http://www.newtheory.org ISSN: 2149-1402

Journal of New Theory

Received: 10.12.2017 *Year*: **2018**, *Number*: **20**, *Pages*: **27-47**
Published: 14.01.2018 *Original Article*

On Distances and Similarity Measures between Two Interval Neutrosophic Sets

Sudip Bhattacharyya[1] <sudip.mathematica@gmail.com>
Bikas Koli Roy[1] <bikashroy235@gmail.com>
Pinaki Majumdar[1,*] <pmajumdar2@rediffmail.com>

[1]*M.U.C Women's College, Mathematics Department, 713104, Burdwan, India.*

Abstract – An Interval Neutrosophic set (INS) is an instance of a Neutrosophic set and also an emerging tool for uncertain data processing in real scientific and engineering applications. In this paper, several distance and similarity measures between two Interval Neutrosophic sets have been discussed. Distances and similarities are very useful techniques to determine interacting segments in a data set. Here we have also shown an application of our similarity measures in solving a multicriteria decision making method based on INS's. Finally, we take an illustrative example from [14] to apply the proposed decision making method. We use the distance as well as the similarity measures between each alternative and ideal alternative to form a ranking order and also to find the best alternative. We compare the obtained results with the existing result in [14] and also reveal the best distance and similarity measure to find the best alternative and also point out the best alternative.

Keywords – *Interval Neutrosophic Set, Distance, Similarity Measure, Multicriteria Decision Making.*

1. Introduction

"As far as the laws of Mathematics refer to reality, they are not certain; and as far they are certain, they do not refer to reality." – Albert Einstein. Uncertainty is a common phenomenon in our daily life; because in our real or daily life we have to take account a lot of uncertainties. From centuries, numerous theories have been developed in both Science and Philosophy to understand and represent the features of uncertainty. Probability theory and stochastic techniques are such theories, which were developed in early eighteenth century and probability was the sole technique to handle a certain type of uncertainty called Randomness. But there are several other kinds of uncertainties, such as vagueness, imprecision, cloudiness, haziness, ambiguity, variety etc. It is generally agreed that the most important invention in the evolution of the concept of uncertainty was made by Zadeh in 1965, when he coined the theory of Fuzzy sets [17], which was a remarkable step to deal with such types of uncertainties, though some ideas presented by him, were borrowed from the envisions of American philosopher Max Black (1937). In his theory, Zadeh introduced the fuzzy sets, which have imprecise boundaries. When A is a fuzzy set and x is an object of A, then the statement ' x is a member of A' is not only either true or false as in crisp sets, but also it is true only to some degree to which x is actually a member of A. The membership degrees are within the closed interval [0,1]. Later, this theory leads to a highly commendable theory of Fuzzy logic, which was applied to engineering such as washing machine or shifting gears of cars with great efficiency. After Zadeh's invention of Fuzzy sets, many other concepts began to develop. In 1986, K. Atanassov [1], introduced the idea of Intuitionistic fuzzy sets (IFS), which is a generalization of Fuzzy sets. The IFS is a set with each member having a degree of belongingness and a degree of non-belongingness as well. There is a restriction that sum of the membership grade and non-membership grade of an element is less or equal to 1. IFS is quite useful to deal with applications like expert systems, information fusion etc., where 'degree of non belongingness' of an object is equally important as the 'degree of belongingness'. Besides IFS, there are other generalizations of Fuzzy sets and intuitionistic fuzzy sets like L-Fuzzy sets, interval valued fuzzy sets, intuitionistic L-Fuzzy sets, interval valued intuitionistic fuzzy sets [11,2] etc.

In 1995, Smarandache [9, 10], introduced a more generalized tool to handle Uncertainty, called as Neutrosophic logic and sets. It is a logic, in which each proposition has a degree of truth (T), a degree of indeterminacy (I), and a degree of falsity (F). Also an element x in a Neutrosophic set (NS) X has a truth membership, an indeterminacy membership and a falsity membership, which are independent and which lies between [0, 1], and sum of them is less or equal to 3. Thus Neutrosophic set is a generalization of fuzzy set [17], interval valued fuzzy set [11], intuitionistic fuzzy set [1], interval valued intuitionistic fuzzy set [2], paraconsistent set [9], dialetheist set [9], paradoxist set [9] and tautological set [9]. Though the NS generalized the above mentioned sets, but the generalization was only from philosophical point of view. For application in engineering and other areas of science, NS needed to be more specific. Further Wang et. al., in 2005, developed an instance of NS, called as single valued Neutrosophic sets (SVNS) [13]. Later they have also introduced the notion of Interval valued neutrosophic sets (INS) [12]. The INS is more capable to handle the uncertain, imprecise, incomplete and inconsistent information that exist in real world. In INS, the degree of truth, indeterminacy and falsity membership of an object are expressed in closed subintervals of [0, 1].

In many problems, it is often needed to compare two sets, which may be fuzzy, intuitionistic fuzzy, vague etc. We are often interested to reveal the similarity or the least degree of similarity of two images or patterns. Distance and similarity measures are the efficient tools to do this. Many authors have done extensive research regarding distance and similarity of fuzzy and intuitionistic fuzzy sets and their interval valued versions [7, 8, 15, 16]. Similarity measures are also a very good tool for solving many decision making problems. The notion of distance and similarity was first introduced in [5,6]. Later Broumi et. al. [3] has defined several other similarity measures on Single valued neutrosophic sets. The notion of similarity of INS is introduced in [4, 14].This paper also deals with distance and similarity of Interval neutrosophic sets. However, in this article, our motive is to establish the best suitable distance and similarity measures by comparing the numerical value of various distances and similarities between two INSs. We are to also point out the best alternative, similar to the ideal alternative in the decision making problem stated and solved by Jun Ye [14], by comparing numerical values of distances and similarities of each alternative with the ideal alternative and also comparing with the existing results [14].

The organization of the rest of this paper is as follows: In section 2, definitions of Fuzzy set, Intuitionistic Fuzzy set, Neutrosophic Set (NS) and Interval valued Neutrosophic set (INS) are given and some operations on NS and INS have been defined and also Set theoretic properties on INS are also given. Several distances and Similarities on INSs are defined in section 3 and 4. A decision making method is established in Interval Neutrosophic setting by means of distance and similarity measures between each alternative and ideal alternative in section 5. In section 6, an illustrative example is adapted from [14], to illustrate the proposed method. Finally a comparative study has been made with the existing results in section 7 and at last section 8 concludes the article.

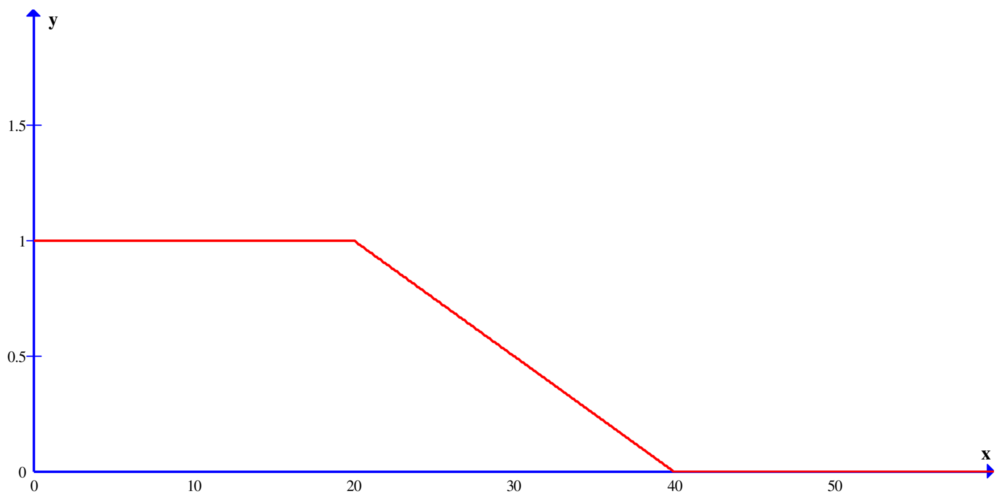
2. Preliminaries

In this section, we give some useful definitions, examples and results which will be used in the rest of this paper.

