# Modelling Gross Domestic Product Series in Turkey[1]-[2]

*Türkiye'nin Gayri Safi Yurt İçi Hasıla Serisinin Modellenmesi*

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**ABSTRACT**

All the studies regarding time series methods are useful only in case the series in interest do not display seasonal patterns. That is why it is of great importance to take the time series properties of the series like seasonal patterns or trends into account while dealing with economic time series data and the research on what form of seasonality exists (deterministic or stochastic) in the data in interest and thus the way of modelling seasonality is also crucial (Türe & Akdi, 2005, p.3). Considering its importance with this respect, in this application, it has been aimed to decide about which seasonal pattern quarterly GDP (Gross Domestic Product) series displays over 1998Q1-2014Q4 for Turkey by recoursing to DHF (Dickey, Hasza and Fuller) and HEGY (Hylleberg, Engle, Granger and Yoo) test procedures and it has been mainly focused on the dummy variable and trigonometric representations of deterministic seasonality.

**Keywords:** Deterministic–stochastic seasonality, dummy variable representation, trigonometric representation, DHF test, HEGY test.

**ÖZET**

Zaman serisi yöntemlerine ilişkin çalışmaların tümü ancak serilerin mevsimsel örüntü sergilememesi durumunda yararlı olabilmektedir. Bu sebeple, iktisadi zaman serisi verileriyle ilgilenirken, serilerin mevsimsel örüntü ve trend gibi zaman serisi özelliklerini göz önünde bulundurmak; verilerde ne tür bir mevsimselliğin olduğunun araştırılması (deterministik veya stokastik) ve dolayısıyla mevsimselliğin modellenme şekli önemlidir. Bu açıdan modellemenin önemi düşünüldüğünde, bu uygulamada DHF (Dickey, Hasza and Fuller) ve HEGY (Hylleberg, Engle, Granger and Yoo) testleri uygulanarak 1998Q1-2014Q4 dönemi için Türkiye'nin çeyreklik GSYİH (gayri safi yurt içi hasıla) serisinin nasıl bir mevsimsel örüntü sergilediğinin belirlenmesi amaçlanmış olup ağırlıklı olarak deterministik mevsimselliğin kukla değişken ve trigonometrik gösterimleri üzerinde durulmuştur.

**Anahtar Kelimeler:** Deterministic-stokastik mevsimsellik, kukla değişken gösterimi, trigonometrik gösterim, DHF testi, HEGY testi.

## 1. INTRODUCTION:

Seasonality could be viewed as deterministic or stochastic. The difference between these two types of seasonality can be explained in that way: While in the deterministic seasonal model shocks die out in the long run, in the stochastic seasonal model shocks have a permanent effect. Therefore, in the stochastic seasonal model a positive shock at time $t$ will not only increase the value of a $y_t$ series, but also the value of $y_{t+s}, y_{t+2s}, \ldots$ etc. (Özcan, 1994, p.64). Taking seasonal differences can remove the seasonal pattern. However, in the case of deterministic seasonal variation which can be modelled as a deterministic function of time plus stationary noise, this transaction is not required. Since a deterministic seasonal pattern that is subject to differencing results in a noninvertible series; in other words, it contains a unit root in the Moving Average (MA) operator. There are some tests relating to testing the presence of deterministic seasonality which are the Canova-Hansen (CH) Test, the Caner Test and the Tam-Reinsel Test. While Canova and Hansen (1995) adopt a nonparametric approach in handling of autocorrelation problem, Caner (1998) and Tam and Reinsel (1997) adopt a parametric approach and the Monte Carlo study conducted by Caner (1998) has revealed that his proposed test with the parametric approach provides better size and power properties than Canova and Hansen. Contrary to the deterministic seasonality, in the case of stochastic seasonality the seasonal differences generate a stationary and invertible process. However, if seasonal differencing is not applied to the series having stochastic seasonality, the series continues to be non-stationary. Therefore, it is of great importance to determine which type of seasonality the series in question displays because non-stationarity and non-invertibility situations create difficulties in parameter estimation and forecasting (Tam & Reinsel, 1997, p.725). As opposed to the conventional unit root tests, in the case of seasonal unit roots taking differences as the number of repeating unit roots in series will remain the series as non-stationary and this application will be able to convert the series into very complex models. In that case, the knowledge of whether unit root in a series is seasonal or not is very crucial (Türe & Akdi, 2005, p.3).

Deterministic seasonality gives a description of varying unconditional mean behaviour with the season of the year. It is the known part of the seasonal cycle when "the process is started" and is limited to time constant seasonal means or time constant growth rates that show differences across quarters/months (Kunst, 2012). When we consider topics pertaining to seasonality, it is mostly convenient to realize the season and the year to which a specific observation $t$ relates in an explicit manner. For this realization, it is preferable to use two subscripts for a variable with the first one referring to the season and the second to the year. From the knowledge of the season in which the initial observation falls, we can infer about the season for all subsequent values of $t$. By making a simple assumption that $t = 1$ corresponds to the first season of a year (that is, the first quarter for quarterly data or January for monthly data as $s = 1$) and $s$ denotes the season in which observation $t$ falls, the series of observations $y_1, y_2, \ldots, y_s, y_{s+1}, \ldots$ could be written in the double subscript notation as

$y_{11}, y_{21}, \ldots\ldots, y_{s1}, y_{12}, \ldots\ldots, y_{s2}, y_{13}, \ldots\ldots$ . Generally, $y_t$ could be written as identical to $y_{s\tau}$, where $s = 1 + [(t-1) \bmod S]$ (that is, $s_t$ is one plus the integer remainder obtained when $t-1$ is divided by S that denotes the number of observations per year) and $\tau = 1 + \text{int}[(t-1)/S]$ which is a notation for the year in which a specific observation $t$ falls with "int" denoting the integer part. In the case of that $y_t$ includes T observations, we will assume that there are exactly $T_\tau$ years of data, so that $T_\tau = T/S$ (Ghysels & Osborn, 2001, pp. 6, 19).

## 2. LITERATURE REVIEW:

The study of Canova and Hansen (1995) presents Lagrange Multiplier (LM) tests of the null hypothesis of no unit roots at seasonal frequencies denoting the presence of deterministic seasonality contrary to the tests of Dickey, Hasza and Fuller (DHF) (1984) and Hylleberg, Engle, Granger and Yoo (HEGY) (1990) tests dealing with the null of presence of seasonal unit roots. They generalize the Kwiatkowski, Phillips, Schmidt, and Shin (KPSS) (1992) test framework.

