ÖHÜ Müh. Bilim. Derg. / OHU J. Eng. Sci.
ISSN: 2564-6605
doi: 10.28948/ngumuh.387248
Ömer Halisdemir Üniversitesi Mühendislik Bilimleri Dergisi, Cilt 7, Sayı 1, (2018), 399-406
Omer Halisdemir University Journal of Engineering Sciences, Volume 7, Issue 1, (2018), 399-406

Araştırma / Research

NUMERICAL FIRST MODE FREQUENCY ANALYSIS OF AXIALLY LAYERED FUNCTIONALLY GRADED TAPERED BEAMS

Savaş EVRAN (ORCID: 0000-0002-7512-5997)[*]

Department of Machine and Metal Technologies, Vocational High School of Canakkale Technical Sciences, Canakkale Onsekiz Mart University, Canakkale, Türkiye

Geliş / Received: *15.09.2017*
Kabul / Accepted: *20.11.2017*

ABSTRACT

The purpose of this numerical work is to evaluate the first mode frequency analysis of the tapered beams with three layers, modelled using functionally graded materials (FGM) in the axially direction, under clamped-clamped (C-C) boundary condition based on finite element software named ANSYS V13. Analyses were conducted using L16 Taguchi orthogonal array design consisting of three control factors and four levels. The layers were determined as the control factors and were considered to be made from aluminum (Al)/monotungsten carbide (WC) systems. The optimum layer combination was carried out according to the analysis of signal-to-noise (S/N) ratio. The importance levels and contribution ratios of the layers on the first mode frequency of the axially layered FG tapered beams were observed by using analysis of variance (ANOVA). Layer 1 and Layer 3 have positive effects on the first mode frequency values. However, Layer 2 has negative influence. In addition, the most effective layers are Layer 1 with 82.17%, Layer 2 with 16.36% and Layer 3 with 1.45%, respectively.

Keywords: Functionally graded materials, tapered layered beam, finite element method

EKSENEL TABAKALI FONKSİYONEL DERECELENDİRİLMİŞ KONİK KİRİŞLERİN SAYISAL İLK MOD FREKANS ANALİZİ

ÖZ

Bu numerik çalışmanın amacı, ANSYS V13 sonlu elemanlar programı kullanarak ankastre-ankastre (C-C) sınır şartı altında eksenel yönde fonksiyonel derecelendirilmiş malzemeden (FDM) modellenmiş üç tabakalı konik kirişlerin birinci mod frekans analizini değerlendirmektir. Analizler üç seviye ve üç kontrol faktöründen oluşan L16 Taguchi ortogonal dizi tasarımı kullanılarak yürütülmüştür. Tabakalar kontrol faktörleri olarak karar verilmiştir ve alüminyum (Al) / monotungsten karbür (WC) sistemlerinden oluştuğu düşünülmektedir. Optimum tabaka kombinasyonu sinyal gürültü oran analizine göre gerçekleştirildi. Eksenel tabakalı FD konik kirişlerin birinci mod frekansı üzerinde tabakaların önem seviyeleri ve katkı oranları varyans analizi (ANOVA) kullanılarak incelenmiştir. Birinci mod frekans değerleri üzerinde Tabaka 1 ve Tabaka 3 pozitif etkilere sahiptir. Ancak Tabaka 2 negatif etkiye sahiptir. Ayrıca en etkili tabakalar sırasıyla %82,17 ile Tabaka 1, %16,36 ile Tabaka 2 ve %1,45 ile Tabaka 3'tür.

Anahtar Kelimeler: Fonksiyonel derecelendirilmiş malzemeler, konik tabakalı kirişler, sonlu elemanlar metodu

[*]Corresponding author / Sorumlu yazar. Tel.: +90 286 218 00 18; e-mail / e-posta: sevran@comu.edu.tr

