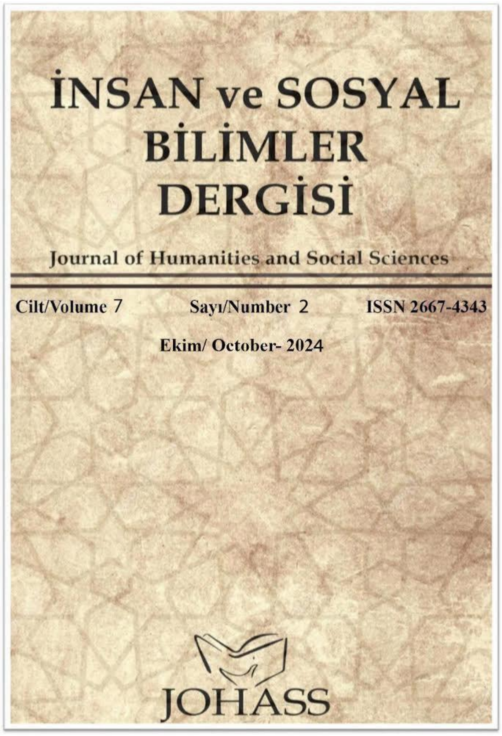
JOURNAL OF HUMAN AND SOCIAL SCIENCES (JOHASS)



https://dergipark.org.tr/tr/pub/johass

The Effectiveness of Feedback Provided by Middle School Mathematics Teachers and Preservice teachers on Student Solutions: An Evaluative Study

Fatmanur ÖZTÜRK[1]
Yıldız Technical University, Faculty of Education, Department of Mathematics and Science Education
Master's Degree
ozturkfatmanur6@gmail.com
Orcid ID: 0000 0002 6033 1552

Zehra TAŞPINAR ŞENER[2]
Yildiz Technical University, Education Faculty, Mathematics and Science Education Department
Assoc. Dr.
zehrataspinarr@gmail.com
Orcid ID: 0000-0001-8914-784X

Article Type: Research Article
Received: 13.08.2024
Accepted: 28.10.2024
Published online: 29.10.2024

Citation: Öztürk, F., & Taşpınar-Şener, Z. (2024). The effectiveness of feedback provided by middle school mathematics teachers and preservice teachers on student solutions: an evaluative study. *Journal of Human and Social Sciences*, *7*(2), 323-352.

The Effectiveness of Feedback Provided by Middle School Mathematics Teachers and Preservice teachers on Student Solutions: An Evaluative Study

Fatmanur ÖZTÜRK[1]

Yıldız Technical University, Faculty of Education, Department of Mathematics and Science Education

Zehra TAŞPINAR ŞENER[2]

Yildiz Technical University, Education Faculty, Mathematics and Science Education Department

Abstract

The aim of this study is to reveal the suggestions of middle school mathematics teachers and preservice mathematics teachers that can both correct errors in students' incorrect solutions and contribute to the advancement of students who have provided correct solutions. The study is based on the case study method. Initially, six problems related to proportional reasoning were posed to middle school students in a public school, and the solutions obtained from the students were classified according to their proportional reasoning strategies. These solutions were then examined by the students' mathematics teachers and by preservice mathematics teachers. Two questions were asked to mathematics teachers and preservice mathematics teachers during these examinations. After the examination, interviews were conducted to discuss the suggestions made to (1) correct the errors in incorrect solutions and (2) advance the proportional reasoning skills of students who obtained correct solutions. The findings indicate that both mathematics teachers and preservice mathematics teachers were adequate in providing suggestions to correct errors in students' incorrect solutions. However, it was also found that both groups had difficulties in making suggestions to further the advancement of students who provided correct solutions.

Keywords: Noticing, mathematics teacher, preservice teacher, proportional reasoning, feedback

Research Article

Received: 13.08.2024
Accepted: 27.10.2024
Published online: 28.10.2024

Corresponding author:
[1] *Master's Degree*
ozturkfatmanur6@gmail.com
Orcid ID: 0000 0002 6033 1552
[2] *Assoc. Dr.*
zehrataspinarr@gmail.com
Orcid ID: 0000-0001-8914-784X

Introduction

In contemporary education, the primary aim of mathematics instruction is to develop individuals who can generate knowledge, think critically, and make informed decisions in various situations (Ministry of National Education [MoNE], 2018). Achieving this goal requires effective mathematics instruction. For mathematics education to be effective, teachers must have a deep understanding of their students' prior knowledge, common misconceptions, learning challenges, and cognitive processes in mathematics (Shulman, 1986; Marks, 1990; MEB, 2018). Therefore, mathematics teachers need to develop and refine the skill of noticing. (Kaiser et al., 2016).

The skill of noticing encompasses the ability to "pay attention to significant interactions or events in a teaching environment and interpret these interactions and events using existing knowledge" (Sherin et al., 2011, p. 5). Van Es and Sherin (2002) added that in addition to noticing and interpreting interactions or events, teachers should also adapt their teaching methods to improve instruction based on what they notice and interpret. This is because simply noticing and interpreting students' learning is not enough to address the difficulties and misconceptions they may have (van Es, 2011, p. 138; Miller, 2011). Therefore, it is vital for teachers to decide how to provide feedback to students regarding their learning errors and misconceptions (Jacobs et al., 2010). Consequently, the skill of noticing allows teachers to offer suggestions to address students' learning errors and misconceptions and to plan their lessons accordingly (van Es, 2011, p. 138; Miller, 2011, p. 51; Choy, 2014; Taylan, 2018; Şermetoğlu & Baki, 2019; Birinci and Baki, 2019).

Jacobs et al. (2010) identified three essential components for effective noticing in mathematics education: attending to children's strategies, interpreting children's mathematical understanding, and deciding how to respond on the basis of children's understandings

1. Attending to Students' Mathematical Strategies: By attending to students' solutions, mathematics teachers can assess the level of their students' mathematical knowledge (Jacobs et al., 2007). This allows teachers to identify what is correct and what is incorrect in the solution strategies used by their students.
2. Interpreting Students' Mathematical Understanding: Through interpreting their students' mathematical understanding, teachers can assess the correctness and incorrectness of the students' solutions and draw conclusions about their learning in mathematics (Sherin et al., 2011, p. 81). This enables teachers to

form ideas about their students' misconceptions and incomplete learning in mathematics.

3. Deciding How to Respond on the basis of Children's Understanding: After evaluating and interpreting their students' learning in mathematics, teachers decide on how to provide feedback. This allows them to develop instructional methods that address students' learning difficulties and misconceptions in mathematics (Jacobs et al., 2010; Miller, 2011, p. 55). Therefore, it is crucial that mathematics teachers have a strong background in both mathematical and pedagogical content knowledge. (Dick, 2013; Tunç-Pekkan and Kılıç, 2015). Moreover, many mathematics teachers assume that once students provide correct solutions, their mathematical learning is complete and do not offer further suggestions for improvement (Özel, 2019). However, providing suggestions to correct errors in incorrect solutions alone is not sufficient for effective noticing; mathematics teachers must also offer suggestions that contribute to the advancement of students who provide correct solutions (Jacobs et al., 2010).

This study examines the decision-making skills of mathematics teachers and pre-service teachers on how to provide feedback on the solution strategies used by students in problems related to proportional reasoning. Proportional reasoning forms the foundation for algebraic topics related to ratio and proportion, as well as for advanced mathematical topics (Lesh et al., 1988). Proportional reasoning involves identifying proportional situations, representing these situations, distinguishing proportional situations from non-proportional ones, and solving problems related to proportional situations (Cramer et al., 1993). Mathematical topics related to ratio and proportion include slope, fractions, percentages, congruence and similarity, probability, histograms, trigonometry, etc. (Van De Walle et al., 2013, p. 348; Dougherty et al., 2016). Thus, proportional reasoning plays a crucial role in mathematical achievement (Ünsal, 2009). However, it is observed that many adults struggle to acquire this skill (Lamon, 2006, p. 3). Among those struggling with proportional reasoning are also mathematics teachers and preservice teachers (Çıkla and Duatepe, 2002; De la Cruz, 2008; Benson, 2009; Bahar, 2016; Kasemsukpipat, 2016; Johnson, 2017; Arıcan, 2018; Boyacı, 2019; Brown, Weiland and Orrill, 2019).

