Investigating The Effect of Artificial Intelligence-Based Tools on Pre-Service Mathematics Teachers' Concept Images

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Abstract

Artificial intelligence (AI) and AI-based tools (AIBTs) are becoming increasingly important in mathematics education because of their potential to enrich learning experiences, provide customized solutions, and increase the effectiveness of learning processes. This case study was conducted to investigate the impact of AIBTs experiences on the mathematical concept images of pre-service mathematics teachers (PMTs). The study examined changes in PMTs' concept images of rational number, equation, quadrilateral, limit, derivative, and continuity concepts. 12 PMTs participated in the study. As a result of the content analysis, the data were categorized under the themes of component, understanding, and association, and the changes in PMTs' concept images after their ChatGPT experiences were examined according to these themes. Despite not being always, the ChatGPT experience had an effect on PMTs' mathematical concept images in general. In particular, PMTs mentioned more components directly related to mathematical concepts, while associating related concepts with different terms decreased. However, there were partial transformations in concept understanding. While subjective knowledge of concepts decreased, objective and directly related knowledge increased, but unnecessary repetition decreased. In conclusion, ChatGPT experiences contributed to the development of PMTs' concept images, but this contribution was not at the same level for all concepts.

Keywords: Artificial intelligence (AI), chatGPT, chatbots, concept images, pre-service mathematics teachers.

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Introduction

In recent years, AI and AIBTs have spread rapidly and become an important part of our daily lives. This rapid spread is driven by various factors, such as fulfilling, facilitating, and accelerating people's needs, which directly and profoundly affect human life (Chen et al., 2020; Luckin et al., 2016). While AIBTs attract attention with their potential to do more work in less time and increase productivity, they also open new horizons with the opportunities they offer (Adamopoulou & Moussiades, 2020; Sindermann et al., 2021). However, the transformation of AI does not only affect working life, but also deeply affects the daily lives and social relationships of individuals as part of society. Many tools and applications that we use every day, such as smartphones, digital assistants, and smart home appliances, are an indication of how integrated AI technologies are in our daily lives (Al Darayseh, 2023; Kim & Kim 2022; Lee & Yeo, 2022; Sindermann et al., 2021). As an important part of today's rapid technological advancements, AI technologies have the potential to provide smarter, more innovative, and more efficient solutions, which could lead to their wider adoption and integration into daily life in the future (Gökçearslan et al., 2024; Hwang & Tu, 2021). In this context, it can be said that recognizing the changes that AI can bring and developing the skills to adapt to these technologies are becoming increasingly important in our time.

Components of AI Technology

AI, machine learning (ML), and deep learning (DL) are closely related concepts that are often confused with each other (Figure 1) (Chen et al., 2020; Theodosiou & Read, 2023). AI is the field of computer science that can recognize data such as text, visual, or audio data, perform tasks such as making inferences and decisions based on this data, and mimic human intelligence (Luckin et al., 2016; Yang et al., 2021). Since AI applications are often based on the process of learning from and making sense of large data sets and the patterns formed by these data, it can be said that the organization and processing of this data is an important place for AI. Big data science is used to organize and transform large amounts of structured or unstructured data into valuable information (Luckin et al., 2016). ML involves the processes of discovering relationships between organized data, searching for patterns, modeling, and making predictions and decisions based on these models (Chen et al., 2020; Luckin et al., 2016). DL, on the other hand, focuses on the ability to learn complex patterns and relationships using multi-layer artificial neural networks (Chen et al., 2020). In this process, DL is a more complex and sophisticated form of ML as it can analyze and interpret large amounts of data using different ML approaches such as supervised, unsupervised, and reinforcement (Theodosiou & Read, 2023). These fields are intertwined and interrelated, and AI is an umbrella term that generally encompasses all of them. The ultimate goal in the development of AI is to create systems that can think and make decisions like humans (Chen et al., 2020; Luckin et al., 2016).

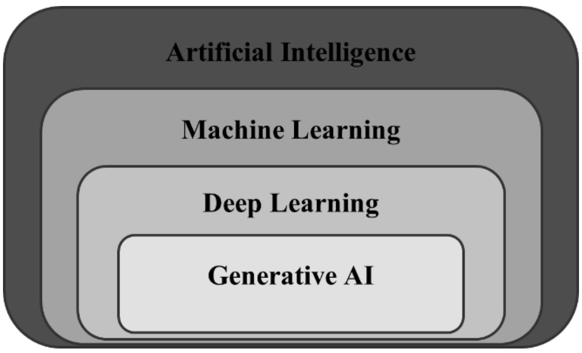

Figure 1. The relationship between AI, ML, DL and generative AI (Adapted from Theodosiou & Read [2023]).

One of the important factors for AI to enter our lives so quickly is AI-based chatbots (AICBs). AICBs are customized AIBTs, and they are generally AI programs that engage in text-based interactions with humans using natural language processing technologies (Deng & Yu, 2023). AICBs have become popular because they offer many benefits to users and developers, such as productivity, entertainment, social communication, and the ability to generate customized and quick responses (Adamopolou &

Moussiades, 2020). Alexa developed by Amazon, Siri developed by Apple, smart assistants on smartphones, chatbots developed by companies to interact with customers are the first examples of AICB, but ChatGPT developed by the company OpenAI has revolutionized AICB (Küchemann et al., 2023; Tapan-Broutin, 2023).

ChatGPT, which was developed using large amounts of text data in its training, is a language model for understanding and generating text-based conversations that can be used in various natural language processing tasks (Küchemann et al., 2023; Lee et al., 2024; Tapan-Broutin, 2023). However, the latest version of ChatGPT, ChatGPT 3.5, has added image processing and generation capabilities, as well as textual content. In addition to ChatGPT, Gemini (formerly known as Bard) developed by Google AI and Copilot developed by Microsoft are increasing competition in the field of AICBs with their important features. Thanks to its ability to generate content, it can create content ranging from poems to stories, from codes to songs, and can also help with writing. With these features, it can be said that the purpose of AICBs in the most general sense is to meet the needs of users by communicating with them through written, verbal or visual means.

