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# ON SUBCLASS OF ANALYTIC FUNCTIONS DEFINED BY $\mathfrak{q}$-ANALOGUE OF MODIFIED TREMBLAY FRACTIONAL DERIVATIVE OPERATOR

ABDULLAH ALSOBOH*, AND WAGGAS GALIB ATSHAN**
*PHILADELPHIA UNIVERSITY, AMMAN, JORDAN. ORCID NUMBER: 0000-0002-5183-2654
**UNIVERSITY OF AL-QADISIYAH, DIWANIYAH, IRAQ. ORCID NUMBER: 0000-0002-7033-8993.

ABSTRACT. In this research, by using the principle of quantum calculus, we introduce a modified fractional derivative operator $\mathcal{T}^{\xi,F}_{\mathfrak{q},\varsigma}$ of the analytic functions in the open unit disc $\diamond = \{\varsigma : \varsigma \in \mathbb{C}, |\varsigma| < 1\}$. The operator $\mathcal{T}^{\xi,F}_{\mathfrak{q},\varsigma}$ can then be used to introduce a new subclass of analytic functions $\mathcal{D}\bigoplus(\vartheta, F, d, \xi, \gamma; \mathfrak{q})$. We present the necessary conditions for functions belonging to the subclass $\mathcal{D}\bigoplus(\vartheta, F, d, \xi, \gamma; \mathfrak{q})$. Furthermore, we discuss a growth and distortion bounds, the convolution condition, and the radii of starlikeness. In addition, we present neighbourhoods problems involving the $\mathfrak{q}$-analogue of a modified Tremblay operator for functions in the introduced class $\mathcal{D}\bigoplus(\vartheta, F, d, \xi, \gamma; \mathfrak{q})$.

## 1. INTRODUCTION

Let $\diamond = \{\varsigma : \varsigma \in \mathbb{C}, |\varsigma| < 1\}$ denote the open unit disc and $\mathcal{A}$ the class of functions $\hbar(\varsigma)$ of the form

$$\hbar(\varsigma) = \varsigma + \sum_{\varkappa=2}^{\infty} a_\varkappa \varsigma^\varkappa, \quad (\varsigma \in \diamond) \tag{1.1}$$

that are analytic in the open unit disc $\diamond$. Furthermore, let $\mathcal{S}$ be the subset of $\mathcal{A}$ consisting of one-to-one (univalent) functions in $\diamond$.

The convolution of functions $\hbar$ as in (1.1) and the function

$$y(\varsigma) = \varsigma + \sum_{\varkappa=2}^{\infty} \gamma_\varkappa \varsigma^\varkappa,$$

is defined by:

$$(\hbar * y)(\varsigma) \quad = \quad \hbar(\varsigma) * y(\varsigma) = \varsigma + \sum_{\varkappa=2}^{\infty} a_\varkappa \gamma_\varkappa \varsigma^\varkappa.$$

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The fractional $\mathfrak{q}$-calculus is an extension of ordinary fractional calculus, and it has become increasingly popular in recent decades due to its wide range of applications in various fields of science and engineering, particularly mathematics ([12], [20]). The concept of fractional $\mathfrak{q}$-calculus was introduced by Al-Salam and Verma [4], Al-Salam [5], and Agrawal [1]. They also explored some basic properties of fractional $\mathfrak{q}$-derivatives. In addition, Isogawa et al. [14] investigated some fundamental properties of fractional $\mathfrak{q}$-derivatives. Several problems involving fractional $\mathfrak{q}$-calculus operators have recently been recognized ([2, 15, 16, 17, 21, 24, 25]). In 2011, Garg and Chanchlani [13] defined a $\mathfrak{q}$-analog of Saigo's fractional integrals. Two authors, Exton [10] and Gasper [11], have written books about $\mathfrak{q}$-calculus.

The following are the notations and definitions again for main terms in $\mathfrak{q}$-calculus, which may be found in Gasper and Rahman [11] and Purohit and Rania [18], as follows:

1) The $\mathfrak{q}$-shifted factorial $(\vartheta, \mathfrak{q})_{\varkappa}$ is defined for $\vartheta \in \mathbb{C}$ and $0 < \mathfrak{q} < 1$ by:

$$(\vartheta; \mathfrak{q})_{\varkappa} = (\mathfrak{q}^{\vartheta}; \mathfrak{q})_{\varkappa} = \begin{cases} \prod\limits_{\imath=0}^{\varkappa-1}(1-\vartheta\mathfrak{q}^{\imath}), & \varkappa > 0 \\ \\ \prod\limits_{\imath=0}^{\infty}(1-\vartheta\mathfrak{q}^{\imath}), & \varkappa \to \infty. \end{cases} \tag{1.2}$$

Equivalently,

$$(\vartheta; \mathfrak{q})_{\varkappa} = \frac{\Gamma_{\mathfrak{q}}(\vartheta+\varkappa)(1-\mathfrak{q})^{\varkappa}}{\Gamma_{\mathfrak{q}}(\vartheta)}, \tag{1.3}$$

where the $\mathfrak{q}$-gamma function (see for example Gasper and Rahman [11]), is given by

$$\Gamma_{\mathfrak{q}}(\vartheta) = \frac{(\mathfrak{q}, \mathfrak{q})_{\infty}}{(\mathfrak{q}^{\vartheta}, \mathfrak{q})_{\infty}(1-\mathfrak{q})^{\vartheta-1}}, \quad \vartheta \neq 0, -1, -2, \dots . \tag{1.4}$$

2) [**?**] For $0 < \mathfrak{q} < 1$. The $\mathfrak{q}$-derivative, also known as the $\mathfrak{q}$-difference operator, of a function $\hbar$ is defined by

$$\partial_{\mathfrak{q}}\hbar(\varsigma) = \begin{cases} \frac{\hbar(\varsigma)-\hbar(\mathfrak{q}\varsigma)}{\varsigma-\mathfrak{q}\varsigma}, & \text{if} \quad \varsigma \neq 0, \\ \\ \hbar'(0), & \text{if} \quad \varsigma = 0, \\ \\ \hbar'(\varsigma), & \text{if} \quad \mathfrak{q} \to 1^{-}, \ \varsigma \neq 0. \end{cases} . \tag{1.5}$$

3) The $\mathfrak{q}$-Jackson's integral of a function $\hbar$ is defined by:

$$\int_0^{\varsigma} \hbar(\gimel)\partial_{\mathfrak{q}}\gimel \quad = \quad \varsigma(1-\mathfrak{q}) \sum_{\varkappa=0}^{\infty} \mathfrak{q}^{\varkappa}\hbar(\mathfrak{q}^{\varkappa}\varsigma),$$

provided that the series converges.