Definition 2.1 (*Type I Fuzzy set*) If X is a collection of objects denoted by x, then a fuzzy set (or type I fuzzy set) A in X *is* a set of ordered pairs: $A = \{(x, \mu_A(x)) \mid x \in X\}$ where $\mu_A(x)$ is called the membership function or grade of membership (also degree of compatibility or degree of truth) of x in A that maps X to the membership space, i.e. $\mu_A : X \to M = [0,1]$. A becomes a crisp set when M contains only two points 0 and 1and μ_A is the characteristic function χ_A of A.

Example 2.2 As an illustration, consider the following example. Let, the set 'P' is the set of people. To each person in 'P' we have to assign a degree of membership in the fuzzy subset YOUTH, which is defined as follows:

$$\text{Youth } (x) = \{\ 1,\ if\ age(x) \le 20,$$
$$(40 - age(x))/20,\ if\ 20 < age(x) \le 40,$$
$$0,\ if\ age(x) > 40\ \}$$



Then the set YOUTH is a fuzzy set of type I or an ordinary fuzzy set.

Definition 2.3 (*Intuitionistic fuzzy set)* Intuitionistic fuzzy sets generalize fuzzy sets, since with membership function μ, a non-membership function ν is also introduced for each object in it.

Let us have a fixed universe $\boldsymbol{X}$. Let $\boldsymbol{A} \subseteq \boldsymbol{X}$. Let us construct the set:

$$A^* = \{(x, \mu_A(x), \nu_A(x)) \mid x \in X \;\&\; 0 \le \mu_A(x) + \nu_A(x) \le 1\}$$

where $\mu_A : X \to [0,1], \nu_A : X \to [0,1]$ and $\forall x \in X$ .We call the set $\boldsymbol{A^*}$ intuitionistic fuzzy set (IFS).

Example 2.4 Let us illustrate the concept of IFS by an example as follows: Let X be the set of all Secondary schools in a district. We assume that, for every school $x \in X$, the number of students qualified in the final exam is known and say it is $P(x)$. Let,

$$\mu_X(x) = {P(x)}\big/{(total\ number\ of\ students)}$$

Take $\nu_X(x) = 1 - \mu_X(x)$, which indicates the part of students couldn't qualify the exam. By Fuzzy set theory, we cannot obtain that how many students have not given the exam. But, if we take $\nu_X(x)$ as the number of students failed to qualify the exam, then we can easily obtain the part of the students, have not given the exam at all and the value will be $1 - \mu_X(x) - \nu_X(x)$. Thus we construct the IFS, $\{(x, \mu_X(x), \nu_X(x)) : x \in X\}$ and obviously $0 \le \mu_X(x) + \nu_X(x) \le 1$

Definition 2.5 (*Neutrosophic set)* Neutrosophic sets (NS) further generalizes the IFS. As in NS, the indeterminacy is explicitly defined and also the truth membership, falsity membership and indeterminacy membership are beyond any restriction. Let X be a collection of objects denoted by $\boldsymbol{x}$. A Neutrosophic set $\boldsymbol{A}$ in $\boldsymbol{X}$ is characterized by a truth membership function $\boldsymbol{T_A}$, an Indeterminacy membership function $\boldsymbol{I_A}$ and a falsity membership function $\boldsymbol{F_A}$, where,

$$T_A(x),\ I_A(x) \text{ and } F_A(x) : X \to [0,1] \text{ and } 0 \le \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \le 3.$$

The NS $\boldsymbol{A}$ in $\boldsymbol{X}$ can be denoted as $A = \{x, T_A(x), I_A(x), F_A(x) : x \in X\}$

Example 2.6 If x_1 be an element of a set A and if we take the probability of x_1 in A is 60%, probability of x_1 not in A is 20% and probability of x_1 in A is undetermined is 10%, then the NS can be denoted as x_1(0.6,0.1,0.2). Also to generalize the example, Take X be the set of 'rainy days'. Consider A be the set "today it will rain heavily." Let according to an observer x_1, probability of heavy raining is 80%, that of not raining is 10%, and also the indeterminacy is 10%. According to another observer x_2, those probabilities are 40%, 50% and 10% respectively. Then NS A in X can be denoted as follows:

$$A = \langle 0.8, 0.1, 0.1 \rangle / x_1 + \langle 0.4, 0.1, 0.5 \rangle / x_2$$

Definition 2.7(*Interval Neutrosophic set*) Let ***X be*** a space of objects, whose elements are denoted by *x*. An INS ***A*** in ***X*** is characterized by a truth-membership function. $T_A(x)$, an indeterminacy-membership function $I_A(x)$ and a falsity-membership function $F_A(x)$. For each point ***x*** in ***X***, we have:

$$T_A(x) = [\inf T_A(x), \sup T_A(x)] \subseteq [0,1],$$
$$I_A(x) = [\inf I_A(x), \sup I_A(x)] \subseteq [0,1],$$
$$F_A(x) = [\inf F_A(x), \sup F_A(x)] \subseteq [0,1]$$

and

$$0 \le \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \le 3, \forall x \in X.$$

When *X* is continuous, an INS *A* can be written as :

$$A = \int_X \langle T(x), I(x), F(x) \rangle / x, \quad x \in X$$

When *X* is discrete, an INS *A* can be written as :

$$A = \sum_{i=1}^{n} \langle T(x_i), I(x_i), F(x_i) \rangle / x_i, \quad x_i \in X$$

Example 2.8 For example, Assume that x_1 is quality, x_2 is trustworthiness and x_3 is price of a book. The values of x_1, $x_2$and x_3 are in [0, 1]. They are obtained from some questionnaires, having options as 'degree of good', 'degree of indeterminacy' and 'degree of bad'. Take *A* and *B* are interval neutrosophic sets of X defined as:

$$A = \langle [0.1, 0.3], [0, 0.2], [0.5, 0.7] \rangle / x_1 + \langle [0.4, 0.5], [0.1, 0.2], [0.6, 0.7] \rangle / x_2 + \langle [0.7, 0.8], [0, 0.3], [0.1, 0.2] \rangle / x_3$$

$$B = \langle [0.2, 0.4], [0.1, 0.3], [0.6, 0.8] \rangle / x_1 + \langle [0.7, 0.9], [0.4, 0.6], [0.2, 0.4] \rangle / x_2 + \langle [0.3, 0.5], [0.2, 0.4], [0.1, 0.3] \rangle / x_3$$

Some operations on Neutrosophic sets

Definition 2.9

(i) **Complement:** Let *A* be a Neutrosophic set. Then *complement* of *A* is denoted by A^c or $\bar{A}$ and is defined by

$$T_{\bar{A}}(x) = F_A(x), I_{\bar{A}}(x) = 1 - I_A(x), F_{\bar{A}}(x) = T_A(x), \forall x \in X$$

(ii) **Containment:** A NS *A* is *contained* in the other NS *B*, denoted as $A \subseteq B$, if and only if:

$$\mathrm{T}_A(\mathrm{x}) \leq \mathrm{T}_B(\mathrm{x})\ ; \mathrm{I}_A(\mathrm{x}) \geq \mathrm{I}_B(\mathrm{x})\ ; \mathrm{F}_A(\mathrm{x}) \geq \mathrm{F}_B(\mathrm{x})\ ; \mathrm{x} \in \mathrm{X}$$

(iii) **Union:** The *union* of two NS *A* and *B* is a NS *C,* written as $C = A \cup B$, whose truth-membership, indeterminacy-membership and falsity membership functions are related to those of *A* and *B* by:

$$\begin{aligned} T_C(x) &= T_A(x) \vee T_B(x), \\ I_C(x) &= I_A(x) \wedge I_B(x), \\ F_C(x) &= F_A(x) \wedge F_B(x), \forall x \in X \end{aligned}$$

