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Middle School Students' Approaches to Optimization: Insights into Mathematical Modeling and Real-World Problem Solving

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Abstract

This study investigates middle school students' perspectives on optimization problems in mathematics, focusing on their problem-solving processes and learning experiences. Optimization involves finding the best solution under specific constraints or maximizing/minimizing an objective function, a concept closely related to mathematical modeling. This process is designed to develop essential skills, such as logical reasoning, prediction, argumentation, and critical thinking, by framing real-world situations as mathematical challenges. Although existing research on students' problem-solving with optimization problems primarily involves high school or university students, recent studies emphasize the necessity of introducing optimization concepts earlier in education. This study administered four optimization problems to 16 middle school students to explore their experiences and opinions. Data were collected through observations, feedback forms, and individual interviews and analyzed using descriptive and content analysis methods. The findings reveal that students' limited prior exposure to optimization problems significantly contributes to their challenges. While students generally understand the optimization problems, their performance varies notably, especially in assumption-making and mathematical calculations. The study underscores the need for systematically integrating optimization problems into the middle school curriculum to enhance students' problem-solving skills and critical thinking. It suggests incorporating mathematical modeling with optimization tasks could improve students' abstraction and problem-solving abilities. Future research could investigate the effects of optimization problems on students' problem-solving skills and mathematical understanding.

Keywords: Mathematical education, mathematical modeling, optimization problems, middle school students.

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Introduction

Across all educational stages—from elementary school to higher education—a central goal of mathematics education is to cultivate students' skills in reasoning, problem-solving, and making connections between mathematical concepts and real-life contexts (MoNE, 2018; NCTM, 2000). These competencies are critical for academic achievement and vital for addressing complex challenges in everyday life and future careers. Mathematical modeling bridges abstract theory and practical application (Lesh & Doerr, 2003), enabling students to translate real-world situations into mathematical representations and devise solutions rooted in mathematical reasoning (Borromeo Ferri, 2018). Through this process, students gain a deeper understanding of the relevance and utility of mathematics beyond traditional problem sets.

Educational frameworks worldwide emphasize the importance of incorporating mathematical modeling into curricula, promoting student engagement in tasks that mirror real-world complexity (MoNE, 2018; CCSSI, 2010). This approach fosters a more applied, experiential form of mathematics learning that develops essential 21st-century skills, including critical thinking, creativity, and problem-solving (English, 2023). Within this framework, optimization problems are particularly valuable, offering practical applications across diverse domains (Taranto et al., 2024). Successfully navigating such problems requires the ability to construct, manipulate, and interpret mathematical models, reinforcing the role of mathematical modeling as a cornerstone of contemporary mathematics education.

Mathematical modeling bridges real-world problems and mathematical concepts, enabling students to enhance their analytical thinking and practical application skills (Borromeo Ferri, 2018; Lesh & Doerr, 2003). It fosters critical competencies such as logical reasoning, abstraction, and problem-solving while also integrating 21st-century skills like creativity and critical thinking (English, 2021; 2023). Through an iterative process involving phases like idealizing, mathematizing, solving, and validating, modeling strengthens students' ability to connect mathematics with real-world contexts (Blum & Leiss, 2007; Kaiser et al., 2013). Within this framework, optimization problems play a significant role by linking mathematical theory to practical applications, requiring students to find the best solutions under specific constraints (Taranto et al., 2024). These problems promote the development of heuristic strategies and approximation methods, even in complex scenarios, and achievements in solving them often depend on constructing and articulating representations, highlighting their educational value (Villegas et al., 2009; Lesh & Doerr, 2003).

The research emphasizes the benefits of optimization problems in motivating students, enhancing modeling competencies, and fostering meta-cognitive and problem-solving skills (Greefrath et al., 2022; Ferrarello et al., 2022). Although often absent in many curricula, these problems encourage reflection, discovery, and the real-world application of mathematical concepts, making them essential in mathematics education (Greefrath et al., 2022; Schuster, 2004). However, existing studies predominantly focus on high school and university contexts, leaving a significant gap in research on middle school students (Colajanni et al., 2023; Raffaele & Gobbi, 2021). This highlights the growing need to introduce optimization concepts earlier in education to build foundational skills in mathematics (Sandefur et al., 2022).

