{18}, x_{19}, x_{20}, x_{21}, x_{22}, x_{23}, x_{24}, x_{25}, x_{26}\right\} \\
\text { for } a_{4}=3, & X_{4}=\left\{x_{27}, x_{28}, x_{29}\right\}
\end{array}
$$

and value of $m_{i}(i=0,1,2,3,4)$ can also choose as,

$$
\begin{array}{ll}
\text { for } i=0, & \\
\text { for } i=1, & \\
m_{0}=\left(m_{0}=0\right)+\left(a_{1}=11\right)=11 \\
\text { for } i=2, & \\
m_{2}=\left(m_{1}=11\right)+\left(a_{2}=6\right)=17 \\
\text { for } i=3, & \\
\text { for } i=4, & \\
\text { for } i=\left(m_{2}=17\right)+\left(a_{3}=9\right)=26 \\
\left.m_{3}=26\right)+\left(a_{4}=3\right)=29 .
\end{array}
$$

Step 4: For $i=1,2,3,4$ values of the $X_{i}$-rotation function $\beta_{i}$ are obtained as follows. Here, we first obtain the values of $\beta_{1}$ as,

$$
\begin{array}{ll}
\beta_{1}\left(x_{1}\right)=x_{4}, & \text { since } 1+r_{1}=1+3=4 \text { because of } 1+3<11 \\
\beta_{1}\left(x_{2}\right)=x_{5}, & \text { since } 2+r_{1}=1+3=5 \text { because of } 2+3<11 \\
\beta_{1}\left(x_{3}\right)=x_{6}, & \text { since } 3+r_{1}=1+3=6 \text { because of } 3+3<11 \\
\beta_{1}\left(x_{4}\right)=x_{7}, & \text { since } 4+r_{1}=1+3=7 \text { because of } 4+3<11 \\
\beta_{1}\left(x_{5}\right)=x_{8}, & \text { since } 5+r_{1}=1+3=8 \text { because of } 5+3<11 \\
\beta_{1}\left(x_{6}\right)=x_{9}, & \text { since } 6+r_{1}=1+3=9 \text { because of } 6+3<11 \\
\beta_{1}\left(x_{7}\right)=x_{10}, & \text { since } 7+r_{1}=1+3=10 \text { because of } 7+3<11 \\
\beta_{1}\left(x_{8}\right)=x_{11}, & \text { since } 8+r_{1}=1+3=11 \text { because of } 8+3=11 \\
\beta_{1}\left(x_{9}\right)=x_{1}, & \text { since }(9+3)(\bmod 11)+0=1 \text { because of } 9+3>11 \\
\beta_{1}\left(x_{10}\right)=x_{2}, & \text { since }(10+3)(\bmod 11)+0=2 \text { because of } 10+3>11 \\
\beta_{1}\left(x_{11}\right)=x_{3}, & \text { since }(11+3)(\bmod 11)+0=3 \text { because of } 11+3>11
\end{array}
$$