In 2010, Purhot and Yadav [18] introduced fractional integral operator and fractional derivative operator by

**Definition 1.1.** [18] *The fractional integral operator* $I^{\vartheta}_{\mathfrak{q},\varsigma}\hbar(\varsigma)$*, which operates on a function* $\hbar(\varsigma)$ *of order* $\vartheta$ $(\vartheta > 0)$*, is defined as follows:*

$$I^{\vartheta}_{\mathfrak{q},\varsigma}\hbar(\varsigma) \quad = \quad \frac{1}{\Gamma_{\mathfrak{q}}(\vartheta)}\int_0^{\varsigma}(\varsigma-\tau\mathfrak{q})_{\vartheta-1}\hbar(\tau)\partial_{\mathfrak{q}}\tau,$$

*Here,* $\hbar(\varsigma)$ *is an analytic function in a simply-connected region of the* $\varsigma$*-plane that includes the origin.*

**Definition 1.2.** [18] *The fractional derivative operator* $D^{\vartheta}_{\mathfrak{q},\varsigma}\hbar(\varsigma)$ *of a function* $\hbar(\varsigma)$ *of order* $\vartheta$ $(0 \le \vartheta < 1)$ *is defined as*

$$D^{\vartheta}_{\mathfrak{q},\varsigma}\hbar(\varsigma) \quad = \quad \partial_{\mathfrak{q}}I^{\vartheta}_{\mathfrak{q},\varsigma}\hbar(\varsigma) = \frac{1}{\Gamma_{\mathfrak{q}}(1-\vartheta)}\partial_{\mathfrak{q}}\int_0^{\varsigma}(\varsigma-\tau\mathfrak{q})_{\vartheta-1}\hbar(\tau)\partial_{\mathfrak{q}}\,\tau.$$

**Definition 1.3.** [18]*(Extended Fractional* $\mathfrak{q}$*-Derivative Operator) Under the hypotheses of Definition 2, the fractional* $\mathfrak{q}$*-derivative for a function* $f(\varsigma)$ *of order* $\vartheta$ *is defined by*

$$D^{\vartheta}_{\mathfrak{q},\varsigma}\hbar(\varsigma) \quad = \quad D^{m}_{\mathfrak{q},\varsigma}I^{m-\vartheta}_{\mathfrak{q},\varsigma}\hbar(\varsigma), \quad (m-1 \le \vartheta < m), \quad m \in \mathbb{N}_0 = \mathbb{N}\cup\{0\}.$$

By virtue of Definitions 1.1, 1.2 and 1.3, we have

$$I^{\vartheta}_{\mathfrak{q},\varsigma}\varsigma^{\varkappa} \quad = \quad \frac{\Gamma_{\mathfrak{q}}(\varkappa+1)}{\Gamma_{\mathfrak{q}}(\varkappa+\vartheta+1)}\varsigma^{\varkappa+\vartheta}, \quad (\varkappa \in \mathbb{N}, \vartheta > 0),$$

and

$$D^{\vartheta}_{\mathfrak{q},\varsigma}\varsigma^{\varkappa} \quad = \quad \frac{\Gamma_{\mathfrak{q}}(\varkappa+1)}{\Gamma_{\mathfrak{q}}(\varkappa-\vartheta+1)}\varsigma^{\varkappa-\vartheta}, \quad (\varkappa \in \mathbb{N}, 0 \le \vartheta < 1).$$

Now, let us define the $\mathfrak{q}$-analogue of the Tremblay operator. The modified $\mathfrak{q}$-Tremblay operator ofor analytic functions in the complex domain is then given by:

**Definition 1.4.** *For* $0 < \gamma \le 1, 0 < \xi \le 1, 0 \le \xi-\gamma < 1, \xi \ge \gamma$ *and* $\hbar \in \mathcal{A}$*. The* $\mathfrak{q}$*-analouge of Tremblay derivative operator can be defined by*

$$\Psi^{\xi,\gamma}_{\mathfrak{q},\varsigma}\hbar(\varsigma) = \qquad \frac{\Gamma_{\mathfrak{q}}(\gamma)}{\Gamma_{\mathfrak{q}}(\xi)}\varsigma^{1-\gamma}D^{\xi-\gamma}_{\mathfrak{q},\varsigma}(\varsigma^{\xi-1}\hbar(\varsigma)).$$

**Definition 1.5.** *Let* $\hbar \in \mathcal{A}$*, the* $\mathfrak{q}$*-analouge of modified Tremblay operator denoted by* $\mathcal{T}^{\xi,\gamma}_{\mathfrak{q},\varsigma} : \mathcal{A} \to \mathcal{A}$ *and defined as*

$$\begin{aligned}
\mathcal{T}^{\xi,\gamma}_{\mathfrak{q},\varsigma}\hbar(\varsigma) &= \frac{[\gamma]_{\mathfrak{q}}}{[\xi]_{\mathfrak{q}}}\Psi^{\xi,\gamma}_{\mathfrak{q},\varsigma}\hbar(\varsigma)\\
&= \frac{\Gamma_{\mathfrak{q}}(\gamma+1)}{\Gamma_{\mathfrak{q}}(\xi+1)}\varsigma^{1-\gamma}D^{\xi-\gamma}_{\mathfrak{q},\varsigma}(\varsigma^{\xi-1}\hbar(\varsigma))\\
&= \varsigma+\sum_{\varkappa=2}^{\infty}\frac{\Gamma_{\mathfrak{q}}(\gamma+1)\Gamma_{\mathfrak{q}}(\varkappa+\xi)}{\Gamma_{\mathfrak{q}}(\xi+1)\Gamma_{\mathfrak{q}}(\varkappa+\gamma)}a_{\varkappa}\varsigma^{\varkappa},
\end{aligned}$$