This study addresses a gap in mathematics education by exploring middle school students' perspectives on optimization problems, focusing on their opinions, perceptions, and experiences to understand how they engage with and relate to these concepts. By investigating these dynamics, the study sheds light on students' attitudes toward mathematics and their interactions with optimization-related ideas. Insights into students' perspectives will inform educators and curriculum designers on effectively integrating real-world optimization problems into mathematics instruction, as Greefrath et al. (2022) highlighted. Additionally, the study emphasizes the role of optimization in fostering critical thinking, problem-solving, and modeling skills, aligning with findings by Lehmann (2024) and Ferrarello et al. (2022) that demonstrate how optimization enhances student engagement and learning outcomes. By advocating for the inclusion of optimization in middle school mathematics curricula, the research bridges the gap between abstract mathematical concepts and practical, real-world applications (Taranto et al., 2024). Moreover, the findings may contribute to preparing students for future academic and STEM career challenges by demonstrating how optimization cultivates essential 21st-century skills such as creativity, logical reasoning, and critical thinking (English, 2021; Sokolowski, 2015). The study seeks to enhance students' appreciation of mathematics and sustain their motivation and long-term interest in the field. It addresses the need for earlier exposure to optimization concepts identified by Sandefur et al. (2022) and Raffaele and Gobbi (2021).

The research questions are as follows.

1. What are middle school students' views on optimization problems in mathematics?
2. What challenges and achievements do middle school students encounter while engaging in optimization problems, and how do they approach the problem-solving process for optimization problems?

Literature Review

Mathematical Modelling

Mathematical modeling bridges abstract theory and practical application, enabling students to translate real-world problems into mathematical terms. Through this process, learners develop analytical thinking skills and enhance their capacity to apply mathematical knowledge in meaningful, context-driven ways. Far from being limited to procedural problem-solving, modeling promotes a more profound understanding by illustrating how mathematical concepts can be leveraged to address tangible, everyday challenges (CCSSI, 2010; MoNE, 2018; NCTM, 2000). In modeling tasks, students engage in logical reasoning, prediction, and critical judgment, fostering their ability to relate mathematics to real-life contexts (Borromeo Ferri, 2017; Lesh & Doerr, 2003; Author, 2023). This strengthens mathematical competencies and supports the acquisition of essential 21st-century skills, including critical thinking, creativity, and problem-solving (English, 2021; 2023).

Recognizing its transformative potential, educational frameworks and scholarly research advocate for systematically integrating mathematical modeling into school curricula and teacher education programs. Borromeo Ferri (2018) emphasizes the importance of modeling to promote experiential and applied learning, a stance echoed by national and international curriculum standards (CCSSI, 2010; MoNE, 2018; NCTM, 2000). These frameworks highlight the need to move beyond theoretical instruction and provide opportunities for students to engage in authentic problem-solving tasks that mirror the complexity of real-world situations. Consequently, the inclusion of modeling not only reinforces mathematical understanding but also aligns with global educational priorities aimed at preparing students for the demands of contemporary life.

Optimization Problems in the Context of Mathematical Modelling

Optimization is an essential mathematical concept with practical applications in various fields, such as engineering (Taranto et al., 2024). Optimization involves finding the best solution or maximizing/minimizing an objective function under certain constraints. Gomez et al. (2006, p. 301) defined optimization as “doing the most with the least.” Another definition of optimization is “the process of finding the most effective or suitable value or condition” (Lockhart & Johnson, 1996). Optimization aims to achieve the “best” design or outcome given a set of constraints. To achieve this goal, mathematical models are necessary. In this context, optimization problems can also be considered mathematical modeling problems. Solving optimization problems involves creating and solving mathematical models, which illustrate the connections between mathematics and real-world situations. This approach emphasizes the practical relevance of mathematical concepts and encourages learners to engage with the subject meaningfully (Taranto et al., 2024). By tackling these problems, students can develop essential problem-solving skills and enhance their ability to construct and interpret mathematical models. Consequently, optimization problems arising from real-life situations are inherently linked to mathematical modeling (Taranto et al., 2024). They can play a valuable role in generating interest in mathematics and supporting the development of critical thinking and modeling competencies.

Therefore, the investigation into the theoretical concepts of optimization addresses critical questions, such as how optimization can foster the development of heuristic strategies and how students employ approximation methods when exact solutions are impractical. In this context, Sokolowski's (2015) study supports the hypothesis that immersing students in mathematical modeling activities can effectively challenge their preconceptions and enhance their understanding of core mathematical principles. Notably, the research identifies that the phases involving hands-on measurements and the creation of value tables were particularly influential in altering students' perspectives. Consequently, this structured data collection and analysis approach led students to uncover fundamental optimization principles, thereby illustrating the benefits of integrating measurement tasks into mathematics education.