In the literature, it is known that different types of problems are used to develop proportional reasoning skills (Cramer et al., 1993; Lamon, 1993). Cramer and Post (1993) identified three types of problems: problems involving numerical comparisons (quantitative comparison), problems that require reasoning without numerical values (qualitative comparison), and problems where three out of four quantities are given and the fourth is to be found (finding the unknown value). Lamon (1993) classified proportional situations into four types of problems based on their content: unite rate, part-part whole, related sets, and scaling problems. Unit rate problems involve creating a ratio between two quantities and deriving a new unit or concept from that ratio. Part-part-whole problems involve creating ratios either between two or more quantities within a whole or between the quantities and the whole. Related sets problems are those where the relationship between two quantities is not explicitly stated in the problem and there is no common relationship independent of the problem. Finally, scaling problems involve two quantities that are in a continuous relationship and increase or decrease by a certain ratio. These problems involve the use of multiplicative relationships (Lesh, Post, & Behr, 1988). Dooren et al. (2010) noted that after learning the concept of ratio, students tend to apply multiplicative reasoning even in non-proportional situations. Consequently, additive relationship problems are also used to help students distinguish between situations that require additive reasoning and those that require multiplicative reasoning (Langrall and Swafford, 2000). All these problems can be solved using different strategies. As proportional reasoning develops, students can use both correct strategies and multiple strategies to solve problems, whereas students with underdeveloped proportional reasoning skills may use incorrect strategies. For this reason, the feedback that mathematics teachers give to their students regarding their students' solutions is important (Jacobs et al., 2010; Şen ve Güler, 2017). The purpose of this study is to examine the decision-making skills of mathematics teachers and mathematics preservice teachers regarding how to provide feedback on students’ solutions to problems related to proportional reasoning. This study aims to answer the following questions:

1. What suggestions do middle school mathematics teachers and preservice mathematics teachers offer to address the errors in students' incorrect solutions?
2. What suggestions do middle school mathematics teachers and preservice mathematics teachers provide to support the further development of students who offer correct solutions?

Method

Model

This study is part of a larger research project described by Jacobs et al. (2010), which encompasses all aspects of the skill of noticing, focusing specifically on the dimension of "deciding how to respond on the basis of children's understanding.' Participants were asked to provide suggestions both for correcting errors in students' incorrect solutions and for supporting the further development of students who provided correct solutions. The study employed a case study model, which is a research method that involves an in-depth examination of one or more cases, environments, social groups, etc. (McMillan, 2000; as cited in Büyüköztürk et al., 2018, p. 23). Since students from both one school and one university were selected, the research was conducted as a case study.

Participants

In this study, to obtain the suggestions of teachers and preservice teachers, data were first collected from the students taught by the participating teachers. For this, 5th, 6th, 7th, and 8th-grade students in a public school in Istanbul were asked to solve six problems related to proportional reasoning. These problems were designed to cover all proportional reasoning problems identified in the literature. The solutions obtained were analyzed based on proportional reasoning strategies, and incorrect strategies were identified. Examples of each strategy were selected and prepared for review by the teachers and preservice teachers.

Five middle school mathematics teachers participated in this study, coded as T1, T2, ..., and these codes were used when presenting the findings. The characteristics of these teachers are shown in Table 1. below.

Table 1

Characteristics of the Middle School Mathematics Teachers Participating in the Study

Teacher	Years of Experience	Classes Taught
T1	2 years	8th grade
T2	6 years	6th grade
T3	16 years	5th grade
T4	2 years	7th and 8th grades
T5	2 years	6th and 7th grades

The other participants in this study were nine senior preservice mathematics teachers from a public university. These preservice teachers had completed courses such as Approaches to Teaching and learning mathematics and special methods of teaching mathematics. They were selected based on their grade points average (GPA) rankings, with three students from the top, three from the middle, and three from the lower ranks. The preservice teachers are coded as P1, P2, ..., and these codes were used when presenting the findings. The GPA rankings of the preservice teachers are as follows: P1, P3, and P5 are in the top ranks; P2, P6, and P7 are in the middle ranks; and P4, P8, and P9 are in the lower ranks.

Data Collection Tool

In this study, a semi-structured interview technique was used. Semi-structured interviews allow for both the asking of pre-prepared questions and, if necessary, the asking of additional questions to obtain more detailed information about the research topic (Büyüköztürk et al., 2018).

During the interviews, participants were asked for their suggestions on how to address the errors in students' incorrect solutions through the question: "What would you do for students who solved the problem incorrectly? What kind of interventions would you make?" Additionally, the participants' suggestions for advancing the abilities of students who provided correct solutions were collected through the question: "What would you do for students who solved the problem correctly to help them advance? For example, how would you modify the problem?"

Data Collection and Analysis

Participants were asked to provide suggestions both for correcting errors in students' incorrect solutions and for supporting the further development of students who provided correct solutions. The data collected from the participants were analyzed using content analysis. Content analysis is a systematic and replicable method of data analysis in which specific rules are applied to code, categorize, and summarize certain words or themes within the text (Büyüköztürk et al., 2018, p. 259).

In analyzing the suggestions made by participants to correct errors in students' incorrect solutions, codes were first created based on the suggestions, and similar suggestions were grouped under a single code. These suggestions were then categorized as effective, low effective, or ineffective. Similarly, in analyzing the suggestions made by participants to support the further development of students who provided correct solutions, codes were created based on the suggestions, and similar suggestions were grouped under a single code. These suggestions were also categorized as effective, low effective, or ineffective.

The criteria used to categorize the suggestions are provided in Table 2 below.

Table 2

Criteria for Suggestions

	Criteria for Effective Suggestions	Criteria for Low-effective Suggestions	Criteria for Ineffective Suggestions
Suggestions for the elimination of errors	Suggestions that will make the student realise the multiplicative or additive relationship in the question and make proportional reasoning	Suggestions to make you realise that you are only doing the question right or wrong More rote-based recommendations Retelling the same question without suggesting anything new, presenting visuals to help them understand the question, etc.	Suggestions that the student will have difficulty in understanding (equation, inverse proportion, etc.).
Suggestions for further improvement	Making changes that will take the problem one step further (adding new variables, having the student determine the variables, etc.), having the problem set up, trying to solve the problem with different strategies	Suggestions given to those who make mistakes	Suggestions for understanding the problem

Validity and Reliability

In qualitative research, the concepts of dependability and transferability are used in place of reliability, while credibility and confirmability replace the concept of validity (Creswell, 2013; Lincoln and Guba, 1985; Merriam, 2009; Patton, 2002). In this study, expert opinions were sought to ensure dependability. Initially, an expert was consulted during the coding of the participants' statements to determine which codes the statements corresponded to (for example, a teacher’s statement being coded as "providing examples from daily life"). Subsequently, meetings were held with three experts to categorize the codes appropriately. Initially, two categories were established: effective and ineffective. During the meeting, it was decided that some codes fell between these two categories, leading to the creation of a category for low-effective suggestions. To ensure transferability, the research process was detailed comprehensively. For credibility and confirmability, in-depth data analysis was conducted, and each code was illustrated with an excerpt in the findings section.

Ethical Committee Approval

The ethics committee permission for the article was obtained by the Yıldız Technical University Publication Ethics Committee with the decision numbered 2021/01 dated 21.03.2021.

Findings

In this part, the suggestions of mathematics teachers and prospective mathematics teachers for the correction of errors for incorrect student solutions and the suggestions for the progress of students who made correct solutions are given. These suggestions were analyzed one by one for each problem and presented in tables. These suggestions were grouped as effective suggestions, low effective suggestions, and ineffective suggestions.

In Table 3.1. below, the suggestions of the teachers and pre-service teachers for eliminating the errors for the incorrect solutions of the students in problem 1 and the suggestions for the further improvement of the students who made correct solutions are given. In addition to these suggestions, the participants who had these suggestions were also included.