AIBTs are designed to help us, but they may not always meet all of our needs due to their various limitations. In order to use these tools for their full potential, it is important to understand their limitations and the implications of these limitations. The most important of these limitations is their ability to understand and express what is written or said. Although they are improving day by day, they may still have difficulty in interacting with people or making incorrect inferences (Kim & Kim, 2022; Park & Manley, 2024). To minimize these limitations in understanding and expression, instructions must be clear and unambiguous. Another limitation relates to issues such as confidentiality and security of data, which are necessary to protect user privacy. Therefore, transparency and accountability are important factors to consider to ensure the trustworthiness of AI systems (Nguyen et al., 2023). Apart from these, dependence on limited data, bias, misinterpretation, generation of data and results that are not real, lack of emotional intelligence, cultural and linguistic deficiencies are some of the other limitations of AIBTs (Akgun & Greenhow, 2022; Gökçearslan et al., 2024; Küchemann et al., 2023; Tapan-Broutin, 2023; Uğraş & Uğraş, 2025). Uğraş et al. (2024) argue that limitations such as reliability, accuracy, and ethics may raise concerns, and therefore the literacy of individuals regarding AI and AIBT should be developed.

The Role of AI-Based Tools in Mathematics Education

Research shows that AIBTs have significant potential to increase efficiency and innovation in various fields (Hwang & Tu, 2021; Deng & Yu, 2023). In education, these tools are characterized by their ability to transform learning processes (Gökçearslan et al., 2024; İpek et al., 2023; Taani & Alabidi, 2024; Uğraş & Uğraş, 2025). AIBTs offer various functions in education, including early identification of students' needs, generating complex and detailed answers, personalizing learning experiences, creating materials, assessing and evaluating, summarizing, and rewriting (İpek et al., 2023). However, despite these possibilities, they also have certain limitations. For example, due to their inability to consistently verify sources of information, AIBTs run the risk of generating misinformation, potentially leading to misconceptions among students (Uğraş & Uğraş, 2025). Although they can provide personalized responses, their limited adaptability to individual learning needs and challenges in solving creative problems may negatively affect the learning process (Wardat et al., 2023). In addition, effective use of AIBTs without teacher guidance can be challenging, and these tools may have limitations in aligning with curricula (Uğraş & Uğraş, 2025). Therefore, caution and necessary measures are essential when using these tools considering their potential to generate incomplete, incorrect, or misleading information (İpek et al., 2023).

Mathematics education is one area that could significantly benefit from this change. This is because mathematics education is a multifaceted process that aims to concretize abstract concepts, develop skills to tackle complex problems, and enhance analytical thinking skills (Jablonka, 2020; Skovsmose, 2020). In other words, mathematics encompasses both computational aspects, such as problem solving, and conceptual understanding, as it seeks to uncover the fundamental principles that govern the relationships between mathematical objects (Skovsmose, 2020). Thus, mathematics education aims to ensure that students effectively learn mathematical concepts and relationships (Jablonka, 2020; Skovsmose, 2020). This includes fostering students' mathematical thinking skills, enabling them to grasp abstract concepts, and strengthening their problem-solving abilities. In this process, the use of various tools, such as concrete materials, models, virtual manipulatives, and dynamic geometry software, can enhance the quality and effectiveness of education (Jablonka, 2020; Skovsmose, 2020). AIBTs are among the tools that have gained prominence in mathematics education and have significant potential to contribute to the field (Hwang, 2022; Park & Manley, 2024; Taani & Alabidi, 2024; Wardat et al., 2023).

AIBTs can make mathematical learning experiences more effective and personalized by addressing individual student needs and helping teachers enrich instructional content (İpek et al., 2023; Taani & Alabidi, 2024; Vaskoglou & Salem, 2020). Exploring how AIBTs can enhance mathematical understanding and skills provides valuable insights for evaluating their potential in mathematics education. AIBTs can provide opportunities to enhance and enrich the learning environment in mathematics education by promoting effective and interactive learning processes. This is because these tools can respond to the individual needs of students with dynamic content, customized learning materials, and content appropriate to the student's level (Celik, 2023; Taani & Alabidi, 2024; Uğraş, 2024).

Furthermore, through their ability to personalize interactive instruction, AIBTs can support the understanding of abstract mathematical concepts in a tangible way by providing tailored responses (Deng & Yu, 2023; Hwang & Tu, 2021). By increasing student motivation and encouraging active participation, these tools can also provide significant benefits to teachers in classroom management, assessment, and feedback processes (Taani & Alabidi, 2024; Tapan-Broutin, 2023). As a result, AIBTs can help make learning environments more dynamic, student-centered, and efficient (Deng & Yu, 2023; Hwang & Tu, 2021; Luckin et al., 2016; Taani & Alabidi, 2024; Zafari et al., 2022). Therefore, AIBTs have the potential to improve mathematics learning and teaching by enhancing students' understanding of mathematical concepts and developing their skills (Gökçearslan et al., 2024; Hwang, 2022; Hwang & Tu, 2021; Mohammed et al., 2022; Samuelson, 2023).

AIBTs also have significant potential to support the professional development of mathematics teachers and PMTs by providing a variety of information, methods, approaches, and content. These tools can enable teachers to enhance their pedagogical competencies, discover new teaching strategies, develop appropriate solutions to students' needs, and implement more effective methods in mathematics education (Al Darayseh, 2023; Taani & Alabidi, 2024; Tapan-Broutin, 2023). Together, these factors can help improve the quality of mathematics instruction, benefiting both teachers and PMTs (Küchemann et al., 2023). Studies generally show a positive relationship between the use of AIBTs in mathematics classrooms and student achievement, demonstrating that AIBTs can positively impact mathematics learning and serve as valuable tools in mathematics education (Hwang, 2022; Hwang & Tu, 2021; Lee et al., 2024; Park & Manley, 2024; Samuelson, 2023; Wardat et al., 2023).

Concept Image in Mathematics Education

While the possibilities offered by AIBTs in mathematics education enrich the learning process, they also raise questions about how students make sense of and construct mathematical concepts. In this context, understanding the aspects of mathematical learning based on mental processes can make the teaching process more effective. Concept image provides an important framework for understanding how a mathematical concept is represented in individuals' minds and the effects of this representation on learning (Tall & Vinner, 1981).

In general terms, the concept image is a cognitive structure that includes all the images, relationships, and processes in the mind about a concept (Tall & Vinner, 1981, p. 152). In this context, Vinner (1983) mentioned the mental picture of the concept in relation to the concept image. A mental picture is a set of visuals, symbols, meanings and relations associated with a concept. These mental picture sets form the individual's concept image of the concept (Vinner, 1983). Concept definition is the words we use to specify, distinguish, or explain a concept (Tall & Vinner, 1981, p. 152). These words are used to express the concept directly rather than indirectly (Vinner, 1983). However, the concept definition can be considered within the concept image because each individual creates his/her own concept image with a concept definition (Tall & Vinner, 1981). Furthermore, considering that concepts consist of cognitive structures or processes, it is possible to speak of images of a concept rather than a single image (Tall & Vinner, 1981; Vinner, 1983).