and for $i=2,3,4$ the values of $\beta_{i}$ are obtained similarly. Hence

| $X_{1}$ | $x_{1}$ | $x_{2}$ | $x_{3}$ | $x_{4}$ | $x_{5}$ | $x_{6}$ | $x_{7}$ | $x_{8}$ | $x_{9}$ | $x_{10}$ | $x_{11}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\beta_{1}$ | $\downarrow$ | $\downarrow$ | $\downarrow$ | $\downarrow$ | $\downarrow$ | $\downarrow$ | $\downarrow$ | $\downarrow$ | $\downarrow$ | $\downarrow$ | $\downarrow$ |
| $X_{1}$ | $x_{4}$ | $x_{5}$ | $x_{6}$ | $x_{7}$ | $x_{8}$ | $x_{9}$ | $x_{10}$ | $x_{11}$ | $x_{1}$ | $x_{2}$ | $x_{3}$ |
| $X_{2}$ | $x_{12}$ | $x_{13}$ | $x_{14}$ | $x_{15}$ | $x_{16}$ | $x_{17}$ |  |  |  |  |  |
| $\beta_{2}$ | $\downarrow$ | $\downarrow$ | $\downarrow$ | $\downarrow$ | $\downarrow$ | $\downarrow$ |  |  |  |  |  |
| $X_{2}$ | $x_{16}$ | $x_{17}$ | $x_{12}$ | $x_{13}$ | $x_{14}$ | $x_{15}$ |  |  |  |  |  |
| $X_{3}$ | $x_{18}$ | $x_{19}$ | $x_{20}$ | $x_{21}$ | $x_{22}$ | $x_{23}$ | $x_{24}$ | $x_{25}$ | $x_{26}$ |  |  |
| $\beta_{3}$ | $\downarrow$ | $\downarrow$ | $\downarrow$ | $\downarrow$ | $\downarrow$ | $\downarrow$ | $\downarrow$ | $\downarrow$ | $\downarrow$ |  |  |
| $X_{3}$ | $x_{25}$ | $x_{26}$ | $x_{18}$ | $x_{19}$ | $x_{20}$ | $x_{21}$ | $x_{22}$ | $x_{23}$ | $x_{24}$ |  |  |
| $X_{4}$ | $x_{27}$ | $x_{28}$ | $x_{29}$ |  |  |  |  |  |  |  |  |
| $\beta_{4}$ | $\downarrow$ | $\downarrow$ | $\downarrow$ |  |  |  |  |  |  |  |  |
| $X_{4}$ | $x_{29}$ | $x_{27}$ | $x_{28}$ |  |  |  |  |  |  |  |  |

and therefore,

$$
\begin{array}{c|llll|llll|llll|lll}
X & x_{1} & x_{2} & \ldots & x_{11} & x_{12} & x_{13} & \ldots & x_{17} & x_{18} & x_{19} & \ldots & x_{26} & x_{27} & x_{28} & x_{29} \\
\beta & \downarrow & \downarrow & & \downarrow & \downarrow & \downarrow & & \downarrow & \downarrow & \downarrow & & \downarrow & \downarrow & \downarrow & \downarrow \\
X & x_{4} & x_{5} & \ldots & x_{3} & x_{16} & x_{17} & \ldots & x_{15} & x_{25} & x_{26} & \ldots & x_{24} & x_{29} & x_{27} & x_{28}
\end{array}
$$

Step 5: Values of the characterization function $\alpha^{-1}$ are obtained as following list:

$$
\begin{array}{c|llll|llll|llll|lll}
X & x_{4} & x_{5} & \ldots & x_{3} & x_{16} & x_{17} & \ldots & x_{15} & x_{25} & x_{26} & \ldots & x_{24} & x_{29} & x_{27} & x_{28} \\
\alpha^{-1} & \downarrow & \downarrow & & \downarrow & \downarrow & \downarrow & & \downarrow & \downarrow & \downarrow & & \downarrow & \downarrow & \downarrow & \downarrow \\
U & \text { ç } & \mathrm{d} & \ldots & \mathrm{c} & \mathrm{~m} & \mathrm{n} & \ldots & \mathrm{l} & \mathrm{u} & \mathrm{u} & \ldots & \mathrm{t} & \mathrm{z} & \mathrm{v} & \mathrm{y}
\end{array}
$$

Step 6: Values of the encryption function $\gamma$ are obtained as following list:

| $U$ | a | b | c | c | d | e | f | g | g | h | l | i | j | k | l | m | n | $\ldots$ | v | y | z |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\gamma$ | $\downarrow$ | $\downarrow$ | $\downarrow$ | $\downarrow$ | $\downarrow$ | $\downarrow$ | $\downarrow$ | $\downarrow$ | $\downarrow$ | $\downarrow$ | $\downarrow$ | $\downarrow$ | $\downarrow$ | $\downarrow$ | $\downarrow$ | $\downarrow$ | $\downarrow$ |  | $\downarrow$ | $\downarrow$ | $\downarrow$ |
| $U$ | c | d | e | f | g | g | h | l | a | b | c | m | n | o | i | j | k | $\ldots$ | z | v | y |

In this example we showed that the plaintext "ankara" is encrypted as "çliçöç" according to the method which can be seen in Figure 1.