Dickey, Hasza and Fuller (DHF) (1984) propose a test called DHF which is the extension of the well-known Dickey-Fuller (DF) procedure for the zero frequency unit root case to seasonal time series. The assumption of DHF test is that the true data generating process (DGP) displays a seasonal autoregressive process of order one or SAR(1) process and thus, seasonal integration is expressed to be tested with the alternative hypothesis of stationary seasonality. One main disadvantage of this test is that it does not allow for unit roots at some but not all of the seasonal frequencies.

Tam and Reinsel (1997) examine the locally best invariant unbiased (LBIU) and point optimal invariant (POI) test procedures for a unit root in the seasonal moving average (SMA) operator for seasonal autoregressive integrated moving average models (SARIMA) and make use of the monthly non-agricultural industry employment series for males age 16-19. The results for conducted simulations have revealed that for this series, seasonality is stochastic and therefore seasonal differencing is appropriate. They also apply their tests to different types of seasonal time series data and find some of these series to have deterministic seasonality.

In order to distinguish stochastic seasonality from deterministic seasonal pattern, Tam and Reinsel (1998) also examine the LBIU and POI tests for a unit root in SMA model in the presence of a deterministic linear trend. The test procedures are applied to the monthly average total ozone data at Boulder, Colorado from 1966 to 1991 and as associated with non-rejection of the null hypothesis, they decide on that modelling seasonality as deterministic is appropriate rather than stochastic.

The article proposed by Caner (1998) suggests a locally best invariant test with the null of seasonal stationarity and the test is derived from the framework of King and Hillier (1985). When compared with the CH test, contrary to it Caner takes the autocorrelation into account in a parametric way and conducted Monte Carlo simulations revealed that his proposed test has better finite sample performance with good power properties than the CH test in an AR type of autocorrelation.

Lenten and Moosa (1999) aim to model the trend and seasonal behaviour of the alcoholic beverages consumption in the U.K. over the period 1964-1995 by means of the univariate version of Harvey's (1989) basic structural time series model. Using quarterly seasonally adjusted data, they have found the consumption of beer and wine to display stochastic seasonality and the consumption of spirits to display deterministic seasonality.

In the study by Lim and McAleer (2000), the presence of stochastic seasonality is examined to clarify the nonstationary quarterly international tourist arrivals from Hong Kong and Singapore to Australia from 1975:Q1 to 1996:Q4 using HEGY (1990) procedure. Since the presence of seasonal unit roots gives an insight into a varying seasonal pattern that is against a constant seasonal pattern, the Box Jenkins Seasonal Autoregressive Integrated Moving Average (SARIMA) process is possible to be a more suitable model for tourist arrivals rather than a deterministic seasonal model with seasonal dummy variables.

Leong (1997) presents an empirical study focusing on the nature of the seasonality and testing for the presence of seasonal unit roots using HEGY procedure for quarterly observed Australian macroeconomic data (total exports, total imports, expenditure-based GDP, retail trade turnover, total unemployed persons and manufacturers' actual sales for clothing and footwear) and finds that although total exports and total imports include seasonal unit roots, other analysed macroeconomic variables do not have a seasonal unit root and it is seen that the variables exhibit deterministic fluctuations besides stochastic seasonality.

Ayvaz (2006) investigates the seasonal behaviours of Gross National Product (GNP), consumption, export and import series in Turkish Economy using HEGY procedure and tries to detect the presence of stochastic or deterministic seasonality for these quarterly data for the period 1989:Q1-2004:Q4. The evidence has shown that consumption series displays stochastic seasonality, GNP and export series include seasonal unit roots at semi-annual and annual frequencies.

Coşar (2006) has tried to examine the seasonal properties of the Turkish consumer price index (CPI) through Beaulieu and Miron's (1993) extension of the classical HEGY test developed by Hylleberg et al. (1990) and the LM-type CH seasonal unit root test procedures with the aim to specify the seasonality accurately in econometric models. In the Coşar's (2006) study, there has been an evidence of both deterministic and nonstationary stochastic seasonality in the CPI series of Turkey.

Gagea (2007) studies the identification methods of the nature of the seasonal component of Romania's quarterly exports between 1990-2006 by using HEGY seasonal unit root testing procedure. Conducted test shows that the seasonal component may be deterministic, stochastic or mixed and since the deterministic seasonal component situation seems to be rather weak; the appropriate filter to eliminate seasonal variations is expressed to be the seasonal difference operator $(1 - L^4)$.

In the paper suggested by Chirico (2012), Italian daily electricity price data in the years 2008-2011 are analysed in order to detect the type of seasonality for the application of ARIMA (Autoregressive Integrated Moving Average) modelling. When HEGY test is performed on the sub-periods 2008-2009 and 2010-2011, it is concluded that 2008-2009 prices are seen to display a random walk movement contrary to 2010-2011 daily prices that do not include such a movement. In addition, the seasonality features non-stochasticity in both sub-periods pointing out to the absence of seasonal unit roots and thus the presence of deterministic seasonality in the short run.

In their study, Gürel and Tiryakioğlu (2012) have analysed the seasonal patterns of the seasonally unadjusted quarterly Turkish Industrial Production Index estimated by the Turkish Statistical Institute (TURKSTAT) and the sub-sectors of the mining industry, the manufacturing industry and electricity, gas and water sectors at constant 1997 prices over the period 1977:1–2008:4 by using the HEGY approach. According to the evidence, the presence of both deterministic and non-stationary stochastic seasonality has been detected in the Turkish manufacturing industry series.

## 3. METHODOLOGY:

### a. Representations of Deterministic Seasonality

There are two representations of deterministic seasonality as Dummy Variable Representation and Trigonometric Representation.

### i. The Dummy Variable Representation

The most frequently used dummy variable representation of seasonality can be expressed as follows:

$$y_t = \sum_{s=1}^{S} \gamma_s \delta_{st} + z_t \ (t = 1,.........,T_\tau) \tag{1}$$

where $y_t$ is a univariate process, $\delta_{st}$ is a seasonal dummy variable that takes the value one in season s (that is, $\delta_{st} = 1$ if $s_t = s$ for s = 1,.....,S) and is zero otherwise and finally the process $z_t$ is a weakly stationary stochastic process with mean zero. Thus, for season s of year $\tau$,

$$E(y_{s\tau}) = \gamma_s, \ s = 1,\ldots\ldots,S \tag{2}$$

This property is of primary interest with respect to implying that the process has a seasonally shifting mean. This time varying mean gives information about nonstationarity of process. Since it is very easy to remove this nonstationarity position of the process so that the deviations $y_t - E(y_t) = z_t$ are weakly stationary, this nonstationarity is often ignored. The disadvantage of this dummy representation in (2) is that it cannot distinguish seasonality from the overall mean when the latter becomes nonzero. The overall mean of $y_t$ is given as $E(y_t) = \mu = \dfrac{1}{S} \sum_{s=1}^{S} \gamma_s$.

