http://communications.science.ankara.edu.tr

Commun.Fac.Sci.Univ.Ank.Ser. A1 Math. Stat.
Volume 74, Number 1, Pages 162–169 (2025)
https://doi.org/10.31801/cfsuasmas.1537286
ISSN 1303-5991 E-ISSN 2618-6470

COMMUNICATIONS SERIES A1

Research Article; Received: August 22, 2024; Accepted: December 26, 2024

Modified fibonomial graphs

Gökçe Dicle KARAAĞAÇ[1] and Semih YILMAZ[2]
[1]Department of Mathematics, Faculty of Engineering and Natural Sciences, Kırıkkale University, 71450, Kırıkkale, TÜRKİYE
[2]Department of Actuarial Sciences, Faculty of Economics and Administrative Sciences, Kırıkkale University, 71450, Kırıkkale, TÜRKİYE

ABSTRACT. In this paper, a new graph type similar to binomial graphs is constructed using fibonomial coefficients. The spectrum of this new graph was obtained, the energy of the graph and the sum of the Laplacian eigenvalues are calculated. In addition, the connectivity feature of the graph is examined and the properties of the vertices forming the graph are revealed.

2020 Mathematics Subject Classification. 05C07, 05C40, 05C50.
Keywords. Graph, adjacency matrix, characteristic polynomial, eigenvalue, eigenvector, graph energy, spectrum, Laplacian matrix, Laplacian eigenvalues, closed walk.

1. Introduction

Binomial graphs were introduced by Peter R. Christopher and John W. Kennedy [1]. For $n \geq 0$, the binomial graph B_n has vertex set V_n and edge set E_n, where $|V_n| = 2^n$ and $\{v_i, v_j\} \in E_n$ if $\binom{i+j}{j} \equiv 1 \,(\text{mod}\, 2)$. The eigenvalues and eigenvectors of the adjancency matrices of the binomial graphs give important information about the closed walks in the binomial graphs. It was shown that the sum of the degrees is of the vertices in the binomial graph B_n is

$$\sum_{j=0}^{2^n-1} \deg(v_j) = \deg(v_0) + \sum_{j=1}^{2^n-1} \deg(v_j) = (2^n+1) + \sum_{j=0}^{n-1} \binom{n}{j} 2^j = 1 + \sum_{j=0}^{n} \binom{n}{j} 2^j = 3^n + 1.$$

Thus, the number of edges in binomial graph B_n is $\frac{1}{2}(3^n+1)$ [1].

The Fibonacci sequence, which has been widely studied, also holds an important place in graph theory. The Fibonacci sequence is defined by the $F_{n+1} = F_n + F_{n-1}$ relation, where $F_1 = 1$ and $F_2 = 1$. Similar to binomial coefficients, fibonomial coefficients are obtained with the help of Fibonacci numbers. Fibonomial coefficients are obtained as follows, for $1 \leq j \leq m$,

$$\begin{bmatrix} m \\ j \end{bmatrix}_F = \frac{F_m \; F_{m-1} \; \ldots \; F_{m-j+1}}{F_1 \; F_2 \; \ldots \; F_j}$$

where $\begin{bmatrix} m \\ 0 \end{bmatrix}_F = 1$ and $\begin{bmatrix} m \\ j \end{bmatrix}_F = 0$ for $m < j$.

If F_m in the numerator of the fraction is replaced by $F_j F_{m-j+1} + F_{j-1} F_{m-j}$, the following equation is obtained

$$\begin{bmatrix} m \\ j \end{bmatrix}_F = F_{m-j+1} \begin{bmatrix} m-1 \\ j-1 \end{bmatrix}_F + F_{j+1} \begin{bmatrix} m-1 \\ j \end{bmatrix}_F$$

[2].

In our work, we frequently use the Kronecker product operation of matrices to obtain the adjacency matrices. Kronecker product make it easier for us to calculate the eigenvalues of graphs. Let A be a

[1] gdkaraagac@gmail.com -Corresponding author; 0000-0003-2691-7274;
[2] syilmaz@kku.edu.tr; 0000-0003-3171-8763.

