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Generalized Pre $\alpha-$Closed Sets in Topology

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Abstaract − In this paper, a new class of sets called generalized pre α-closed sets are introduced and studied in topological spaces, which are properly placed between the class of pre closed and the class of generalized star pre closed (g*p-closed) sets.

Keywords − *Closed sets, gpα-closed sets, gpα-open sets.*

1 Introduction

The concept of stronger forms of open sets and closed sets were introduced by Stone[17], which were called as regular open and regular closed sets respectively. Levine[10]introduced the generalized closed sets in topology as generalization of closed sets. The concept of Levine[10] opened the flood gates of research in weaker forms of closed sets in general topology. Many researchers like [1], [2], [4], [7], [12], [13], [14], [16], [18], [19] and others have studied many weaker forms of closed sets in topological spaces. Recently, Benchalli et al.[3] and Jafari et al.[8] studied $\omega\alpha$-closed and pre g*-closed sets. The aim of this paper is to continue the study of generalization of closed sets namely generalized pre α-closed(briefly gpα-closed) set using α-open [16] in topological spaces. Also, we introduce the concept of gpα-closure, gpα-interior and gpα-neighborhood in topological spaces.

2 Preliminaries

Throughout this paper, spaces X and Y(or (X,τ) and (Y,σ)) denote topological spaces, in which no separation axioms are assumed unless explicitly stated. The

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following definitions are useful in the sequel.

Definition 2.1. A subset A of a topological space X is called a

1. semi-open [9] if $A \subseteq cl(int(A))$ and semi-closed set if $int(cl(A)) \subseteq A$.
2. pre-open set [14] if $A \subseteq int(cl(A))$ and pre-closed set if $cl(int(A)) \subseteq A$.
3. α-open set [16] if $A \subseteq int(cl(int(A)))$ and α-closed set if $cl(int(cl(A))) \subseteq A$.
4. semi-preopen set [1] if $A \subseteq cl(int(cl(A)))$ and semi-preclosed set if $int(cl(int(A))) \subseteq A$.

Definition 2.2. A subset A of a topological space X is called a

1. generalized closed (briefly g-closed)[10](briefly ω-closed[18], pre $g^\star$-closed[8]) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open(resp. semi-open, $\omega\alpha$-open) in X.
2. generalized preclosed (briefly gp-closed)[13],(resp. generalized pre regular closed (briefly gpr-closed[7])), if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open(resp. regular open) in X.
3. generalized semi-pre closed(briefly gsp-closed)[5], if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.
4. semi generalized closed(briefly sg-closed)[4] (resp.generalized semi-closed[2]), if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open (resp. open) in X.
5. $\omega\alpha$-closed [3], if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is ω-open in X.

3 Generalized Pre α Closed Sets

In this section, the concept of generalized pre α closed set is introduced and studied some of its properties in topological spaces.

Definition 3.1. In a topological space X, a subset A of X is called generalized pre α-closed (briefly gpα-closed) if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is α-open in X.

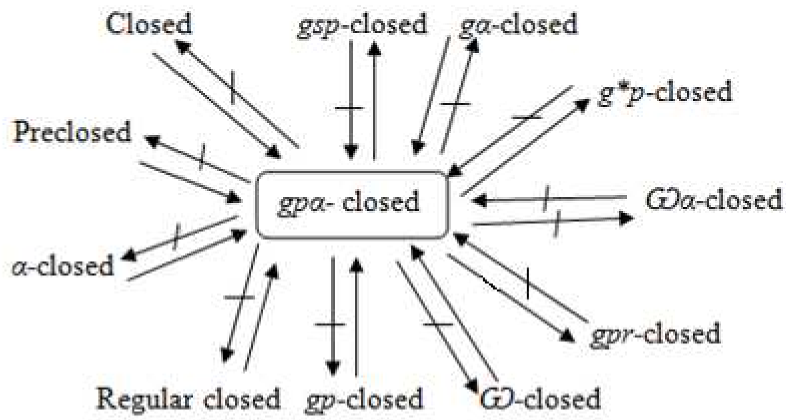
The compliment of gpα-closed is gpα-open in X. The family of all gpα-closed sets in X is denoted by Gp$\alpha C\,(X)$.

Example 3.2. Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{a\}, \{a, b\}\}$. Then the family of gp$\alpha$-closed sets in X is given by Gp$\alpha C\,(X) = \{X, \phi, \{b\}, \{c\}\,\{b, c\}\}$.

Remark 3.3. From the definition 3.1, it is clear that every pre closed set is gpα-closed but not conversely.