The deterministic seasonal effect for season s denoted by $m_s$ is found by subtracting this overall mean. That is, $m_s = \gamma_s - \mu$. It is very clear from this equation that when observations are summed over a year, there will be no deterministic seasonality. Since this equation comes with a restriction of $\sum_{s=1}^{S} m_s = 0$. If the level of the series (here denoted as $\mu$) is isolated from the seasonal component, it will take the form of

$$y_t = \mu + \sum_{s=1}^{S} m_s \delta_{st} + z_t \tag{3}$$

This equation can be reformulated in a way to include a trend component that is unchanged over the seasons by putting $\mu_0 + \mu_1 t$ instead of $\mu$. A further reformulation is realized by writing separate trends for each season:

$$y_t = \mu_0 + \mu_1 t + \sum_{s=1}^{S} (m_{0s} + m_{1s} t)\delta_{st} + z_t \tag{4}$$

Here, we encounter again with same restrictions that are $\sum_{s=1}^{S} m_{0s} = \sum_{s=1}^{S} m_{1s} = 0$ mentioned above. However, this type of trending deterministic seasonality has such an implication that observations for seasons of the year diverge over time and that is why it may seem unrealistic for many applications. For both (3) and (4) processes, each observation deviates from its respective seasonal mean with a constant variance over both $s$ and $\tau$ as implied by stationarity for $z_t = y_t - E(y_t) = y_s - E(y_{s\tau})$. This result points out to that when we have a deterministic seasonal process, the observations cannot wander too far from their underlying mean (Ghysels & Osborn, 2001, pp. 20-21).

ii. **The Trigonometric Representation**

A deterministic seasonal process with period S can also be equivalently written in terms of sines and cosines corresponding to (3) as follows:

$$y_t = \mu + \sum_{k=1}^{S/2} \left[ \alpha_k \cos\left(\frac{2\pi kt}{S}\right) + \beta_k \sin\left(\frac{2\pi kt}{S}\right) \right] + z_t \qquad (5)$$

where

$$\sum_{s=1}^{S} \delta_{st} m_s = \sum_{k=1}^{S/2} \left[ \alpha_k \cos\left(\frac{2\pi kt}{S}\right) + \beta_k \sin\left(\frac{2\pi kt}{S}\right) \right] \quad \text{for } t = 1,........,T \qquad (6)$$

with

$$\alpha_k = \frac{2}{S} \sum_{s=1}^{S} m_s \cos\left(\frac{2\pi kj}{s}\right), \quad k = 1,2,........,\frac{S}{2}, \quad \alpha_{S/2} = \frac{1}{S} \sum_{s=1}^{S} m_s \cos(\pi j) \quad (7)$$

$$\beta_k = \frac{2}{S} \sum_{s=1}^{S} m_s \sin\left(\frac{2\pi kj}{s}\right), \quad k = 1,2,........,\frac{S}{2} - 1 \qquad (8)$$

Thus, both dummy variable representation and trigonometric representation will be the same. However, the trigonometric representation is seen to be more useful in separating seasonality from the overall mean $\mu$ than the dummy variable representation. In equation (6), $\alpha_k$ is considered only for $k = 1,2,........,\frac{S}{2}$ and $\beta_k$ only for $k = 1,2,........,\frac{S}{2} - 1$. Because, $\beta_{S/2}$ multiplies a sine term that is always zero. This representation provides a basis to spectral analysis of seasonality and seasonal adjustment (see Hannan, Terrel & Tuckwell, 1970).

If we took the case of quarterly data, as an implication of equation (5) the seasonal dummy variable coefficients of equation (1) are connected with the deterministic components of the trigonometric representation in the following way:

$$\begin{aligned} \gamma_1 &= \mu + \beta_1 - \alpha_2 & \gamma_3 &= \mu - \beta_1 - \alpha_2 \\ \gamma_2 &= \mu - \alpha_1 + \alpha_2 & \gamma_4 &= \mu + \alpha_1 + \alpha_2 \end{aligned} \qquad (9)$$

For quarterly data (that is, S = 4), the trigonometric components can easily be expressed in a clear way:

For $k=1, \cos\left(\dfrac{2\pi t}{4}\right) = \cos\left(\dfrac{\pi t}{2}\right) = 0,-1,0,1,....$ So, $\alpha_1 = \dfrac{1}{2}\displaystyle\sum_{s=1}^{4} m_s \cos\left(\dfrac{s\pi}{2}\right) = \dfrac{1}{2}(-m_2 + m_4)$

$k=2, \cos\left(\dfrac{4\pi t}{4}\right) = \cos(\pi t) = -1,+1,-1,+1,....$ $\alpha_2 = \dfrac{1}{4}\displaystyle\sum_{s=1}^{4} m_s \cos(s\pi) = \dfrac{1}{4}(-m_1 + m_2 - m_3 + m_4)$

$k=1, \sin\left(\dfrac{2\pi t}{4}\right) = \sin\left(\dfrac{\pi t}{2}\right) = 1,0,-1,0,....$ $\beta_1 = \dfrac{1}{2}\displaystyle\sum_{s=1}^{4} m_s \sin\left(\dfrac{s\pi}{2}\right) = \dfrac{1}{2}(m_1 - m_3)$

$k=2, \sin\left(\dfrac{4\pi t}{4}\right) = 0$

with $\alpha_1$ and $\beta_1$ denoting the annual wave and $\alpha_2$ denoting the half-year component (Kunst, 2012).

As seen above, the coefficients $\alpha_1$ and $\beta_1$ are related with the spectral frequency $\dfrac{\pi}{2}$, because they multiply $\cos\left(\dfrac{\pi t}{2}\right)$ and $\sin\left(\dfrac{\pi t}{2}\right)$ respectively for $t = 1,2,........$ (through the values 1, 0, -1, 0; it can be inferred that $\alpha_1$ and $\beta_1$ have a half-cycle every two periods and a full cycle every four periods even though $\alpha_1$ is associated with the second and fourth quarters while $\beta_1$ is associated with the first and third quarters). By the same logic, it is obvious that $\alpha_2$ is related with the spectral frequency $\pi$, since it multiplies $\cos(\pi t)$ for $t = 1,2,........$ in (4.6). Also because the terms of $\cos(\pi t)$ alternate between -1 and 1, $\alpha_2$ displays a full cycle every two periods. In the case of quarterly data, these spectral frequencies also mean the seasonal frequencies; since any deterministic seasonal pattern over the four quarters of the year can be specified as a linear function of terms at these $\dfrac{\pi}{2}$ and $\pi$ frequencies, such that $\alpha_1 \cos(t\pi/2) + \beta_1 \sin(t\pi/2) + \alpha_2 \cos(t\pi)$. By construction of these functions, in an essential manner the seasonal pattern sums to zero over any four sequential values of t.