1. INTRODUCTION

In the Sendai area in Japan, functionally graded materials (FGMs) was presented by some scientists who work in materials science fields in 1984 [1]. Since then, it had been made an effort to improve materials having high resistant using FGMs [2]. FGMs are generally used as a mixture of ceramic and metal materials and so it may be obtained the resist high-temperature environments while preserving toughness [3]. Metal materials have excellent strength and toughness but ceramic materials have excellent characteristics in term of heat resistance [2]. The concept named FGM was continued to be used for different areas such as dental implants [4], sensor and energy applications [5], thermal, wear and corrosion barriers [6] etc. In the following years, the concept was used for analysis of the beams. In the literature, it can be seen many studies consisting of free vibration response of beams formed using FGM. Aydogdu and Taskin [7] presented a study consisting of free vibration behavior of FG beam under simply supported edges. Kapuria et al. [8] published a study about bending and free vibration analysis of FG beams made of layers through thickness direction. They presented theoretical model and used its experimental validation based on Al/SiC and Ni/Al_2O_3 systems. Rajasekaran [9] observed the free vibration behavior of rotating tapered Timoshenko beams consisting of FGM in axial direction for different boundary conditions and they also used differential transformation and quadrature methods for analysis. Shahba and Rajasekaran [10] presented a study including the free vibration and stability analysis of FG Euler–Bernoulli beams in axial direction and the beam has tapered geometry. Shahba et al. [11] published a study about the free vibration and stability behaviors of tapered Timoshenko beams created using FGM using a finite element approach according to classical and non-classical boundary conditions. Fang and Zhou [12] evaluated the free vibration behavior of rotating tapered Timoshenko beams with FGM in axial direction. Akgöz and Civalek [13] published a study about the free vibration behavior of tapered Bernoulli–Euler micro beams formed using FGM in axial direction. In addition, they used the modified couple stress theory. Huang and Li [14] reported a work based on the free vibration analysis of beams formed using FGM in axial direction according to non-uniform cross-section. Sina et al. [15] proposed an analytical method based on free vibration behavior of beams created using FGM. Pradhan and Chakraverty [16] presented a study consisting of the free vibration behavior of Euler and Timoshenko beams which having FGM based on various sets of boundary conditions. Alshorbagy et al. [17] investigated the free vibration response of a beam modelled using FGM and they used finite element method. Wattanasakulpong et al. [18] presented a study about the free vibration behavior of FG beams which having layers based on experimental validation. Şimşek [3] evaluated the fundamental frequency behavior of FG beams using various boundary conditions and various higher-order beam theories were used for analysis. Huang et al. [19] presented a study about the free vibration behavior of Timoshenko beams formed using FGM in the axial direction according to non-uniform cross-section. Hein and Feklistova [20] observed the free vibration behaviors of non-uniform and axially beams consisting of FGM under different boundary conditions and varying cross-sections. They also used Euler-Bernoulli theory and Haar matrices. Liu et al. [21] observed the free vibration behaviors of exponential FG beams based on single delamination. Mashat et al. [22] presented a study about the free vibration behavior of layered beams consisting of FGM according to various theories and finite elements. Thai and Vo [23] presented a study about the bending and free vibration behavior of beams consisting of FGM and they also used different higher-order shear deformation beam theories. Evran and Yılmaz [27] carried out the influences of layer positions on first mode frequency of axially layered beams with three layers under clamped-free boundary condition using finite element software ANSYS. Yilmaz and Evran [28] analyzed the free vibration behavior for first mode of layered FG short beams made of Al and SiC materials through the axial direction for clamped-free boundary condition and they used experimental and numerical analysis. In this numerical work, the first mode free vibration behavior of the layered FG tapered beams made of Al/WC systems in axial direction based on clamped-clamped boundary condition was carried out using finite element software ANSYS V13 Mechanical APDL and L16 Taguchi orthogonal array design.

2. MATERIAL AND METHODS

2.1. Material

The layers of the axially layered functionally graded tapered beams consist of aluminium (Al) and monotungsten carbide (WC) materials. Al and WC were used as metal and ceramic respectively. Mechanical properties of the materials were shown in Table 1. In the Table, it is seen that the Young's modulus and density of WC material are higher than Al material.

Table 1. Mechanical properties [24]

Properties	Symbol	Constituents	
		Al	WC
Young's modulus (GPa)	E	70	696
Poisson's ratio	υ	0.3	0.3
Density (kg/m^3)	ρ	2707	15,600

The layers consisting of different volume fractions of Al and WC materials have various mechanical properties. The mechanical properties such as Young's modulus, Poisson's ratio and density for layers were calculated using a simple rule of mixture of composite materials. The effective material properties P_{Ef} of the layers can be obtained according to Equation 1 [2],

$$P_{Ef} = \sum_{j=1} P_j V_{f_j} \tag{1}$$

where, P_j and V_{fj} refer to the mechanical properties and volume fraction of the constituent material j respectively. The sum of the volume fractions of all the constituent materials is determined as one, based on Equation 2 [2].

$$\sum_{j=1} V_{f_j} = 1 \tag{2}$$

According to Equation 1 and 2, the mechanical properties of the layers were calculated and were given in Table 2.