Concept definitions and concept images of many mathematical concepts have been investigated in mathematics education. Studies have generally been interested in uncovering participants' concept images of mathematical concepts. Prihandhika et al. (2022) showed that PMTs' concept images of the concept of derivative are still limited to functional representations, and they usually use the concept of derivative to solve procedural problems. When engineering and mathematics students' concept images of the concept of derivative are compared, mathematics students generally perceive the derivative as a tangent, while engineering students perceive it as a rate of change (Bingolbali & Monaghan, 2008). A study comparing PMTs' concept images of fractions and rational numbers showed that they mostly have a part-whole image for fractions, a ratio image for rational numbers, and an image that fractions cannot be negative (Macit & Nacar, 2019). In addition, studies have been conducted to examine the accuracy of participants' knowledge of mathematical concepts by examining their concept images. Köğce (2015) stated in his study that PMTs had difficulty in defining rational numbers, with one in two PMTs misidentifying the set of rational numbers and not knowing the necessary conditions for a rational number. He also found that they had difficulty in defining the concept of equation and that they often used incomplete or incorrect expressions such as 'a system for finding the unknown using the known', 'algebraic expressions with variables or unknowns', 'finding the unknown', 'mathematical equations or inequalities'. Sulastri et al. (2021) examined the concept images of limit and found that the formal definition of limit is not fully understood and, as a result, limit is defined as an unreachable function.

The study of concept images in mathematics education can highlight how individuals perceive and internalize mathematical concepts, revealing both strengths and limitations in their understanding. In this context, we think that the integration of advanced tools such as AIBTs into mathematics education may offer promising solutions for the development of accurate and comprehensive concept images. This is because AIBTs have the potential to influence how mathematical concepts are mentally understood and represented through dynamic explanations, personalized feedback, and the ability to provide contextualized examples (Taani & Alabidi, 2024). This connection provides the basis for examining the role of AIBTs in shaping PMTs' concept images, an area that brings together the fields of mathematics education and artificial intelligence to address critical pedagogical challenges.

Significance and purpose of the study

AIBTs such as ChatGPT have the potential to contribute to education in several areas, such as analyzing students' needs, providing personalized learning experiences, and providing feedback (İpek et al., 2023). In mathematics, it is found that AIBTs can be useful for problem solving, explaining complex mathematical relationships, making sense of abstract mathematical concepts, and relating mathematical concepts to everyday life (Gökçearslan et al., 2024; Hwang, 2022; Hwang & Tu 2021; Mohammed et al., 2022; Samuelson, 2023). It can be said that these opportunities offered by AIBTs in mathematics education are basically related to mathematical concepts and concept images of these concepts. This is because mathematics learning is related to the mental representations associated with mathematical concepts and the concept images that encompass them (Tall & Vinner, 1981). From this point of view, we can talk about the potential of AIBTs to affect the concept images of mathematical concepts. In this context, the aim of our study is to investigate the effect of AIBTs on concept images of mathematical concepts. The study was conducted with PMTs because they are the individuals who will effectively use mathematical concepts in both learning and teaching processes. Mathematical concepts and concept images of these concepts have a critical importance not only for pre-service teachers' own learning processes, but also for the knowledge and skills they will transfer to their students in the future. In this context, the research question of our study is: "How do AIBTs affect PMTs' concept images of mathematical concepts?

Understanding how AIBTs influence the formation and development of concept images of mathematical concepts can provide valuable insights for both teacher education and the integration of technology into learning environments. Therefore, this study is important in that it focuses on the intersection of artificial intelligence and mathematics education and investigates the potential effects of AIBTs on mathematics education. It is also expected to provide a perspective on how AIBTs can support or enhance the teaching and learning of mathematical concepts.

Method

The research has a case study design, which is one of the qualitative research methods. Case study is a qualitative research design that aims to comparatively examine and interpret one or several well-defined situations based on the data obtained through various data collection methods (Creswell, 2012; Yıldırım & Şimşek, 2016; Yin, 2012). In the present study, "the effect of ChatGPT on PMTs' mathematical concept images" was investigated. This situation to be investigated was determined as the unit of analysis of the study. For this reason, the research has a holistic single-case design. The holistic single case design is a case study design in which the unit of analysis determined within a context is examined in detail as a whole (Yin, 2012).

Participants

The study was conducted in a state university in the Central Anatolian region of Turkey in the fall semester of 2023-2024. The students were selected among the students of the Faculty of Education, Elementary Mathematics Teaching undergraduate program of the university. Convenience sampling was used to select the participants. Convenience sampling is a sampling method in which the participants are selected from the environment that is close and easily accessible (Yıldırım & Şimşek, 2016). In addition, it was determined as a prerequisite that the students who would participate in the study had taken or were taking the "Computer Assisted Mathematics Teaching" course. This was because it was believed that the knowledge and skills acquired by the students in this course would facilitate the use of AIBTs in accordance with the objectives of the research. In this case, in order to fully evaluate the research results, a total of 12 PMTs (5 female, 7 male) participated in the study, four each from the second, third, and fourth years. First-year students were not included in the study because they were not currently enrolled in the Computer Assisted Mathematics Teaching course. In addition, the PMTs' participation in the study was voluntary.

Data Collection Instruments

Two data collection instruments were used in the study. The first is the Mathematical Concepts Form (MCF), which contains mathematical concepts. The MCF contains a total of six mathematical concepts and two questions related to each concept. Three of the concepts are "rational number", "equation", "quadrilateral", which are important and widely used in secondary school mathematics. The other three questions are "limit", "derivative", and "continuity", which are commonly used concepts in calculus classes. For each concept in MCF, the questions were "define the concept" and "what do you know about the concept, explain? ChatGPT-3.5, developed by OpenAI, was used as the AIBT in the study. The participants conducted their research on the concepts in MCF using ChatGPT. The other data collection tool is the interview form used in semi-structured interviews. The interview form elicited the participants' experiences with ChatGPT. The form contained two questions: "Did ChatGPT give you information about mathematical concepts that you did not know? What were these questions?" and "What do you think about the usability of ChatGPT in the mathematics classroom?

Data Collection Process

Prior to the data collection process, the participants were trained together on how to use ChatGPT. The training included information about what ChatGPT is, how it works, and what prompts it provides.

Then, all participants were asked to simultaneously learn a mathematical concept from ChatGPT that would not be used in the study. At this stage, the aim was for all participants to gain knowledge and experience about ChatGPT. The data collection process was carried out in five stages (Table 1).