Example 3.4. Let $X = \{a, b, c, d, e\}$ and $\tau = \{X, \phi, \{a\}, \{c, d\}, \{a, c, d\}\}$. Then the subset A=$\{a, e\}$ of X is gpα-closed but not pre closed in X.

Remark 3.5. From the definition 3.1 and from [1,3,4,5,7,12,18,19,20],we have the following implications. However converse implications are not true in general.



Example 3.6. Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{a\}, \{b, c\}\}$. Then the subset A=$\{a, b\}$ is gpα-closed but not closed, regular closed and $\omega\alpha$-closed in X and B=$\{c\}$ is gpα-closed but not α-closed.

Example 3.7. Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{a, b\}\}$. Then the subset A=$\{a\}$ is gpα-closed but not gα-closed in X and the subset B=$\{a, b\}$ is gpr-closed but not gpα-closed set in X.

Example 3.8. Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{a\}, \{a, b\}\}$. Then the subset $A = \{b\}$ is gpα-closed but not ω-closed in X and $B = \{a, c\}$ is $\omega\alpha$-closed, gp-closed,gsp-closed and g*p-closed but not gpα-closed in X.

From the above observations, the class of gpα-closed sets are properly placed between the class of preclosed and g*p-closed sets.

Remark 3.9. The following examples show that semi-closed(resp. semi-preclosed) and gpα-closed sets are independent of each other.

Example 3.10. Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$. Then the subset $A = \{b\}$ is semi-closed (resp.semi-pre closed) but not gpα-closed.

Example 3.11. In Example 3.7, the subset $A = \{a\}$ is gpα-closed but not semi-closed and semi-pre closed in X.

Remark 3.12. From the following examples it is clear that gpα-closed and sg-closed (resp. g-closed, gs-closed) sets are independent of each other.

Example 3.13. Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$. Then the subset A=$\{a\}$ is sg-closed, gs-closed but not gpα-closed in X.

Example 3.14. In Example 3.7, the subset A={b} is gpα-closed but not gs-closed, sg-closed and g-closed in X.

Example 3.15. Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{a\}\}$. Then the subset A={a, c} is g-closed but not gpα-closed in X.

Remark 3.16. From the examples 3.6 and 3.7, the $\omega\alpha$-closed and gpα-closed sets are independent of each other.

Theorem 3.17. If A is gpα-closed in X, then $pcl(A) - A$ does not contain any non-empty α-closed set in X.

Proof. Let F be a α-closed set in X contained in $pcl(A) - A$. Then $F \subseteq X - A$ and $A \subseteq X - A$. A subset A is α-closed and $X - F$ is α-open in X, then $pcl(A) \subseteq X - F$. So, $F \subseteq X - pcl(A)$. Therefore $F \subseteq (pcl(A) - A) \cap (X - cl(A)) = \phi$. Hence, $pcl(A) - A$ does not contain any non-empty α-closed set in X.

Theorem 3.18. Let A and B are gpα-closed sets, then $A \cup B$ is gpα-closed.

Proof. Let U be an α-open set in X such that $A \subseteq U$ and $B \subseteq U$. Since A and B are gpα-closed sets, then $pcl(A) \subseteq U$ and $pcl(B) \subseteq U$. But $pcl(A \cup B) = pcl(A) \cup pcl(B) \subseteq U$, so $pcl(A \cup B) \subseteq U$. Hence $A \cup B$ is gpα-closed.

Theorem 3.19. If A is gpα-closed set and $A \subseteq B \subseteq pcl(A)$, then B is gpα-closed.

Proof. Let U be an α-open in X such that $B \subseteq U$. Then $A \subseteq B$ implies that $A \subseteq U$. Since A is gpα-closed, then $pcl(A) \subseteq U$. But $B \subseteq pcl(A)$, so $pcl(B) \subseteq pcl(A)$. Then $pcl(B) \subseteq U$. Hence B is gpα-closed.

Theorem 3.20. If A is α-open and gpα-closed set of X, then A is preclosed.

Proof. Let $A \subseteq A$, where A is α-open. Then $pcl(A) \subseteq A$ as A is gpα-closed. But $A \subseteq pcl(A)$ is always true. Therefore $A = pcl(A)$. Hence A is preclosed.

Theorem 3.21. Let $A \subseteq Y \subseteq X$ and suppose that A is gpα-closed in X, then A is gpα-closed relative to Y.

Proof. Consider $A \subseteq Y \cap G$, where G is open and so α-open in X. Since$A$ is gpα-closed in X, $A \subseteq G$ which implies $pcl(A) \subseteq G$. That is $Y \cap pcl(A) \subseteq Y \cap G$, where $Y \cap pcl(A)$ is the pre-closure of A. Thus A is gpα-closed relative to Y.