(iv) **Intersection:** The *intersection* of two NS *A* and *B* is a NS *C*, denoted as *C*=*A*∩*B*, whose truth-membership, indeterminacy-membership and falsity membership functions are related to those of *A* and *B* by:

$$\begin{aligned} T_C(x) &= T_A(x) \wedge T_B(x), \\ I_C(x) &= I_A(x) \vee I_B(x), \\ F_C(x) &= F_A(x) \vee F_B(x), \forall x \in X \end{aligned}$$

Some operations on Interval Neutrosophic set

The notion of IVNS was defined by Wang et. al. [13]. Here we give some definitions and examples of IVNS

Definition 2.10 (Complement): Let *A* be an Interval Neutrosophic set. Then *complement* of A is denoted by A^c or $\bar{A}$ and is defined by:

$$\begin{aligned} &T_{\bar{A}}(x) = F_A(x), \\ &\inf I_{\bar{A}}(x) = 1 - \sup I_A(x), \\ &\sup I_{\bar{A}}(x) = 1 - \inf I_A(x), \\ &F_{\bar{A}}(x) = T_A(x) \end{aligned}$$

Example 2.11 Let A be the interval valued Neutrosophic set defined in *example 2.8*. Then

$$\begin{aligned} \bar{A} = &\langle [0.5, 0.7], [0.8, 1.0], [0.1, 0.3] \rangle / x_1 + \\ &\langle [0.6, 0.7], [0.8, 0.9], [0.4, 0.5] \rangle / x_2 + \\ &\langle [0.1, 0.2], [0.7, 1.0], [0.7, 0.8] \rangle / x_3 \end{aligned}$$

Definition 2.12 (Containment) A INS *A* is *contained* in the other INS *B*, denoted as $A \subseteq B$, if and only if:

$$\inf T_A(x) \le \inf T_B(x),\ \sup T_A(x) \le \sup T_B(x);$$
$$\inf I_A(x) \ge \inf I_B(x),\ \sup I_A(x) \ge \sup I_B(x);$$
$$\inf F_A(x) \ge \inf F_B(x),\ \sup F_A(x) \ge \sup F_B(x); \forall\ x \in X$$

Two interval neutrosophic sets A and B are ***equal***, written as A = B, if and only if $A \subseteq B$ and $B \subseteq A$

Example 2.13 Let A and B be two INS defined in *example 3.1.4*, then it can be easily observed that those INSs do not satisfy all the required properties for containment of A in B. So here $A \not\subset B$.

Definition 2.14 (Union): The *union* of two INS *A* and *B* is a INS *C,* written as $C = A \cup B$, whose truth-membership, indeterminacy-membership and falsity membership functions are related to those of *A* and *B* by:

$$\inf T_C(x) = \max(\inf T_A(x), \inf T_B(x)),$$
$$\sup T_C(x) = \max(\sup T_A(x), \sup T_B(x)),$$
$$\inf I_C(x) = \min(\inf I_A(x), \inf I_B(x)),$$
$$\sup I_C(x) = \min(\sup I_A(x), \sup I_B(x)),$$
$$\inf F_C(x) = \min(\inf F_A(x), \inf F_B(x)),$$
$$\sup F_C(x) = \min(\sup F_A(x), \sup F_B(x)), \forall x \in X$$

Example 2.15: Consider two INS A and B defined in example 2.8. Then their union C =A∪B is

$$C = \langle [0.2,0.4],[0,0.2],[0.5,0.7] \rangle / x_1 + \langle [0.7,0.9],[0.1,0.2],[0.2,0.4] \rangle / x_2 + \langle [0.7,0.8],[0,0.3],[0.1,0.2] \rangle / x_3$$

Definition 2.16 (Intersection) The *intersection* of two INS *A* and *B* is a INS *C*, denoted as *C*=*A*∩*B*, whose truth-membership, indeterminacy-membership and falsity membership functions are related to those of *A* and *B* by:

$$\inf T_C(x) = \min(\inf T_A(x), \inf T_B(x)),$$
$$\sup T_C(x) = \min(\sup T_A(x), \sup T_B(x)),$$
$$\inf I_C(x) = \max(\inf I_A(x), \inf I_B(x)),$$
$$\sup I_C(x) = \max(\sup I_A(x), \sup I_B(x)),$$
$$\inf F_C(x) = \max(\inf F_A(x), \inf F_B(x)),$$
$$\sup F_C(x) = \max(\sup F_A(x), \sup F_B(x)), \forall x \in X$$

Example 2.17 Take A and B be two INS defined in *example 2.8*. Then their intersection C=A∩B is as follows:

$$C = \langle [0.1,0.3],[0.1,0.3],[0.6,0.8] \rangle / x_1 + \langle [0.4,0.5],[0.4,0.6],[0.6,0.7] \rangle / x_2 + \langle [0.3,0.5],[0.2,0.4],[0.1,0.3] \rangle / x_3$$

Set theoretical properties

Here we will give some properties of set-theoretic operators defined on interval neutrosophic sets.

Let, A, B and C be three INSs. Then the properties satisfied by A, B and C are as follows:

Property 1 ***(Commutativity)***

$$A \cup B = B \cup A$$
$$A \cap B = B \cap A$$

Property 2 ***(Associativity)***

$$A \cup (B \cup C) = (A \cup B) \cup C$$
$$A \cap (B \cap C) = (A \cap B) \cap C$$

Property 3 ***(Distributivity)***

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$
$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

Property 4 ***(Idempotency)***

$$A \cup A = A, A \cap A = A.$$

Property 5 $A \cap \Phi = \Phi, A \cup X = X$, Where Φ and X are respectively Null set and absolute INS defined below:

$$\inf T_\Phi = \sup T_\Phi = 0,$$
$$\inf I_\Phi = \sup I_\Phi = \inf F_\Phi = \sup F_\Phi = 1,$$
$$\inf T_X = \sup T_X = 1,$$
$$\inf I_X = \sup I_X = \inf F_X = \sup F_X = 0$$

Property 6

$A \cup \Phi = A, A \cap X = A,$ Where Φ and X are defined above.

Property 7 ***(Absorption)***

$$A \cup (A \cap B) = A, A \cap (A \cup B) = A$$

Property8 (Involution)

$$\overline{\overline{A}} = A$$

Here, we notice that by the definitions of complement, union and intersection of interval neutrosophic set as defined previously, INS satisfies the most properties of crisp set, fuzzy set and intuitionistic fuzzy set. Also, it does not satisfy the principle of excluded middle, same as fuzzy set and intuitionistic fuzzy set.

3. Distance Measure

In this section, we investigate several distance measures for two INS's A and B. Also, we take the weights of the element x_i (i= 1, 2,….,n) into account. In the following, we consider some weighted distance measures between INSs. For this we take $w=\{w_1,w_2,….,w_n\}$ as the weight vector of the element x_i (i =1,2,….,n) and also $w_i \in [0,1], \forall i = 1,2,.....,n$ .We adopt some distance and similarity measures from [15] and extend those in INS setting as follows:

a. *Hamming Distance :*

$$\begin{aligned} d_1(A,B) = \frac{1}{6}\sum_{i=1}^{n} [&|\inf T_A(x_i) - \inf T_B(x_i)| + |\sup T_A(x_i) - \sup T_B(x_i)| + \\ &|\inf I_A(x_i) - \inf I_B(x_i)| + |\sup I_A(x_i) - \sup I_B(x_i)| + \\ &|\inf F_A(x_i) - \inf F_B(x_i)| + |\sup F_A(x_i) - \sup F_B(x_i)|] \end{aligned}$$

b. *Normalized Hamming Distance :*

$$\begin{aligned} d_2(A,B) = \frac{1}{6n}\sum_{i=1}^{n} [&|\inf T_A(x_i) - \inf T_B(x_i)| + |\sup T_A(x_i) - \sup T_B(x_i)| + \\ &|\inf I_A(x_i) - \inf I_B(x_i)| + |\sup I_A(x_i) - \sup I_B(x_i)| + \\ &|\inf F_A(x_i) - \inf F_B(x_i)| + |\sup F_A(x_i) - \sup F_B(x_i)|] \end{aligned}$$