2.2. Methods

Numerical results obtained using ANSYS software were determined according to L16 Taguchi orthogonal array design with three control factors and four levels in order to obtain the optimum layers arrangements of the layered FG tapered beams in axial direction. The mechanical properties of the contents in the layers were considered as control factors. Each layer was determined using different WC/Al systems, from 4% WC/96% Al to 48% WC/52% Al, based on 4% WC increasing. Therefore 16 analyses were performed. The control factors and their levels were tabled in Table 2.

Table 2. Control factors and their levels

Levels	Mechanical Properties	Control Factors		
		Layer 1	Layer 2	Layer 3
Level 1	E (GPa)	95.04	195.20	295.36
	ρ (kg/m^3)	3222.72	5285.60	7348.48
Level 2	E (GPa)	120.08	220.24	320.40
	ρ (kg/m^3)	3738.44	5801.32	7864.20
Level 3	E (GPa)	145.12	245.28	345.44
	ρ (kg/m^3)	4254.16	6317.04	8379.92
Level 4	E (GPa)	170.16	270.32	370.48
	ρ (kg/m^3)	4769.88	6832.76	8895.64

From Table 2, it can be seen that the layers are different from each other and the increasing of the WC and the decreasing of the Al in the layers increases mechanical properties of the layers. In order to detect optimum layers and layer arrangements in the beams, 16 analyses results are determined using Minitab R15 software according to “larger is the better characteristic” as $(S/N)_{HB}$ as given Equation 3 [26].

$$(S/N)_{HB} = -10.\log\left(n^{-1}\sum_{i=1}^{n}(y_i^2)^{-1}\right) \tag{3}$$

in which, n represents number of analysis in a trial and y_i demonstrates observed i^{th} data.

2.2.1. Numerical Analysis

The axially layered FG tapered beams were modeled using finite element software named ANSYS under clamped-clamped (C-C) boundary condition and a linear (2-node) beam element named BEAM188 in three dimensions which has 6 degrees of freedom at each note was used for modelling. The element is related to Timoshenko Beam theory. Comprehensive information for the beam element can be obtained from ANSYS help menu. The eigenvalue extraction was performed according to block lanczos method. Mesh operation for lines was made according to global element size and the number of element divisions (NDIV) for each layer was taken as 70 value. In the analyses, problem dimensionality was determined as 3-D. Analysis type is modal and degrees of freedom are UX, UY, UZ, ROTX, ROTY and ROTZ. The beam consists of three layers and their shape was demonstrated in Figure 1. The mechanical properties of the layers for numerical analyses were taken from Table 2. The small and big cross sections of the axially layered FG tapered beams were considered as 10x10 mm^2 and 40x40 mm^2 respectively. The axially beam length was assumed as 120 mm and the layer length in the axial direction were equal with each other as 40 mm.

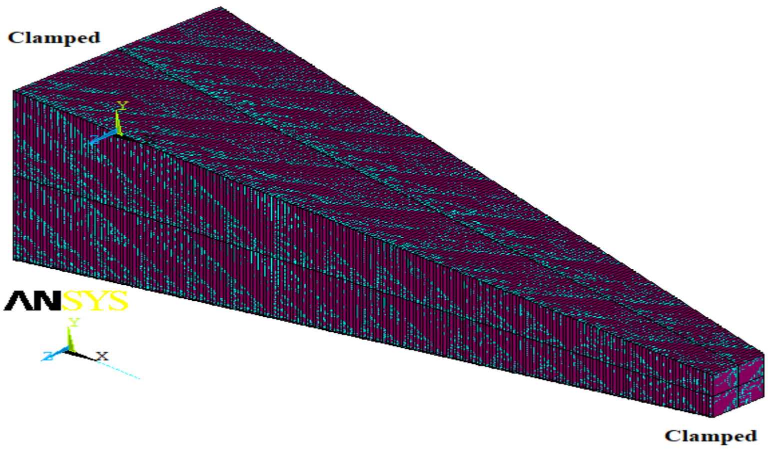

Figure 1. Axially layered FG tapered beam with C-C boundary condition

3. RESULTS AND DISCUSSION

In this numerical study, the effects of the Al/WC systems on the first mode frequency of the axially layered functionally graded tapered beams were detected. Analyses were observed using Finite Element Method based on ANSYS V13 Mechanical APDL software. Analysis results obtained were conducted using Minitab R 15 software according to L16 Taguchi orthogonal array design which having three control factors and four levels. 16 analyses results and their S/N ratios values calculated using Equation 3 were tabulated in Table 3.