Figure 1: Encryption of "ankara" by FCEA

### 2.3 FCC-Decryption Algorithm

Assume that $U$ is a character set and $X$ is an indexed character set. Then, an algorithm of the FCC-Decryption is set up as follows:

## Algorithm of FCC-Decryption:

Step 1: Use the $\left\{X_{i}: i \in I_{p}\right\}$ and the module $m_{i}$ for each $X_{i}$,
Step 2: Find the $\beta_{i}^{-1}\left(x_{t}\right)$ for $x_{t} \in X, t \in I_{a_{i}}$ and $i \in I_{p}$,
Step 3: Find the $\alpha^{-1}(x)$ for $x \in U$,
Step 4: Find the $\gamma^{-1}(x)$ for $x \in U$.
Example 20. Let us consider the result of Example 19 where

$$
U=\{c, \mathrm{~d}, \mathrm{e}, \mathrm{f}, \mathrm{~g}, \mathrm{~g}, \mathrm{~h}, \mathrm{l}, \mathrm{a}, \mathrm{~b}, \mathrm{c}, \mathrm{~m}, \mathrm{n}, \mathrm{i}, \mathrm{j}, \mathrm{k}, \mathrm{l}, \mathrm{u}, \ddot{\mathrm{u}}, \mathrm{o}, \ddot{\mathrm{o}}, \mathrm{p}, \mathrm{r}, \mathrm{~s}, \mathrm{~s}, \mathrm{t}, \mathrm{z}, \mathrm{v}, \mathrm{y}\}
$$

and

$$
X=\left\{x_{1}, x_{2}, x_{3}, x_{4}, x_{5}, x_{6}, x_{7}, x_{8}, x_{9}, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}, x_{17}, \ldots, x_{29}\right\}
$$

Then,
Step 1: In Example 19, for $a_{i}(i=1,2,3,4)$ the fragments of $X, X_{i}$, and was obtained as

$$
\begin{array}{ll}
\text { for } a_{1}=11, & X_{1}=\left\{x_{1}, x_{2}, x_{3}, x_{4}, x_{5}, x_{6}, x_{7}, x_{8}, x_{9}, x_{10}, x_{11}\right\} \\
\text { for } a_{2}=6, & X_{2}=\left\{x_{12}, x_{13}, x_{14}, x_{15}, x_{16}, x_{17}\right\} \\
\text { for } a_{3}=9, & X_{3}=\left\{x_{18}, x_{19}, x_{20}, x_{21}, x_{22}, x_{23}, x_{24}, x_{25}, x_{26}\right\} \\
\text { for } a_{4}=3, & X_{4}=\left\{x_{27}, x_{28}, x_{29}\right\}
\end{array}
$$

and value of $m_{i}(i=0,1,2,3,4)$ was also choosen as,

$$
\begin{array}{ll}
\text { for } i=0, & m_{0}=0 \\
\text { for } i=1, & m_{1}=\left(m_{0}=0\right)+\left(a_{1}=11\right)=11 \\
\text { for } i=2, & m_{2}=\left(m_{1}=11\right)+\left(a_{2}=6\right)=17 \\
\text { for } i=3, & m_{3}=\left(m_{2}=17\right)+\left(a_{3}=9\right)=26 \\
\text { for } i=4, & m_{4}=\left(m_{3}=26\right)+\left(a_{4}=3\right)=29
\end{array}
$$

Step 2: For $i=1,2,3,4$ values of the $X_{i}$-rotation function $\beta_{i}^{-1}$ are obtained as follows. Here, we first obtain the values of $\beta_{1}^{-1}$ as,

$$
\begin{aligned}
& \beta_{1}^{-1}\left(x_{1}\right)=x_{9}, \\
& \beta_{1}^{-1}\left(x_{2}\right)=x_{10}, \\
& \beta_{1}^{-1}\left(x_{3}\right)=x_{11}, \\
& \beta_{1}^{-1}\left(x_{4}\right)=x_{1}, \\
& \beta_{1}^{-1}\left(x_{5}\right)=x_{2}, \\
& \beta_{1}^{-1}\left(x_{6}\right)=x_{3}, \\
& \beta_{1}^{-1}\left(x_{7}\right)=x_{4}, \\
& \beta_{1}^{-1}\left(x_{8}\right)=x_{5}, \\
& \beta_{1}^{-1}\left(x_{9}\right)=x_{6}, \\
& \beta_{1}^{-1}\left(x_{10}\right)=x_{7}, \\
& \beta_{1}^{-1}\left(x_{11}\right)=x_{8},
\end{aligned}
$$