c. *Euclidean distance :*

$$\begin{aligned} d_3(A,B) = \{\frac{1}{6}\sum_{i=1}^{n} [&|\inf T_A(x_i) - \inf T_B(x_i)|^2 + |\sup T_A(x_i) - \sup T_B(x_i)|^2 + \\ &|\inf I_A(x_i) - \inf I_B(x_i)|^2 + |\sup I_A(x_i) - \sup I_B(x_i)|^2 + \\ &|\inf F_A(x_i) - \inf F_B(x_i)|^2 + |\sup F_A(x_i) - \sup F_B(x_i)|^2]\}^{1/2} \end{aligned}$$

d. Normalized Euclidean distance :

$$d_4(A,B)=\{\frac{1}{6n}\sum_{i=1}^{n}[|\inf T_A(x_i)-\inf T_B(x_i)|^2+|\sup T_A(x_i)-\sup T_B(x_i)|^2+ |\inf I_A(x_i)-\inf I_B(x_i)|^2+|\sup I_A(x_i)-\sup I_B(x_i)|^2+ |\inf F_A(x_i)-\inf F_B(x_i)|^2+|\sup F_A(x_i)-\sup F_B(x_i)|^2]\}^{1/2}$$

e. Hausdroff distance :

$$d_5(A,B)=\sum_{i=1}^{n}\max[|\inf T_A(x_i)-\inf T_B(x_i)|,|\sup T_A(x_i)-\sup T_B(x_i)|, |\inf I_A(x_i)-\inf I_B(x_i)|,|\sup I_A(x_i)-\sup I_B(x_i)|, |\inf F_A(x_i)-\inf F_B(x_i)|,|\sup F_A(x_i)-\sup F_B(x_i)|]$$

f. Normalized Hausdroff distance :

$$d_6(A,B)=\frac{1}{n}\sum_{i=1}^{n}\max[|\inf T_A(x_i)-\inf T_B(x_i)|,|\sup T_A(x_i)-\sup T_B(x_i)|, |\inf I_A(x_i)-\inf I_B(x_i)|,|\sup I_A(x_i)-\sup I_B(x_i)|, |\inf F_A(x_i)-\inf F_B(x_i)|,|\sup F_A(x_i)-\sup F_B(x_i)|]$$

g. Weighted Hamming Distance :

$$d_7(A,B)=\frac{1}{6}\sum_{i=1}^{n}w_i[|\inf T_A(x_i)-\inf T_B(x_i)|+|\sup T_A(x_i)-\sup T_B(x_i)|+ |\inf I_A(x_i)-\inf I_B(x_i)|+|\sup I_A(x_i)-\sup I_B(x_i)|+ |\inf F_A(x_i)-\inf F_B(x_i)|+|\sup F_A(x_i)-\sup F_B(x_i)|]$$

h. Weighted normalized Hamming distance :

$$d_8(A,B)=\frac{1}{6n}\sum_{i=1}^{n}w_i[|\inf T_A(x_i)-\inf T_B(x_i)|+|\sup T_A(x_i)-\sup T_B(x_i)|+ |\inf I_A(x_i)-\inf I_B(x_i)|+|\sup I_A(x_i)-\sup I_B(x_i)|+ |\inf F_A(x_i)-\inf F_B(x_i)|+|\sup F_A(x_i)-\sup F_B(x_i)|]$$

i. Weighted Euclidean distance :

$$d_9(A,B)=\{\frac{1}{6}\sum_{i=1}^{n}w_i[|\inf T_A(x_i)-\inf T_B(x_i)|^2+|\sup T_A(x_i)-\sup T_B(x_i)|^2+ |\inf I_A(x_i)-\inf I_B(x_i)|^2+|\sup I_A(x_i)-\sup I_B(x_i)|^2+ |\inf F_A(x_i)-\inf F_B(x_i)|^2+|\sup F_A(x_i)-\sup F_B(x_i)|^2]\}^{1/2}$$

j. Weighted normalized Euclidean distance

$$d_{10}(A,B)=\{\frac{1}{6n}\sum_{i=1}^{n} w_i[|\inf T_A(x_i)-\inf T_B(x_i)|^2+|\sup T_A(x_i)-\sup T_B(x_i)|^2+ |\inf I_A(x_i)-\inf I_B(x_i)|^2+|\sup I_A(x_i)-\sup I_B(x_i)|^2+ |\inf F_A(x_i)-\inf F_B(x_i)|^2+|\sup F_A(x_i)-\sup F_B(x_i)|^2]\}^{1/2}$$

k. Weighted Hausdroff distance :

$$d_{11}(A,B)=\sum_{i=1}^{n} w_i \max[|\inf T_A(x_i)-\inf T_B(x_i)|,|\sup T_A(x_i)-\sup T_B(x_i)|, |\inf I_A(x_i)-\inf I_B(x_i)|,|\sup I_A(x_i)-\sup I_B(x_i)|, |\inf F_A(x_i)-\inf F_B(x_i)|,|\sup F_A(x_i)-\sup F_B(x_i)|]$$

l. Weighted normalized Hausdroff distance:

$$d_{12}(A,B)=\frac{1}{n}\sum_{i=1}^{n} w_i \max[|\inf T_A(x_i)-\inf T_B(x_i)|,|\sup T_A(x_i)-\sup T_B(x_i)|, |\inf I_A(x_i)-\inf I_B(x_i)|,|\sup I_A(x_i)-\sup I_B(x_i)|, |\inf F_A(x_i)-\inf F_B(x_i)|,|\sup F_A(x_i)-\sup F_B(x_i)|]$$

m. Euclidean Hausdroff distance :

$$d_{13}(A,B)=\{\sum_{i=1}^{n} \max[|\inf T_A(x_i)-\inf T_B(x_i)|^2,|\sup T_A(x_i)-\sup T_B(x_i)|^2, |\inf I_A(x_i)-\inf I_B(x_i)|^2,|\sup I_A(x_i)-\sup I_B(x_i)|^2, |\inf F_A(x_i)-\inf F_B(x_i)|^2,|\sup F_A(x_i)-\sup F_B(x_i)|^2]\}^{1/2}$$

n. Weighted Euclidean Hausdroff distance :

$$d_{14}(A,B)=\{\sum_{i=1}^{n} w_i \max[|\inf T_A(x_i)-\inf T_B(x_i)|^2,|\sup T_A(x_i)-\sup T_B(x_i)|^2, |\inf I_A(x_i)-\inf I_B(x_i)|^2,|\sup I_A(x_i)-\sup I_B(x_i)|^2, |\inf F_A(x_i)-\inf F_B(x_i)|^2,|\sup F_A(x_i)-\sup F_B(x_i)|^2]\}^{1/2}$$

o. Normalized Euclidean Hausdroff Distance :

$$d_{15}(A,B)=\{\frac{1}{n}\sum_{i=1}^{n} \max[|\inf T_A(x_i)-\inf T_B(x_i)|^2,|\sup T_A(x_i)-\sup T_B(x_i)|^2, |\inf I_A(x_i)-\inf I_B(x_i)|^2,|\sup I_A(x_i)-\sup I_B(x_i)|^2, |\inf F_A(x_i)-\inf F_B(x_i)|^2,|\sup F_A(x_i)-\sup F_B(x_i)|^2]\}^{1/2}$$

p. Normalized Weighted Euclidean Hausdroff Distance :

$$d_{16}(A,B) = \{\frac{1}{n}\sum_{i=1}^{n} w_i \max[|\inf T_A(x_i) - \inf T_B(x_i)|^2, |\sup T_A(x_i) - \sup T_B(x_i)|^2, |\inf I_A(x_i) - \inf I_B(x_i)|^2, |\sup I_A(x_i) - \sup I_B(x_i)|^2, |\inf F_A(x_i) - \inf F_B(x_i)|^2, |\sup F_A(x_i) - \sup F_B(x_i)|^2]\}^{1/2}$$