$m \times n$ matrix and B be a $r \times s$ matrix. Then, the Kronecker product between A and B is the block matrix $A \otimes B = [a_{i,j}B]$, where $A \otimes B$ is a $mr \times ns$ matrix [6].

The $n \times n$ adjacency matrix $A(G)$ of a graph G with n vertices is a binary matrix. The non-diagonal entry $a_{i,j}$ of the adjacency matrix A is 1 if the i and j vertices are adjacent, and 0 otherwise. Also the loop at vertex v_i in the graph corresponds to the diagonal element a_{ii} in the adjacency matrix.

The eigenvalues of a graph are important in determining the algebraic properties of the graph. Additionally, the sequence of these eigenvalues gives the spectrum of the graph. We denote the spectrum of graph G by $\Lambda(G)$. Let $\lambda_1, \lambda_2, ..., \lambda_n$ denote the eigenvalues of the graph G, so that the spectrum of G is

$$\Lambda(G) = \{\lambda_1, \lambda_2, ..., \lambda_n\}$$

[7]. Let G be a graph with n vertices. The energy of the graph G is obtained by summing up all absolute values of the eigenvalues of the graph. Also, the energy of the graph G is denoted by $E(G)$. Gutman [3], who has worked on the energy of graphs for many years, gave the definition of the energy of non-simple graphs as follows:

$$E(G) = \sum_{i=1}^{n} \left| \lambda_i - \frac{S}{n} \right|,$$

where

$$S = tr(A(G)) = \sum_{i=1}^{n} \lambda_i.$$

We also want to talk about the sum of the Laplacian eigenvalues of a graph. For this we must first define the Laplacian matrix. If $D(G) = diag(d_1, d_2, ..., d_n)$ is the diagonal matrix associated to G, where $d_i = \deg(v_i)$ for all $i = 1, 2, 3, ..., n$ the matrix $L(G) = D(G) - A(G)$ is called the Laplacian matrix and its spectrum is called the Laplacian spectrum of the graph G [9]. If $\mu_1, \mu_2, ..., \mu_n$ denote the eigenvalues of $L(G)$, then the sum of the Laplacian eigenvalues of G is defined as

$$S(G) = tr(L(G)) = \sum_{i=1}^{n} \mu_i$$

[4]. Similar to the modified binomial coefficients defined by Shiro Ando [5], the following relationship is used to define modified fibonomial coefficients,

$$\left\langle \begin{matrix} n \\ k \end{matrix} \right\rangle_F = F_{n+1} \cdot \begin{bmatrix} n \\ k \end{bmatrix}_F,$$

where

$$\left\langle \begin{matrix} n \\ 0 \end{matrix} \right\rangle_F = \left\langle \begin{matrix} n \\ n \end{matrix} \right\rangle_F = F_{n+1},$$

F_{n+1} is $n+1$ *th* Fibonacci number and

$$\left\langle \begin{matrix} n \\ k \end{matrix} \right\rangle_F = \left\langle \begin{matrix} n \\ n-k \end{matrix} \right\rangle_F.$$

Modified fibonomial coefficients can be represented by a triangle similar to Pascal's triangle in Figure 1. Using the modified fibonomial coefficients we obtained, we construct a new type of graph and call it modified fibonomial graphs.

1
1 1
2 2 2
3 6 6 3
5 15 30 15 5
8 40 120 120 40 8
13 104 520 780 520 104 13
21 273 2184 5460 5460 2184 273 21

FIGURE 1. Modified fibonomial triangle

2. Modified Fibonomial Graphs

For each nonnegative integer n, we define the modified fibonomial graph $\mathcal{F}_n$ to has vertex set $V_n = \{v_j : j = 0, 1, 2, ..., 3.2^n - 1\}$ and the edge set

$$E_n = \left\{ \{v_i, v_j\} : \left\langle \begin{array}{c} i+j \\ j \end{array} \right\rangle_F \equiv 1 \,(\mathrm{mod}\, 2) \right\}.$$