*where* $0 < \gamma \le 1$*,* $0 < \xi \le 1$*,* $0 \le \xi-\gamma < 1$ *and* $\xi \ge \gamma$*.*

**Remark.** *We can conclude that, when we choose the parameters* $q, \xi$ *and* $\gamma$*, the operator* $\mathcal{T}^{\xi,\gamma}_{\mathfrak{q},\varsigma}$ *can lead to other operators results. Examples are presented for further illustration.*

1) *For* $\xi = 1$ *and* $\gamma = 1-\vartheta$*, we get the operator* $\Omega^{\vartheta}_{\mathfrak{q},\varsigma}$ *studied by Purohit and Rania* [18].
2) *For* $\mathfrak{q} \to 1^-$*, then* $\mathcal{T}^{\xi,\gamma}_{\mathfrak{q},\varsigma}\hbar(\varsigma) = \mathcal{T}^{\xi,\gamma}_{\varsigma}f$ *the modified Tremblay operator studied by Esa et.al* [8].

*3) For $\xi = 1$, $\gamma = 1$ and $\mathfrak{q} \to 1^-$ we get the Tremblay operator $\mathcal{T}_{\varsigma}^{\xi,\gamma} f$ syudied by Tremblay* [23].

Various authors, such as Alb Lupaș and Oros [3], Purohit and Rania [17], Atshan et al. [6], Seoudy and Aouf [22], Frasin and Darus [9], Ramadan and Darus [19], Elhaddad and Darus [7], and others, have conducted studies on different subfamilies of normalized analytic functions. These publications have introduced a novel subclass $\mathcal{D}\bigoplus(\vartheta, F, d, \xi, \gamma; \mathfrak{q})$ of $\mathcal{A}$. This subclass incorporates the operator $\mathcal{T}_{\mathfrak{q},\varsigma}^{\xi,\gamma}\hbar(\varsigma)$ and is represented as follows:

**Definition 1.6.** *The class of functions $\mathcal{D}\bigoplus(\vartheta, F, d, \xi, \gamma; \mathfrak{q})$ is denoted by $\hbar \in \mathcal{A}$ and satisfies the inequality:*

$$\left|\frac{1}{d}\left(\frac{\varsigma\partial_{\mathfrak{q}}\left(\mathcal{T}_{\mathfrak{q},\varsigma}^{\xi,\gamma}\hbar(\varsigma)\right) + \vartheta\varsigma^2\partial_{\mathfrak{q}}^2\left(\mathcal{T}_{\mathfrak{q},\varsigma}^{\xi,\gamma}\hbar(\varsigma)\right)}{(1-\vartheta)(\mathcal{T}_{\mathfrak{q},\varsigma}^{\xi,\gamma}\hbar(\varsigma) + \vartheta\varsigma\partial_{\mathfrak{q}}\left(\mathcal{T}_{\mathfrak{q},\varsigma}^{\xi,\gamma}\hbar(\varsigma)\right)} - 1\right)\right| < F, \tag{1.6}$$

*where $\varsigma \in \diamond$, $d \in \mathbb{C} \setminus \{0\}$, $0 < F \leq 1$, $0 \leq \vartheta \leq 1$, $0 < \gamma \leq 1$, $0 < \xi \leq 1$, $0 \leq \xi - \gamma < 1$, and $\xi \geq \gamma$.*

## 2. Main Results

This section examines the conditions that must be met for equation (1.6) to yield the function $\hbar$ in the class $\mathcal{D}\bigoplus(\vartheta, F, d, \xi, \gamma; \mathfrak{q})$. It also highlights the significance of these criteria for functions in this class. Furthermore, it presents growth and distortion bounds, $\mathfrak{q}$-raddi of stralikness of order $\lambda$ $(0 \leq \lambda < 1)$, and the neighborhood problems for the class $\mathcal{D}\bigoplus(\vartheta, F, d, \xi, \gamma; \mathfrak{q})$. The necessary and sufficient conditions for functions $\hbar \in \mathcal{D}\bigoplus(\vartheta, F, d, \xi, \gamma; \mathfrak{q})$ are first discussed in our theorem.

**Theorem 2.1.** *Let the function $\hbar$ as is in* (1.1) *belong to the class $\mathcal{D}\bigoplus(\vartheta, F, d, \xi, \gamma; \mathfrak{q})$ if and only if the following inequality holds:*

$$\sum_{\varkappa=2}^{\infty} \frac{\Gamma_{\mathfrak{q}}(\gamma+1)\Gamma_{\mathfrak{q}}(\varkappa+\xi)}{\Gamma_{\mathfrak{q}}(\xi+1)\Gamma_{\mathfrak{q}}(\varkappa+\gamma)}\Big([\varkappa-1]_{\mathfrak{q}}(\vartheta([\varkappa]_{\mathfrak{q}} - \mathfrak{q}) + \mathfrak{q}(1+\vartheta F|d|)) + F|d|\Big)|a_{\varkappa}| \leq F|d|. \tag{2.1}$$