Some other distances between two INS's are given as follows

We consider 'p' as a positive integer in the following.

q. $$d_{17}(A,B) = \{\frac{1}{6}\sum_{i=1}^{n}[|\inf T_A(x_i) - \inf T_B(x_i)|^p + |\sup T_A(x_i) - \sup T_B(x_i)|^p + |\inf I_A(x_i) - \inf I_B(x_i)|^p + |\sup I_A(x_i) - \sup I_B(x_i)|^p + |\inf F_A(x_i) - \inf F_B(x_i)|^p + |\sup F_A(x_i) - \sup F_B(x_i)|^p]\}^{1/p}, \quad \forall p > 0$$

r. $$d_{18}(A,B) = \{\frac{1}{6}\sum_{i=1}^{n} w_i[|\inf T_A(x_i) - \inf T_B(x_i)|^p + |\sup T_A(x_i) - \sup T_B(x_i)|^p + |\inf I_A(x_i) - \inf I_B(x_i)|^p + |\sup I_A(x_i) - \sup I_B(x_i)|^p + |\inf F_A(x_i) - \inf F_B(x_i)|^p + |\sup F_A(x_i) - \sup F_B(x_i)|^p]\}^{1/p}, \quad \forall p > 0$$

s. $$d_{19}(A,B) = \{\frac{1}{6n}\sum_{i=1}^{n}[|\inf T_A(x_i) - \inf T_B(x_i)|^p + |\sup T_A(x_i) - \sup T_B(x_i)|^p + |\inf I_A(x_i) - \inf I_B(x_i)|^p + |\sup I_A(x_i) - \sup I_B(x_i)|^p + |\inf F_A(x_i) - \inf F_B(x_i)|^p + |\sup F_A(x_i) - \sup F_B(x_i)|^p]\}^{1/p}, \quad \forall p > 0$$

t. $$d_{20}(A,B) = \{\frac{1}{6n}\sum_{i=1}^{n} w_i[|\inf T_A(x_i) - \inf T_B(x_i)|^p + |\sup T_A(x_i) - \sup T_B(x_i)|^p + |\inf I_A(x_i) - \inf I_B(x_i)|^p + |\sup I_A(x_i) - \sup I_B(x_i)|^p + |\inf F_A(x_i) - \inf F_B(x_i)|^p + |\sup F_A(x_i) - \sup F_B(x_i)|^p]\}^{1/p}, \quad \forall p > 0$$

u. $$d_{21}(A,B) = \{\sum_{i=1}^{n} \max[|\inf T_A(x_i) - \inf T_B(x_i)|^p, |\sup T_A(x_i) - \sup T_B(x_i)|^p, |\inf I_A(x_i) - \inf I_B(x_i)|^p, |\sup I_A(x_i) - \sup I_B(x_i)|^p, |\inf F_A(x_i) - \inf F_B(x_i)|^p, |\sup F_A(x_i) - \sup F_B(x_i)|^p]\}^{1/p}, \quad \forall p > 0$$

v. $d_{22}(A,B)=\{\sum_{i=1}^{n} w_i \max[|\inf T_A(x_i)-\inf T_B(x_i)|^p, |\sup T_A(x_i)-\sup T_B(x_i)|^p,$

$|\inf I_A(x_i)-\inf I_B(x_i)|^p, |\sup I_A(x_i)-\sup I_B(x_i)|^p,$

$|\inf F_A(x_i)-\inf F_B(x_i)|^p, |\sup F_A(x_i)-\sup F_B(x_i)|^p]\}^{1/p}, \quad \forall p>0$

w. $d_{23}(A,B)=\{\frac{1}{n}\sum_{i=1}^{n} \max[|\inf T_A(x_i)-\inf T_B(x_i)|^p, |\sup T_A(x_i)-\sup T_B(x_i)|^p,$

$|\inf I_A(x_i)-\inf I_B(x_i)|^p, |\sup I_A(x_i)-\sup I_B(x_i)|^p,$

$|\inf F_A(x_i)-\inf F_B(x_i)|^p, |\sup F_A(x_i)-\sup F_B(x_i)|^p]\}^{1/p}, \quad \forall p>0$

x. $d_{24}(A,B)=\{\frac{1}{n}\sum_{i=1}^{n} w_i \max[|\inf T_A(x_i)-\inf T_B(x_i)|^p, |\sup T_A(x_i)-\sup T_B(x_i)|^p,$

$|\inf I_A(x_i)-\inf I_B(x_i)|^p, |\sup I_A(x_i)-\sup I_B(x_i)|^p,$

$|\inf F_A(x_i)-\inf F_B(x_i)|^p, |\sup F_A(x_i)-\sup F_B(x_i)|^p]\}^{1/p}, \quad \forall p>0$

Properties of Distance Measure

The above defined distance $d_k(A,B)$ (k=1, 2, 3, …)between INSs A and B satisfies the following properties (D1–D3) :

D1: $d_k(A,B) \geq 0$;

D2: $d_k(A,B)=0$ if and only if $A=B$

D3: $d_k(A,B) = d_k(B,A)$;

It can be easily shown that the distances as defined above satisfy the said properties.

4. Algorithm

Now we present an algorithm to solve a decision making problem in Interval Neutrosophic Sets by means of distance and similarity measures in INSs.
Let $\{A_i : i=1,2,...,m\}$ be a set of alternatives and $\{C_j : j=1,2,...,n\}$ be a set of criteria. Assume that the weight of the criterion C_j is $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$. In this case the INS A_i can be denoted as follows:

$$A_i = \{\langle C_j, (T_{A_i}(C_j), I_{A_i}(C_j), F_{A_i}(C_j))\rangle : C_j \in C\},$$

where

$$T_{A_i}(C_j) = [\inf T_{A_i}(C_j), \sup T_{A_i}(C_j)] \in [0,1],$$
$$I_{A_i}(C_j) = [\inf I_{A_i}(C_j), \sup I_{A_i}(C_j)] \in [0,1],$$
$$F_{A_i}(C_j) = [\inf F_{A_i}(C_j), \sup F_{A_i}(C_j)] \in [0,1],$$

and $0 \le \sup T_{A_i}(C_j) + \sup I_{A_i}(C_j) + \sup F_{A_i}(C_j) \le 3$, i=1,2,…,m and j=1,2,…,n.
Now let us consider an INS denoted as:

$$\alpha_{ij} = ([a_{ij}, b_{ij}], [c_{ij}, d_{ij}], [e_{ij}, f_{ij}])$$

where

$$[a_{ij}, b_{ij}] = [\inf T_{A_i}(C_j), \sup T_{A_i}(C_j)],$$
$$[c_{ij}, d_{ij}] = [\inf I_{A_i}(C_j), \sup I_{A_i}(C_j)],$$
$$[e_{ij}, f_{ij}] = [\inf F_{A_i}(C_j), \sup F_{A_i}(C_j)]$$

Now, an INS is derived from the evaluation of an alternative A_i with respect to a criterion C_j, by means of score law and data processing. Therefore, we can introduce an interval neutrosophic decision matrix $D = (\alpha_{ij})_{m\times n}$.

The evaluation criteria are generally taken of two kinds, benefit criteria and cost criteria. Let ***B*** be a collection of benefit criteria and ***P*** be a collection of cost criteria. Then we define an ideal INS for a benefit criterion in the ideal alternative A^* as:

$$\alpha^*_j = ([a^*_j, b^*_j], [c^*_j, d^*_j], [e^*_j, f^*_j]) = ([1,1],[0,0],[0,0]) \text{ for } j \in B$$

and for a cost criterion, we define the ideal alternative A^{**} as:

$$\alpha^{**}_j = ([a^{**}_j, b^{**}_j], [c^{**}_j, d^{**}_j], [e^{**}_j, f^{**}_j]) = ([0,0],[1,1],[1,1]) \text{ for } j \in P.$$

Although, the ideal alternative doesn't exist in real world, it is only used to identify the best alternative in decision set.