(9) can also be represented in a different notation as:

$$\Gamma = R.B, \tag{10}$$

where $\Gamma = (\gamma_1, \gamma_2, \gamma_3, \gamma_4)'$, $B = (\mu, \alpha_1, \beta_1, \alpha_2)'$ and $R = \begin{bmatrix} 1 & 0 & 1 & -1 \\ 1 & -1 & 0 & 1 \\ 1 & 0 & -1 & -1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$ (11)

This 4x4 non-singular matrix handles the one-to-one relationship between the dummy variable representation expressed in (1) and the trigonometric representation (5) for the quarterly case. Equation (10) can also be applied for data at sampled other frequencies. For instance, if we take monthly data with S=12, then the seasonal frequencies become $\frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{5\pi}{6}$ and $\pi$. For monthly data, it is also possible to express any deterministic seasonal pattern by using the trigonometric cosine and sine functions at these spectral frequencies. However, recall one more time that the representation holds the overall mean $\mu$ separate from the deterministic component with the latter necessarily summing to zero over any twelve successive values of t. Now return to the general case of S seasons. There are some good properties concerned with matrix R in (11). When $\mu$ is included in the vector B, the matrix R becomes a square matrix and must be non-singular because there is a one-to-one relationship between the seasonal dummy and trigonometric representations. The columns of the matrix R are orthogonal to each other meaning that when the vector $R_i$ represents the $i$th column, so that $R = (R_1,......, R_S)$, then $R_i^{'} R_j = 0, i \neq j$. This quality of R assures that $R'R = D$ is a diagonal matrix. So, if the $i$th diagonal element of D is shown as $d_i$, then

$$R^{-1} = \begin{bmatrix} \frac{1}{d_1} R_1^{'} \\ \frac{1}{d_2} R_2^{'} \\ \vdots \\ \frac{1}{d_s} R_S^{'} \end{bmatrix} \qquad (12)$$

so that the inverse of R becomes the transpose of itself. For instance, let's verify this in the quarterly case:

$$R^{-1} = \begin{bmatrix} 0.25 & 0.25 & 0.25 & 0.25 \\ 0 & -0.5 & 0 & 0.5 \\ 0.5 & 0 & -0.5 & 0 \\ -0.25 & 0.25 & -0.25 & 0.25 \end{bmatrix} \qquad (13)$$

Thus, it should be noted that the first column of R includes a vector of ones. In this case, $R_1^{'} R_1 = d_1 = S$ and consequently, each element of the first row of $R^{-1}$ corresponds to 1/S and this inverse provides us a definitional relationship $\mu = (1/S) \sum_{s=1}^{S} \gamma_s$. It is also remarkable to say that equations (7) and (8) - which describe the coefficients $\alpha_k$

and $\beta_k$ as cosine and sine functions - efficiently reveal the elements of $R^{-1}$. Sometimes, it is very practical to identify the overall mean with the zero spectral frequency. So, $\mu$ can be expressed with respect to trigonometric functions as $\alpha_0 \cos(2\pi kt/S)$ with $k=0$ and (5) becomes equivalent to

$$y_t = \sum_{k=0}^{S/2} \left[ \alpha_k \cos\left(\frac{2\pi kt}{S}\right) + \beta_k \sin\left(\frac{2\pi kt}{S}\right) \right] + z_t \qquad (14)$$

since sin(0)=0. It is realized that the overall mean $\mu$ has a spectral interpretation, so it is convenient to write it as $\mu$ and therefore use the representation (5) in preference to (9).

Even if not explicitly stated here, it is obvious that the trend coefficients in the seasonally varying trend model can also be expressed by using a trigonometric representation and then with suitable definitions of the elements of $\Gamma, R$ and B in (10), the relationships between the trend coefficients in the dummy variable and trigonometric representations can easily be observed (Ghysels & Osborn, 2001, pp. 21-24).

### b. Dickey-Hasza-Fuller Test

One of the simplest testing procedures for seasonal integration possibly belongs to the one proposed by Dickey, Hasza and Fuller (1984) and modified by Osborn et al. (1988), denoted DHF. It can be regarded as a generalization of the Augmented Dickey Fuller test (ADF) and it is the first test of the null hypothesis $y_t \sim SI(1)$. Using DHF test for seasonal integration is identical to testing for stochastic seasonality. Supposing that the process is known to be a SAR(1) $[y_t = \phi_s y_{t-s} + \varepsilon_t]$, then the DHF test can be parameterized as

$$\Delta_s y_t = \alpha_s y_{t-s} + \varepsilon_t \qquad (15)$$

where $\alpha_s = -(1-\phi_s)$. Here the null hypothesis of seasonal integration is $\alpha_s = 0$ and the alternative of a stationary stochastic seasonal process implies $\alpha_s < 0$ (Baltagi, 2001, p. 661).

The asymptotic distribution of the DHF statistic is nonstandard, but it has a similar type to the DF *t* distribution. It is very well known that the DF *t* statistic is not symmetric about zero. The distribution for the DHF *t*-statistic depends on S which represents the frequency with which observations are made within each year.

In Charemza and Deadman (1992), it is shown that for a series measured $s$ times for each year, this test is built on the Student-t statistic for the OLS estimate of the parameter $\delta$ in the following regression:

$$\Delta_s y_t = \delta . z_{t-s} + \sum_{i=1}^{k} \delta_i . \Delta_s y_{t-i} + \varepsilon_t \qquad (16)$$

where the variable $z_{t-s}$ is constructed in that way: first, the regression of $\Delta_s y_t$ (where, $\Delta_s y_t = y_t - y_{t-s}$) is run on its own lagged values which are lagged up to k periods and the following equation is estimated:

$$\Delta_s y_t = \sum_{i=1}^{k} \lambda_i . \Delta_s y_{t-i} + \xi_t \qquad (17)$$