Table 1.
Data Collection Process

Time	Process	Operation
Week 1	ChatGPT Tutorial	Collectively in one session
Week 1	Written reply to MCF (S1)	Collectively in one session
Week 2	ChatGPT experiences	Collectively in one session
Week 3	Written re-reply to MCF (S2)	Collectively in one session
Week 4	Semi-structured interview	Individual

In the first phase, each participant answered the MCF in writing. In the second phase, they were asked to make inquiries about the mathematical concepts in the forms via ChatGPT. There were no restrictions in this questioning process. In the third phase, they answered the MCF again. Finally, semi-structured interviews were conducted about their experiences with ChatGPT. Each of these phases was conducted in different sessions and over a period of 4 weeks. All steps of the data collection process took place in the computer lab. However, the participants only had access to the computer during the ChatGPT experiences. Except for the interviews, each session was conducted collectively at one time. Interviews were conducted individually and lasted 5-10 minutes. Participants' ChatGPT accounts were created separately for each participant by the researcher.

Data Analysis

Content analysis was used to examine and analyze the collected data. The purpose of content analysis is to organize the collected data in a detailed and systematic way and to reveal and define the relationships between the data with an inductive approach (Yıldırım & Şimşek, 2016). The data collected in this context were first made suitable for research. The interviews were transcribed, the participants' correspondence with ChatGPT was collected in a single file, and the MCFs completed by the participants before and after their ChatGPT experiences were collected. All the organized data were put into one file so that the data could be compared.

Each participant's data was compiled into separate Excel files, and content analysis was performed in MAXQDA qualitative analysis software. No coding framework was created prior to content analysis. The words and word groups that appeared to be meaningful from the participants' statements about the related concepts were first coded in vivo. After the initial coding, the in vivo codes that were thought to be related were grouped together and the groups were coded. As a result of the examination of the codes obtained, the codes that were thought to be related were grouped and themes were obtained. This process was done separately for each concept.

In order to compare the participants' concept images before and after their ChatGPT experience and to examine possible changes as a whole, the MCFs completed by the participants before the ChatGPT experience were labeled Session 1 (S1), and the MCFs completed after the ChatGPT experience were labeled Session 2 (S2). While the participants were labeled from P1 to P12, the form data of these users were created by writing the label of the application file at the end of the user label. For example, the label P1S2 was used for the data of participant 1 in session 2. Interview data were used to support the MCF data.

For example, P7 used the following expression for the concept of derivative in Session 2 (P7S2): "The rate we get as a result of this change gives us the derivative. When we take the derivative of a graph, it gives us the slope for that point. In this statement, “The rate we obtain as a result of this change” was coded as change rate; “a graph” was coded as graphic; “that for point” was coded as one point; “the slope” was coded as slope. Since the concept of derivative is associated with the concepts of slope and graph, these codes were classified under the association theme. Since the change rate is a part of the derivative process and point is an important component of the derivative, these codes were classified under the component theme.

Content analyses were conducted separately for the MCF data from S1 and S2. To determine the possible changes in the concept images of the ChatGPT experiences, they were combined into a single visual graph. The common codes in the visuals were linked to both sessions, while the codes observed in one session were linked to the corresponding session. While the codes were linked to the sessions, the themes in which the codes were found were indicated in color. Accordingly, codes belonging to the component theme were colored orange, codes belonging to the understanding theme were colored purple, and codes belonging to the association theme were colored green.

Validity and Reliability

There are different perspectives for analyzing the validity and reliability of qualitative data (Creswell, 2012). In this study, validity and reliability were analyzed based on the criteria established by Lincoln and Guba (1985). Lincoln and Guba (1985) assessed the validity and reliability of data through the concepts of credibility, transferability, dependability, and confirmability. In this study, the content of the data collection instruments, the data collection process, the data organization, and the data analysis process were explained in detail. The codes and themes obtained in the data analysis were described in detail and supported by direct references. In addition, the data were re-coded by the researcher approximately one month after the initial coding. The codes that differed between the codings were evaluated separately and finalized. For example, in the first coding, the statement of P11 "... refers to a fraction, it can carry the meaning of part-whole" was coded as "part-whole relationship" in the first coding, while it was coded as "fraction" in the second coding. Afterwards, it was seen that coding the expression as a fraction was more appropriate, and the final version of the coding was made. A more sincere and reliable research environment was created by explicitly telling the participants that the research process did not correspond to any success or grade. These actions aimed to increase the credibility, transferability, reliability, and confirmability of the study.

Results

Content analysis of participants' concept images was conducted separately for each concept. After the initial coding in the content analysis, the codes that were thought to be related were grouped. The codes for each concept were grouped under the themes of component, understanding, and association. Codes that were thought to be used in defining or constructing the related concept were grouped under the “component” theme, codes that were thought to have a special meaning for the participant about the concept were grouped under the “understanding” theme, and codes that associated the concept with other concepts were grouped under the “association” theme. For example, sample codes and user expressions belonging to these themes for the concept of rational number are shown in Table 2.

Table 2.
Examples of Themes, Codes and Participant Statements Related to the Concept of Rational Numbers

Theme	Code	Participant Statement
Component	Can be written as a/b	*All numbers that can be written as a/b. (P11S1)*
Understanding	Mathematical term	*A rational number is a mathematical term for all numbers between two integers. (P10S2)*
Association	Rate	*It is the rate of two numbers. (P5S1)*

To further examine the changes in participants' concept images before and after their ChatGPT experiences, the codes for each concept were visualized as diagrams for S1 and S2. The diagrams reflect the cognitive structures or mental models that students form about the related concepts. The common codes in S1 and S2 were visualized in relation to both documents, while the others were associated only with the relevant document. In addition, the frequency of the codes in the related document is shown on the graph. In order to increase comprehensibility, the number of common and non-common codes was limited to a maximum of five codes. To distinguish the themes of the codes in the diagrams, the codes belonging to the component theme were colored orange, the codes belonging to the understanding theme were colored purple, and the codes belonging to the association theme were colored green. In selecting these codes, the codes with the highest frequency were prioritized.

Results Related to the Concept of Rational Number

The codes associated with the concept of rational numbers are shown in Figure 2. When Figure 2 is analyzed, it is seen that both S1 and S2 associate rational numbers with other number sets such as integers and real numbers. In addition, rational numbers are associated with the concept of "fraction" and "infinity", emphasizing "write as a/b" and "denominator is non-zero". For example, P11 and P7 statements are given below.