Table 3

Suggestions Offered by Participants for Problem 1

		Effective Suggestions	Low Effective Suggestions	Ineffective Suggestions	No Suggestion
Problem 1: Additive Problem	Suggestions for the elimination of errors	Giving Examples from Daily Life (T1- T2- T3- T4- P1-P2-P6-P9) Making Realize The Additive-Multiplicative Relationship (T5- P7 P8) Visualization (P5)		Teaching formulating equations (T4- P3-P4-P6)	
	Suggestions for further improvement	Adding a New Variable- Asking Different Questions Regarding the Problem (T2- P4-P6)		Giving Examples from Daily Life (T1) Changing variables in the problem (T4- T3-T5-P1-P2- P3-P5-P6-P8-	P7

	P9)

When examining Table 3, it is observed that for Problem 1, suggestions such as providing examples from daily life, highlighting additive-multiplicative relationships, using visualization, and teaching the strategy of equation formulation were made to address student errors. For example, T1, who suggested providing examples from daily life, explained as follows:

"In general, what the student is failing to analyze is the age difference. 'If there is a 3-year difference between you and me, it will still be a 3-year difference 10 years later.' Because the student cannot reason this out, I would provide a simple example. For instance, 'What is the age difference between you and your sibling? 5 years. What will it be in 20 years? It will still be 5 years.' If I can explain that the difference does not change, I can help the student solve this problem better."

T5, who suggested highlighting additive-multiplicative relationships, stated:

"In the first instance, we can explain to the student that the concept of multiples is related to multiplication, while the increase in age is related to addition."

P5, who suggested using visualization, explained:

"I could create a timeline with units, maybe on paper. Then, on the timeline, I would arrange each age with a difference and advance Ayşe's age by 18 years, and similarly, create a timeline for the father, advancing his age in the same way, so they can observe the difference."

T4, who suggested teaching the formulating equation, stated:

"I would explain that this problem can be solved with an equation. I would say Ayşe is x, and her father is 3x. Or if the class level is 6 or 5, I would use 'apple' and say 'three apples.'"

To advance the skills of students who provided correct solutions, suggestions such as introducing a new variable, asking different questions related to the problem, providing examples from daily life, and changing the variables in the problem were made.

T2, who suggested introducing a new variable and asking different questions related to the problem, explained:

"I would introduce 3 people instead. I would change the question and ask, 'How old will the father be?' I would make it a bit more challenging, for example, by asking how the sum of their ages changes. Then I would diversity the problem. As I said, I could add three people: the father, Ayşe, Ali, etc., and develop the questions further."

T1, who suggested providing examples from daily life, explained:

"I could ask the problem in a similar way. It could be more personal, like instead of Ayşe and her father, I could use you as an example. For instance, I could say 'you and your sibling,' to make it more concrete. By using examples from their own life, family members, I can create a more permanent understanding, by using examples from their own life."

As seen, T1's suggestion is focused on helping the student better understand and concretize the problem. However, since the student has already understood the problem and provided the correct solution, T1's suggestion was categorized as ineffective. P3, who suggested changing the variables in the problem, explained:

"Instead of just saying Ayşe's age is twice her father's current age minus 15, we could introduce other variables, like 2x-15, or add, subtract, halve, or quarter the amounts. We could add such things that require the four basic operations."

Based on P3's statements, it is seen that the problem is shifted from measuring proportional reasoning ability to equation solving. This suggestion was also categorized as ineffective.

Table 4 below presents the suggestions provided by participants for Problem 2.

Table 4

Suggestions Provided by Participants for Problem 2

		Effective Suggestions	Low Effective Suggestions	Ineffective Suggestions	No Suggestion
Problem 2: Missing Value and Part-Part-Whole Problem	Suggestions for the elimination of errors	Simplifying The Problem (T1-P1-P2-P7-P9) Making Realize The Additive-Multiplicative Relationship (P8) Using Unit Rates (T5-P3-P6) Concretization (P5)	Making the Problem Read Again- Retelling the Topic (T3- P8)	Making Distinguish Direct Proportion and Inverse Proportion (T2- P4)	T4
	Suggestions for further improvement	Changing variables in the problem (P3) Adding A New Variable (P4-P5) Directing to A Different Strategy (P8-P1)		Changing the numbers in the problem (T1-T4-T5-T2-P2-P6-P9) Asking Similar Problems (T3-P5)	P7

When examining Table 4, it is observed that suggestions such as simplifying the problem, using unit rate, providing examples from daily life, highlighting additive-multiplicative relationships, concrete representation, rereading the problem, reexplaining the topic, and distinguishing between direct and inverse proportion were made. T1, who suggested simplifying the problem, explained:

"I could show this in a more concrete way. How can I put it? I could use an example, or ask them to do something. For instance, as homework, 'You do this.' Or we could use an example with pizza. 'If I use this many ingredients for 1 pizza, how much would I need for 2 pizzas?' and so on to make it more concrete."

T5, who suggested using unit rate, explained:

"Before explaining proportion, I would first explain ratio, focusing on the value corresponding to one part, and then compare it with direct proportion. We would focus more on example solutions in this context. We compare and try to explain that these mean the same thing. This is one of the common problems we encounter in class. For instance, while teaching 6th graders last year, I explained that finding the value corresponding to one part and comparing it with the number of parts is logically the same as direct proportion. I would offer this as a solution."

P5, who suggested concretization, explained:

"Maybe we could make the problem more concrete so that the student can see the proportion. We could think of it as a drawing or prepare it from cardboard. For example, after mixing four onions with 8 cups of water, we could prepare eight solid plates, and the student can observe how much onion and water is allocated to each person."

P3's suggestion to change the variables in the problem explained:

"While preparing 20 meals, for example, how much oil is needed? How much should remain? During the meal, two, for instance, were dropped and ruined. So, how much is needed now? I think about playing with the

numbers here. Can the student identify the necessary relationships and carry out the operations? The ingredients could increase or decrease."

P8's suggestion to direct students to different strategies explained:

"Some students tried to calculate based on one person, finding the amount per person and then multiplying by the number of people. Instead, I would ask them to use proportion and solve it that way as well."

T1's suggestion to change the numbers in the problem explained:

"The student already understands this. I can change the numbers and give larger ones, increasing the complexity of the operations. Because they have already grasped the concept. I think we can change the problem's numbers and provide more numbers to develop their operational skills, as they have

As it can be seen, T1 made a suggestion that would enable the student to develop his/her calculation skills instead of developing his/her proportional reasoning.

The explanation of T3 who suggested asking similar problems is given below:

'If he/she does this, the child has already settled the subject. Then I continue with different examples related to this.'

In the statement of T3, again, there is no suggestion that will carry the student to an advanced stage.

Table 5 includes suggestions from teachers and prospective teachers regarding solutions to the third problem.

Tablo 5

Suggestions Provided by Participants for Problem 3

		Effective Suggestions	Low Effective Suggestions	Ineffective Suggestions	No suggestion
Problem 3: Quantitive Comparison and Well-Chunked Measures Problem	Suggestions for the elimination of errors	Simplifying The Problem (T3-P1-P9) Using Unit Rate (T4-T5-T1-T2- P2-P7-P9)	Concretization (P2-P5-P6) Explaining Division (T1-T2- P1-P4-P8)	Using Fraction Strategy (P2-P8)	P3
	Suggestions for further improvement	Adding A New Variable (T2-T5-P2) Directing to A Different Strategy (T3-P1-P9)		Changing The Numbers in The Problem (T1-T5-P6) Asking Similar Problems (T4-T5-P3-P4-P5-P7-P8)	

According to Table 5, the suggestions for this problem include simplifying the problem, using unit rate, concrete representation, explaining division, and using the equivalent fractions strategy. Below are the explanations of T3, who suggested simplifying the problem:

"We would just change the numbers, or maybe instead of a race, it could be something else like painting or reading books. These could be changed. But more simply, because when the numbers are more evenly divisible, without decimals, the child will understand better."