3.1. Effects of Al and WC Contents

The average values of the first mode frequency for each layer at level 1, level 2, level 3 and level 4 for raw results were plotted in Figure 2. The effects of WC contents on the first mode frequency of the axially layered functionally graded tapered beams were demonstrated in Figure 2. It can be seen from Figure 2 that the Al/WC systems in the Layer 1 and Layer 3 has positive effects on the first mode frequency. The increasing of the WC contents and the decreasing of the Al contents for Layer 1 and Layer 3 increase the first mode frequency. The increasing of the WC contents and the decreasing of the Al contents in Layer 2 decrease the first mode frequency.

Table 3. First mode frequency results

Runs	Control Factors			Results	S/N Ratio
	Layer 1	Layer 2	Layer 3	λ (Hz)	η (dB)
1	4%WC/96%Al	20%WC/80%Al	36%WC/64%Al	7280.7	77.2435
2	4%WC/96%Al	24%WC/76%Al	40%WC/60%Al	7184.6	77.1281
3	4%WC/96%Al	28%WC/72%Al	44%WC/56%Al	7095.9	77.0201
4	4%WC/96%Al	32%WC/68%Al	48%WC/52%Al	7013.8	76.9191
5	8%WC/92%Al	20%WC/80%Al	40%WC/60%Al	7672.9	77.6992
6	8%WC/92%Al	24%WC/76%Al	36%WC/64%Al	7493.4	77.4936
7	8%WC/92%Al	28%WC/72%Al	48%WC/52%Al	7469.7	77.4661
8	8%WC/92%Al	32%WC/68%Al	44%WC/56%Al	7310.9	77.2794
9	12%WC/88%Al	20%WC/80%Al	44%WC/56%Al	8004.7	78.0669
10	12%WC/88%Al	24%WC/76%Al	48%WC/52%Al	7892.3	77.9441
11	12%WC/88%Al	28%WC/72%Al	36%WC/64%Al	7651.0	77.6744
12	12%WC/88%Al	32%WC/68%Al	40%WC/60%Al	7558.9	77.5692
13	16%WC/84%Al	20%WC/80%Al	48%WC/52%Al	8290.8	78.3719
14	16%WC/84%Al	24%WC/76%Al	44%WC/56%Al	8129.6	78.2014
15	16%WC/84%Al	28%WC/72%Al	40%WC/60%Al	7955.8	78.0137
16	16%WC/84%Al	32%WC/68%Al	36%WC/64%Al	7771.4	77.8100
Average Mean ($\bar{T}_\lambda$)				7611.025	

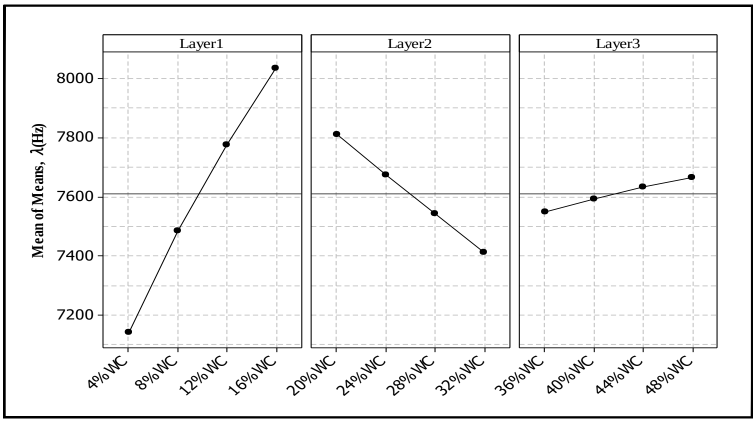

Figure 2. The effects of the WC contents on the first mode frequency

3.2. Analysis of Optimum Layers

Average results and their S/N ratio values of the first mode frequency for each level of each layer on the axially layered functionally graded tapered beams were listed in Table 4. According to "larger is better characteristic", optimum layer levels were determined as fourth levels of the Layer 1 and Layer 3 and first level of Layer 2. These values were indicated by the mark (*). In addition, differences between maximum and minimum average values for each layer were given as delta. According to delta values, maximum values were obtained for Layer 1, Layer 2 and Layer 3 respectively. These results can be seen using rank values.