Definition 3.22. [11] For a topological space X, the kernel of a subset A of X is defined as the intersection of all open supersets of A and denoted by ker(A) or $A^\wedge$.

Definition 3.23. A subset A of X is called p star-closed (briefly $p^\star$-closed), if $A = pcl(A) \cap A^\wedge$ and its compliment is $p^\star$-open.

Theorem 3.24. For a subset A of X, the following are equivalent:

(i) A is preclosed.

(ii) A is gpα-closed and $A = pcl(A) \cap U$, for some open set U.

(iii) A is gpα-closed and p^*-closed.

Proof. $(i) \rightarrow (ii)$ Every preclosed set is gpα-closed and $A = pcl(A)$ and X is open. Then $A = X \cap A$, implies that $A = pcl(A) \cap X$.
$(ii) \rightarrow (iii)$ Let $A = pcl(A) \cap U$, where U is some open set. Then $A \subseteq pcl(A)$ and $A \subseteq U$. But $A \subseteq ker(\mathrm{A}) \subseteq U$. So,$A \subseteq ker(\mathrm{A}) \subseteq pcl(A)$ implies $A \subseteq pcl(A) \cap U = A$. Then we have $A = ker(\mathrm{A}) \cap pcl(A)$. Hence A is p^*-closed.
$(iii) \rightarrow (i)$ Let A be gpα-closed, by definition, $pcl(A) \subseteq A$, wherever $A \subseteq U$ and U is α-open. Then $pcl(A) \subseteq ker(\mathrm{A}) \subseteq U$, therefore $A = ker(\mathrm{A}) \cap pcl(A)$ and hence A is preclosed.

Theorem 3.25. For each $x \in X$, $\{x\}$ is α-closed or $X - \{x\}$ is gpα-closed in X.

Proof. Let $\{x\}$ be α-closed, then the proof is completed. Suppose $\{x\}$ is not α-closed in X, then $X - \{x\}$ is not α-open and only α-open set containing $X - \{x\}$ is space X itself. Therefore $pcl(X - \{x\}) \subseteq X$ and hence $X - \{x\}$ is gpα-closed in X.

4 gpα-Closure and gpα-Interior

In this section we introduce gpα-closure and gpα-interior of a subset A of X by using the gpα-closed and gpα-open sets also studied their properties.

Definition 4.1. A subset A of X, the intersection of all gpα-closed sets containing A is called the gpα-closure of A and is denoted by gp$\alpha - cl(A)$.

That is gp$\alpha - cl(A) = \cap\{G : A \subseteq G$, G is gp$\alpha$ -closed in X $\}$.

Definition 4.2. A subset A of X, gpα-interior of A and denoted by gp$\alpha - int(A)$, defined as gp$\alpha - int(A) = \cup\{G : G \subseteq A$, G is gp$\alpha$-open in X $\}$.

Remark 4.3. If $A \subseteq X$, then

(i) $A \subseteq$ gp$\alpha - cl(A) \subseteq cl(A)$

(ii) $int(A) \subseteq$ gp$\alpha - int(A) \subseteq A$.

Theorem 4.4. If A and B are subsets of X, then

(i) gp$\alpha - cl(X) = X$ and gp$\alpha - cl(\phi) = \phi$.

(ii) $A \subseteq$ gp$\alpha - cl(A)$

(iii) If B is any gpα-closed set containing A, then gp$\alpha - cl(A) \subseteq B$

(iv) If $A \subseteq B$, then gp$\alpha - cl(A) \subseteq$ gp$\alpha - cl(B)$

(v) gp$\alpha - cl(A)$=gpα-cl(gp$\alpha - cl(A)$)

(vi) gp$\alpha - cl(A \cup B)$=gp$\alpha - cl(A)\cup$ gp$\alpha - cl(B)$

Proof. (i),(ii), (iii) and (iv) follows from the definition 4.1.
(v) Let E be gpα-closed set containing A. Then by definition 4.1, gp$\alpha - cl(A) \subseteq E$. Since, E is gpα-closed and contains gp$\alpha - cl(A)$ and is contained in every gpα-closed set containing A, it follows gp$\alpha - cl($gp$\alpha - cl(A)) \subseteq$ gp$\alpha - cl(A)$. Therefore gp$\alpha - cl($gp$\alpha - cl(A)$)=gp$\alpha - cl(A)$.