The adjacency matrix of $\mathcal{F}_n$ is defined as $A(\mathcal{F}_n) = [a_{i,j}]$, where

$$a_{i,j} \equiv \left\langle \begin{array}{c} i+j \\ j \end{array} \right\rangle_F (\mathrm{mod}\, 2).$$

Since $\left\langle \begin{array}{c} 0 \\ 0 \end{array} \right\rangle_F = 1$, the modified fibonomical graph F_n has only one loop at vertex v_0. $|V_n| = 3.2^n$ and for $j = 0, 1, ..., n$, F_n has $\binom{n+1}{j}$ vertices of degree 2^j and the vertex v_0 of degree $2^{n+1} + 1$. From here $|E_n| = \frac{1}{2}\left(3^{n+1} + 1\right)$.

$\mathcal{F}_0$, $\mathcal{F}_1$ and $\mathcal{F}_2$ modified fibonomial graphs and their adjacency matrices are given as follows.

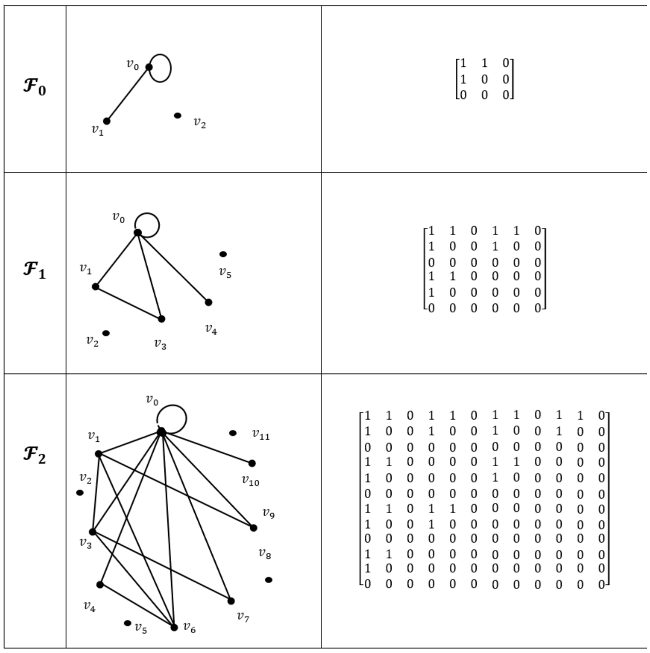

Figure 2. $\mathcal{F}_0$, $\mathcal{F}_1$ and $\mathcal{F}_2$ modified fibonomial graphs and their adjacency matrices

Here, by the Kronecker product, we obtain the adjacency matrices of the modified fibonomial graphs as follows. If we take $\mathcal{F} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$, then

$$A(F_0) = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A(\mathcal{F}_1) = \begin{bmatrix} 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} = \mathcal{F} \otimes A(\mathcal{F}_0)$$

and

$$A\left(\mathcal{F}_2\right)=\left[\begin{array}{cccccccccccc} 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]=\mathcal{F}\otimes A\left(\mathcal{F}_1\right)$$

From here, for each $n \geq 1$, the adjacency matrix of the modified fibonomial graph $\mathcal{F}_n$ is

$$A\left(\mathcal{F}_n\right)=\left[\begin{array}{cc} A\left(\mathcal{F}_{n-1}\right) & A\left(\mathcal{F}_{n-1}\right) \\ A\left(\mathcal{F}_{n-1}\right) & 0 \end{array}\right]=\mathcal{F}\otimes A\left(\mathcal{F}_{n-1}\right).$$

3. Eigenvalues of Modified Fibonomial Graphs

The eigenvalues of a graph, which provide information about the spectral structure of the graph, are calculated with the help of the adjacency matrix of the graph. The eigenvalues of the adjacency matrix of a graph are defined as the eigenvalues of the graph and so they are just the roots of the equation $\wp(\mathcal{F}_n;x)=0$. Since $A\left(G\right)$ is symmetric, its eigenvalues are all real. We denote them by $\lambda_1,\lambda_2,...,\lambda_n$ and the set of all eigenvalues is the spectrum of G, denoted by $Spec(G)$ [8].