*Proof.* Suppose $\hbar$ belongs to the set $\mathcal{A}$ and that inequality (2.1) is satisfied. Consequently, we arrive at the following expression:

$$\begin{aligned}
&\left|\frac{\varsigma\partial_{\mathfrak{q}}\left(\mathcal{T}_{\mathfrak{q},\varsigma}^{\xi,\gamma}\hbar(\varsigma)\right) + \vartheta\varsigma^2\partial_{\mathfrak{q}}^2\left(\mathcal{T}_{\mathfrak{q},\varsigma}^{\xi,\gamma}\hbar(\varsigma)\right)}{(1-\vartheta)\left(\mathcal{T}_{\mathfrak{q},\varsigma}^{\xi,\gamma}\hbar(\varsigma)\right) + \vartheta\varsigma\partial_{\mathfrak{q}}\left(\mathcal{T}_{\mathfrak{q},\varsigma}^{\xi,\gamma}\hbar(\varsigma)\right)} - 1\right| \\
&= \left|\frac{\varsigma + \sum_{\varkappa=2}^{\infty}\frac{\Gamma_{\mathfrak{q}}(\gamma+1)\Gamma_{\mathfrak{q}}(\varkappa+\xi)}{\Gamma_{\mathfrak{q}}(\xi+1)\Gamma_{\mathfrak{q}}(\varkappa+\gamma)}[\varkappa]_{\mathfrak{q}}a_{\varkappa}\varsigma^{\varkappa} + \vartheta\left(\sum_{\varkappa=2}^{\infty}\frac{\Gamma_{\mathfrak{q}}(\gamma+1)\Gamma_{\mathfrak{q}}(\varkappa+\xi)}{\Gamma_{\mathfrak{q}}(\xi+1)\Gamma_{\mathfrak{q}}(\varkappa+\gamma)}[\varkappa]_{\mathfrak{q}}[\varkappa-1]_{\mathfrak{q}}a_{\varkappa}\varsigma^{\varkappa}\right)}{(1-\vartheta)\left(\varsigma + \sum_{\varkappa=2}^{\infty}\frac{\Gamma_{\mathfrak{q}}(\gamma+1)\Gamma_{\mathfrak{q}}(\varkappa+\xi)}{\Gamma_{\mathfrak{q}}(\xi+1)\Gamma_{\mathfrak{q}}(\varkappa+\gamma)}a_{\varkappa}\varsigma^{\varkappa}\right) + \vartheta\left(\varsigma + \sum_{\varkappa=2}^{\infty}\frac{\Gamma_{\mathfrak{q}}(\gamma+1)\Gamma_{\mathfrak{q}}(\varkappa+\xi)}{\Gamma_{\mathfrak{q}}(\xi+1)\Gamma_{\mathfrak{q}}(\varkappa+\gamma)}[\varkappa]_{\mathfrak{q}}a_{\varkappa}\varsigma^{\varkappa}\right)} - 1\right| \\
&= \left|\frac{\sum_{\varkappa=2}^{\infty}\frac{\Gamma_{\mathfrak{q}}(\gamma+1)\Gamma_{\mathfrak{q}}(\varkappa+\xi)}{\Gamma_{\mathfrak{q}}(\xi+1)\Gamma_{\mathfrak{q}}(\varkappa+\gamma)}[\varkappa-1]_{\mathfrak{q}}(\mathfrak{q} + \vartheta([\varkappa]_{\mathfrak{q}} - \mathfrak{q}))a_{\varkappa}\varsigma^{\varkappa}}{\varsigma + \sum_{\varkappa=2}^{\infty}\frac{\Gamma_{\mathfrak{q}}(\gamma+1)\Gamma_{\mathfrak{q}}(\varkappa+\xi)}{\Gamma_{\mathfrak{q}}(\xi+1)\Gamma_{\mathfrak{q}}(\varkappa+\gamma)}(1 + \mathfrak{q}\vartheta[\varkappa-1]_{\mathfrak{q}})a_{\varkappa}\varsigma^{\varkappa}}\right|
\end{aligned}$$

$$\leq \quad \frac{\sum\limits_{\varkappa=2}^{\infty}\frac{\Gamma_{\mathfrak{q}}(\gamma+1)\Gamma_{\mathfrak{q}}(\varkappa+\xi)}{\Gamma_{\mathfrak{q}}(\xi+1)\Gamma_{\mathfrak{q}}(\varkappa+\gamma)}[\varkappa-1]_{\mathfrak{q}}(\mathfrak{q}+\vartheta([\varkappa]_{\mathfrak{q}}-\mathfrak{q}))|a_{\varkappa}||\varsigma|^{\varkappa-1}}{1-\sum\limits_{\varkappa=2}^{\infty}\frac{\Gamma_{\mathfrak{q}}(\gamma+1)\Gamma_{\mathfrak{q}}(\varkappa+\xi)}{\Gamma_{\mathfrak{q}}(\xi+1)\Gamma_{\mathfrak{q}}(\varkappa+\gamma)}(1+\mathfrak{q}\vartheta[\varkappa-1]_{\mathfrak{q}})|a_{\varkappa}||\varsigma|^{\varkappa-1}} < F|d|.$$

When we consider values of $\varsigma$ on the real axis and let $\varsigma \to 1^-$, we obtain

$$\sum_{\varkappa=2}^{\infty}\frac{\Gamma_{\mathfrak{q}}(\gamma+1)\Gamma_{\mathfrak{q}}(\varkappa+\xi)}{\Gamma_{\mathfrak{q}}(\xi+1)\Gamma_{\mathfrak{q}}(\varkappa+\gamma)}\Big([\varkappa-1]_{\mathfrak{q}}(\vartheta([\varkappa]_{\mathfrak{q}}-\mathfrak{q})+\mathfrak{q}(1+\vartheta F|d|))+F|d|\Big)|a_{\varkappa}| < F|d|. \tag{2.2}$$

Conversely, suppose $\hbar \in \mathcal{D}\bigoplus(\vartheta, F, d, \xi, \gamma; \mathfrak{q})$, we obtain the following inequality

$$\left|\frac{1}{d}\left(\frac{\varsigma\partial_{\mathfrak{q}}\left(\mathcal{T}_{\mathfrak{q},\varsigma}^{\xi,\gamma}\hbar(\varsigma)\right)+\vartheta\varsigma^2\partial_{\mathfrak{q}}^2\left(\mathcal{T}_{\mathfrak{q},\varsigma}^{\xi,\gamma}\hbar(\varsigma)\right)}{(1-\vartheta)(\mathcal{T}_{\mathfrak{q},\varsigma}^{\xi,\gamma}\hbar(\varsigma)+\vartheta\varsigma\partial_{\mathfrak{q}}\left(\mathcal{T}_{\mathfrak{q},\varsigma}^{\xi,\gamma}\hbar(\varsigma)\right)}-1\right)\right| > -F, \tag{2.3}$$

$$\Re e\left\{\frac{\varsigma\partial_{\mathfrak{q}}\left(\mathcal{T}_{\mathfrak{q},\varsigma}^{\xi,\gamma}\hbar(\varsigma)\right)+\vartheta\varsigma^2\partial_{\mathfrak{q}}^2\left(\mathcal{T}_{\mathfrak{q},\varsigma}^{\xi,\gamma}\hbar(\varsigma)\right)}{(1-\vartheta)\left(\mathcal{T}_{\mathfrak{q},\varsigma}^{\xi,\gamma}\hbar(\varsigma)\right)+\vartheta\varsigma\partial_{\mathfrak{q}}\left(\mathcal{T}_{\mathfrak{q},\varsigma}^{\xi,\gamma}\hbar(\varsigma)\right)}-1+F|d|\right\} > 0$$