"Numbers that can be written as a/b are called rational numbers." (P11S1)

"can be written as a/b such that ∀a∈N and b∈N-{0}." (P7S2)

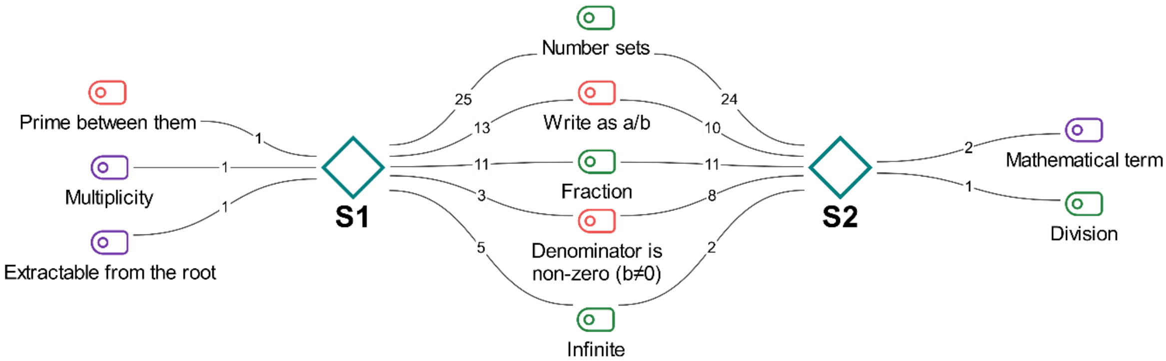

Figure 2. Codes and Frequencies of S1 and S2 Related to the Concept of Rational Number

Although the frequencies of the common codes are generally close to each other, it is noteworthy that the participants mentioned "the denominator being different from zero" more in S2. When the non-common codes were analyzed, it was observed that the frequencies of these codes were quite low. When these codes were examined, while they were associated with "being prime", "multiplicity" and "going beyond the root" for rational numbers, they were associated with "mathematical expression" and "division" after the ChatGPT experiences. However, while there were no codes belonging to the theme of understanding in the common codes, the codes belonging to the theme of understanding in the non-common codes were more than the codes belonging to the theme of association and component.

These findings indicate a change in understanding after the ChatGPT experiences, as can be understood from codes such as associating with the concept of "infinity" and "extractable form the root". In addition, technical details such as "denominator is non-zero" and "mathematical term" were mentioned more. P7 stated, "For example, I could not clearly remember the difference between rational numbers and irrational numbers. When I asked ChatGPT, he gave me examples to eliminate the difficulty in remembering.". However, as can be understood from the frequencies of "fraction", "write as a/b" and "number set" codes, it is seen that the changes in concept images are not radical and they tend to refer to similar concepts.

Results Related to the Concept of Equation

The codes belonging to the concept of equation and the diagram of the frequencies of these codes are given in Figure 3. When Figure 3 is analyzed, the codes "unknown" and "two expressions" come to the forefront in S1 and S2. These codes were the most frequently used expressions related to the concept of equation both before and after the ChatGPT experiences. In addition, while the frequencies of the other common codes are quite close to each other except for the "two expressions" code, the frequency of the "two expressions" code is significantly higher in S2 compared to S1.

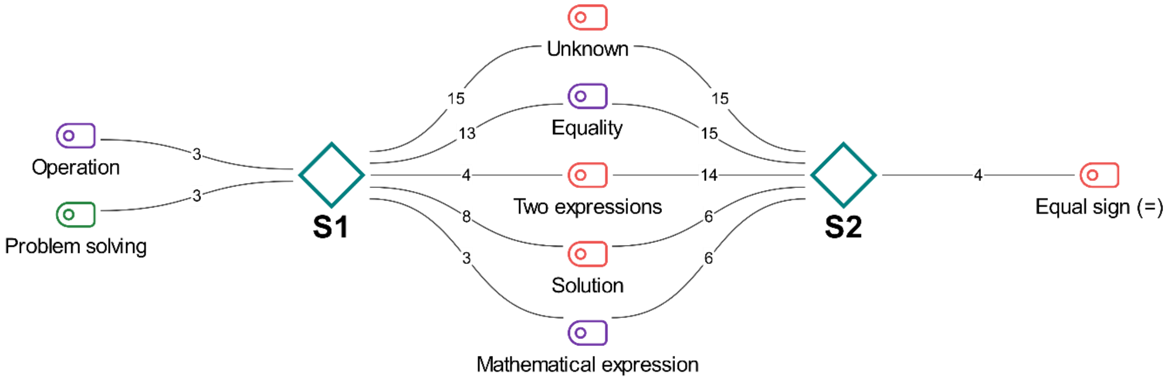

Figure 3. Codes and Frequencies of S1 and S2 Related to the Concept of Equation

When the non-common codes were analyzed, “operation” and “problem solving” codes were observed in S1, while “equals sign” code was observed in S2. However, the frequencies of these codes were not high. In addition, it is a remarkable finding that while the concept of equality was mentioned in SP1 for the concept of equation, the equal sign was not emphasized at all.

“Equal must be used. The two sides cannot be different. It must contain at least one unknown expression.” (P6S2)

Looking at the diagram as a whole, codes from the component and association themes were generally observed in relation to the concept of equation. Only in S1, “problem solving” was used for the theme of understanding. The fact that the frequencies of the common codes except "two expressions" are quite close to each other and the frequencies of the non-common codes are low shows that there is not much change in the concept images of the equation concept after the ChatGPT experience. However, the increase in the codes of "two expressions" and "equal sign" shows that the participants mentioned more important components of the equation after the ChatGPT experience.

Results Related to the Concept of Quadrilateral

The codes of the quadrilateral concept and the diagram of the frequencies of these codes are given in Figure 4. Looking at Figure 4, it was found that while the concept of quadrilateral was associated more with quadrilateral types in S1, the concept of quadrilateral was associated more with quadrilateral components such as "side" and "angle" in S2. However, although not with the same frequency as in S1, associations were made with quadrilateral types in S2. Comparing the common codes in S1 and S2, the association with "quadrilateral types" decreased after the ChatGPT experience, while the frequencies of quadrilateral components such as "edge" and "angle" increased.