Now if we denote the ideal alternative as the INS E, then by the distance measures $d_k(E, A_i)$, (i =1,2,….,m), (k=1,2,…,24) and the similarity measures $s_k(E, A_i)$, (i=1,2,….,m), (k=1,2,…,21) (as defined in previous section), between each alternative A_i and the ideal alternative E *(For benefit criteria* $E = A^*$ *and for cost criteria* $E = A^{**}$*)*, the ranking order of all alternatives can be determined and the best one can be easily identified as well.

5. Problem

To illustrate the above algorithm we take a multi-criteria decision making problem of alternatives to apply the proposed decision making method.

We adapt the required problem from the article by Jun Ye [14], stated as follows:

There is an investment company, which wants to invest a sum of money in the best option. There is a panel with four possible alternatives to invest the money:

(1) A_1 is a car company; (2) A_2 is a food company; (3) A_3 is a computer company; (4) A_4 is an arms company.

The investment company must take a decision according to the following three criteria:

(1) C_1 is the risk analysis; (2) C_2 is the growth analysis; (3) C_3 is the environmental impact analysis, where C_1 and C_2 are benefit criteria and C_3 is a cost criterion. The weight vector of the criteria is given by :w = (0.35, 0.25, 0.40). The four possible alternatives are to be evaluated under the above three criteria by corresponding to the INSs, as shown in the following interval neutrosophic decision matrix *D:*

$$D = \begin{bmatrix} \langle [0.4,0.5],[0.2,0.3],[0.3,0.4] \rangle & \langle [0.4,0.6],[0.1,0.3],[0.2,0.4] \rangle & \langle [0.7,0.9],[0.2,0.3],[0.4,0.5] \rangle \\ \langle [0.6,0.7],[0.1,0.2],[0.2,0.3] \rangle & \langle [0.6,0.7],[0.1,0.2],[0.2,0.3] \rangle & \langle [0.3,0.6],[0.3,0.5],[0.8,0.9] \rangle \\ \langle [0.3,0.6],[0.2,0.3],[0.3,0.4] \rangle & \langle [0.5,0.6],[0.2,0.3],[0.3,0.4] \rangle & \langle [0.4,0.5],[0.2,0.4],[0.7,0.9] \rangle \\ \langle [0.7,0.8],[0.0,0.1],[0.1,0.2] \rangle & \langle [0.6,0.7],[0.1,0.2],[0.1,0.3] \rangle & \langle [0.6,0.7],[0.3,0.4],[0.8,0.9] \rangle \end{bmatrix}$$

Now we measure the distances and also the similarities between each alternative A_i and the ideal alternatives E, as defined earlier.

To calculate the Hamming distance between E and A_1 we take :

$$d_1(E, A_1) = \frac{1}{6}\sum_{i=1}^{n} \begin{matrix} [|\inf T_E(x_i) - \inf T_{A_1}(x_i)| + |\sup T_E(x_i) - \sup T_{A_1}(x_i)| + \\ |\inf I_E(x_i) - \inf I_{A_1}(x_i)| + |\sup I_E(x_i) - \sup I_{A_1}(x_i)| + \\ |\inf F_E(x_i) - \inf F_{A_1}(x_i)| + |\sup F_E(x_i) - \sup F_{A_1}(x_i)|] \end{matrix}$$

=1/6[|1-0.4|+|1-0.5|+|0-0.2|+|0-0.3|+|0-0.3|+|0-0.4|+|1-0.4|+|1-0.6|+|0-0.1|+|0-0.3|+|0-0.2|+|0-0.4|+|0-0.7|+|0-0.9|+|1-0.2|+|1-0.3|+|1-0.4|+|1-0.5|] =1.4167

Similarly, $d_1(E, A_2) = 0.9$, $d_1(E, A_3) = 1.25$ and $d_1(E, A_4) = 0.86$.

In this way, the obtained results are presented in tabular form as follows:

For Distance measurement

Distance	Obtained Results	Rank of Alternatives (descending order)	Best alternative obtained
$d_1(E,A_i)$	$A_1 = 1.4167$ $A_2 = 0.9$ $A_3 = 1.25$ $A_4 = 0.86$	$A_1 > A_3 > A_2 > A_4$	A_4
$d_2(E,A_i)$	$A_1 = 0.4722$ $A_2 = 0.3$ $A_3 = 0.4167$ $A_4 = 0.2867$	$A_1 > A_3 > A_2 > A_4$	A_4
$d_3(E,A_i)$	$A_1 = 0.8990$ $A_2 = 0.5916$ $A_3 = 0.7450$ $A_4 = 0.6245$	$A_1 > A_3 > A_4 > A_2$	A_2
$d_4(E,A_i)$	$A_1 = 0.5190$ $A_2 = 0.3416$ $A_3 = 0.4301$ $A_4 = 0.3606$	$A_1 > A_3 > A_4 > A_2$	A_2
$d_5(E,A_i)$	$A_1 = 2.1$ $A_2 = 1.5$ $A_3 = 2.0$ $A_4 = 1.4$	$A_1 > A_3 > A_2 > A_4$	A_4
$d_6(E,A_i)$	$A_1 = 0.7000$ $A_2 = 0.5000$ $A_3 = 0.6667$ $A_4 = 0.4667$	$A_1 > A_3 > A_2 > A_4$	A_4
$d_7(E,A_i)$	$A_1 = 0.4975$ $A_2 = 0.3100$ $A_3 = 0.4233$ $A_4 = 0.3042$	$A_1 > A_3 > A_2 > A_4$	A_4
$d_8(E,A_i)$	$A_1 = 0.1658$ $A_2 = 0.1033$ $A_3 = 0.1411$ $A_4 = 0.1014$	$A_1 > A_3 > A_2 > A_4$	A_4
$d_9(E,A_i)$	$A_1 = 0.5428$ $A_2 = 0.3545$ $A_3 = 0.4401$ $A_4 = 0.3800$	$A_1 > A_3 > A_4 > A_2$	A_2
$d_{10}(E,A_i)$	$A_1 = 0.3134$ $A_2 = 0.2047$ $A_3 = 0.2541$ $A_4 = 0.2194$	$A_1 > A_3 > A_4 > A_2$	A_2
$d_{11}(E,A_i)$	$A_1 = 0.7200$ $A_2 = 0.5200$ $A_3 = 0.6900$ $A_4 = 0.4850$	$A_1 > A_3 > A_2 > A_4$	A_4
$d_{12}(E,A_i)$	$A_1 = 0.2400$ $A_2 = 0.1733$ $A_3 = 0.2300$ $A_4 = 0.1617$	$A_1 > A_3 > A_2 > A_4$	A_4
$d_{13}(E,A_i)$	$A_1 = 1.2369$ $A_2 = 0.9000$ $A_3 = 1.1747$ $A_4 = 0.8602$	$A_1 > A_3 > A_2 > A_4$	A_4
$d_{14}(E,A_i)$	$A_1 = 0.7348$ $A_2 = 0.5404$ $A_3 = 0.7000$ $A_4 = 0.5172$	$A_1 > A_3 > A_2 > A_4$	A_4
$d_{15}(E,A_i)$	$A_1 = 0.7141$ $A_2 = 0.5196$ $A_3 = 0.6782$ $A_4 = 0.4966$	$A_1 > A_3 > A_2 > A_4$	A_4
$d_{16}(E,A_i)$	$A_1 = 0.4242$ $A_2 = 0.3120$ $A_3 = 0.4041$ $A_4 = 0.2986$	$A_1 > A_3 > A_2 > A_4$	A_4
$d_{17}(E,A_i)$	For p = 6 $A_1 = 0.033700$ $A_2 = 0.005336$ $A_3 = 0.013387$ $A_4 = 0.009309$	$A_1 > A_3 > A_4 > A_2$	A_2
	For p = 10 $A_1 = 0.00888288$ $A_2 = 0.00059184$ $A_3 = 0.00240292$ $A_4 = 0.00114518$	$A_1 > A_3 > A_4 > A_2$	A_2