Lemma 1. *[1] Let matrix A be an $n\times n$ square matrix with eigenvalues $\lambda_1,\lambda_2,...,\lambda_n$ and matrix B be an $m\times m$ square matrix with eigenvalues $\mu_1,\mu_2,...,\mu_m$ then the eigenvalues of the $nm\times nm$ matrix $A\otimes B$ are $\lambda_i\mu_j$ for $1\leq i\leq n$ and $1\leq j\leq m$.*

Theorem 1. *Let $\varphi=\frac{1+\sqrt{5}}{2}$. For each nonnegative integer n, the modified fibonomial graphs $\mathcal{F}_n$ has 3.2^n eigenvalues. More precisely, it has eigenvalue 0 with multiplicity 2^n and $(-1)^j.\varphi^{n+1-2j}$ with multiplicity $\binom{n+1}{j}$ for each $j=0,1,2,...,n+1$. Then we can write the spectrum of the modified fibonomial graph $\mathcal{F}_n$ as follows,*

$$\Lambda(\mathcal{F}_n)=\{0^{2^n},\left((-1)^j.\varphi^{n+1-2j}\right)^{\binom{n+1}{j}}:j=0,1,2,...,n+1\}.$$

Proof. Since $\mathcal{F}=\left[\begin{array}{cc} 1 & 1 \\ 1 & 0 \end{array}\right]$, the characteristic polynomial of $\mathcal{F}$ is

$$\wp(\mathcal{F};x)=x^2-x-1$$

so that $\Lambda(\mathcal{F})=\{\varphi,-\varphi^{-1}\}$. Additionally, since

$$A(\mathcal{F}_0)=\left[\begin{array}{ccc} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right],$$

the characteristic polynomial of $\mathcal{F}_0$ is

$$\wp(\mathcal{F}_0;x)=-x^3+x^2+1$$

so that $\Lambda(\mathcal{F}_0)=\{0,\varphi,-\varphi^{-1}\}$. Since

$$A(\mathcal{F}_1)=\mathcal{F}\otimes A(\mathcal{F}_0),\ \Lambda(\mathcal{F}_1)=\{0,0,-1,-1,\varphi^2,\varphi^{-2}\}$$

is obtained. We know that since $A(\mathcal{F}_n)=\mathcal{F}\otimes A(\mathcal{F}_{n-1})$, it is easy to see that the spectrum of $\mathcal{F}_n$ can be written in the form

$$\Lambda(\mathcal{F}_n)=\{0^{2^n},\left((-1)^j.\varphi^{n+1-2j}\right)^{\binom{n+1}{j}}:j=0,1,2,...,n+1\}.$$

□

4. Energy of the Modified Fibonomial Graphs

In this section, we calculate the energy of the non-simple the modified fibonomial graph of $\mathcal{F}_n$.

Theorem 2. *Let $\mathcal{F}_n$ be a modified fibonomial graph with 3.2^n vertices. If $E(\mathcal{F}_n)$ denotes the energy of the modified fibonomial graph $\mathcal{F}_n$, then*

$$E(\mathcal{F}_n) = \sum_{i=1}^{3.2^n} \left| \lambda_i - \frac{1}{3.2^n} \right|.$$

Proof. Gutman introduced the energy of a graph as

$$E(G) = \begin{cases} \sum_{i=1}^{n} |\lambda_i| & \text{if } G \text{ is a simple graph,} \\ \sum_{i=1}^{n} \left|\lambda_i - \frac{S}{n}\right| & \text{otherwise} \end{cases}$$

in his previous studies. Here we used the abbreviation $S = tr\left(A(G)\right) = \sum_{i=1}^{n} \lambda_i$. Summing up all the eigenvalues of $\mathcal{F}_0$ yields $S = 1$.