As seen in Appendix 1, in the third problem, it is difficult for the student to reason since the relationship between quantities is not an exact multiple. T3 noticed this and suggested simplifying the problem so that the relationship is an exact multiple, and this suggestion was evaluated as effective.

Below are the explanations of T4, who suggested using unit rate:

"In these questions, I would first say that we need to find out how much one lap takes. Then, 'We can decide who is faster in this way,' I would say."

Below are the explanations of P2, who suggested concretization

"Maybe I could explain it with an animation because I don't think the student who gave the first answer understood it at all. With the help of a demonstration, I would perhaps bring two students to the board, create the scenario, and explain the question environment in that way."

As seen in P2's statements, the focus is more on helping the student understand the problem rather than promoting proportional thinking. While this suggestion might be effective for some students, it was considered likely ineffective for students who do not yet understand proportional reasoning.

Below are the explanations of P2, who suggested using the fractions strategy:

"I would have them think of it like equivalent fractions, either equalizing to 50 minutes or finding out how many laps they ran in 24 minutes."

P2's statements indicate that students who already have issues with proportional reasoning may not understand why they need to equalize to 50 minutes or 24 laps, so this suggestion was categorized as ineffective.

To help advance the skills of students who provided correct solutions, participants suggested introducing a new variable, directing them to different solution strategies, changing the numbers in the problem, and asking similar problems. Below are the explanations of T2, who suggested introducing a new variable:

"As I mentioned earlier, maybe I would increase the difficulty by adding a third person, like Tarık, Ozan, Ali, and then write something for Ali and ask, 'Who ran faster?' By increasing the number of people, I could raise the question to a higher level."

Below are the explanations of P9, who suggested directing students to different solution strategies:

"For example, in the second solution, the student reached the correct solution, but how did they use strategy? They added three laps and another three laps, which made one and a half. I would give numbers that would prevent this kind of thinking, so that they cannot reach a whole number or set up the ratio. I would make them set up a proportion calculation instead of guessing."

Finally, T4's explanation, who suggested asking similar problems, is as follows:

"I would solve different questions related to this topic."

Table 6 shows the suggestions made by the participants for solutions to the fourth problem.

Tablo 6

Suggestions Provided by Participants for Problem 4

Prob	Effective Suggestions	Low Effective	Ineffective	No suggestion

			Suggestions	Suggestions	
	Suggestions for the elimination of errors	Concretization (T1- T3-T4-T5- P1-P2-P3-P4-P5-P6-P7) Giving Numeric Values (T2)	Making the Problem Read Again (P8) Giving Similar Examples (P9)	Explaining Inverse Proportion (T5)	
	Suggestions for further improvement	Adding A New Variable (P3-P6)	Giving Numeric Values (T2-T5-P1-P2-P4-P9)	Asking Similar Problems (T1-T4-P5-P7-P8)	T3

According to Table 6 the participants suggested concrete representation, providing numerical values, rereading the problem, providing similar examples, and explaining inverse proportion. Below are the explanations of T1, who suggested concretization;

"I would go to the teachers' lounge, take two glasses of tea, and teach by doing. Which one is sweeter? I would put two sugars in the small glass and one in the large glass. It can be easily learned through this hands-on experience."

Below are the explanations of T2, who suggested providing numerical values:

"I would give an example from daily life or present it more numerically. For instance, 'Would the size of the glass be large?' I would guide them towards B in a more concrete way by establishing a ratio."

Below are the explanations of P8, who suggested rereading the problem:

"I would advise the student to read the question more carefully."

Below are the explanations of T5, who suggested explaining inverse proportion:

"For students who made mistakes, we could first explain the concept of inverse proportion here."

To help advance the skills of students who provided correct solutions, the participants suggested introducing a new variable, providing numerical values, and asking similar problems. Below are the explanations of P3, who suggested adding a new variable:

"I would add a new variable. In the second version of the tea, the bitterness came from the ratio of the tea's strength. If we brew tea with twice the strength, how many sugars would be needed to maintain the same sweetness ratio as before? I added a third variable like this."

Below are the explanations of T1, who suggested asking similar problems:

"I could diversify the question types. Similar question types, but I could give different examples. It might be a different object or a different example rather than tea."

As seen, while P3 suggested making the problem more difficult to help students move to a higher level, T1 suggested providing similar examples by changing the context of the problem.

Table 7 includes the suggestions made by the participants for solutions to the fifth problem.

Table 7

Suggestions Provided by Participants for Problem 5

Pr ob le		Effective Suggestions	Low Effective Suggestions	Ineffective Suggestions	Participants who did not make a

				suggestion
Suggestions for the elimination of errors	Using Unit Rate (T2-P8) Concretization (T5-T1-P7) Visualization (T3- T4-T1- P1-P3-P4-P5-P6-P7-P9) Simplifying The Problem (P1)	Explaining The Topic (P2-P3)	Using Fraction Strategy (P8)	
Suggestions for further improvement	Adding A New Variable (T1) Directing to A Different Strategy (T2-T3-T5-P1)		Asking Similar Problems (T4-T5-P3-P5-P6-P8-P9) Changing The Numbers in The Problem (P2)	P4-P7

According to Table 7, the participants suggested using unit rate, concretization, visualization, simplifying the problem, explaining the topic, and using the equivalent fractions strategy to address incorrect solutions. Below are the explanations of T2, who suggested using unit rate:

"I would guide them to unit rate. If 7 people ate 3 pizzas... I would direct them to 3/7 and 1/3 unit rates to find out which is more."

Below are the explanations of T5, who suggested concrete representation:

"For instance, we can select seven girls and three boys from the class and represent them as a whole pizza. Here, we could say there are three pizzas. To distribute them, what do we need to do? First, we need to explain division because some students performed subtraction here. The basic operation in proportion is multiplication. We need to explain that proportion can be done with multiples or multiplication. With the support of materials, we can explain this situation more easily."

Below are the explanations of T3, who suggested visualization:

"We could explain this, for example, by drawing a picture. We could equalize it. If this one eats this much, and that one eats this much, 'Who eats less? Who eats more?' I would try to explain it visually for those who couldn't do it. I think they would understand better that way."

Below are the explanations of P1, who suggested simplifying the problem:

"I could change the numbers. Because here, for instance, the 3/7 pizza ratio might be difficult for students. I would make them perform operations where the answers are whole numbers."

Below are the explanations of P3, who suggested explaining the topic:

"*We have two components, right? One component will be all pizzas, and the other will be all people. I need to emphasize who is giving what to whom. They have errors in division, fundamentally. Some of them have incorrect division from elementary school. First, I would correct those basic deficiencies and then try to progress."*

P3's statements indicate that while they identified the deficiencies in students' division skills, it is unlikely that this suggestion would completely resolve the problem they are experiencing. Below are the explanations of P8, who suggested using the equivalent fractions strategy:

"I would suggest finding the common multiples of 7 and 3, thinking of it as 21 people. Then, how many pizzas would 21 people get? How many pizzas would 21 people get? This is what I would suggest."

To help advance the skills of students who provided correct solutions, the participants suggested introducing a new variable, directing them to different solution strategies, asking similar problems, and changing the numbers in the problem. Below are the explanations of T1, who suggested introducing a new variable:

"I could add a different factor. For example, not just pizza, but maybe pizza and a drink. I could say, 'There's this much drink. Who drinks how much?' and thus increase the complexity."

Below are the explanations of T2, who suggested directing students to different solution strategies:

"To help them advance, I would ask for different solution methods. Instead of changing the problem, I would say, 'How else would you do it? What other solution strategies would you use?' This way, I would have a different method."

Finally, below are the explanations of T4, who suggested asking similar problems:

"I would change the problem. Instead of pizza, I would use bread, or something else."

Below are the explanations of P2, who suggested changing the numbers in the problem:

"I could change the numbers. For example, I would say that 3 boys ordered 2 pizzas."

Table 8 includes the suggestions made by the participants for solutions to the sixth problem.