Then, use the OLS estimates of $\lambda_1, \lambda_2, ........, \lambda_k$ (denoted as $\hat{\lambda}s$) to create the variable $z_t$ from $y_t, y_{t-1}, ......, y_{t-k}$ as $z_t = y_t - \sum_{i=1}^{k} \hat{\lambda}_i . y_{t-i}$ and substitute the lagged value of $z_t$ expressed as $z_{t-s}$ into (16), estimate the equation and compute the Student-t statistic for $\delta$ (however it should be noted that here instead of $\Delta_s z_t$ proposed actually by Dickey, Hasza and Fuller (1984), Charemza and Deadman (1992) covered the view adopted by Osborn et al. (1988) and used $\Delta_s y_t$ as the dependent variable in equation (16). The critical values for the test are available in Dickey, Hasza and Fuller (1984). Here, the null hypothesis implies the presence of a seasonally integrated process and the alternative hypothesis says about either absence or nonexistence of stochastic seasonality which can be removed by using s-differences. In the case of significantly negative estimate of $\delta$, the null hypothesis may be rejected in favour of the alternative hypothesis. If it is not rejected, we need to consider the order of nonseasonal differencing required for achieving stationarity; since it is not common to face with higher orders of seasonal differencing and general expectations for most economic series are in the direction of that they are $I(0,0), I(0,1)$ or $I(d,1)$ so that using s-differences once at most is expected to eliminate seasonal nonstationarity. Therefore, if we cannot reject the null hypothesis [$(\delta = 0)$ in (16)] saying that the variable is $I(0,1)$ (or $SI(1,1)$), for the next step we need to consider whether the variable is $I(1,1)$ (or $SI(2,1)$), instead of $I(0,1)$ with the former standing for the new null hypothesis and the latter the new alternative one. For these new hypotheses, the model which should be established and estimated like ADF test is given as:

$$\Delta\Delta_s y_t = \delta . \Delta_s y_{t-1} + \sum_{i} \delta_i . \Delta\Delta_s y_{t-i} + \varepsilon_t , \qquad (18)$$

Here in the same way whether $\delta$ is significantly negative or not is examined. So, if the null that the variable is $I(1,1)$ cannot be rejected, then this expression becomes the next alternative hypothesis for the null which then says that the variable is $I(2,1)$ (or $SI(3,1)$), for the following equation:

$$\Delta\Delta\Delta_s y_t = \delta.\Delta\Delta_s y_{t-1} + \sum_i \delta_i.\Delta\Delta\Delta_s y_{t-i} + \varepsilon_t, \tag{19}$$

and so on. It should be noted that the constructed $z_t$ variable is used only for the DHF test, so not used for testing the order of nonseasonal integration (Charemza & Deadman, 1992, pp. 136-140)).

## c. HEGY Test

The analysis of seasonal unit roots is fundamentally conducted with the most popular approach developed by Hylleberg et al. (1990) called HEGY. one apparent advantage of HEGY procedure over DHF is that it enables to test for unit roots at each frequency separately without maintaining that there are unit roots at some or all other frequencies (Ghysels, Lee & Noh, 1994, p. 416). Hylleberg et al. (1990) have introduced a factorization of the seasonal differencing polynomial $\Delta_4 = (1-L)^4$ for quarterly data using lag operator L, where $L^j y_t = y_{t-j}$ and developed a testing procedure for seasonal unit roots that could be estimated by OLS in the following way:

$$\Delta_4 y_t = \sum_{i=1}^4 \alpha_i D_{i,t} + \sum_{i=1}^4 \pi_i Y_{i,t-1} + \sum_{i=1}^k c_i \Delta_4 y_{t-i} + \varepsilon_t \tag{20}$$

where k is the number of lagged terms included to ensure that residuals are white noise, the $D_{i,t}$ are seasonal dummy variables and the $Y_{i,t}$ variables are constructed from the series on $y_t$ as:

$$Y_{1,t} = (1+L)(1+L^2).y_t = y_t + y_{t-1} + y_{t-2} + y_{t-3} \tag{21}$$

$$Y_{2,t} = -(1-L)(1+L^2).y_t = -y_t + y_{t-1} - y_{t-2} + y_{t-3} \tag{22}$$

$$Y_{3,t} = -(1-L)(1+L).y_t = -y_t + y_{t-2} \tag{23}$$

$$Y_{4,t} = -(L)(1-L)(1+L).y_t = Y_{3,t-1} = -y_{t-1} + y_{t-3} \tag{24}$$

(Charemza & Deadman, 1992, p. 141).

The null hypothesis of the HEGY test is that the variable in question is seasonally integrated. Hence, if the null hypothesis of stochastic seasonality is true rather than deterministic seasonality, in this case in equation (20) all the $\alpha_i s$ will be equal to each

other and all the $\pi_i s$ will be equal to zero. In the case of different $\alpha_i s$ and at least one of the $\pi_i s$ that is nonzero, there exists a combination of both deterministic and stochastic seasonality. Critical values of these tests are provided in the Hylleberg et al. (1990) paper. The hypotheses to be tested are:

$$
\begin{array}{lll}
\textbf{1)} \ \ H_0: \pi_1 = 0 & \textbf{2)} \ \ H_0: \pi_2 = 0 & \textbf{3)} \ \ H_0: \pi_3 = \pi_4 = 0 \\
H_1: \pi_1 < 0 & H_1: \pi_2 < 0 & H_1: \pi_3 \neq \pi_4 \neq 0 \quad\quad (25)\\
\text{(t statistic)} & \text{(t statistic)} & \text{(F statistic)}
\end{array}
$$

Here, $\quad H_A : \pi_1 = 0 \quad\quad\quad \rightarrow$ the existence of nonseasonal unit root

$\quad\quad\quad H_B : \pi_2 = 0 \quad\quad\quad \rightarrow$ the existence of biannual unit root

$\quad\quad\quad H_C : \pi_3 = \pi_4 = 0 \rightarrow$ the existence of annual unit root

As seen in (25), the first two hypotheses $H_A$ and $H_B$ are tested by using one-sided $t$ tests against the hypothesis that $\pi_i < 0$. The other hypothesis which is $H_C$ is tested with an F test. For a series to include no seasonal unit roots, both $\pi_2 = 0$ and the joint F test which is $\pi_3 = \pi_4 = 0$ should be rejected. That is, $\pi_2$ and either $\pi_3$ or $\pi_4$ should be different from zero. On the other hand, in conclusion to find out that a series is stationary and thus includes no unit roots at all, we must establish that each of the $\pi$'s is different from zero (in other words, each of the $t$ test of $\pi_1 = \pi_2 = 0$ and the joint F test of $\pi_3 = \pi_4 = 0$ should be rejected in order to have a stationary series) (Hylleberg et al., 1990, pp. 221-223). In Table 1, a summary of frequencies has been given for quarterly data:

Table 1 *Long Run and Seasonal Frequencies for Seasonal Unit Root Tests in Quarterly Data*

| Frequency | Period | Cycles/year | Root | Filter | Tested hypothesis $H_0$ : Unit Root |
|---|---|---|---|---|---|
| **0** <br> **Long run** | $\infty$ | 0 | 1 | $(1-L)$ | $\pi_1 = 0$ |
| $\dfrac{\pi}{2}, \dfrac{3\pi}{2}$ <br> **Annual** | $4; \dfrac{4}{3}$ | 1; 3 | $\pm i$ | $(1+L^2)$ | $\pi_3 \cap \pi_4 = 0$ |
| $\pi$ <br> **Semiannual** | 2 | 2 | -1 | $(1+L)$ | $\pi_2 = 0$ |

*Note.* The information on first five columns have been obtained from Dıaz-Emparanza & López-de- Lacalle (2006, p.7).