Since $A(\mathcal{F}_n) = \mathcal{F} \otimes A(\mathcal{F}_{n-1})$ and trace of the Kronecker product of matrices is the product of traces, sum of eigenvalues of $\mathcal{F}_n$ is

$$\sum_{i=1}^{3.2^n} \lambda_i = \sum_{j=0}^{n} (\frac{\varphi^2 - 1}{\varphi}).(-1)^j.\varphi^{n-2j} = 1.$$

□

Remark 1. *Since $\lim_{n\to\infty} \frac{1}{3.2^n} = 0$, the energy of $\mathcal{F}_n$ tends to*

$$\sum_{i=1}^{3.2^n} |\lambda_i|$$

when $n \to \infty$. Since all non-zero eigenvalues are in the form $(-1)^j.\varphi^{n+1-2j}$ with multiplicity $\binom{n+1}{j}$, we have

$$E(F_n) \cong \sum_{i=1}^{3.2^n} |\lambda_i| = \sum_{k=1}^{n+1} \left| \binom{n+1}{k} \varphi^{n+1-k} \varphi^{-k} \right| = \left(\varphi + \varphi^{-1}\right)^{n+1} = (\sqrt{5})^{n+1}$$

5. Connectivity of the Modified Fibonomial Graphs

Theorem 3. *In the modified fibonomial graphs, the vertex v_{3k+2} is isolated for all $k = 0, 1, 2, ..., 2^n - 1$.*

Proof. The adjancency matrix of $\mathcal{F}_n$ is defined as $A(\mathcal{F}_n) = [a_{i,j}]$, where

$$a_{i,j} = \left\langle \begin{matrix} i+j \\ j \end{matrix} \right\rangle_F (\bmod 2)$$

Here we must show that

$$a_{3k+2,l} \equiv 0 \; for \; k = 0, 1, 2, ..., 2^n - 1 \; and \; l = 0, 1, 2, ..., 3.2^n - 1.$$

Since

$$\left\langle \begin{matrix} 3k+l+2 \\ l \end{matrix} \right\rangle_F = F_{3(k+1)+l} \cdot \begin{bmatrix} 3k+l+2 \\ l \end{bmatrix}_F,$$

we can write

$$\begin{aligned} \left\langle \begin{matrix} 3k+l+2 \\ l \end{matrix} \right\rangle_F &= (F_{l+1} \cdot F_{3(k+1)} + F_l \cdot F_{3k+2}) \begin{bmatrix} 3k+l+2 \\ l \end{bmatrix}_F \\ &= (F_{l+1} \cdot F_{3(k+1)} + F_l \cdot F_{3k+2}) \frac{F_{3k+l+2} \cdot F_{3k+l+1} \ldots F_{3(k+1)}}{F_1 \cdot F_2 \ldots F_l} \\ &= (F_{l+1}.F_{3(k+1)}) \frac{F_{3k+l+2} \cdot F_{3k+l+1} \ldots F_{3(k+1)}}{F_1 \cdot F_2 \ldots F_l} \\ &\quad + F_l \cdot F_{3k+2} \frac{F_{3k+l+2} \cdot F_{3k+l+1} \ldots F_{3(k+1)}}{F_1 \cdot F_2 \ldots F_l} \end{aligned}$$

Since $F_{3(k+1)}$ is even, we have

$$\left\langle \begin{matrix} 3k+l+2 \\ l \end{matrix} \right\rangle_F \equiv 0 (\bmod 2)$$

$$a_{3k,l} \equiv 1 \; and \; a_{3k+1,l} \equiv 1 \; for \; k = 0,1,2,...,2^n-1 \; and \; l = 0,1,2,...,3.2^n-1$$

$$\begin{aligned}
\left\langle \begin{matrix} 3k+l \\ l \end{matrix} \right\rangle_F =& F_{3k+l+1} \, . \left[\begin{matrix} 3k+l \\ l \end{matrix} \right]_F \\
=& (F_{3k} \, . \, F_{l+2} + F_{3k+1} \, . \, F_{l+1}) \left[\begin{matrix} 3k+l \\ l \end{matrix} \right]_F \\
=& (F_{3k} \, . \, F_{l+2} + F_{3k+1} \, . \, F_{l+1}) \frac{F_{3k+l} \, . \, F_{3k+l-1} \, ... \, F_{3k+1}}{F_1 \, . \, F_2 \, ... \, F_l} \\
=& F_{3k} F_{l+2} . \frac{F_{3k+l} \, . \, F_{3k+l-1} \, ... \, F_{3k+1}}{F_1 \, . \, F_2 \, ... \, F_l} + F_{3k+1} \, . \, F_{l+1} . \frac{F_{3k+l} \, . \, F_{3k+l-1} \, ... \, F_{3k+1}}{F_1 \, . \, F_2 \, ... \, F_l} \\
\equiv& 1 \, (\bmod 2)
\end{aligned}$$