Table 8

Suggestions Provided by Participants for Problem 6

		Effective Suggestions	Low Effective Suggestions	Ineffective Suggestions	Participants who did not make a suggestion
Problem 6: Missing Value and Scaling Problem	Suggestions for the elimination of errors	Simplifying The Problem (T1-T3-P1) Using Geometry Related Software (T5- P2-P5-P7)	Explaining the Similarity (T2-T4-T3-P3-P4-P6-P7-P8-P9)		
	Suggestions for further improvement	Making Set Up The Problem (T2) Making Find Different Relationships in The Problem (T5- P3-P4-P7)		Changing the Shapes in the Problem (T4-P9) Multiplying the Shapes in the Problem (T1-P1-P2-P5-P6-P8)	T3

According to Table 8, the participants suggested simplifying the problem, using geometry software, and explaining similarity to address incorrect solutions. Below are the explanations of T1, who suggested simplifying the problem:

"*For this question, the student needs to see this. For example, we gave it as 12 by 18, but since it's not an exact multiple, it results in a fraction like 2/3. First, I think I should show it as an exact multiple. For instance, if this side is 2 cm, the other side should be 4 cm. It should be twice as long, so I would show that the long side is proportional to the long side and the short side is proportional to the short side. I would show that the ratio is fixed, for example, it could be 2 or 3 or 5 times as long... First, I would show that it's a fixed number, and then I would explain that it can also be given as a fraction, not just as an exact multiple. Then I would explain the solution."*

Below are the explanations of T5, who suggested using geometry software:

"We can explain it visually. Generally, we could use dynamic software models for this. Using physical materials might be difficult because as 12 and 18 grow, they don't increase proportionally. We could use visual software support to show that as 12/18 grows, it increases at the same rate."

Below are the explanations of T2, who suggested explaining similarity:

"I would tell the student that there is a relationship because the short side is similar to the long side. 'What kind of relationship is there between the long sides?' The short sides must also have the same relationship because of the similarity. I could have them establish the same relationship. For example, if one side is 15 times longer, the other side must also be 15 times longer. Additionally, I could ask the student what might happen when the short side is proportionate to the long side to maintain the same proportion."

To help advance the skills of students who provided correct solutions, the participants suggested making students formulate problems, finding different relationships in the problem, changing the shapes in the problem, and increasing the number of shapes in the problem. Below are the explanations of T2, who suggested making students formulate problems:

"I could ask the student to create two similar rectangles. This way, I could make them both formulate a problem and solve the problem they created. I would say, 'Create a similar problem like this.'"

Below are the explanations of T5, who suggested finding different relationships in the problem:

"We could have them establish a proportion related to the areas. We could create awareness about how the areas change. For example, if they find the question mark using the relationship 'if it's 10 by 12, then what will it be at 18?' (T5 wrote it down). So, we created option A here. This option A made the student find the question mark. After finding the question mark, we could have the student calculate the areas and observe the change in the areas. After observing this change, for instance, we could help the student realize that while the ratio constant in similarity for lengths is a single factor, for areas, it becomes squared. This could help the student get ahead, especially in similar questions."

Below are the explanations of T4, who suggested changing the shapes in the problem:

"I would apply the same question to different quadrilaterals, pentagons, and polygons. I would give triangles, pentagons."

Finally, below are the explanations of T1, who suggested increasing the number of shapes in the problem:

"I could increase the number of shapes. Instead of two shapes, I could include three. This way, the student could build the logic before jumping into the calculation."

As seen, the suggestions of T1 and T3 are more aimed at helping students who couldn't solve the problem to better understand and reinforce the concept. Therefore, these suggestions were categorized as ineffective.

Discussion and Results

In this study, feedback given by teachers and preservice teachers based on 6 different problem solutions involving proportional reasoning were examined. In this section, the results obtained from the feedback for each problem were discussed.

In first problem, according to Table 3, the participants provided effective suggestions for addressing the errors in this problem, such as giving examples from daily life, highlighting additive-multiplicative relationships, and using visualization. By relating mathematics to daily life, students can create connections between mathematics and their own lives (Stylianides and Stylianides, 2008). This, in turn, can increase students' motivation for the mathematics course and, consequently, enhance their success in mathematics (Singletary, 2012). Therefore, it can be said that providing examples from daily life can be effective in addressing students' errors. Participants who suggested highlighting additive-multiplicative relationships can be said to have made an effective suggestion because the inability to distinguish between additive and multiplicative relationships is likely to cause errors in this problem, and addressing this issue can help correct students' errors in this area (Langrall and Swafford, 2000). Teaching through concrete representation can also help students better understand the topic in mathematics and minimize their errors, making this suggestion effective as well (Temel et al., 2015). Additionally, some participants suggested teaching the strategy of equation formulation, but this suggestion is not effective for this problem. This is because none of the errors in this problem stem from a lack of understanding of equations. The effective suggestions made for advancing the students who provided correct solutions to Problem 1 include introducing a new variable and asking different questions related to the problem. Introducing a new variable contributes to students' mathematical progress by increasing the number of objects or people they need to consider. Asking additional questions involving different relationships can also contribute to students' mathematical progress by encouraging them to engage in different types of mathematical thinking. However, providing examples from daily life and changing the variables in the problem are not effective suggestions for advancing students' progress in mathematics. Providing examples from daily life is generally effective when used to address students' errors (Stylianides and Stylianides, 2008), so it is unlikely to contribute significantly to students' progress. Participants who suggested changing the variables in the problem believed that this would help develop students' arithmetic skills, but since the problem does not involve significant changes in numbers and operations, students may not progress in mathematics by performing similar operations. For this problem, most mathematics teachers and preservice mathematics teachers made effective suggestions for addressing students' errors. However, most mathematics teachers and preservice mathematics teachers did not make effective suggestions for advancing students who provided correct solutions.

According to Table 4, the effective suggestions made by participants to address students' errors in this problem include simplifying the problem, using unit rate, giving examples from daily life, highlighting additive-multiplicative relationships, and using concretization. Participants who suggested simplifying the problem aimed to adjust the numbers in the problem so that they are exact multiples, thus guiding students toward proportional solutions. Indeed, they are likely to achieve their goal, as studies by Artut and Pelen (2015), Steinthorsdottir (2006), and Degrande et al. (2019) have found that middle school students are more likely to arrive at proportional solutions in problems where the numbers are set as exact multiples. Using unit rate is also effective in addressing students' errors, as unit rate is an effective method for teaching proportional situations (Cramer et al., 1989; Van de Walle et al., 2013, p. 458). Some participants aimed to address errors by having students reread the problem or re-explain the topic. These participants identified the cause of the errors as students not reading the problem carefully or not fully understanding the topic. Although this possibility is low, it still exists, so these suggestions were categorized as low-effective. However, they did not provide detailed information on how to re-explain the topic. Simply re-explaining the topic may not be sufficient for students to better understand it. Some participants suggested distinguishing between direct and inverse proportion, but none of the errors in this problem were caused by students' inability to distinguish between direct and inverse proportion. Therefore, this suggestion will not address students' errors in this problem. The effective suggestions made by participants to advance students who provided correct solutions in this problem include changing the variables in the problem, introducing a new variable, and directing students to different strategies. Participants who suggested changing the variables aimed to transform this proportional problem into a fixed relationship problem by adding a new variable, thus encouraging students to think about a different mathematical situation. Distinguishing between proportional and non-proportional situations can help students advance in proportional reasoning (Modestou and Gagatsis, 2010). Therefore, it can be said that this suggestion will be effective in advancing students' progress. In fixed relationship problems, as one variable in the problem increases or decreases, the other remains constant (Dooren et al., 2010). Participants who suggested introducing a new variable aimed to transform this proportional problem into a non-proportional one by adding a new variable, making it an effective suggestion for advancing students' progress in mathematics. Participants who suggested directing students to different strategies aimed to change the problem so that students who used the unit rate strategy would be directed toward the cross-multiplication strategy. This suggestion will also contribute to the advancement of students who provided correct solutions, as the cross-multiplication strategy is considered more advanced than the unit rate strategy (Langrall and Swafford, 2000). Most participants, however, made suggestions that were not effective in advancing students' progress in mathematics, such as changing the numbers in the problem and asking similar problems. The numbers used in the problems in the study were not exact multiples of each other anyway, so, changing the numbers in the problem will not contribute to students' progress in mathematics. Asking similar problems will not change the mathematical structure of the problem, so students will not be encouraged to explore different mathematical relationships. Therefore, this suggestion will not contribute to students' progress in mathematics. For this problem, most mathematics teachers and preservice mathematics teachers made effective suggestions for addressing students' errors. However, most mathematics teachers and preservice mathematics teachers did not make effective suggestions for advancing students who provided correct solutions.