This need to complete

$$\Re e\left\{\frac{\varsigma+\sum_{\varkappa=2}^{\infty}\frac{\Gamma_{\mathfrak{q}}(\gamma+1)\Gamma_{\mathfrak{q}}(\varkappa+\xi)}{\Gamma_{\mathfrak{q}}(\xi+1)\Gamma_{\mathfrak{q}}(\varkappa+\gamma)}[\varkappa]_{\mathfrak{q}}(1+\vartheta[\varkappa-1]_{\mathfrak{q}})a_{\varkappa}\varsigma^{\varkappa}}{\varsigma+\sum_{\varkappa=2}^{\infty}\frac{\Gamma_{\mathfrak{q}}(\gamma+1)\Gamma_{\mathfrak{q}}(\varkappa+\xi)}{\Gamma_{\mathfrak{q}}(\xi+1)\Gamma_{\mathfrak{q}}(\varkappa+\gamma)}(1+\vartheta\mathfrak{q}[\varkappa-1]_{\mathfrak{q}})a_{\varkappa}\varsigma^{\varkappa}}-1+F|d|\right\} > 0$$

or

$$\Re e\left\{\frac{F|d|\varsigma+\sum\limits_{\varkappa=2}^{\infty}\frac{\Gamma_{\mathfrak{q}}(\gamma+1)\Gamma_{\mathfrak{q}}(\varkappa+\xi)}{\Gamma_{\mathfrak{q}}(\xi+1)\Gamma_{\mathfrak{q}}(\varkappa+\gamma)}\Big(\ [\varkappa-1]_{\mathfrak{q}}(\vartheta([\varkappa]_{\mathfrak{q}}-\mathfrak{q})+\mathfrak{q}(1+\vartheta F|d|))+F|d|\ \Big)a_{\varkappa}\varsigma^{\varkappa}}{\varsigma+\sum\limits_{\varkappa=2}^{\infty}\frac{\Gamma_{\mathfrak{q}}(\gamma+1)\Gamma_{\mathfrak{q}}(\varkappa+\xi)}{\Gamma_{\mathfrak{q}}(\xi+1)\Gamma_{\mathfrak{q}}(\varkappa+\gamma)}(1+\vartheta\mathfrak{q}[\varkappa-1]_{\mathfrak{q}})a_{\varkappa}\varsigma^{\varkappa}}\right\} > 0.$$

The inequality can be expressed as follows, taking into account the real part of the expression $-e^{i\theta}$: $\Re e\left\{-e^{i\theta}\right\} \geq |e^{i\theta}| = -1$.

$$\frac{F|d|\mathsf{r}-\sum\limits_{\varkappa=2}^{\infty}\frac{\Gamma_{\mathfrak{q}}(\gamma+1)\Gamma_{\mathfrak{q}}(\varkappa+\xi)}{\Gamma_{\mathfrak{q}}(\xi+1)\Gamma_{\mathfrak{q}}(\varkappa+\gamma)}\Big(\ [\varkappa-1]_{\mathfrak{q}}(\vartheta([\varkappa]_{\mathfrak{q}}-\mathfrak{q})+\mathfrak{q}(1+\vartheta F|d|))+F|d|\ \Big)a_{\varkappa}\mathsf{r}^{\varkappa}}{\mathsf{r}-\sum\limits_{\varkappa=2}^{\infty}\frac{\Gamma_{\mathfrak{q}}(\gamma+1)\Gamma_{\mathfrak{q}}(\varkappa+\xi)}{\Gamma_{\mathfrak{q}}(\xi+1)\Gamma_{\mathfrak{q}}(\varkappa+\gamma)}(1+\vartheta\mathfrak{q}[\varkappa-1]_{\mathfrak{q}})a_{\varkappa}\mathsf{r}^{\varkappa}} > 0.$$

By employing the mean value theorem for the limit as $\mathsf{r}$ approaches $1^-$, we derive the inequality 2.1. Thus, we have concluded the proof of Theorem 2.1. □

**Corollary 2.2.** *Assuming that the function $\hbar$ is of the form* (1.1) *and belongs to the class $\mathcal{D}\bigoplus(\vartheta, F, d, \xi, \gamma; \mathfrak{q})$, then the following inequality can be expresse*

$$|a_{\varkappa}| \leq \frac{F|d|\Gamma_{\mathfrak{q}}(\xi+1)\Gamma_{\mathfrak{q}}(\varkappa+\gamma)\Big([\varkappa-1]_{\mathfrak{q}}(\vartheta([\varkappa]_{\mathfrak{q}}-\mathfrak{q})+\mathfrak{q}(1+\vartheta F|d|))+F|d|\Big)}{\Gamma_{\mathfrak{q}}(\gamma+1)\Gamma_{\mathfrak{q}}(\varkappa+\xi)}, \tag{2.4}$$

*for $\varkappa \geq 2$.*

The following result will provide bounds on the growth and distortion of functions in the class $\mathcal{D}\bigoplus(\vartheta, F, d, \xi, \gamma; \mathfrak{q})$.