$$\begin{aligned}
\left\langle \begin{matrix} 3k+l+1 \\ l \end{matrix} \right\rangle_F =& F_{3k+l+2} \, . \left[\begin{matrix} 3k+l+1 \\ l \end{matrix} \right]_F \\
=& (F_{3k} \, . \, F_{l+3} + F_{3k+1} \, . \, F_{l+2}) \left[\begin{matrix} 3k+l+1 \\ l \end{matrix} \right]_F \\
=& (F_{3k} \, . \, F_{l+3} + F_{3k+1} \, . \, F_{l+2}) \frac{F_{3k+l+1} \, . \, F_{3k+l} \, ... \, F_{3k+2}}{F_1 \, . \, F_2 \, ... \, F_l} \\
=& F_{3k} F_{l+3} . \frac{F_{3k+l+1} \, . \, F_{3k+l} \, ... \, F_{3k+2}}{F_1 \, . \, F_2 \, ... \, F_l} + F_{3k+1} \, . \, F_{l+2} . \frac{F_{3k+l+1} \, . \, F_{3k+l+} \, ... \, F_{3k+2}}{F_1 \, . \, F_2 \, ... \, F_l} \\
\equiv& 1 \, (\bmod 2)
\end{aligned}$$

Consequently the vertex v_{3k+2} is the isolated vertex for all $k = 0,1,...,2^n-1$. □

Theorem 4. *The modified fibonomial graph $\mathcal{F}_n$ contains exactly one loop, the one on the vertex v_0.*

Proof. It is clear that $a_{0,0} = 1$.

Now let's take $i \neq 0$. We must show that $a_{i,i} = 0$.

$$\left\langle \begin{matrix} 2i \\ i \end{matrix} \right\rangle_F = F_{2i+1} \left[\begin{matrix} 2i \\ i \end{matrix} \right]_F$$

where similar to the central binomial coefficient, $\left[\begin{matrix} 2n \\ n \end{matrix} \right]_F$ can be taken as the central fibonomial coefficient. Since the central fibonomical coefficients are always even,

$$\left\langle \begin{matrix} 2i \\ i \end{matrix} \right\rangle_F \equiv 0 \, (\bmod 2) \, .$$

□

Corollary 1. *Let $\mathcal{F}_n$ be a modified fibonomial graphs with 3.2^n vertices, for each nonnegative integer n. The number of isolated vertices in this graph is 2^n.*

Proof. In the modified fibonomial graphs, the vertex v_{3k+2} is isolated for all $k = 0,1,2,...,2^n-1$.

Then vertices $v_2, v_5, ..., v_{3.2^n-1}$ are isolated.

Then the number of isolated vertices is obtained as 2^n. □

Corollary 2. *In the modified fibonomial graphs, the degree of vertex v_0 is $2^{n+1}+1$.*

Proof. In the modified fibonomial graphs, the vertex v_0 is connected to

$$2^{n+1}-1$$

different vertices. The vertex v_0 is connected to every vertice except isolated vertices.

For all

$$k=0,1,2,...,2^n-1,\ a_{0,3k}=1 \text{ and } a_{0,3k+1}=1$$

In that case,

$$\deg(v_0)=3.2^n-1-2^n+2=2^{n+1}+1.$$

□

6. Sum of the Laplacian Eigenvalues of the Modified Fibonomial Graphs

To obtain the sum of Laplacian eigenvalues of modified fibonomial graphs, we first examined the Laplacian matrices of modified fibonomial graphs. The Laplacian matrices of the modified fibonomial graphs $\mathcal{F}_0$, $\mathcal{F}_1$ and $\mathcal{F}_2$ are given below.