According to Table 5, the effective suggestions made by participants to address students' errors in this problem include simplifying the problem and using unit rate. The suggestions categorized as low effective include concretisation and explaining division. Explaining division was only suggested for students who made errors in division with remainders. Since participants indicated that division would be explained in a straightforward manner, this suggestion is likely to be ineffective in addressing students' errors in division. Although concrete representation has been mentioned in the first paragraph as an effective way to help students better understand mathematical concepts and minimize errors, the concrete representation suggested for this problem is less effective in addressing errors. This is because participants suggested simulating the problem, but the two individuals selected for the simulation may not maintain the same speeds as those in the problem, and even if they run at a constant speed, it is unlikely they will complete their laps within the specified time. Some participants suggested using the equivalent fractions strategy to address errors in this problem, but this suggestion is not effective in correcting students' errors. Although the numbers in the problem are not exact multiples of each other, the equivalent fractions strategy is one of the most advanced strategies in proportional reasoning, making it difficult for students who have made errors to understand the problem or the concept using this strategy (Noelting, 1980; Langrall and Swafford, 2000). The effective suggestions made by participants to advance students who provided correct solutions in this problem include introducing a new variable and directing students to different strategies. The ineffective suggestions include changing the numbers in the problem and asking similar problems. For this problem, most mathematics teachers and preservice mathematics teachers made effective suggestions for addressing students' errors. While most mathematics teachers made effective suggestions for advancing students who provided correct solutions, most preservice mathematics teachers did not.

According to Table 6, the effective suggestions made by participants to address errors in Problem 4 include concrete representation and providing numerical values. The suggestions categorized as low effective include rereading the problem and providing similar examples. Providing similar examples may not be effective in addressing errors in this problem, as students who do not understand the problem are likely to also not understand similar examples. Explaining inverse proportion was suggested by only one participant and is not effective in addressing errors in this problem, as none of the errors in this problem were caused by a lack of understanding of inverse proportion. The only effective suggestion made by participants to advance students who provided correct solutions in this problem was introducing a new variable. The suggestion categorized as low effective was providing numerical values, as this suggestion does not take the problem to a different level and may not contribute to students' progress. Asking similar problems will not contribute to the advancement of students who provided correct solutions, as it does not change the mathematical structure of the problem. In this problem, almost all mathematics teachers and preservice mathematics teachers made effective suggestions for addressing students' errors. However, except for two preservice mathematics teachers, the other participants did not make effective suggestions for advancing students who provided correct solutions.

According to Table 7, the effective suggestions made by participants to address students' errors in Problem 5 include using unit rate, concrete representation, visualization, and simplifying the problem. Like concrete representation, visualization, when used in teaching, helps students better understand a mathematical concept and minimize errors (Temel et al., 2015). The suggestion categorized as loe effective in addressing students' errors in this problem is explaining the topic. The suggestion to use the equivalent fractions strategy will not correct students' errors in this problem. The effective suggestions made by participants to advance students who provided correct solutions in this problem include introducing a new variable and directing students to different solution strategies. The ineffective suggestions include asking similar problems and changing the numbers in the problem. For this problem, most mathematics teachers and preservice mathematics teachers made effective suggestions for addressing students' errors. While almost all mathematics teachers made effective suggestions for advancing students who provided correct solutions, almost all preservice mathematics teachers did not.

According to Table 8, the effective suggestions made by participants to address students' errors in Problem 6 include simplifying the problem and using geometry software. Using geometry software is effective in addressing students' errors in this problem, as it allows students to visualize similarity. The suggestion to explain similarity has a low impact on addressing students' errors in this problem. The effective suggestions made by participants to advance students who provided correct solutions in this problem include making students formulate problems and finding different relationships in the problem. Since problem formulation is a high-level skill, making students formulate problems can contribute to their mathematical progress. Participants who suggested finding different relationships in the problem aimed to have students discover the relationship between the similarity ratio of the sides and the ratio of the areas of the shapes, thus providing students who solved the problem correctly with new knowledge. Therefore, this suggestion is also effective in advancing the mathematical progress of students who provided correct solutions. The suggestions to change the shapes in the problem and to increase the number of shapes in the problem are ineffective in advancing students who provided correct solutions, as these actions would create a similar problem. Therefore, these suggestions will not contribute to the mathematical progress of students who provided correct solutions. Most mathematics teachers made effective suggestions for addressing students' errors in this problem. Nearly half of the preservice mathematics teachers made effective suggestions for addressing students' errors in this problem. However, most mathematics teachers and preservice mathematics teachers did not make effective suggestions for advancing students who provided correct solutions.

In general, it can be concluded that mathematics teachers are sufficiently capable of making suggestions to address students' errors in incorrect solutions. This finding is consistent with the results found by Jacobs et al. (2010). In their study, Jacobs et al. (2010) found that only mathematics teachers with more than four years of experience were able to make suggestions to address students' errors, while novice mathematics teachers, those with two years of experience, and preservice mathematics teachers struggled to make such suggestions. However, this result contradicts the findings of LaRochelle (2018) and Chick (2010). LaRochelle (2018) found that mathematics teachers struggled to make suggestions to address students' errors in their study on the noticing skills of mathematics teachers and preservice mathematics teachers. Chick (2010) found similar results in their study, concluding that mathematics teachers had difficulty making suggestions to address students' errors.

When it comes to preservice mathematics teachers, it can also be concluded that they are sufficiently capable of making suggestions to address students' errors in incorrect solutions. This result contradicts the findings of Bahar (2019), LaRochelle (2018), and Jacobs et al. (2010). In their study, Jacobs et al. (2010) found that preservice mathematics teachers, novice mathematics teachers, and those with two years of experience struggled to make suggestions to address students' errors. Bahar (2019) also concluded that preservice mathematics teachers were insufficient in making suggestions to address students' errors in their study on the pedagogical content knowledge of preservice mathematics teachers regarding proportional reasoning. This result is consistent with the findings of Özel (2019), who found that preservice mathematics teachers were sufficiently capable of making suggestions to address students' errors in incorrect solutions in their study on the noticing skills of preservice mathematics teachers.

In general, it can be concluded that mathematics teachers and preservice mathematics teachers are insufficient in making suggestions to advance the mathematical progress of students who provided correct solutions. Taylan (2016) found that preservice mathematics teachers were insufficient in making suggestions to advance the mathematical progress of students who provided correct solutions. The reason for this result is that most of the suggestions they provided for students who provided correct solutions were in the form of exercises. This finding is consistent with the results found by Özel (2019), who concluded that most of the suggestions provided by preservice mathematics teachers for advancing the progress of students who provided correct solutions were in the form of exercises.

Recommendations

The recommendations related to this research are provided below:

Mathematics teachers struggle to make suggestions that contribute to the mathematical progress of students who provide correct solutions. In-service training can be provided to mathematics teachers on this topic.

Preservice mathematics teachers also struggle to make suggestions that contribute to the mathematical progress of students who provide correct solutions. A curriculum could be prepared on this topic within mathematics education courses such as Approaches to Teaching and Learning Mathematics and Special Methods of Teaching Mathematics for preservice mathematics teachers.

The suggestions made by preservice mathematics teachers who have not taken any mathematics education courses and those who have successfully completed most of the mathematics education courses can be examined to see how they address students' errors in incorrect solutions.

The suggestions made by preservice mathematics teachers who have not taken any mathematics education courses and those who have successfully completed most of the mathematics education courses can be examined to see how they contribute to the mathematical progress of students who provide correct solutions.