Table 4. Response results for S/N ratios and means

Level	S/N Ratios (dB)			Means (Hz)		
	Layer 1	Layer 2	Layer 3	Layer 1	Layer 2	Layer 3
1	77.08	77.85*	77.56	7144	7812*	7549
2	77.48	77.69	77.60	7487	7675	7593
3	77.81	77.54	77.64	7777	7543	7635
4	78.10*	77.39	77.68*	8037*	7414	7667*
Delta	1.02	0.45	0.12	893	399	118
Rank	1	2	3	1	2	3

3.3. Analysis of Variance

In order to determine the significant layers and percent contributions of the layers on responses, Analysis of Variance (ANOVA) based on raw data was carried out for S = 8.00026, R-Sq = 99.98% and R-Sq(adj) = 99.96%. The ANOVA results were given in Table 5.

It can be seen in Table 5 that all the layers have significant effects on the first frequency because of the p value < 0.05. In addition, percent contributions of the Layer 1, Layer 2 and Layer 3 on responses are 82.17%, 16.36% and 1.45%. Therefore, the most effective layers are Layer 1, Layer 2 and Layer 3 respectively.

Table 5. ANOVA results

Source	DF	Seq SS	Adj MS	F	P	% Contributions
Layer 1	3	1770490	590163	9220.70	0.000	82.17
Layer 2	3	352490	117497	1835.76		16.36
Layer 3	3	31348	10449	163.26		1.45
Error	6	384	64			0.02
Total	15	2154711				

3.4. Analysis of Estimated Optimum First Mode Frequency

In order to determine the optimum first mode frequency value, the optimum layer levels such as fourth levels of the Layer 1 and Layer 3 and first level of the Layer 2 was used. Estimated mean of the first mode frequency was calculated using Equation 4 [25].

$$\mu_{\lambda_{Hz}} = \bar{T}_{\lambda} + (\overline{Layer1}_4 - \bar{T}_{\lambda}) + (\overline{Layer2}_1 - \bar{T}_{\lambda}) + (\overline{Layer3}_4 - \bar{T}_{\lambda}) \quad (4)$$

in which, the average values for the fourth levels of the Layer 1 and Layer 3 are 8037 Hz for $\overline{Layer1}_4$ and 7667 Hz for $\overline{Layer3}_4$ respectively. The average value at the first level of the Layer 2 is 7812 Hz for $\overline{Layer2}_1$. All these values mentioned were taken from Table 4. In addition, the average value of the first mode frequencies ($\bar{T}_{\lambda}$) is 7611.025 Hz and was taken from Table 3. According to Equation 4, $\mu_{\lambda_{Hz}}$ was found as 8293.95 Hz. 95% confidence intervals of verification analyses ($CI_{\lambda Hz}$) was determined according to Equation 5 [26].

$$CI_{\lambda Hz} = \left(F_{\alpha;1;n_2} V_{error}\left[\frac{1}{n_{eff}} + \frac{1}{R}\right]\right)^{1/2} \quad (5)$$

where, α was taken as 0.05 and n_2 is error value of DF was found as 6 and so $F_{0.05;1;6} = 5.9874$ was taken from the list including F values [25]. $V_{error} = 64$ was taken from Table 5. R represents the repetition number for verification analyses and was determined as 1. n_{eff} refers to effective repetition number and was calculated as shown in Equation 6 [26],

$$n_{eff} = N/[1+T_{DF}] \quad (6)$$

in which, N is defined as total number of analyses and N=16. $T_{DF} = 9$ refers to the sum of degrees of freedom for meaningful control parameters and was taken from Table 5. Therefore n_{eff} was calculated as 1.6 and $CI_{\lambda Hz}$ value was calculated as 24.9537. Estimated optimum first mode frequency at 95% confidence intervals was obtained as given in Equation 7 [26],

$$Mean\ \mu_{\lambda_{Hz}} - CI_{\lambda Hz} < \mu_{\lambda_{Hz}} < CI_{\lambda Hz} + Mean\ \mu_{\lambda_{Hz}} \quad (7)$$

The observed results based on estimation and ANSYS were listed in Table 6.