$$
\text { since } 1+m_{1}-r_{1}=1+11-3=9 \text { because of } 1+11-3<11
$$

since $2+m_{1}-r_{1}=2+11-3=10$ because of $2+11-3<11$
since $3+m_{1}-r_{1}=3+11-3=11$ because of $3+11-3=11$
since $(4+11-3)(\bmod 11)+0=1$ because of $4+11-3>11$
since $(5+11-3)(\bmod 11)+0=2$ because of $5+11-3>11$
since $(6+11-3)(\bmod 11)+0=3$ because of $6+11-3>11$
since $(7+11-3)(\bmod 11)+0=4$ because of $7+11-3>11$
since $(8+11-3)(\bmod 11)+0=5$ because of $8+11-3>11$
since $(9+11-3)(\bmod 11)+0=6$ because of $9+11-3>11$
since $(10+11-3)(\bmod 11)+0=7$ because of $10+11-3>11$
since $(11+11-3)(\bmod 11)+0=8$ because of $11+11-3>11$
and for $i=2,3,4$ the values of $\beta_{i}^{-1}$ are obtained similarly. Hence,

$$
\begin{array}{c|lllllllllll}
X_{1} & x_{1} & x_{2} & x_{3} & x_{4} & x_{5} & x_{6} & x_{7} & x_{8} & x_{9} & x_{10} & x_{11} \\
\beta_{1}^{-1} & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\
X_{1} & x_{9} & x_{10} & x_{11} & x_{1} & x_{2} & x_{3} & x_{4} & x_{5} & x_{6} & x_{7} & x_{8} \\
X_{2} & x_{12} & x_{13} & x_{14} & x_{15} & x_{16} & x_{17} & & & & \\
\beta_{2}^{-1} & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & & & & \\
X_{2} & x_{14} & x_{15} & x_{16} & x_{17} & x_{12} & x_{13} & & & & \\
X_{3} & x_{18} & x_{19} & x_{20} & x_{21} & x_{22} & x_{23} & x_{24} & x_{25} & x_{26} \\
\beta_{3}^{-1} & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \\
X_{3} & x_{20} & x_{21} & x_{22} & x_{23} & x_{24} & x_{25} & x_{26} & x_{18} & x_{19} & \\
X_{4} & x_{27} & x_{28} & x_{29} & & & & & & & \\
\beta_{4}^{-1} & \downarrow & \downarrow & \downarrow & & & & & & & \\
X_{4} & x_{29} & x_{27} & x_{28} & & & & & & &
\end{array}
$$

and therefore,

$$
\begin{array}{c|llll|llll|llll|lll}
X & x_{1} & x_{2} & \ldots & x_{11} & x_{12} & x_{13} & \ldots & x_{17} & x_{18} & x_{19} & \ldots & x_{26} & x_{27} & x_{28} & x_{29} \\
\beta^{-1} & \downarrow & \downarrow & & \downarrow & \downarrow & \downarrow & & \downarrow & \downarrow & \downarrow & & \downarrow & \downarrow & \downarrow & \downarrow \\
X & x_{9} & x_{10} & \ldots & x_{8} & x_{14} & x_{15} & \ldots & x_{13} & x_{20} & x_{21} & \ldots & x_{19} & x_{29} & x_{27} & x_{28}
\end{array}
$$

Step 3: Values of the characterization function $\alpha^{-1}$ are obtained as following list:

$$
\begin{array}{c|llll|llll|llll|lll}
X & x_{9} & x_{10} & \ldots & x_{8} & x_{14} & x_{15} & \ldots & x_{13} & x_{20} & x_{21} & \ldots & x_{19} & x_{29} & x_{27} & x_{28} \\
\alpha^{-1} & \downarrow & \downarrow & & \downarrow & \downarrow & \downarrow & & \downarrow & \downarrow & \downarrow & & \downarrow & \downarrow & \downarrow & \downarrow \\
U & \mathrm{a} & \mathrm{~b} & \ldots & 1 & \mathrm{i} & \mathrm{j} & \ldots & \mathrm{n} & \mathrm{o} & \ddot{\mathrm{o}} & \ldots & \text { ü } & \mathrm{v} & \mathrm{y} & \mathrm{z}
\end{array}
$$