A study could be conducted to observe how the teaching experience of mathematics teachers affects their ability to make suggestions that contribute to the mathematical progress of students who provide correct solutions.

A study could be conducted to observe how the general academic performance of preservice mathematics teachers affects their ability to make suggestions that contribute to the mathematical progress of students who provide correct solutions.

Ethical Committee Approval

The ethics committee permission for the article was obtained by the Yıldız Technical University Publication Ethics Committee with the decision numbered 2021/01 dated 21.03.2021.

References

Arican, M. (2019). Preservice mathematics teachers' understanding of and abilities to differentiate proportional relationships from nonproportional relationships. *International Journal of Science and Mathematics Education*, *17*(7), 1423-1443. https://doi.org/10.1007/s10763-018-9931-x

Artut, P. D., & Pelen, M. S. (2015). 6th grade students' solution strategies on proportional reasoning problems. *Procedia-Social and Behavioral Sciences*, *197*, 113-119. https://doi.org/10.1016/j.sbspro.2015.07.066

Bahar, G. (2016). *Measurıng pre-service teachers' mathematics content knowledge and mathematics pedagogical content knowledge on ratio and proportion* [Master's thesis, Bosphorus University]. YÖK Tez Merkezi.

Bart, W., Post, T., Behr, M., Lesh, R. (1994). A diagnostic analysis of a proportional reasoning test item: An introduction to the properties of a semi-dense item. *Focus on Learning Problems in Mathematics,* 16(3), 1-11.

Ben-Chaim, D., Fey, J.T., Fitzgerald, W.M., Benedetto, C., Miller, J (1998). Proportional Reasoning among 7th Grade Students with Different Curricular Experiences. *Educational Studies in Mathematics* 36**,** 247–273. https://doi.org/10.1023/A:1003235712092

Benson, S. L. D. (2009). *The influence of studying students' proportional reasoning on middle school mathematics teachers' content and pedagogical content knowledge* [PhD Thesis, University of Houston]. ProQuest.

Birinci, M., & Baki, M. (n.d.). Bir Ortaokul Matematik Öğretmeninin Mesleki Gelişiminden Yansımalar: Kesir Öğretiminde Fark Etme Becerisinin İşe Koşulması. *İlköğretim Online*1141-1156. https://doi.org/10.17051/ilkonline.2019.610853

Boyacı, H. S. (2019) *The proportional reasoning ability of pre-service mathematics teachers: a mixed method study*. [Master's thesis, Bosphorus University]. YÖK Tez Merkezi.

Brown, R.E., Weiland, T. & Orrill, C.H. (2019). Mathematics Teachers' Use of Knowledge Resources When Identifying Proportional Reasoning Situations. *International Journal of Science and Mathematics Education* 18**,** 1085–1104 https://doi.org/10.1007/s10763-019-10006-3

Büyüköztürk, Ş., Çakmak, E. K., Akgün, Ö. E., Karadeniz, Ş., Demirel, F. (2018) *Bilimsel Araştırma Yöntemleri.* Pegem Akademi.

Chick, H. (2010). Aspects of Teachers' Knowledge for Helping Students Learn about Ratio. *Mathematics Education Research Group of Australasia*.

Choy, B. H. (2014). Noticing Critical Incidents in a Mathematics Classroom. *Mathematics Education Research Group of Australasia*.

Cramer, K. & Post, T. (1993, May). Connecting Research To Teaching Proportional Reasoning. *Mathematics Teacher*, 86(5), 404-407.

Cramer, K., Behr, M., & Bezuk, N. (1989, October). Proportional Relationships and Unit Rates. *Mathematics Teacher*, 82 (7), 537-544.

Cramer, K., Post, T., & Currier, S. (1993). Learning and Teaching Ratio and Proportion: Research Implications. D. Owens (Ed.), *Research Ideas For the Classroom* (pp. 159-178). NY: Macmillan Publishing Company.

Çıkla, O. A., & Duatepe, A. (2002). İlköğretim matemetik öğretmen adaylarının orantısal akıl yürütme becerileri üzerine niteliksel bir çalışma. *Hacettepe Üniversitesi Eğitim Fakültesi Dergisi*, *23*(23).

De la Cruz, J. A. (2008). *A look at middle school mathematics teachers' instruction related to proportional reasoning: A cognitively guided perspective* [PhD Thesis, University of Virginia]. ProQuest.

Degrande, T., Verschaffel, L., & Van Dooren, W. (2019). To add or to multiply? An investigation of the role of preference in children's solutions of word problems. *Learning and Instruction*, *61*, 60-71. https://doi.org/10.1016/j.learninstruc.2019.01.002Get rights and content

Dick, L. K. (2013). *Preservice student teacher professional noticing through analysis of their students' work* [PhD Thesis, North Carolina State University]. ProQuest.

Dooren, W. V., Bock, D. D., & Verschaffel, L. (2010). From addition to multiplication… and back: The development of students' additive and multiplicative reasoning skills. *Cognition and Instruction*, *28*(3), 360-381.

Dougherty, B., Bryant, D.P., Bryant, B.R. & Shin, M. (2016) Helping Students with Mathematics Difficulties Understand Ratio and Proportion. *Teaching Exceptional Children, Vol. 49, No. 2, pp. 96–105. Copyright 2017 The Author(s).* https://doi.org/10.1177/0040059916674897

Dündar, S., Temel, H., & Gündüz, N. (2015). Matematik Eğitiminde Görselleştirme ve Somutlaştırma Üzerine Bir Değerlendirme. *Dicle Üniversitesi Ziya Gökalp Eğitim Fakültesi Dergisi*(*24*), 339-362. https://doi.org/10.14582/DUZG

Jacobs, V. R., Franke, M. L., Carpenter, T. P., Levi, L., & Battey, D. (2007). Professional Development Focused on Children's Algebraic Reasoning in Elementary School. *Journal for Research in Mathematics Education JRME*, *38*(3), 258-288. https://doi.org/10.2307/30034868

Jacobs, V. R., Lamb, L. L. C., & Philipp, R. A. (2010). Professional Noticing of Children's Mathematical Thinking. *Journal for Research in Mathematics Education*, *41*(2), 169-202. https://doi.org/10.5951/jresematheduc.41.2.0169

Johnson, K. (2017, October 5-8). *A Study of Pre-Service Teachers Use of Representations in Their Proportional Reasoning* [Conference session]. The Annual Meeting of The North American Chapter of the International Group for the Psychology of Mathematics Education, Indianapolis, IN, United States of America.

Kaiser, G., Blömeke, S., König, J., Busse, A., Döhrmann, M., & Hoth, J. (2017). Professional competencies of (prospective) mathematics teachers—Cognitive versus situated approaches. *Educational Studies in Mathematics*, *94*, 161-182. https://doi.org/10.1007/s10649-016-9713-8

Kasemsukpipat, W. (2016). *An Exploration of Thai Pre-service Mathematics Teachers' Knowledge: Cases of Subject Matter Knowledge, Knowledge of Students' Understanding, and Knowledge of Instructional Strategies in Ratio and Proportion* [PhD Thesis, University of Wyoming]. ProQuest.

Lamon, S. J. (1993). Ratio and proportion: Connecting content and children's thinking. *Journal for research in mathematics education*, *24*(1), 41-61.

Lamon, S. J. (2006). *Teaching fractions and ratios for understanding: Essential content knowledge and instructional strategies for teachers*. Routledge.

Langrall, C. W., & Swafford, J. (2000). Three Balloons for Two Dollars: Developing Proportional Reasoning. *Mathematics Teaching in the Middle School*, *6*(4), 254-261. https://doi.org/10.5951/MTMS.6.4.0254

LaRochelle, R. M. (2018). *Secondary teachers' professional noticing of students' mathematical thinking*. [PhD Thesis, University of California]. ProQuest.

Lesh, R., Post, T., & Behr, M. (1988). Proportional Reasoning. J. Hiebert & M. Behr (Ed.) *Number Concepts and Operations in the Middle Grades*(pp. 93-118). Reston, VA: Lawrence Erlbaum & National Council of Teachers of Mathematics.