(vi) Since gp$\alpha - cl(A) \subseteq$ gp$\alpha - cl(A \cup B)$ and gp$\alpha - cl(B) \subseteq$ gp$\alpha - cl(A \cup B)$ implies that gp$\alpha - cl(A)\cup$ gp$\alpha - cl(A \cup B)$. Let x be any point in X such that $x \notin$ gp$\alpha - cl(A)\cup$ gp$\alpha - cl(B)$, then there exist gpα-closed sets E and$F$, such that $A \subseteq E$ and $B \subseteq F$, $x \notin E$ and $x \notin F$ implies that $x \notin E \cup F$, $A \cup B \subseteq E \cup F$ and $E \cup F$ is gpα-closed. Thus $x \notin$ gp$\alpha - cl(A \cup B)$, gp$\alpha - cl(A \cup B) \subseteq$ gp$\alpha - cl(A)\cup$ gp$\alpha(B)$. Hence, we conclude that gp$\alpha - cl(A \cup B)$=gp$\alpha - cl(A)\cup$ gp$\alpha - cl(B)$.

Theorem 4.5. Let A and B be subsets of X, then

$$gp\alpha - cl(A \cap B) \subseteq gp\alpha - cl(A) \cap gp\alpha - cl(B)$$

Remark 4.6. The equality of Theorem 4.5 does not hold in general as seen from the following example.

Example 4.7. Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{a\}, \{a, b\}\}$ be a topology on X. For subsets of X, $A = \{a\}$ and $B = \{b\}$. The gp$\alpha - cl(A) = \{a, c\}$ and gp$\alpha - cl(B) = \{b, c\}$, then gp$\alpha - cl(\{A \cap B\} = \phi$. Hence

$$gp\alpha - cl(A) \cap gp\alpha - cl(B) \nsubseteq gp\alpha(A \cap B)$$

Remark 4.8. If $A \subseteq X$ and A is gpα-closed, then gp$\alpha - cl(A)$ is smallest gpα-closed subset of X containing A.

Theorem 4.9. For any $x \in X$, $x \in$ gp$\alpha - cl(A)$ if and only if $A \cap V \neq \phi$ for every gpα-open set V containing x.

Proof. Let $x \in$ gp$\alpha - cl(A)$. Suppose there exists gpα-open set V containing x, such that $A \cap V \neq \phi$, then $A \subseteq X - V$, where $X - V$ is gpα-closed set. So, that gp$\alpha - cl(A) \subseteq X - V$. This implies that $x \notin$ gp$\alpha - cl(A)$, which contradicts to the fact that $x \in$ gp$\alpha - cl(A)$. Hence $A \cap V \neq \phi$ for every open set containing x.
Conversely, let $x \notin$ gp$\alpha - cl(A)$, then there exists gpα-closed set G containing A, such that, $x \notin G$. Then $x \in X - F$ is gpα-open. Also $(X - F) \cap A = \phi$, which is contradiction. Hence, $x \in gp\alpha - cl(A)$.

Theorem 4.10. Let A be subset of X, then gp$\alpha - int(A)$ is the largest gpα-open subset of X contained in$A$, if A is gpα-open.

The converse of the above theorem need not be true as seen from following example.

Example 4.11. In the example 3.7, the subset $A = \{b, c\}$ of X, then gpα-int(A)=$\{b\}$ is gpα-open in (X, τ), but A is not gpα-open in X.

Theorem 4.12. Let A and B be subsets of X, then

(i) gp$\alpha - int(X) = X$ and gp$\alpha - int(\phi) = \phi$.

(ii) gp$\alpha - int(A) \subseteq A$.

(iii) If B is any gpα-open set contained in A, then $B \subseteq$ gp$\alpha - int(A)$.

Proof. *(i)* and *(ii)* follows from the definition 4.2.
(iii) Suppose B is any gpα-open set contained in A. Let $x \in B$,since B is gpα-open set contained in A. Then $x \in$ gpα-int(A). Hence, $B \subseteq$ gpα-int(A).

Remark 4.13. For any subset of X, $int(A) \subseteq$ gp$\alpha - int(A)$

5 gpα-Neighborhoods and gpα-Limit points

In this section we define the gpα-neighborhood, gpα-limit points and gpα-derived set of a set and study some of their basic properties.

Definition 5.1. A subset N of X is said to be gpα-neighborhood of a point $x \in X$, if there exists an gpα-open set G containing x, such that $x \in G \subseteq N$.

Definition 5.2. Let (X, τ) be a topological space and A be a subset of X. A subset N of X is said to be gpα-neighborhood of A if there exists an gpα-open set G such that $A \in G \subseteq N$.

The collection of all gpα-neighborhood of $x \in X$ is called the gpα-neighborhood system and denoted by gp$\alpha N(x)$.