**Theorem 2.3.** *The following inequalities hold true for any function $\hbar$ in the class $\mathcal{D}\bigoplus(\vartheta, F, d, \xi, \gamma; \mathfrak{q})$ when $|\varsigma| = \mathrm{r} < 1$:*

$$\mathrm{r} - \frac{F|d|(1+\gamma)\mathrm{r}^2}{(1+\xi)(\vartheta+\mathfrak{q}+\mathfrak{q}\vartheta F|d|+F|d|)} \leq |\hbar(\varsigma)| \leq \mathrm{r} + \frac{F|d|(1+\gamma)\mathrm{r}^2}{(1+\xi)(\vartheta+\mathfrak{q}+\mathfrak{q}\vartheta F|d|+F|d|)}, \tag{2.5}$$

*and*

$$1 - \frac{F|d|(1+\mathfrak{q})(1+\gamma)\mathrm{r}}{(1+\xi)(\vartheta+\mathfrak{q}+\mathfrak{q}\vartheta F|d|+F|d|)} \leq |\partial_{\mathfrak{q}}\hbar(\varsigma)| \leq 1 + \frac{F|d|(1+\mathfrak{q})(1+\gamma)\mathrm{r}}{(1+\xi)(\vartheta+\mathfrak{q}+\mathfrak{q}\vartheta F|d|+F|d|)}. \tag{2.6}$$

*These inequalities are sharp by the function*

$$\hbar(\varsigma) = \varsigma + \frac{F|d|(1+\gamma)}{(1+\xi)(\vartheta+\mathfrak{q}+\mathfrak{q}\vartheta F|d|+F|d|)}\varsigma^2.$$

*Proof.* Given $\hbar \in \mathcal{D}\bigoplus(\vartheta, F, d, \xi, \gamma; \mathfrak{q})$ from (2.1) and since

$$\frac{\Gamma_{\mathfrak{q}}(\gamma+1)\Gamma_{\mathfrak{q}}(\varkappa+\xi)}{\Gamma_{\mathfrak{q}}(\xi+1)\Gamma_{\mathfrak{q}}(\varkappa+\gamma)}\Big([\varkappa-1]_{\mathfrak{q}}\Big(\vartheta([\varkappa]_{\mathfrak{q}}-\mathfrak{q})+\mathfrak{q}(1+\vartheta F|d|)\Big)+F|d|\Big)$$

is increasing and positive for $\varkappa \geq 2$, then we have

$$\begin{aligned}&\frac{1+\xi}{1+\gamma}\Big(\vartheta+\mathfrak{q}+\mathfrak{q}\vartheta F|d|+F|d|\Big)\sum_{\varkappa=2}^{\infty}a_{\varkappa} \leq \\ &\frac{\Gamma_{\mathfrak{q}}(\gamma+1)\Gamma_{\mathfrak{q}}(\varkappa+\xi)}{\Gamma_{\mathfrak{q}}(\xi+1)\Gamma_{\mathfrak{q}}(\varkappa+\gamma)}\Big([\varkappa-1]_{\mathfrak{q}}\Big(\vartheta([\varkappa]_{\mathfrak{q}}-\mathfrak{q})+\mathfrak{q}(1+\vartheta F|d|)\Big)+F|d|\Big)\sum_{\varkappa=2}^{\infty}a_{\varkappa} \\ &\leq F|d|,\end{aligned}$$

which is equivalent to,

$$\sum_{\varkappa=2}^{\infty}a_{\varkappa} \leq \frac{F|d|(1+\gamma)}{(1+\xi)(\vartheta+\mathfrak{q}+\mathfrak{q}\vartheta F|d|+F|d|)}. \tag{2.7}$$

We can acquire this through the utilization of the properties of the modulus function

$$\begin{aligned}|\hbar(\varsigma)| &= \left|\varsigma + \sum_{\varkappa=2}^{\infty}a_{\varkappa}\varsigma^{\varkappa}\right| \\ &\leq |\varsigma| + \sum_{\varkappa=2}^{\infty}|a_{\varkappa}||\varsigma|^{\varkappa} \\ &\leq \mathrm{r} + \mathrm{r}^2\sum_{\varkappa=2}^{\infty}|a_{\varkappa}| \\ &\leq \mathrm{r} + \frac{F|d|(1+\gamma)\mathrm{r}^2}{(1+\xi)(\vartheta+\mathfrak{q}+\mathfrak{q}\vartheta F|d|+F|d|)}, \quad \text{by (2.7)}.\end{aligned}$$

and

$$\begin{aligned}|\hbar(\varsigma)| &= \left|\varsigma + \sum_{\varkappa=2}^{\infty}a_{\varkappa}\varsigma^{\varkappa}\right| \geq |\varsigma| - \sum_{\varkappa=2}^{\infty}|a_{\varkappa}||\varsigma|^{\varkappa} \\ &\geq \mathrm{r} - \mathrm{r}^2\sum_{\varkappa=2}^{\infty}|a_{\varkappa}| \geq \mathrm{r} - \frac{F|d|(1+\gamma)\mathrm{r}^2}{(1+\xi)(\vartheta+\mathfrak{q}+\mathfrak{q}\vartheta F|d|+F|d|)}, \quad \text{by (2.7)}.\end{aligned}$$

Now, by applying the Jackson's derivative of (1.5) with respect to $\varsigma$, we get:

$$\begin{aligned} |\partial_{\mathfrak{q}}\hbar(\varsigma)| &= \left|1+\sum_{\varkappa=2}^{\infty}[\varkappa]_{\mathfrak{q}}a_{\varkappa}\varsigma^{\varkappa-1}\right| \leq 1+\sum_{\varkappa=2}^{\infty}[\varkappa]_{\mathfrak{q}}|a_{\varkappa}||\varsigma|^{\varkappa} \\ &\leq \mathsf{r}+[2]_{\mathfrak{q}}\mathsf{r}^2\sum_{s=2}^{\infty}|a_{\varkappa}| \leq \mathsf{r}+\frac{F|d|(1+\mathfrak{q})(1+\gamma)}{(1+\xi)(\vartheta+\mathfrak{q}+\mathfrak{q}\vartheta F|d|+F|d|)}\mathsf{r}^2. \end{aligned}$$

In other hand,

$$\begin{aligned} |\partial_{\mathfrak{q}}\hbar(\varsigma)| &= \left|1+\sum_{\varkappa=2}^{\infty}[\varkappa]_{\mathfrak{q}}a_{\varkappa}\varsigma^{\varkappa-1}\right| \geq 1-\sum_{\varkappa=2}^{\infty}[\varkappa]_{\mathfrak{q}}|a_{\varkappa}||\varsigma|^{\varkappa} \\ &\geq \mathsf{r}-[2]_{\mathfrak{q}}\mathsf{r}^2\sum_{\varkappa=2}^{\infty}|a_{\varkappa}| \geq \mathsf{r}-\frac{F|d|(1+\mathfrak{q})(1+\gamma)}{(1+\xi)(\vartheta+\mathfrak{q}+\mathfrak{q}\vartheta F|d|+F|d|)}\mathsf{r}^2. \end{aligned}$$

□

The neighbourhoods problems of the class $\mathcal{D}\bigoplus(\vartheta,F,d,\xi,\gamma;\mathfrak{q})$ will now be determined.