## 4. APPLICATION:

In this application, it has been aimed to decide about which seasonal pattern GDP series displays over 1998Q1-2014Q4 by recoursing to different tests. Quarterly Turkish real

GDP series (expenditure based) has been taken in millions of national currency (at constant 1998 prices). Data for GDP have been obtained from CBRT. In order to linearize exponential growth, the logarithm of the series has been taken (namely, lngdp). The raw and logarithmic real GDP series have been graphed in Figure 1:
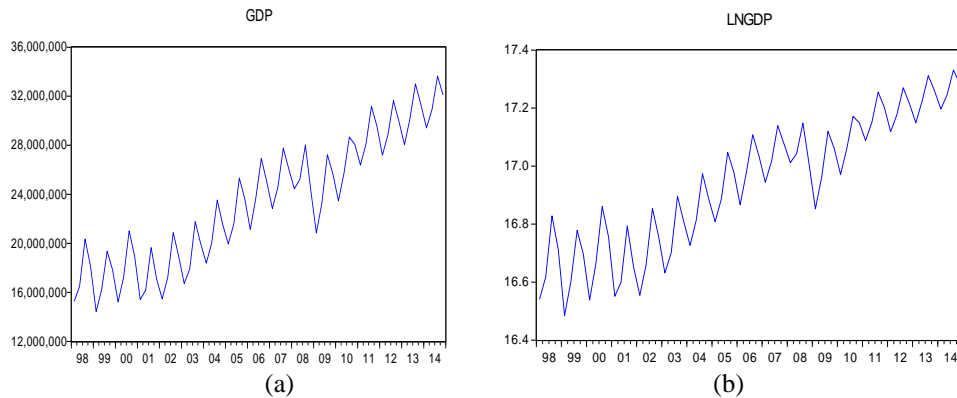


(a)                                                          (b)

*Figure 1*. Graphs of original (a) and logarithmic (b) gdp series

Logarithmic graph (b) in Figure 1 is the indicator for an upward trend implying that this series is not stationary (it includes a unit root) under the given period. In addition, the presence of seasonal components can be easily detected from this graph. In that case, in order to remove the growth trend from the series, the first difference of the logarithmic GDP series can be taken in the form of $\Delta \ln gdp = \ln gdp - \ln gdp(-1)$. The graph of first-differenced lngdp series in Figure 2 implies once again the presence of some seasonal pattern of Turkish real gdp series.

We can also compare four seasons for lngdp series. It is clear to see from the Figure 3 that the seasonal peak is observed in the third quarter. Quarters two and four seem as if they yield approximately the same amount of output. The difference between the four seasons can be clearly seen from the Figure 3: Seasonal mean in quarter 1 is the lowest, while the mean for quarter 2 and 4 are in the middle and that of quarter 3 is the highest.

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On the other hand, it can be said that Figure 4 implies that a seasonal deterministic model may seem not to be suitable for Turkish real GDP series over the given period because of not having a time constant mean for all of the four quarters.

In Figure 5, first-differenced lngdp series (namely, FD) has been graphed. For Figure 5, it can be said that quarterly means may be accepted as stationary and with this first-

differencing, the growth trend effect has been removed from the gdp series. Depending on these, a seasonal deterministic model with time constant means for all of the four quarters may be accepted as a suitable one for the first differenced real gdp series. Thus, primarily it has been aimed to adopt a seasonal deterministic model for this transformed gdp series.
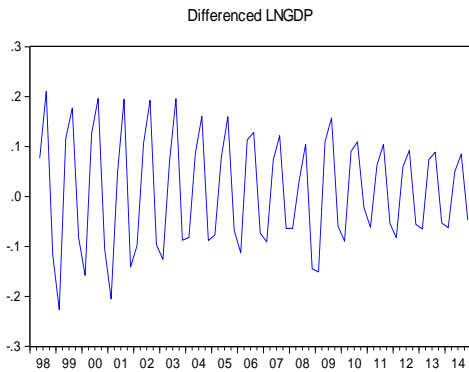


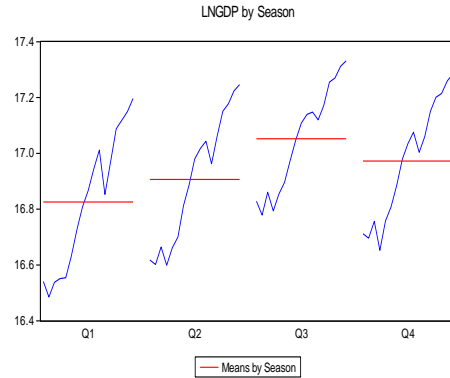*Figure 2.* Graph of first-differenced lngdp series



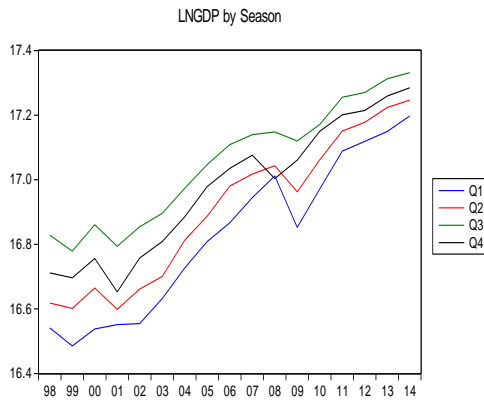*Figure 3.* Seasonal means of lngdp series