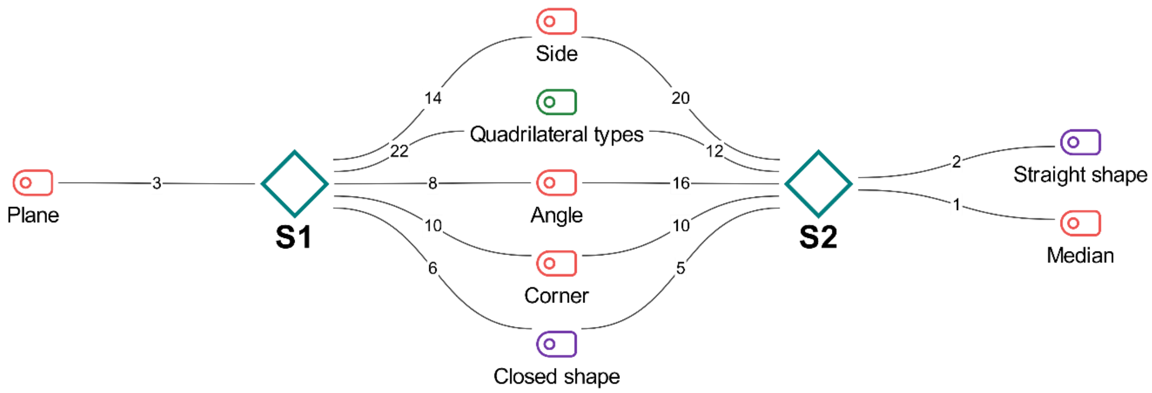

Figure 4. Codes and Frequencies of S1 and S2 Related to the Concept of Quadrilateral

When the non-common codes were analyzed, the codes "plane" in S1, "straight shape" and "bisector" in S2 were identified. At the same time, the frequencies of these codes were significantly lower than the common codes. When analyzing the whole diagram within the framework of themes, it is remarkable that the images of the quadrilateral concept are formed within the framework of the components of the quadrilateral. Moreover, in the association theme, there were only associations with quadrilateral types, while in the understanding theme, the codes "closed shape" and "straight shape" were identified. For example, P5's quadrilateral data in S1 and S2 are given below.

"It is a closed shape with four sides. There are types such as square, rectangle, equilateral, quadrilateral, etc. The sum of its interior angles is 360." (P5S1)

"It is a geometric shape with four sides and four angles. Quadrilaterals are generally classified according to their side lengths and interior angles." (P5S2)

After the ChatGPT experience, more changes were observed in participants' concept images of the quadrilateral concept compared to other concepts. After the ChatGPT experience, the components of quadrilaterals such as "side" and "angle" were mentioned more, while the types of quadrilaterals were mentioned less. However, it is noteworthy that the concept of "plane" was used only in S1.

Results Related to the Concept of Limit

As a result of the content analysis, the codes belonging to the concept of limit and the diagram of the frequencies of these codes are given in Figure 5. The analysis of Figure 5 shows that the concept of limit is generally associated with the concept of "approach". This situation applies to both sessions. However, it is noteworthy that among the common codes, there are more codes belonging to the component theme, and the frequencies of these codes belonging to the component theme increase in S2. When the common codes are evaluated together, in S1 the limit was evaluated as the approach of a point from the right and left, while in S2 it was evaluated as the approach of a function to a point. In addition, the concept of limit was associated with “indefiniteness” in both sessions.

“The value given by approaching a point from its right and left.” (P1S1)

“Limit is a concept in mathematics that expresses how close the function value gets when the independent variable of a function approaches a definite value.” (P9S2)

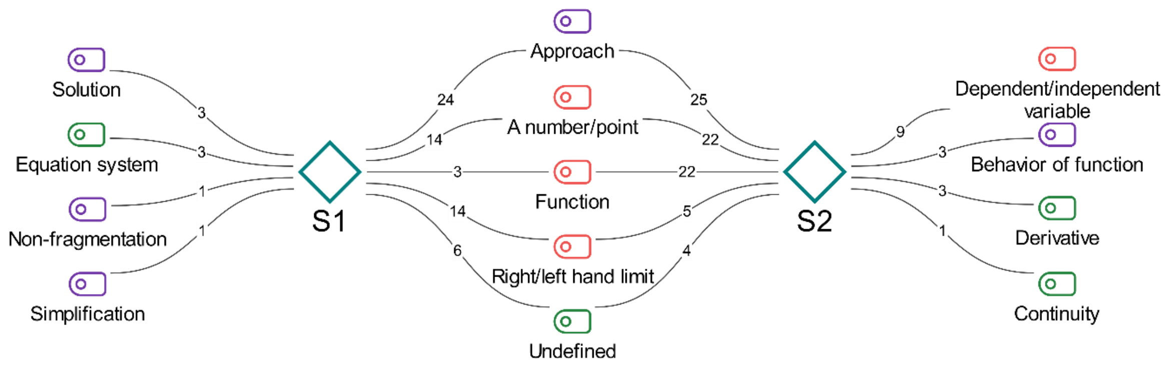

Figure 5. Codes and Frequencies of S1 and S2 Related to the Concept of Limit

When analyzing the non-common codes, although their frequency is not high, the excess of codes belonging to the theme of understanding in S1 draws attention. Here we can see that the concept of limit means "solution", "simplification" and "continuity" for the participants. In S2, the code "dependent/independent variable" stands out due to its high frequency. In addition, in S2, while limit was associated with "derivative" and "continuity", it was observed that participants had an understanding of "function behavior" related to limit. This situation shows that the concept of limit is treated in a closer relationship with the concept of function after the use of ChatGPT.

“It expresses the behavior of a function as a variable approaches a certain point.” (P1S2)

After the ChatGPT experience, the participants' evaluation of the concept of limit as a function and its association with dependent/independent variables came to the fore significantly. In addition, it is noteworthy that the number of codes belonging to the theme of understanding was higher before the ChatGPT experience, but decreased significantly after the ChatGPT experience. In addition, the decrease in the association with right/left limit in S2 shows that the ChatGPT experience masked the participants' experiences with limit applications in the past.

Results Related to the Concept of Derivative

The codes of the concept of derivative and the diagram of the frequencies of these codes are shown in Figure 6. Analyzing Figure 6, it can be seen that the concept image related to the concept of derivative is more diversified in S2. When the common codes are analyzed, it can be seen that the participants associated the concept of derivative with the slope of the most frequently used function, point, and graph components. In addition, although the frequencies of the common codes are close to each other, the frequency of the "rule" code is quite remarkable in S1 compared to S2. For example, the participant expressions belonging to the "rule" code are given below.