**Definition 2.1.** *Let $\hbar\in\mathcal{A}$ and $\sigma>0$. We define the $(m,\sigma,\mathfrak{q})$-neighbourhood of $\hbar$ as follows:*

$$\mathcal{N}_{\sigma,\mathfrak{q}}(\hbar)=\left\{g\in\mathcal{A}: g(\varsigma)=\varsigma+\sum_{\varkappa=2}^{\infty}b_{\varkappa}\varsigma^{\varkappa} \text{ and } \sum_{\varkappa=2}^{\infty}[\varkappa]_{\mathfrak{q}}|a_{\varkappa}-b_{\varkappa}|\leq\sigma\right\}. \tag{2.8}$$

*In particular, for the identity functions $e(\varsigma)=z$, we have*

$$\mathcal{N}_{\sigma,\mathfrak{q}}(e)=\left\{g\in\mathcal{A}: g(\varsigma)=\varsigma+\sum_{\varkappa=2}^{\infty}b_{\varkappa}\varsigma^{\varkappa} \text{ and } \sum_{\varkappa=2}^{\infty}[\varkappa]_{\mathfrak{q}}|b_{\varkappa}|\leq\sigma\right\}. \tag{2.9}$$

**Definition 2.2.** *A function $\hbar\in\mathcal{A}$ belong to the class $\mathcal{D}^{\nu}\bigoplus(\vartheta,F,d,\xi,\gamma;\mathfrak{q})$ if there exists a function $\pounds\in\mathcal{D}\bigoplus(\vartheta,F,d,\xi,\gamma;\mathfrak{q})$ such that*

$$\left|\frac{\hbar(\varsigma)}{\pounds(\varsigma)}-1\right|\leq 1-\nu, \quad 0\leq\psi<1, \quad (\varsigma\in\diamond). \tag{2.10}$$

**Theorem 2.4.** *$f\,\pounds\in\mathcal{D}\bigoplus(\vartheta,F,d,\xi,\gamma;\mathfrak{q})$ and*

$$\nu = 1-\frac{[1+\xi]_{\mathfrak{q}}\Big((\vartheta+\mathfrak{q}+\mathfrak{q}\vartheta F|d|)+F|d|\Big)}{[1+\xi]_{\mathfrak{q}}\Big((\vartheta+\mathfrak{q}+\mathfrak{q}\vartheta F|d|)+F|d|\Big)-F|d|[1+\gamma]_{\mathfrak{q}}},$$

*then*

$$\mathcal{N}_{\sigma,q}(\pounds)\subseteq\mathcal{D}^{\nu}\bigoplus(\vartheta,F,d,\xi,\gamma;\mathfrak{q}).$$

*Proof.* Let $\hbar\in\mathcal{N}_{\sigma,q}(\pounds)$, we find from (2.8) that

$$\sum_{\varkappa=2}^{\infty}[\varkappa]_{\mathfrak{q}}|a_{\varkappa}-b_{\varkappa}| \leq \sigma,$$

which implies the coefficient inequality

$$\sum_{\varkappa=2}^{\infty}|a_{\varkappa}-b_{\varkappa}| \leq \frac{\sigma}{1+\mathfrak{q}}.$$

Since $\pounds \in \mathcal{D}\bigoplus(\vartheta, F, d, \xi, \gamma; \mathfrak{q})$, and Using relation (2.1) of Theorem **??**, we have

$$\begin{aligned} & \frac{\Gamma_{\mathfrak{q}}(\gamma+1)\Gamma_{\mathfrak{q}}(2+\xi)}{\Gamma_{\mathfrak{q}}(\xi+1)\Gamma_{\mathfrak{q}}(2+\gamma)}\Big((\vartheta+\mathfrak{q}+\mathfrak{q}\vartheta F|d|)+F|d|\Big)\sum_{\varkappa=2}^{\infty}|b_{\varkappa}| \\ \leq \quad & \sum_{\varkappa=2}^{\infty}\frac{\Gamma_{\mathfrak{q}}(\gamma+1)\Gamma_{\mathfrak{q}}(\varkappa+\xi)}{\Gamma_{\mathfrak{q}}(\xi+1)\Gamma_{\mathfrak{q}}(\varkappa+\gamma)}\Big([\varkappa-1]_{\mathfrak{q}}(\vartheta([\varkappa]_{\mathfrak{q}}-\mathfrak{q})+\mathfrak{q}(1+\vartheta F|d|))+F|d|\Big)|b_{\varkappa}| \leq F|d|, \end{aligned}$$

for ($\varkappa \geq 2$), which implies

$$\sum_{\varkappa=2}^{\infty}|b_{\varkappa}| \leq \frac{F|d|[1+\gamma]_{\mathfrak{q}}}{[1+\xi]_{\mathfrak{q}}\Big((\vartheta+\mathfrak{q}+\mathfrak{q}\vartheta F|d|)+F|d|\Big)}, \tag{2.11}$$

and so

$$\begin{aligned} \left|\frac{\hbar(\varsigma)}{\pounds(\varsigma)}-1\right| \quad & < \quad \frac{\sum_{\varkappa=2}^{\infty}|a_{\varkappa}-b_{\varkappa}|}{1-\sum_{\varkappa=2}^{\infty}b_{\varkappa}} \\ & \leq \quad \frac{\sigma}{1+\mathfrak{q}}.\left(\frac{[1+\xi]_{\mathfrak{q}}\Big((\vartheta+\mathfrak{q}+\mathfrak{q}\vartheta F|d|)+F|d|\Big)}{[1+\xi]_{\mathfrak{q}}\Big((\vartheta+\mathfrak{q}+\mathfrak{q}\vartheta F|d|)+F|d|\Big)-F|d|[1+\gamma]_{\mathfrak{q}}}\right) \\ & = \quad 1-\nu. \end{aligned}$$

Thus, for given $\nu$ and by Definition 2.1, we have $\hbar \in \mathcal{D}^{\nu}\bigoplus(\vartheta, F, d, \xi, \gamma; \mathfrak{q})$.