Distance	Obtained Results	Rank of Alternatives (descending order)	Best alternative obtained
$d_{18}(E,A_i)$	For p = 6 $A_1 = 0.01317057$ $A_2 = 0.00210276$ $A_3 = 0.00524945$ $A_4 = 0.00624732$	$A_1 > A_4 > A_3 > A_2$	A_2
	For p = 10 $A_1 = 0.00353154$ $A_2 = 0.00023634$ $A_3 = 0.00093445$ $A_4 = 0.00045777$	$A_1 > A_3 > A_4 > A_2$	A_2
$d_{19}(E, A_i)$	For p = 6 $A_1 = 0.06740000$ $A_2 = 0.01067160$ $A_3 = 0.02677516$ $A_4 = 0.01861966$	$A_1 > A_3 > A_4 > A_2$	A_2
	For p = 10 $A_1 = 0.02960961$ $A_2 = 0.00197280$ $A_3 = 0.00800974$ $A_4 = 0.00381729$	$A_1 > A_3 > A_4 > A_2$	A_2
$d_{20}(E, A_i)$	For p =6 $A_1 = 0.02634115$ $A_2 = 0.00420552$ $A_3 = 0.01049890$ $A_4 = 0.01249464$	$A_1 > A_4 > A_3 > A_2$	A_2
	For p = 10 $A_1 = 0.01177182$ $A_2 = 0.00078782$ $A_3 = 0.00311483 A_4 = 0.00152592$	$A_1 > A_3 > A_4 > A_2$	A_2
$d_{21}(E, A_i)$	For p = 6 $A_1 = 0.1041255$ $A_2 = 0.0209735$ $A_3 = 0.0659030$ $A_4 = 0.0204123$	$A_1 > A_3 > A_2 > A_4$	A_4
	For p = 10 A_1=0.03607716 $A_2 = 0.00284572$ A_3 =0.01365982 $A_4 = 0.00283582$	$A_1 > A_3 > A > A_4$	A_4
$d_{22}(E, A_i)$	For p = 6 $A_1 = 0.0400950$ $A_2 = 0.0082528$ $A_3 = 0.0249901$ $A_4 = 0.0080200$	$A_1 > A_3 > A_2 > A_4$	A_4
	For p =10 $A_1 = 0.4975$ $A_2 = 0.3100$ $A_3 = 0.4233$ $A_4 = 0.3042$	$A_1 > A_2 > A_3 > A_4$	A_4
$d_{23}(E, A_i)$	For p = 6 $A_1 = 0.208251$ $A_2 = 0.041947$ $A_3 = 0.131806$ $A_4 = 0.040824$	$A_1 > A_3 > A_2 > A_4$	A_4
	For p = 10 $A_1 = 0.208251$ $A_2 = 0.041947$ $A_3 = 0.131806$ $A_4 = 0.040824$	$A_1 > A_3 > A_2 > A_4$	A_4
$d_{24}(E, A_i)$	For p = 6 $A_1 = 0.0801900$ $A_2 = 0.0165057$ $A_3 = 0.0499803$ $A_4 = 0.0160401$	$A_1 > A_3 > A_2 > A_4$	A_4
	For p = 10 $A_1 = 0.0475990$ $A_2 = 0.0037873$ $A_3 = 0.0126310$ $A_4 = 0.0037757$	$A_1 > A_3 > A_2 > A_4$	A_4

For similarity measurement

Similarity	Obtained Results	Rank of Alternatives (descending order)	Best alternative obtained
$s_1(E,A_i)$	$A_1 = 0.6678$ $A_2 = 0.7634$ $A_3 = 0.7026$ $A_4 = 0.7668$	$A_4 > A_2 > A_3 > A_1$	A_4
$s_2(E,A_i)$	$A_1 = 0.8342$ $A_2 = 0.8967$ $A_3 = 0.8589$ $A_4 = 0.8986$	$A_4 > A_2 > A_3 > A_1$	A_4
$s_3(E,A_i)$	$A_1 = 0.6481$ $A_2 = 0.7383$ $A_3 = 0.6944$ $A_4 = 0.7246$	$A_2 > A_4 > A_3 > A_1$	A_2
$s_4(E,A_i)$	$A_1 = 0.6866$ $A_2 = 0.7953$ $A_3 = 0.7459$ $A_4 = 0.7806$	$A_2 > A_4 > A_3 > A_1$	A_2
$s_5(E,A_i)$	$A_1 = 0.5814$ $A_2 = 0.6579$ $A_3 = 0.5917$ $A_4 = 0.6734$	$A_4 > A_2 > A_3 > A_1$	A_4
$s_6(E,A_i)$	$A_1 = 0.7600$ $A_2 = 0.8267$ $A_3 = 0.7700$ $A_4 = 0.8383$	$A_4 > A_2 > A_3 > A_1$	A_4
$s_7(E,A_i)$	$A_1 = 0.52778$ $A_2 = 0.70000$ $A_3 = 0.60555$ $A_4 = 0.71111$	$A_4 > A_2 > A_3 > A_1$	A_4
$s_8(E,A_i)$	For p = 6 $A_1 = 0.99321146$ $A_2 = 0.99974531$ $A_3 = 0.99926028$ $A_4 = 0.99928211$	$A_2 > A_4 > A_3 > A_1$	A_2
	For p =10 $A_1 = 0.99905556$ $A_2 = 0.99999644$ $A_3 = 0.99998544$ $A_4 = 0.99997679$	$A_2 > A_3 > A_4 > A_1$	A_2
$s_9(E,A_i)$	For p = 6 $A_1 = 0.99191449$ $A_2 = 0.99970251$ $A_3 = 0.99918473$ $A_4 = 0.99914266$	$A_2 > A_3 > A_4 > A_1$	A_2
	For p = 10 $A_1 = 0.99886727$ $A_2 = 0.99999574$ $A_3 = 0.99998329$ $A_4 = 0.99997215$	$A_2 > A_3 > A_4 > A_1$	A_2
$s_{10}(E,A_i)$	For p = 6 $A_1 = 0.92097654$ $A_2 = 0.98738343$ $A_3 = 0.96956463$ $A_4 = 0.97780405$	$A_2 > A_4 > A_3 > A_1$	A_2
	For p = 10 $A_1 = 0.97881070$ $A_2 = 0.99858191$ $A_3 = 0.99439330$ $A_4 = 0.99725333$	$A_2 > A_4 > A_3 > A_1$	A_2
$s_{11}(E,A_i)$	$A_1 = 0.66245024$ $A_2 = 0.74828426$ $A_3 = 0.67495694$ $A_4 = 0.76380514$	$A_4 > A_2 > A_3 > A_1$	A_4
$s_{12}(E,A_i)$	$A_1 = 0.61290322$ $A_2 = 0.70459388$ $A_3 = 0.62601626$ $A_4 = 0.98138033$	$A_4 > A_2 > A_3 > A_1$	A_4
$s_{13}(E,A_i)$	For p = 6 $A_1 = 0.9326000$ $A_2 = 0.9893284$ $A_3 = 0.9732248$ $A_4 = 0.9813803$	$A_2 > A_4 > A_3 > A_1$	A_2
	For p = 10 $A_1 = 0.97039038$ $A_2 = 0.99802719$ $A_3 = 0.99199025$ $A_4 = 0.99618270$	$A_2 > A_4 > A_3 > A_1$	A_2