Table 6. Comparison of optimum results

Units	Estimated Result	ANSYS Result	Predicted Confidence Intervals for 95% Confidence Level
Hz	8293.95	8290.8	$8268.99 < \overline{\lambda_{Hz}} < 8318.90$
dB	78.3752	78.3719	$77.1281 < \overline{S/N_{dB}} < 78.4013$

4. CONCLUSIONS

In this numerical study, the optimum layers and their important levels on the free vibration behavior of the layered functionally graded tapered beams in the axially direction were investigated. Layers were made from metal (Al) and ceramic (WC) materials. Analyses were performed under clamped-clamped boundary condition using ANSYS finite element software and L16 Taguchi orthogonal array with three control factors and four levels. The results obtained according to this study can be summarized as follows:

1) The increasing of the WC% contents and the decreasing of the Al% contents in the Layers increase the mechanical properties of the Layers.
2) The average value of the first mode frequencies was found as 7611.025 Hz for the layered FG tapered beams under C-C boundary condition according to Taguchi L16 orthogonal array design.
3) The first mode frequency increases with Layer 1 and Layer 3 and decreases with the Layer 2.
4) The increasing ratio of the first mode frequency of the Layer 1 is higher than Layer 3.
5) The optimum layers for first mode frequency were obtained as fourth levels of the Layer 1 and Layer3 and first level of the Layer 2. Therefore, beam configuration with optimum layers was obtained as (16%WC/84%Al-20%WC/80%Al-48%WC/52%Al).
6) According to ANOVA results, the most effective Layers on the first mode frequency are Layer 1 with 82.17%, Layer 2 with 16.36% and Layer 3 with 1.45%, respectively.
7) The layers have significant effects on the first mode frequency because of the p value < 0.05.
8) Predicted result and ANSYS result obtained using optimum layers are found as 8293.95 Hz and 8290.8 Hz respectively.
9) Although the mechanical properties of the Layer 2 increase with increasing of the layer levels, Layer 2 decreases the first mode frequency of the axially layered FG tapered beams under C-C boundary condition.
10) Mechanical properties of the materials for the axially layered FG tapered beams with clamped-clamped boundary condition have an important effect on the first mode frequency.
11) The layers with the clamped end increase the first mode frequency.

REFERENCES

[1] KOIZUMI, M., "FGM Activities in Japan". Composites Part B: Engineering, 28(1), 1-4, 1997.
[2] SHEN, H.S., Functionally Graded Materials: Nonlinear Analysis of Plates and Shells, CRC Press, Boca Raton, London, New York, 2009.
[3] ŞIMŞEK, M., "Fundamental Frequency Analysis of Functionally Graded Beams by using Different Higher-Order Beam Theories". Nuclear Engineering and Design, 240(4), 697-705, 2010.
[4] WATARI, F., YOKOYAMA, A., SASO, F., UO, M., KAWASAKI, T., "Fabrication and Properties of Functionally Graded Dental Implant". Composites Part B: Engineering, 28(1), 5-11, 1997.
[5] MÜLLER, E., DRAŠAR, Č., SCHILZ, J., KAYSSER, W.A., "Functionally Graded Materials for Sensor and Energy Applications". Materials Science and Engineering: A, 362(1), 17-39, 2003.
[6] SCHULZ, U., PETERS, M., BACH, F.W., TEGEDER, G., "Graded Coatings for Thermal, Wear and Corrosion Barriers". Materials Science and Engineering: A, 362(1), 61-80, 2003.
[7] AYDOGDU, M., TASKIN, V., "Free Vibration Analysis of Functionally Graded Beams with Simply Supported Edges". Materials & Design, 28(5), 1651-1656, 2007.
[8] KAPURIA, S., BHATTACHARYYA, M., KUMAR, A.N., "Bending and Free Vibration Response of Layered Functionally Graded Beams: A theoretical model and its experimental validation". Composite Structures, 82(3), 390-402, 2008.
[9] RAJASEKARAN, S., "Free Vibration of Centrifugally Stiffened Axially Functionally Graded Tapered Timoshenko Beams using Differential Transformation and Quadrature Methods". Applied Mathematical Modelling, 37(6), 4440-4463, 2013.
[10] SHAHBA, A., RAJASEKARAN, S., "Free Vibration and Stability of Tapered Euler–Bernoulli Beams Made of Axially Functionally Graded Materials". Applied Mathematical Modelling, 36(7), 3094-3111, 2012.
[11] SHAHBA, A., ATTARNEJAD, R., MARVI, M.T., HAJILAR, S., "Free Vibration and Stability Analysis of Axially Functionally Graded Tapered Timoshenko Beams with Classical and Non-Classical Boundary Conditions". Composites Part B-Engineering, 42(4), 801-808, 2011.
[12] FANG, J.S., ZHOU, D., "Free Vibration Analysis of Rotating Axially Functionally Graded Tapered Timoshenko Beams". International Journal of Structural Stability and Dynamics, 16(5), 1-19, 2016.
[13] AKGÖZ, B., CIVALEK, Ö., "Free Vibration Analysis of Axially Functionally Graded Tapered Bernoulli–