“The derivative of unknowns of degree 1 is the coefficient of the unknown. The derivative of a constant number is 0.” (P4S1)

“It is similar to minimizing a function. It is reducing the exponent by 1 and writing the exponent as a coefficient.” (P12S1)

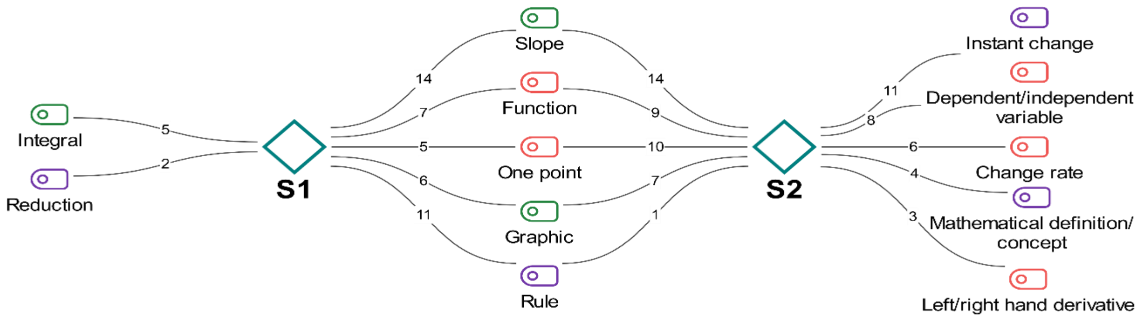

Figure 6. Codes and Frequencies of S1 and S2 Related to the Concept of Derivative

Analyzing the non-common codes, while there are fewer codes in S1, both the number of codes and the frequency of codes are quite high in S2. This is a striking result that distinguishes the concept of derivation from other concepts. Moreover, among the non-common codes in S2, no code belonging to the theme of association was observed. When analyzing the whole diagram, codes belonging to the theme of association and component were observed more frequently in S1, while codes belonging to the theme of component were observed mostly in S2. However, it can be said that the codes belonging to the theme of understanding are less than the codes belonging to other themes in both S1 and S2, both in terms of number and frequency.

It can be seen that the ChatGPT experiences were effective in enriching the participants' concept images of the concept of derivative and in enabling them to associate the concept of derivative with more mathematical concepts. A significant decrease in the rule-based explanation of the concept of derivative supports this situation. However, this is not always the case. P11's statement "I gained new information with ChatGPT, but it also reminded me of the information I had forgotten." summarizes this situation.

Results Related to the Concept of Continuity

As a result of the content analysis, the codes belonging to the concept of continuity and the diagram of the frequencies of these codes are shown in Figure 7. In Figure 7, it is noteworthy that there are more codes belonging to the component theme in the concept of continuity. However, it is observed that in the total of both sessions related to continuity, the participants had the most understanding of breakage/rupture. Comparing the frequencies of the common codes in S1 and S2, a decrease is observed in the frequencies of the codes belonging to the themes of understanding and association, while an increase is observed in the frequencies of the codes belonging to the component theme, except for the code defined/undefined.

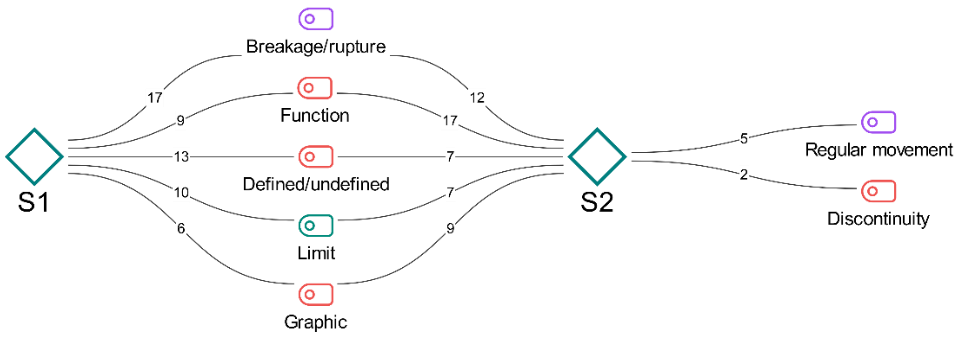

Figure 7. Codes and Frequencies of S1 and S2 Related to the Concept of Continuity

When the non-common codes were analyzed, two codes belonging to the theme of understanding emerged, namely regular movement and discontinuity. However, unlike the codes in other concepts, all of the codes in S1 were also observed in S2. The fact that the frequencies of the common codes are close and that all codes in S1 are also present in S2 indicates that the concept image for the concept of continuity is not differentiated. However, the fact that the concept of continuity was associated more with function and that it was linked to the concepts of "regular movement" and "discontinuity" may indicate that ChatGPT experiences had a partial effect on continuity concepts, although they did not affect them much.

Conclusion and Discussion

This study investigated whether AIBTs cause a change in PMTs' concept images of mathematical concepts. In this context, possible changes in participants' concept images were discussed for each concept according to the themes obtained from the content analysis. Then, from a general perspective, the effects of ChatGPT experiences on participants' concept images were evaluated.

The codes obtained from the content analysis were categorized into three themes: components, understanding, and association. When examining the components theme, it was observed that certain codes such as "equal sign" for the concept of equation, "median" for the concept of quadrilateral, "dependent/independent variable" for the concept of limit, and "rate of change," "dependent/independent variable," and "right/left derivative" for the concept of derivative were absent in S1 but appeared in S2. In addition, an analysis of the frequencies of common codes within the component theme revealed that the frequencies were generally higher in S2 than in S1. This suggests that participants may have focused on more components of the respective concept following their ChatGPT experiences. Consistent with this finding, Küchemann et al. (2023) noted that AIBTs such as ChatGPT could contribute to PMTs' conceptual understanding by helping them overcome difficulties and grasp the fundamental properties of mathematical concepts. According to Rane (2023), when used as a teaching tool, ChatGPT can be effective in teaching algebraic equations or functions. In addition, it can support mathematical learning processes by solving mathematical problems, summarizing and interpreting mathematical texts, explaining mathematical concepts, and reasoning about these concepts (Taani & Alabidi, 2024). These findings suggest that by providing foundational knowledge about a mathematical concept, ChatGPT has the potential to influence the concept images associated with mathematical concepts.