Similarity	Obtained Results	Rank of Alternatives (descending order)	Best alternative obtained
$s_{14}(E,A_i)$	For p = 6 $A_1 = 0.97365884$ $A_2 = 0.99579447$ $A_3 = 0.98950109$ $A_4 = 0.98750536$	$A_2 > A_3 > A_4 > A_1$	A_2
	For p = 10 $A_1 = 0.98822817$ $A_2 = 0.99921217$ $A_3 = 0.99688516$ $A_4 = 0.99847407$	$A_2 > A_4 > A_3 > A_1$	A_2
$s_{15}(E,A_i)$	$A_1 = 0.29661016$ $A_2 = 0.35238095$ $A_3 = 0.37168141$ $A_4 = 0.50000000$	$A_4 > A_3 > A_2 > A_1$	A_4
$s_{16}(E,A_i)$	$A_1 = 0.300$ $A_2 = 0.550$ $A_3 = 0.450$ $A_4 = 0.483$	$A_2 > A_4 > A_3 > A_1$	A_2
$s_{17}(E,A_i)$	$A_1 = 0.43708609$ $A_2 = 0.65384615$ $A_3 = 0.54193548$ $A_4 = 0.66666666$	$A_4 > A_2 > A_3 > A_1$	A_4
$s_{18}(E,A_i)$	$A_1 = 0.20283243$ $A_2 = 0.37547646$ $A_3 = 0.26990699$ $A_4 = 0.38997923$	$A_4 > A_2 > A_3 > A_1$	A_4
$s_{19}(E,A_i)$	$A_1 = 0.18945738$ $A_2 = 0.30270010$ $A_3 = 0.22405482$ $A_4 = 0.33782415$	$A_4 > A_2 > A_3 > A_1$	A_4
$s_{20}(E,A_i)$	$A_1 = 0.4125$ $A_2 = 0.6375$ $A_3 = 0.5250$ $A_4 = 0.6500$	$A_4 > A_2 > A_3 > A_1$	A_4
$s_{21}(E,A_i)$	$A_1 = 0.140625$ $A_2 = 0.222500$ $A_3 = 0.183750$ $A_4 = 0.226250$	$A_4 > A_2 > A_3 > A_1$	A_4

7. Comparative study with existing work

Hence we compare the results given in [14] and the results obtained in previous section (section 6). In the article [14], the authors have used the similarity measures $\boldsymbol{s_1(A,B)}$and $\boldsymbol{s_3(A,B)}$ (as stated in the section 4, where $\boldsymbol{A}$ is the ideal alternative $\boldsymbol{E}$ and $\boldsymbol{B}$ is the alternative to be measured), to obtain the best alternatives. Using $\boldsymbol{s_1(A,B)}$ the best alternative obtained is $\boldsymbol{A_4}$ and using $\boldsymbol{s_3(A,B)}$ the best alternative is $\boldsymbol{A_2}$. Also the similarity measure of $\boldsymbol{A_4}$ with ideal alternative is ***0.9600*** and the same of $\boldsymbol{A_2}$ is ***0.9323***. However, we have measured using various numbers of similarities and distances as well, between the alternatives and the ideal alternative, to obtain the best alternative. According to the results, $\boldsymbol{A_4}$ is the best alternative (in both distances and similarity measures) when the distance or the similarity is in linear form i.e. Hamming distance, Hausdroff distance and their related distance and similarity measures, etc. (except $\boldsymbol{d_{21}(A,B)}$ and its related distance measures, where though they are not linear, the best alternative obtained is $\boldsymbol{A_4}$). Otherwise the best alternative is $\boldsymbol{A_2}$ (except $\boldsymbol{s_{16}(A,B)}$, where being linear similarity measure, the best alternative given is $\boldsymbol{A_2}$). Now, one can decide the best alternative considering the alternative obtained as best alternative according to numerical value in most number of cases in both distance and similarity measures and also this decision can be made considering more distance and similarities besides those defined in this paper. So, we suggest that, according to the number of cases, $\boldsymbol{A_4}$ can be taken as the best alternative.

8. Conclusion

In this article, at first we have defined various distances $d_k(A,B)$, (k =1,2,…,24) and similarity measures $s_k(A,B)$, (k =1,2,…,21) , between two Interval Neutrosophic sets. Then we have shown an application of these distances and similarities in solving a multicriteria decision-making problem. A method, for the solution of this type of problems, has been established by means of distance and similarity measures between each alternative and the respective ideal alternative. Then, as an illustrative example, a problem from [14] has been reconsidered and applying our distance and similarity measures, the ranking order of all alternatives has been calculated and stated in tabular form and the best alternative has also been identified as well. Finally we have made a comparison between the existing result in [14] and the results obtained in this article and finally conclude that the result obtained in this paper is more precise and more specific. The proposed similarity measures are also useful in real life applications of science and engineering such as medical diagnosis, pattern recognitions etc. Furthermore, the proposed techniques, based on distance and similarity measures, can be more useful for decision makers as it extend the existing decision making methods.

Acknowledgements

The authors are highly indebted to the reviewers and editor-in-chief for their valuable comments which have helped to rewrite the paper in its present form.

References

[1] K. Atanassov, Intuitionistic Fuzzy Sets, Fuzzy Sets and Systems, 20(1986) 87-96.

[2] K. Atanassov and G.Gargov, Interval valued intuitionistic fuzzy sets, Fuzzy Sets and Systems 31 (1989), 343–349.

[3] S. Broumi and F. Smarandache, Several Similarity Measures of Neutrosophic Sets, Neutrosophic Sets and Systems, Vol. 1, 2013, 54- 61.

[4] S. Broumi, F. Smarandache, New distance and similarity measures of Interval Neutrosophic Sets, IEEE Conference publication (2014).

[5] P. Majumdar, S.K. Samanta, On Similarity and Entropy of Neutrosophic Sets, Journal of Intelligent and Fuzzy systems, 2014, vol 26, no. 3,2014, 1245-1252

[6] P. Majumdar, *Neutrosophic Sets and its applications to decision making*, Computational *Intelligence for Big Data Analysis: Frontier Advances & Applications*, Springer- Verlag/Heidelberg, D.P. Acharjee at. al. editors, 2015, 97-115

[7] C. P. Pappis, N. I. Karacapilidis, A comparative assessment of measures of similarity of fuzzy values. Fuzzy Sets and Systems, 56:171174, 1993.

[8] G. A. Papakostas, A. G. Hatzimichailidis, V.G. Kaburlasos, Distance and Similarity measures between Intuitionistic Fuzzy sets : A comparative analysis from a pattern recognition point of view, Pattern Recognition Letters, vol. 34, issue 14,2013,1609-1622.

[9] F. Smarandache, A Unifying Field in Logics. Neutrosophy: Neutrosophic Probability, Set and Logic, American Research Press, Rehoboth (1999).

[10] F. Smarandache,, Neutrosophic set: A generalization of the intuitionistic fuzzy set, International Journal of Pure and Applied Mathematics 24 (2005), 287–297.

[11] I. Turksen, Interval valued fuzzy sets based on normal forms, Fuzzy Sets and Systems 20 (1986), 191–210.
[12] H. Wang., F. Smarandache, Y.Q. Zhang,, and R. Sunderraman,., Interval neutrosophic sets and logic: Theory and applications in computing, Hexis, Phoenix, AZ, 2005.
[13] H. Wang, et al., Single valued neutrosophic sets, Proc. Of 10^{th} Int. conf. on Fuzzy Theory and Technology, Salt Lake City, Utah, July 21-26 (2005)
[14] Jun Ye, Similarity measures between interval neutrosophic sets and their applications in multicriteria decision-making, Journal of Intelligent & Fuzzy Systems 26 (2014) 165–172.
[15] Jun Ye, Multicriteria fuzzy decision-making method based on a novel accuracy function under interval-valued intuitionistic fuzzy environment, Expert Systems with Applications 36 (2009), 6899–6902.
[16] Jun Ye, Multicriteria group decision-making method using the distance-based similarity measures between intuitionistic trapezoidal fuzzy numbers, International Journal of General Systems 41 (2012), 729–739.
[17] L. A. Zadeh, Fuzzy Sets, Information and Control, 8(1965) 338-353.