Finally, we establish the radii of starlikeness of order $\lambda$ for functions in the class $\mathcal{D}^{\lambda}\bigoplus(\vartheta, F, d, \xi, \gamma; \mathfrak{q})$. □

**Theorem 2.5.** *Let $\hbar \in \mathcal{A}$ from the class $\mathcal{D}^{\lambda}\bigoplus(\vartheta, F, d, \xi, \gamma; \mathfrak{q})$. The function $\hbar$ univalent starlike of order $\lambda$, $0 \leq \lambda < 1$ and $|\varsigma| < \mathrm{r}_0$, where*

$$\mathrm{r}_0 = \inf_k \left\{\frac{(1-\lambda)\Gamma_{\mathfrak{q}}(\gamma+1)\Gamma_{\mathfrak{q}}(\varkappa+\xi)\Big([\varkappa-1]_{\mathfrak{q}}(\vartheta([\varkappa]_{\mathfrak{q}}-\mathfrak{q})+\mathfrak{q}(1+\vartheta F|d|))+F|d|\Big)}{F|d|([2]_{\mathfrak{q}}-\lambda)\Gamma_{\mathfrak{q}}(\xi+1)\Gamma_{\mathfrak{q}}(\varkappa+\gamma)}\right\}^{\frac{1}{n-1}}. \tag{2.12}$$

*Proof.* We show that

$$\left|\frac{\varsigma\partial_{\mathfrak{q}}(\hbar(\varsigma))}{\hbar(\varsigma)}-1\right| \quad \leq \quad 1-\lambda, \quad (|\varsigma| < r_0).$$

Considering that

$$\left|\frac{\varsigma\partial_{\mathfrak{q}}(\hbar(\varsigma))}{\hbar(\varsigma)}-1\right| \quad = \quad \left|\frac{\sum\limits_{\varkappa=2}^{\infty}([\varkappa]_{\mathfrak{q}}-1)a_{\varkappa}\varsigma^{\varkappa-1}}{1+\sum\limits_{\varkappa=2}^{\infty}a_{\varkappa}\varsigma^{\varkappa-1}}\right| \leq \frac{\sum\limits_{\varkappa=2}^{\infty}([\varkappa]_{\mathfrak{q}}-1)a_{\varkappa}|\varsigma|^{\varkappa-1}}{1-\sum\limits_{\varkappa=2}^{\infty}a_{\varkappa}|\varsigma|^{\varkappa-1}},$$

to prove the theorem, we must show that

$$\frac{\sum\limits_{\varkappa=2}^{\infty}([\varkappa]_{\mathfrak{q}}-1)a_{\varkappa}|\varsigma|^{\varkappa-1}}{1-\sum\limits_{\varkappa=2}^{\infty}a_{\varkappa}|\varsigma|^{\varkappa-1}} \quad \leq \quad 1-\lambda,$$

which equivalent to

$$\sum_{\varkappa=2}^{\infty}([\varkappa]_{\mathfrak{q}}-\lambda)a_{\varkappa}|\varsigma|^{\varkappa-1} \quad \leq \quad 1-\lambda,$$

and applying Theorem **??**, we have

$$|\varsigma| \leq \left\{ \frac{(1-\lambda)\Gamma_{\mathfrak{q}}(\gamma+1)\Gamma_{\mathfrak{q}}(\varkappa+\xi)\Big([\varkappa-1]_{\mathfrak{q}}(\vartheta([\varkappa]_{\mathfrak{q}}-\mathfrak{q})+\mathfrak{q}(1+\vartheta F|d|))+F|d|\Big)}{F|d|([2]_{\mathfrak{q}}-\lambda)\Gamma_{\mathfrak{q}}(\xi+1)\Gamma_{\mathfrak{q}}(\varkappa+\gamma)} \right\}^{\frac{1}{\varkappa-1}}.$$

Hence, the proof is complete. □

## 3. Conclusion

In this article, we introduce a new class of normalized analytic functions called $\mathcal{D}\bigoplus(\vartheta, F, d, \xi, \gamma; \mathfrak{q})$, which is associated with the modified $\mathfrak{q}$-Tremblay operator on the open unit disk $\diamond$. We investigate the necessary conditions for functions belonging to the subclass $\mathcal{D}\bigoplus(\vartheta, F, d, \xi, \gamma; \mathfrak{q})$, as well as the growth and distortion bounds, the convolution condition, the radii of starlikeness, and the neighborhood problems involving the $\mathfrak{q}$-analogue of a modified Tremblay operator for functions in this class.

Our results extend and generalize some of the known results in the literature on analytic functions. We believe that our findings will have useful applications in various areas of mathematics, such as complex analysis, geometric function theory, and applied mathematics.

In summary, this article contributes to the ongoing research in the field of complex analysis by providing a more profound understanding of the theory and applications of analytic functions. The results obtained in this article have the potential for future generalization through the utilization of post-quantum calculus and other q-analogues of the fractional derivative operator. Additionally, further research may be conducted to explore additional subclasses and their respective properties.

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Abdullah Alsoboh,
Philadelphia University, Amman, Jordan, phone: 00962770959697
*Email address*: aalsoboh@philadelphia.edu.jo

Waggas Galib Atshan,
University of Al-Qadisiyah, Diwaniyah, Iraq, phone: 009647830724519
*Email address*: Waggas.galib@qu.edu.iq