When the codes related to the association theme were analyzed, a decrease in code frequencies was observed after the ChatGPT experiences, although no significant differences were found. It was found that the number of non-overlapping codes remained the same before and after the ChatGPT experiences, and their frequencies were similar. This might suggest that ChatGPT experiences are not very effective in helping PMTs associate a mathematical concept with others. However, Poola and Bozic (2023) emphasized that ChatGPT facilitates making connections between different mathematical concepts and topics. Frieder et al. (2023), when they examined the potential of ChatGPT as a teaching assistant, stated that although it generally understood the relevant problem, it may be insufficient in producing the correct solution. This contradiction suggests that the information provided by ChatGPT may vary depending on its intended purpose or how it is used. In this study, PMTs conducted research within a limited framework to gain knowledge about specific mathematical concepts. As a result, it can be argued that ChatGPT responses were limited to the prompts provided. This is because the responses generated by ChatGPT are directly tied to the prompts provided (Taani & Alabidi, 2024).

For the understanding theme, the number and frequency of codes related to the concept of limit decreased, while those for other concepts remained relatively consistent. Despite the similar numbers and frequencies, notable differences were observed in the codes within the understanding theme. This suggests that the ChatGPT experience may have led to a change in the PMTs' understanding of concepts. For example, for the concept of limit, "solution" was emphasized in S1, while "behavior of the function" was emphasized in S2. For the concept of equation, "operation" was emphasized in S1, while "mathematical expression" was emphasized in S2. For the concept of derivative, "rule" was salient in S1, whereas "instantaneous change" and "mathematical definition" were salient in S2. These results suggest that AIBTs may influence mathematical concept understanding and contribute to the development of concept images. Wardat et al. (2023) found that ChatGPT could provide multiple supports for teachers and students in mathematics and geometry. However, they also cautioned against accepting their responses without validation, as AI tools may misinterpret context, provide incorrect or incomplete information, and overlook pedagogical sensitivities (İpek et al., 2023). Therefore, it is crucial for teachers and students to critically evaluate the accuracy of these tools' outputs and make necessary adjustments (İpek et al., 2023; Wardat et al., 2023).

Another important consideration regarding ChatGPT's impact on concept images is whether it provided participants with new knowledge about mathematical concepts or simply reminded them of existing knowledge. The interview data revealed that some participants reported gaining new information from ChatGPT, while others stated that it helped them recall forgotten knowledge. This is consistent with the observation that AIBTs such as ChatGPT can help users recall and associate knowledge (Deng & Yu, 2023). Furthermore, ChatGPT can support learning performance by providing personalized content and guidance tailored to individual learning needs (Li et al., 2023; Uğraş & Uğraş, 2025). In this context, ChatGPT can be seen as playing two different roles: a source of new knowledge and a reminder. While in its role as a reminder, it may not have a direct impact on concept images, its role as a source of new knowledge suggests a direct impact on concept images. In either case, ChatGPT can be considered as a facilitator of learning because of the positive contributions it makes through its responses. Whether providing new information or serving as a reminder, ChatGPT was observed to play a facilitating role for PMTs (Imran & Almusharraf, 2023; Rahman & Watanobe, 2023).

In summary, although ChatGPT experiences were not always impactful, they generally influenced PMTs' mathematical concept images. Research suggests that AIBTs can improve the quality of learning by personalizing learning, providing targeted and timely feedback, and supporting educational processes (Al Darayseh, 2023; Chen et al., 2020; Ulaşan, 2023). The components directly related to mathematical concepts in ChatGPT were particularly effective in this regard. This highlights the potential of AIBTs to provide quick and effective support in educational contexts (Hwang & Tu, 2021; Ouyang & Jiao, 2021). Prior to their ChatGPT experiences, participants relied more on subjective knowledge from personal experience, whereas after using ChatGPT, they tended to explain concepts more directly and formally, reducing associative reasoning. This suggests that ChatGPT experiences generally influenced mathematical concept images, as partial transformations in understanding were observed. Given that a concept image includes all mental representations related to a concept (Tall & Vinner, 1981), it can be concluded that ChatGPT experiences influenced participants' concept images.

In addition to contributing to concept knowledge, AIBTs can enhance PMTs' teaching skills and pedagogical competencies by providing customized recommendations based on their needs. A study by Lee et al. (2024) demonstrated the promising applicability of ChatGPT in mathematics education at both the university and K-12 levels. The research highlighted the significant benefits of integrating AIBTs into educational processes for both teachers and students. These tools provide a variety of information, methods, approaches, and content to support the professional development of mathematics teachers and PMTs (Hwang, 2022; Hwang & Tu, 2021; Mohammed et al., 2022; Samuelson, 2023). This diversity and flexibility can help teachers and PMTs increase their professional competence and confidence, ultimately contributing to improved educational quality.

Suggestions

In this section, some suggestions for the teaching process and researchers are presented based on the results obtained. First of all, the research results show that AICBs such as ChatGPT are generally effective on PMTs' mathematical concept images. Therefore, AIBTs can be included in the professional development programs of PMTs. Because these tools can play an important role in improving PMTs' pedagogical competencies and teaching skills. In addition, this study can also be conducted with mathematics teachers and students. Thus, the effectiveness of AICBs on concept images can be examined by comparing the results according to the study groups.

The results of the study showed that ChatGPT was generally effective on concept images. However, there is no clear answer as to whether this effectiveness is in terms of presenting new knowledge or reminding existing knowledge. In this context, studies can be conducted to further investigate the role and effects of ChatGPT and similar AICBs on the understanding of mathematical concepts. In addition, while the effects of AICBs on concept images were analyzed, the limitations of these tools were not included in the study. Previous studies have also mentioned the potential of AICBs to provide incomplete or misleading information (Park & Manley, 2024; Wardat et al., 2023). For this reason, it is suggested that the limitations of AICBs in the concept teaching process should be investigated. Finally, technological developments are occurring rapidly. At the time of this study, ChatGPT 3.5 had just been released, and this version was chosen because it was the most advanced one. However, new tools are being introduced day by day and existing ones are being improved. The subject of this study can be repeated with new tools other than ChatGPT 3.5 and the results can be compared.

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Ethics statement: In this study, I declare that the rules stated in the "Higher Education Institutions Scientific Research and Publication Ethics Directive" are complied with and that I do not take any of the actions based on "Actions Against Scientific Research and Publication Ethics".

Author Contributions: All processes of the study (conceptualization, data curation, formal analysis, investigation, methodology, software, resources, validation, visualization, writing – original draft, writing – review & editing) are carried out by the author.

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Data Availability Statement: The raw data used in the study are available from the author on request.

Conflict of Interest: There is no conflict of interest with any financial, personal, or institutional relationships in any process of this study.

Generative Artificial Intelligence Statement: ChatGPT was used for language editing in some parts of the manuscript. The author critically reviewed and edited all AI-generated content to ensure accuracy and originality.

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