Euler Microbeams based on The Modified Couple Stress Theory". Composite Structures, 98, 314-322, 2013.

[14] HUANG, Y., LI, X.F., "A New Approach for Free Vibration of Axially Functionally Graded Beams with Non-Uniform Cross-Section". Journal of Sound and Vibration, 329(11), 2291-2303, 2010.

[15] SINA, S.A., NAVAZI, H.M., HADDADPOUR, H., "An Analytical Method for Free Vibration Analysis of Functionally Graded Beams". Materials & Design, 30(3), 741-747, 2009.

[16] PRADHAN, K.K., CHAKRAVERTY, S., "Free Vibration of Euler and Timoshenko Functionally Graded Beams by Rayleigh–Ritz Method". Composites Part B: Engineering, 51, 175-184, 2013.

[17] ALSHORBAGY, A.E., ELTAHER, M.A., MAHMOUD, F.F., "Free Vibration Characteristics of A Functionally Graded Beam by Finite Element Method". Applied Mathematical Modelling, 35(1), 412-425, 2011.

[18] WATTANASAKULPONG, N., GANGADHARA PRUSTY, B., KELLY, D.W., HOFFMAN, M., "Free Vibration Analysis of Layered Functionally Graded Beams with Experimental Validation". Materials & Design, 36, 182-190, 2012.

[19] HUANG, Y., YANG, L.E., LUO, Q.Z., "Free Vibration of Axially Functionally Graded Timoshenko Beams with Non-Uniform Cross-Section". Composites Part B: Engineering, 45(1), 1493-1498, 2013.

[20] HEIN, H., FEKLISTOVA, L., "Free Vibrations of Non-Uniform and Axially Functionally Graded Beams using Haar Wavelets". Engineering Structures, 33(12), 3696-3701, 2011.

[21] LIU, Y., XIAO, J., SHU, D., "Free Vibration of Exponential Functionally Graded Beams with Single Delamination". Procedia Engineering, 75, 164-168, 2014.

[22] MASHAT, D.S., CARRERA, E., ZENKOUR, A.M., AL KHATEEB, S.A., FILIPPI, M., "Free Vibration of FGM Layered Beams By Various Theories and Finite Elements". Composites Part B: Engineering, 59, 269-278, 2014.

[23] THAI, H.T., VO, T.P., "Bending and Free Vibration of Functionally Graded Beams using Various Higher-Order Shear Deformation Beam Theories". International Journal of Mechanical Sciences, 62(1), 57-66, 2012.

[24] BERNARDO, G.M.S., DAMÁSIO, F.R., SILVA, T.A.N., LOJA, M.A.R., "A Study on the Structural Behaviour of FGM Plates Static and Free Vibrations Analyses". Composite Structures, 136, 124-138, 2016.

[25] ROY, R.K., A Primer on the Taguchi Method, Van Nostrand Reinhold, New York, USA, 1990.

[26] ROSS, P.J., Taguchi Techniques for Quality Engineering, (2nd Edition), McGraw-Hill International Book Company, New York, USA, 1996.

[27] EVRAN, S , YILMAZ, Y ., "The Effects of Layer Arrangements on Fundamental Frequency of Layered Beams In Axial Direction". SAÜ Fen Bilimleri Enstitüsü Dergisi 21(5), 968-977, 2017.

[28] YILMAZ Y, EVRAN S., "Free Vibration Analysis of Axially Layered Functionally Graded Short Beams using Experimental and Finite Element Methods. Science and Engineering of Composite Materials", 23(4), 453-460, 2016.