Marks, R. (1990). Pedagogical content knowledge: From a mathematical case to a modified conception. *Journal of teacher education*, *41*(3), 3-11. https://doi.org/10.1177/002248719004100302

Miller, K. F. (2011). Situation awareness in teaching: What educators can learn from video-based research in other fields. M.G. Sherin, V. R. Jacobs ve R. A. Randolph (Ed.), *Mathematics teacher noticing*: *Seeing through teachers' eyes* (pp. 51-65). Routledge.

Milli Eğitim Bakanlığı (2018). *İlkokul ve Ortaokul Matematik Dersi Öğretim Programı*, https://mufredat.meb.gov.tr/Dosyalar/201813017165445-MATEMAT%C4%B0K%20%C3%96%C4%9ERET%C4%B0M%20PROGRAMI%202018v.pdf

Misailidou, C., & Williams, J. (2003). Diagnostic assessment of children's proportional reasoning. *The Journal of Mathematical Behavior*, *22*(3), 335-368. https://doi.org/10.1016/S0732-3123(03)00025-7

Modestou, M., & Gagatsis, A. (2010). Cognitive and metacognitive aspects of proportional reasoning. *Mathematical Thinking and Learning*, *12*(1), 36-53. https://doi.org/10.1080/10986060903465822

Noelting, G. (1980). The development of proportional reasoning and the ratio concept part I-differentiation of stages. *Educational Studies in Mathematics*, 217-253.

Özel, Z. (2019). *Investigation into prospective middle school mathematics teachers' noticing of students' algebraic thinking within the context of pattern generalization* [Master's thesis, Middle East Technical University]. YÖK Tez Merkezi.

Sherin M. G., Jacobs V. R., & Philipp R. A. (2011). Situating the study of teacher noticing. G. Sherin, V. R. Jacobs ve R. A. Randolph (Ed.), *Mathematics teacher noticing:Seeing through teachers' eyes* (pp. 3-13). Routledge.

Sherin, M. G., Russ, R. S., & Colestock, A. A. (2011). Accessing mathematics teachers' in-the-moment noticing. M.G. Sherin, V. R. Jacobs ve R. A. Randolph (Ed.), *Mathematics teacher noticing: Seeing through teachers' eyes* (pp. 79-94). Routledge

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational researcher*, *15*(2), 4-14. https://doi.org/10.3102/0013189X015002004

Singletary, L. M. (2012). *Mathematical connections made in practice: An examination of teachers' belief and practices* [PhD Thesis, University of Georgia]. Google Akademik.

Steinthorsdottir, O. B. (2006, July 16-21). *Proportional reasoning: Variable influencing the problems difficulty level and one's use of problem solving strategies* [Conference session]. 30th Conference of the International Group for the Psychology of Mathematics Education, Prague, Czech Republic.

Stylianides, A. J., & Stylianides, G. J. (2008). Studying the implementation of tasks in classroom settings: High-level mathematics tasks embedded in "real-life" contexts. *Teaching and Teacher Education, 24, 859-875.* https://doi.org/10.1016/j.tate.2007.11.015

Şen, C., & Güler, G. (2017). Effect of strategy teaching for the solution of ratio problems on students' proportional reasoning skills. *Malaysian Online Journal of Educational Sciences*, *5*(2), 1-15.

Şermetoğlu, H., & Baki, M. (2019). Oran ve Orantı Konusu Öğretim Sürecinin Bir Matematik Öğretmeninin Fark Etme Becerisi Bağlamında İncelenmesi. *Turkish Journal of Computer and Mathematics Education*, *10*(2), 394-425. https://doi.org/10.16949/turkbilmat.461124

Taylan, R. D. (2018). The relationship between pre-service mathematics teachers' focus on student thinking in lesson analysis and lesson planning tasks. *International Journal of Science and Mathematics Education*, *16*(2), 337-356. https://doi.org/10.1007/s10763-016-9778-y

Tunç-Pekkan, Z., & Kılıç, H. (2015, February 4-8). *Mathematical opportunities: Noticing and acting* [Conference session]. *9th Congress of the European Society for Research in Mathematics Education,* Prague, Czech Republic. https://hal.science/CERME9/public/CERME9_Proceedings_2015.pdf

Ünsal, A. (2009). *İlköğretim 7.sınıf öğrencilerinin orantısal akıl yürütme becerilerinin başarı, tutum ve değişkenleri açısından incelenmesi: Bolu ili örneği.* [Master's thesis, Abant İzzet Baysal University]. YÖK Tez Merkezi.

Van De Walle, J. A., Karp, K. S., & Bay-Williams, J. M. (2013). *İlkokul ve Ortaokul Matematiği* (S. Durmuş, Tr.), Ankara: Nobel Akademik Yayıncılık

Van Es, E. A. (2011). A framework for learning to notice student thinking. M.G. Sherin, V. R. Jacobs ve R. A. Randolph (Ed.), *Mathematics teacher noticing: Seeing through teachers' eyes* (pp. 134-151). Routledge

Van Es, E. A., & Sherin, M. G. (2002). Learning to notice: Scaffolding new teachers' interpretations of classroom interactions. *Journal of technology and teacher education*, *10*(4), 571-596.

Appendix

Proportional Reasoning Problems

Class:

1-) Ayşe's father's current age is 3 times Ayşe's age. If Ayşe's current age is 18 years old, what is her father's current age? When Ayşe is 36, how old will her father be? Explain how you found it. (Adapted from Bethea(2003))

2-)

4 onions 8 glasses of water 2 tablespoons of oil ½ cup chopped celery

A soup recipe for 8 people is given in the table above. A housewife who wants to prepare this soup for 6 people with the same taste,

a-)How many onions does it need?

b-)How much water does it need?

c-)How much oil does it need?

d-)How much chopped celery does he need?

Explain how you found your answers. (Adapted from Hines and McMahon (2005)).

3-) Tarık and Ozan ran in the park, circling the perimeter of the park. Tarık ran 8 laps in 25 minutes and Ozan ran 3 laps in 10 minutes. Who ran faster? Explain how you found it. (Adapted from Heller et al. (1989))

4-) Tufan drank his breakfast tea in a larger glass than yesterday, with less sugar. The taste of this tea is compared to yesterday's tea;

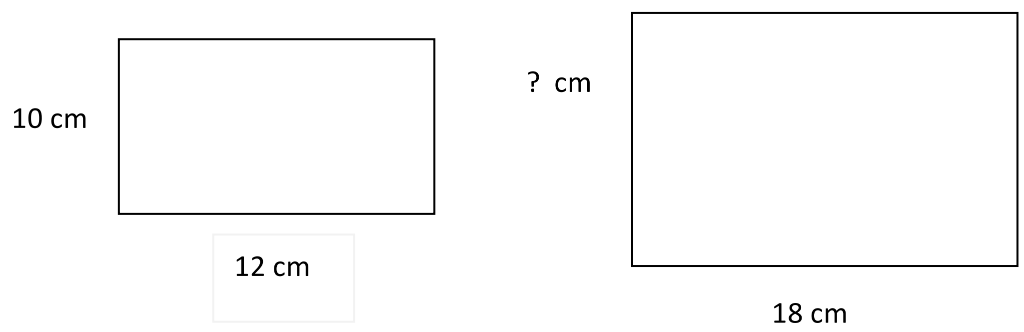
a) it is with more sugar

b) it is with less sugar c) it is the same d) the information given is insufficient.

Write and explain which option is correct. (Taken from Duatepe and Akkuş (2006)).

5-) In a pizzeria, 7 girls are sitting at one of the tables and 3 boys are sitting at another table. The table where 7 girls are sitting has ordered 3 pizzas, and the table where 3 boys are sitting has ordered 1 pizza. Accordingly, which table has the highest amount of pizza per person? Explain how you found it. (Adapted from Lamon (1993))

6-)



The two rectangles above are similar rectangles. Accordingly, what is the length of the short side of the larger rectangle, in cm? Explain how you found it (adapted from Bethea (2003)).