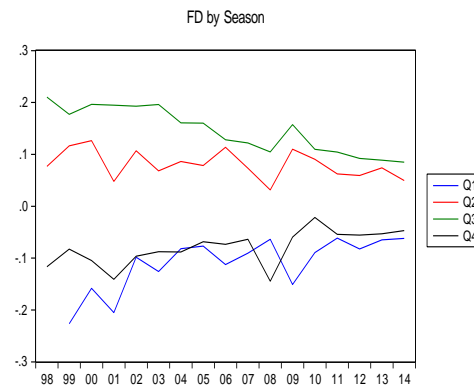*Figure 4.* Graph of lngdp series



*Figure 5.* Graph of first-differenced lngdp series

As mentioned in section 3.1., there are two representations of a deterministic seasonal model: Dummy variable representation and trigonometric representation. Firstly, by using the most frequently used dummy variable representation which is shown as in Equation (1), we are trying to investigate about the presence of deterministic seasonality. This analysis has been executed for the first-differenced real gdp series (dependent variable: dlngdp=lngdp-lngdp(-1)). Application results of (1)
have been presented in Table 2. Accrding to the results in Table 2, all the seasonal dummy variables from D1 to D4 for each of the four quarters have been found to be highly significant. R-squared value of 0.884956 reveals that the explanatory power of

the model is very good as a measure of goodness of fit since it is very close to 1. In addition, DW statistic (1.972045) that is close to 2 shows that there is almost no autocorrelation problem. Therefore, it can be concluded that a dummy variable representation as a seasonal deterministic model can be appropriate for Turkish GDP series.

Table 2 *Dummy Variable Representation of GDP Series*

| Dependent Variable: DLNGDP | | | | |
|---|---|---|---|---|
| Variable | Coefficient | Std. Error | *t*-statistic | Prob. |
| D1 | -0.109426 | 0.009930 | -11.01963 | 0.0000 |
| D2 | 0.080682 | 0.009634 | 8.375044 | 0.0000 |
| D3 | 0.145937 | 0.009634 | 15.14865 | 0.0000 |
| D4 | -0.079901 | 0.009634 | -8.293962 | 0.0000 |
| R-squared: 0.884956 | | Adjusted R-squared: 0.879478 | | DW stat: 1.972045 |

Now, let us have a look at the trigonometric representation for GDP series. Recall that trigonometric representation had been given in Equation (5) as

$$y_t = \mu + \sum_{k=1}^{S/2}\left[\alpha_k \cos\left(\frac{2\pi kt}{S}\right) + \beta_k \sin\left(\frac{2\pi kt}{S}\right)\right] + z_t$$

and the relationship between the parameters of dummy variable and trigonometric representation can be associated as in Equation (9) as follows:

$$\gamma_1 = \mu + \beta_1 - \alpha_2 \qquad \gamma_3 = \mu - \beta_1 - \alpha_2$$
$$\gamma_2 = \mu - \alpha_1 + \alpha_2 \qquad \gamma_4 = \mu + \alpha_1 + \alpha_2$$

Equation (9) can also be represented in a different notation as in Equation (10) ( $\Gamma = R.B$ ). Here $\Gamma = (\gamma_1, \gamma_2, \gamma_3, \gamma_4)'$ matrix is composed of the seasonal means in the dummy variable representation for any season s.

By looking at the Table 2, $\Gamma$ matrix which gives the seasonal means in the dummy variable representation can be expressed as,

$$\Gamma = \begin{pmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \\ \gamma_4 \end{pmatrix} = \begin{pmatrix} -0.109426 \\ 0.080682 \\ 0.145937 \\ -0.079901 \end{pmatrix}$$

Given R in quarterly case in (11), and the matrix of the parameters, $B$ that is associated with the trigonometric representation can be calculated as:

$$B = \begin{pmatrix} \mu \\ \alpha_1 \\ \beta_1 \\ \alpha_2 \end{pmatrix} = R^{-1}\Gamma = \begin{bmatrix} 0.25 & 0.25 & 0.25 & 0.25 \\ 0 & -0.5 & 0 & 0.5 \\ 0.5 & 0 & -0.5 & 0 \\ -0.25 & 0.25 & -0.25 & 0.25 \end{bmatrix} \begin{pmatrix} -0.109426 \\ 0..080682 \\ 0.145937 \\ -0.079901 \end{pmatrix} = \begin{pmatrix} 0.009323 \\ -0.0802915 \\ -0.1276815 \\ -0.0089325 \end{pmatrix}$$

Now, let us verify this result by calculating seasonal means matrix $\Gamma$ :

$\gamma_1 = \mu + \beta_1 - \alpha_2 = 0.009323 + (-0.1276815) - (-0.0089325) = -0.109426$

$\gamma_2 = \mu - \alpha_1 + \alpha_2 = 0.009323 - (-0.0802915) + (-0.0089325) = 0.080682$

$\gamma_3 = \mu - \beta_1 - \alpha_2 = 0.009323 - (-0.1276815) - (-0.0089325) = 0.145937$

$\gamma_4 = \mu + \alpha_1 + \alpha_2 = 0.009323 + (-0.0802915) + (-0.0089325) = -0.079901$

And now we can verify the value of overall mean $\mu$ of $y_t$ in (1). As mentioned before, the expected value of $y_t$ had been given in the form of $E(y_t) = \mu = \dfrac{1}{S}\sum_{s=1}^{S}\gamma_s$ .

Thus, the overall mean is calculated as:

$$E(y_t) = \frac{1}{4}(-0.109426 + 0.080682 + 0.145937 - 0.079901) = 0.009323 \ .$$

After all these, let us calculate the deterministic seasonal effect for season s which is denoted by $m_s$ and is found by using the formula $m_s = \gamma_s - \mu$ and verify that summation of deterministic seasonal effects $\sum_{s=1}^{S}m_s = 0$ are zero:

$$m_1 = \gamma_1 - \mu = -0.109426 - 0.009323 = -0.118749$$
$$m_2 = \gamma_2 - \mu = 0.080682 - 0.009323 = 0.071359$$
$$m_3 = \gamma_3 - \mu = 0.145937 - 0.009323 = 0.136614$$
$$m_4 = \gamma_4 - \mu = -0.079901 - 0.009323 = -0.08922$$

With a summation of deterministic seasonal effects getting to zero that is shown as $m_1 + m_2 + m_3 + m_4 = -0.118749 + 0.071359 + 0.136614 - 0.08922 = 0$ , these deterministic seasonal effects can be used to assess and verify the parameters $\alpha_1, \alpha_2$ and $\beta_1$ (which had been found in B matrix) in that way:

$$\alpha_1 = \frac{1}{2}\sum_{s=1}^{4} m_s \cos\left(\frac{s\pi}{2}\right) = \frac{1}{2}(-m_2 + m_4) = \frac{1}{2}(-0.071359 + (-0.08922)) = -0.0802915$$

$$\alpha_2 = \frac{1}{4}\sum_{s=1}^{4} m_s \cos(s\pi) = \frac{1}{4}(-m_1 + m_2 - m_3 + m_4)$$

$$= \frac{1}{4}(-(-0.118749) + 0.071359 - 0.136614 + (-0.08922)) = -0.0089325$$

$$\beta_1 = \frac{1}{2}\sum_{s=1}^{4} m_s \sin\left(\frac{s\pi}{2}\right) = \frac{1}{2}(m_1 - m_3) = \frac{1}{2}(-0.118749 - 0.136614) = -0.1276815$$