Theorem 5.3. If $N \subseteq X$ is gpα-open if it is a gpα-neighborhood of each of its points.

Proof. Let $x \in N$. Since N is gpα-open such that $x \in N \subseteq N$. Also x is an arbitrary point of N, it follows that N is a gpα-neighborhood of each of its points.

Remark 5.4. The converse of the above theorem need not to be true as seen from following example.

Example 5.5. Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{b\}\}$. A subset $A = \{b, c\}$ is gpα-neighborhood of each of its points b and c but A is not gpα-open.

Theorem 5.6. If A be subset of X and $x \in$ gp$\alpha - cl(A)$ if and only if any gpα-neighborhood N of x in X, $N \cap A \neq \phi$.

Proof. Suppose there is a gpα-neighborhood of N of x in X, such that $N \cap A = \phi$. Then there exist an gpα-open set G of X, such that $x \in G \subseteq N$. So, $G \cap A = \phi$ and $x \in X - G$. This implies gp$\alpha - cl(A) \in X - G$ and therefore $x \notin$ gp$\alpha - cl(A)$, which contradicts to the fact that $x \in$ gp$\alpha - cl(A)$. Hence, $N \cap A \neq \phi$.
Conversely, let us assume that $x \notin$ gp$\alpha - cl(A)$, there exists a gpα-closed set G of X, such that $A \subseteq G$ and $x \notin G$. So, $x \in X - G$ and $X - G$ is gpα-open in X. It becomes a gpα-neighborhood of x in X. Since $A \cap (X - G) = \phi$, which leads to a contradiction. Hence, $x \in$ gp$\alpha - cl(A)$.

Definition 5.7. A point $x \in X$ is called a gpα-limit point of a subset A of X, if and only if every gpα-neighborhood of x contains a point of A distinct from x. That is $[N - \{x\}] \cap A \neq \phi$ for each gpα-neighborhood of N of x.

Equivalently, if and only if every gpα-open set G containing x contains a point of A other than x.
In topological space (X, τ), the set of all gpα-limit points of A is called a gpα-derived set of A and is denoted by gp$\alpha - d(A)$.

Theorem 5.8. Let A and B be subsets of X,then

(i) gp$\alpha - d(\phi) = \phi$.

(ii) If $A \subseteq B$, then gp$\alpha - d(A) \subseteq$ gp$\alpha - d(B)$.

(iii) If $x \in$ gp$\alpha - d(A)$, then $x \in$ gp$\alpha - d\,[A - \{x\}]$.

(iv) gp$\alpha - d(A \cap B)$=gp$\alpha - d(A)\cap$ gp$\alpha - d(B)$.

(v) gp$\alpha - d(A \cap B) \subseteq$ gp$\alpha d(A) \cap -d(B)$.

Proof. (i) and (ii) follows from the definition 5.7.
(iii)Let $x \in gp\alpha - d(A)$. By definition 5.7, every gpα-open set G containing x contains at least one point other than x. Hence, $x \in$ gp$\alpha - d\,[A - \{x\}]$, that is x is gpα-limit point of $[A - \{x\}]$. Thus $x \in$ gp$\alpha - d\,[A - \{x\}]$.
(iv) We know that $A \subseteq A \cup B$ and $B \subseteq A \cup B$. From (ii) gp$\alpha - d(A)\cup$ gp$\alpha - d(B) \subseteq$ gp$\alpha - d(A \cup B)$. In other way, suppose $x \notin$ (gp$\alpha - d(A)\cup$ gp$\alpha - d(B)$), then $x \notin$ gp$\alpha - d(A)$ and $x \notin$ gp$\alpha - d(B)$, hence there exists gpα-open sets U and V each containing x, such that $U \cap (A - \{x\}) = \phi$ and $V \cap (B - \{x\}) = \phi$. Then $(U \cap V) \cap (A - \{x\}) = \phi$ and $U \cap V) \cap (B - \{x\}) = \phi$. On combining $(U \cap V) \cup ((A \cup B) - \{x\}) = \phi$. Therefore $x \notin$ gp$\alpha - d(A \cup B)$. Hence, gp$\alpha - d(A \cup B)$=gp$\alpha - d(A)\cup$ gp$\alpha - d(B)$.
(v) Since $A \cap B \subseteq A$ and $A \cap B \subseteq B$, from (ii) gp$\alpha - d(A \cap B) \subseteq$ gp$\alpha - d(A)$ and gp$\alpha - d(A \cap B) \subseteq$ gp$\alpha - d(B)$. Consequently, gp$\alpha - d(A \cap B) \subseteq$ gp$\alpha - d(A)\cap$ gp$\alpha - d(B)$.

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