$$L(F_0)=\begin{bmatrix} 2 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$L(F_1)=\begin{bmatrix} 4 & -1 & 0 & -1 & -1 & 0 \\ -1 & 2 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & -1 & 0 & 2 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$L(F_2)=\begin{bmatrix} 8 & -1 & 0 & -1 & -1 & 0 & -1 & -1 & 0 & -1 & -1 & 0 \\ -1 & 4 & 0 & -1 & -1 & 0 & -1 & -1 & 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & -1 & 0 & 4 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 2 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & -1 & 0 & -1 & -1 & 0 & 4 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & -1 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Theorem 5. *Let L_n be an $n\times n$ Laplacian matrix with eigenvalues $\mu_1,\mu_2,...,\mu_n$ of the modified fibonomial graph $\mathcal{F}_n$. The sum of the Laplacian eigenvalues of the modified fibonomial graph $\mathcal{F}_n$ is*

$$S(\mathcal{F}_n)=\sum_{i=1}^{n}\mu_i=3^{n+1}$$

Proof. We know that the sum of the eigenvalues of the Laplacian matrix equal to sum of the diagonal etries of the Laplacian matrix.

$L(F_n)=D(F_n)-A(F_n)$ and the $a_{0,0}$ diagonal entry of the adjacency matrix is 1 and all other diagonal entry are 0. Also, $\mathcal{F}_n$ has $\binom{n+1}{j}$ vertices of degree 2^j, for $j=0,1,2,...,n$ and the vertex v_0 of degree $2^{n+1}+1$. Thus,

$$S(\mathcal{F}_n)=tr\left(L\left(F_n\right)\right)=\sum_{j=0}^{n+1}\binom{n+1}{j}2^j=3^{n+1}$$

□

7. Conclusion

In this article, we first examined the spectrum of the newly defined modified fibonomial graphs and tried to determine their relationships with similar graphs. Similar studies can be done on new graphs obtained using generalized Fibonacci numbers.

Author Contribution Statements The authors contributed equally to this work.

Declaration of Competing Interests The authors declare no competing interests.

References

[1] Christopher, P. R., Kennedy, J. W., Binomial graphs and their spectra, *The Fibonacci Quarterly*, 35 (1997), 48-53. DOI: 10.1080/00150517.1997.12429027

[2] Akbulak, M., Kale, A., Öteleş, A., On the connectivity properties and energy of fibonomial graphs, *Discrete Applied Mathematics*, 169 (2014), 1-8. DOI: 10.1016/j.dam.2013.12.007

[3] Gutman, I., The energy of a graph, *Ber. Math. Statist. Sekt. Forschungsz. Graz*, 103 (1978), 1-22.

[4] Pirzada, S., Ganie, H. A., On the Laplacian eigenvalues of a graph and Laplacian energy, *Linear Algebra and Its Applications*, 486 (2015), 454-468. DOI:10.1016/j.laa.2015.08.032

[5] Ando, S., Sato, D., Translatable and rotatable configurations which give equal product, equal GCD and equal LCM properties simultaneously, *Applications of Fibonacci Numbers*, 3 (1988), 15-26. DOI:10.1007/978-94-009-1910-5_3

[6] Vellanki, L. N., Madhu, J., Musa, D., On the Kronecker product of matrices and their applications to linear systems Via modified QR- algorithm, *International Journal Of Engineering And Computer Science*, 10(6) (2021), 25352-25359. DOI:10.18535/ijecs/v10i6.4600

[7] Godsil, C. D., Holton, D. A., McKay, B., The spectrum of a graph, *Combinatorial Mathematics V.(Proceedings of the Fifth Australian Conference, Held at the Royal Melbourne Institute of Technology, August 24 - 26, 1976.)*, Springer, 1977, 91-117.

[8] Lee, S., Yeh, Y., On eigenvalues and eigenvectors of graphs, *Journal of Mathematical Chemistry*, 12(1) (1993), 121-135. DOI:10.1007/BF01164630

[9] Merris, R., Laplacian graph eigenvectors, *Linear Algebra and its Applications*, 278 (1998), 22l-236. DOI: 10.1016/S0024-3795(97)10080-5