Table 3 *DHF Test Results for Quarterly GDP Series*

| Dependent Variable: D4Z | | | | |
|---|---|---|---|---|
| Variable | Coefficient | Std. Error | *t*-statistic | Prob. |
| D1 | 0.264860 | 0.378433 | 0.699884 | 0.4869 |
| D2 | 0.251822 | 0.380409 | 0.661977 | 0.5108 |
| D3 | 0.252065 | 0.383114 | 0.657936 | 0.5133 |
| D4 | 0.262699 | 0.381165 | 0.689201 | 0.4936 |
| LNGDP(-4) | -0.014497 | 0.022497 | **-0.644416** | 0.5220 |
| D4Z(-1) | 1.024267 | 0.126176 | 8.117793 | 0.0000 |
| D4Z(-2) | -0.339303 | 0.123893 | -2.738689 | 0.0083 |
| R-squared: 0.645741 | | Adjusted R-squared: 0.607095 | | DW stat: 2.030687 |

In Table 3, DHF test results have been presented for quarterly GDP series. As recalled from section 3.2., DHF test can be parameterized as in Equation (15):

$$\Delta_s y_t = \alpha_s y_{t-s} + \varepsilon_t$$

Here the null hypothesis of seasonal integration is $\alpha_s = 0$ and the alternative of a stationary stochastic seasonal process implies $\alpha_s < 0$ (Baltagi, 2001, p. 661). Here, the dependent variable has been given as D4Z ( $LNGDP_t - LNGDP_{t-4}$ ). LNGDP(-4) variable represents $y_{t-4}$ in Equation (15). Dummy variables, first and second lagged values of the dependent variable D4Z (which are D4Z(-1) and D4Z(-2)) have been added into the DHF test regression as shown in Table 3 and lags have been determined in a way to get white noise residuals (firstly, it has been started from the Lag 1 and lags have been increased by one until the autocorrelation and heteroscedasticity problems are resolved). Here, critical *t*-value of the DHF test statistic has been taken as equal to the ADF test statistic. Thus, ADF critical value that is -1.95 has been used. According to this critical value, since *t*-value of LNGDP(-4) variable which is -0.644416 is very small in absolute value when compared to the critical value -1.95, it is concluded that the null hypothesis cannot be rejected (where the null hypothesis is $H_0 : y_t \sim SI(1)$ (Seasonal integration of order one, meaning that simultaneous existence of all four roots in

quarterly series), while the alternative one is $H_1 : y_t$ is a stationary stochastic seasonal process). Therefore, DHF test results show that GDP series has a seasonal integration of order one process. Based on this result, it can be said that GDP series can also be modelled as a SARIMA model.

Table 4 presents HEGY test results for quarterly GDP series. As is seen clearly, first and second lagged values of the dependent variable have been added into the regression in order to get white-noise residuals. Here, the null hypothesis for HEGY test means that all four roots are simultaneously equal to zero (simultaneous existence of four roots, that is $\pi_1 = \pi_2 = \pi_3 = \pi_4 = 0$). The hypotheses to be tested in the HEGY test equation have been given in Equation (5.46). In Table 4, coefficients for Z11, Z21, Z31,

Table 4 *HEGY Test Results for Quarterly GDP Series*

| Dependent Variable: D4Z | | | | |
|---|---|---|---|---|
| Variable | Coefficient | Std. Error | *t*-statistic | Prob. |
| C | 0.094024 | 0.345823 | 0.271886 | 0.7868 |
| D1 | 0.003348 | 0.032161 | 0.104109 | 0.9175 |
| D2 | 0.062682 | 0.040941 | 1.531031 | 0.1318 |
| D3 | 0.088378 | 0.029407 | 3.005341 | 0.0041 |
| Z11 | -0.001789 | 0.005081 | **-0.352000** | 0.7263 |
| Z21 | -0.364257 | 0.159460 | **-2.284324** | 0.0265 |
| Z31 | -0.187522 | 0.091489 | **-2.049658** | 0.0455 |
| Z41 | -0.214609 | 0.090537 | **-2.370408** | 0.0215 |
| D4Z(-1) | 0.542410 | 0.175555 | 3.089690 | 0.0032 |
| D4Z(-2) | -0.212251 | 0.125955 | -1.685133 | 0.0980 |
| R-squared: 0.729045 | | Adjusted R-squared: 0.682149 | | DW stat: 2.037153 |

Z41 give $\pi_1, \pi_2, \pi_3, \pi_4$ values. In order to decide about seasonal integration of order one, all of the four hypotheses ($\pi_1 = 0, \pi_2 = 0, \pi_3 = 0, \pi_4 = 0$) have to be accepted separately. For T=100 observations, critical HEGY values have been obtained from Hylleberg et al. (1990, pp. 226-227) for constant, seasonal dummies and no trend models at 5% significance level. These critical values are -2.95, -2.94, -3.44 and -1.96 respectively for $\pi_1, \pi_2, \pi_3$ and $\pi_4$. When *t*-statistics for $\pi_1, \pi_2, \pi_3$ and $\pi_4$ are compared to the critical values, it is concluded that $\pi_1 = 0, \pi_2 = 0, \pi_3 = 0$ hypotheses cannot be rejected among four hypotheses. Only $\pi_4 = 0$ hypothesis is rejected. In other saying, we can mention about the presence of unit roots at 0, ½ and ¼ frequencies. However, there is no ¾ frequency unit root in the series. Hence, since not all four unit roots exist according to the HEGY test results (the presence of all of the four roots is not accepted), it can be said that GDP series cannot be described by a seasonal integration of order one process. Therefore, the results for DHF test and HEGY test have differed.

## 5. CONCLUSION:

When looked at the seasonal deterministic model representations and DHF and HEGY test results, the general result can be expressed as modelling first-differenced real GDP series as a seasonal deterministic model would be more suitable compared to a SARIMA model. Even though the results for dummy variable representation are positive for first-differenced GDP series, Figure 3 and Figure 4 imply that a seasonal deterministic model for GDP may not be suitable. Nevertheless, it is not certain to say about the seasonal pattern of GDP series, since DHF and HEGY test results also differ. According to the final results, it can be said that GDP series can be represented in both deterministic and stochastic structures depending on this uncertainty.

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