Step 4: Values of the decryption function $\gamma^{-1}$ are obtained as following list:

| $U$ | c | d | e | f | g | g | h | l | a | b | c | m | n | o | i | j | k | $\ldots$ | z | v | y |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\gamma^{-1}$ | $\downarrow$ | $\downarrow$ | $\downarrow$ | $\downarrow$ | $\downarrow$ | $\downarrow$ | $\downarrow$ | $\downarrow$ | $\downarrow$ | $\downarrow$ | $\downarrow$ | $\downarrow$ | $\downarrow$ | $\downarrow$ | $\downarrow$ | $\downarrow$ | $\downarrow$ |  | $\downarrow$ | $\downarrow$ | $\downarrow$ |
| $U$ | a | b | c | c | d | e | f | g | g | h | l | i | j | k | l | m | n | $\ldots$ | v | y | z |

In this example we showed that the ciphertext "çliçöç" is decrypted as "ankara".

## 3 FC Encryption Program Codes

In this section, the FCC-method is programmed by using C\# as follows:

```
private static string[] alf_tex()
{
    string[] alphabet = { "a", "b",..,"Y", "Z"}; //caharacter set
    return alpabet;
}
private static int _rnd(int bas, int bit)
```

```
{
    Random rnd = new Random();
    int deger = rnd.Next(bas, bit);
    return deger;
}
private static string _key(int alphabet_number)
{
    string key = "";
    int n = alphabet_number;
    int a,r;
    do
    {
        if (n >= 6)
        {
            a = _rnd(3, n - 3); //Fragment key
        }
        else
        {
            a = n;
        }
        r = _rnd(2, a);//Rotation key
        key += a.ToString() + "," + r.ToString() + ' -';
        n = n - a;
    }
    while (n > 0);
    return key;
}
private void btn_creat_alphabet_Click(object sender, EventArgs e)
{
    //key function
    if (rdsifre.Checked)
    {
        string key = _key(alp_tex().Length);
        txtanahtar.Text = key;
    }
    int alfabe_sayac = 1;
    string[] fragment = key.Substring(0,key.Length-1).Split('-');
    string[][] U = new string[fragment.Length][];//(Açık U)
    string[] [] SU = new string[fragment.Length][];//(encrypted U)
    //be divided into sets of the alphabet
    for (int j = 0; j < parca.Length; j++)
    {
        string[] parca_a = fragment[j].ToString().Split(',');
```

```
    int P = int.Parse(fragment_a[0]); //fragment key
    U[j] = new string[alp_tex().Length + 1];
    SU[j] = new string[alp_tex().Length + 1];
    for (int x = 0; x < P; x++)
    {
        U[j][alfabe_sayac] = alp_tex()[alphabet_sayac - 1];
        alphabet_sayac++;
    }
    }
    //encrypting alphabet
    int m = 0;
    int index = 1;
    for (int j = 0; j < U.Length; j++)
    {
        string[] parca_a = fragment[j].ToString().Split(',');
        int P = int.Parse(fragment_a[0]); //fragment key
        int R = int.Parse(fragment_a[1]); //rotation key
        m += P;
        for (int x = 0; x < P; x++)
        {
            int k = 0;
            if ((index + R) <= m) //rotation function
            {
                k = index + R;
            }
            else if ((index + R) > m)
            {
                k = ((index + R) % m) + (m - P);
            }
            SU[j][index] = U[j][k];
            index++;
            }
    }
}
```


## 4 Conclusions

In this work, a cipher method so called fragmented Caesar cipher method is defined. A mathematical modeling and then a computer programs of the method have done. The method is based on the basic logic of Caesar cipher. The classical Caesar cipher is a type of substitution cipher in which each letter is replaced by a letter some
fixed number of positions in the alphabet. The Fragmented Caesar cipher has more possibility then the classical Caesar cipher because of the fragmented alphabet is used to cipher. We finally give an example to show the cipher method is working successfully.

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[^0]:    ${ }^{* *}$ Edited by Oktay Muhtaroğlu (Area Editor)

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