Moreover, the study (Sokolowski, 2015) suggests that the difficulties students encounter with optimization problems are often tied to understanding the underlying mathematical concepts rather than issues with the mathematization process itself. These findings, therefore, reinforce the argument for a unified structure that combines scientific and mathematical modeling. This integration is particularly relevant given that optimization problems are crucial in STEM education, as they focus on developing formal models to tackle challenges across various fields, including economics, engineering, logistics, and transportation (Taranto et al., 2024). By engaging students in constructing and solving mathematical models, optimization problems demonstrate the practical relevance of mathematics in real-world scenarios. Thus, this connection enhances students' problem-solving and modeling skills and fosters a more profound interest in mathematics, making optimization a vital component of the broader STEM curriculum.

Villegas et al. (2009) found that success in solving optimization problems is closely linked to the ability to construct, use, and effectively articulate representations. This insight is particularly relevant to mathematical modeling, where the process involves translating real-life situations into mathematical models, manipulating these models, and interpreting results in the context of the original problem (Lesh & Doerr, 2003). Building on the findings of Villegas et al. (2009), which emphasize the critical role of constructing, utilizing, and articulating representations in solving optimization problems, this study delves into middle school students' perspectives on optimization. By exploring their opinions, perceptions, and experiences with these concepts, the study aims to illuminate the connections between students' attitudes toward optimization and their broader mathematical learning experiences. It also seeks to provide insights into how students interpret and engage with optimization-related ideas, further enhancing our understanding of their relationship with mathematics.

Mathematical modeling typically follows a cyclical process (Blum & Leiss, 2007; Geiger, 2011; Lesh & Lehrer, 2003). The modeling process (figure 1) is characterized by several phases: (a) idealizing, structuring, and simplifying; (b) mathematizing, translating into mathematical language, and formulating; (c) mathematical work, operating, and solving; and (d) interpreting and validating (Kaiser et al., 1996, Blum, 1996). The ability to skillfully navigate these phases is crucial for effective modeling, as it enables students to move fluidly between real-world scenarios and their mathematical abstractions, ensuring that models accurately reflect problems and lead to viable solutions (Lesh & Doerr, 2003). This modeling cycle highlights that while modeling skills are multifaceted and iterative, they are essential for translating mathematical theory into practical problem-solving strategies.

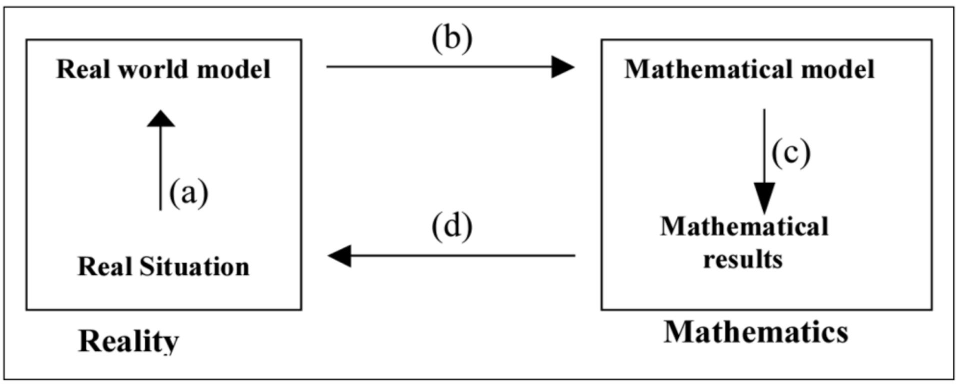

Figure 1. Modeling Cycle (from Kaiser, 1996, p. 68; Blum, 1996, p. 18)

Optimization problems hold substantial educational value and can significantly enhance students' mathematical learning across different educational levels. Taranto et al. (2021) demonstrated that implementing a modeling pathway involving optimization tasks is manageable for typical higher secondary students and leads to positive outcomes across multiple domains. Their findings revealed increased motivation—as students willingly worked through breaks—and the development of meta-cognitive skills and productive work habits. Notably, the pedagogical goal of helping students better understand the world around them was also achieved. These results support the inclusion of optimization problems in regular mathematics instruction, even if not on a daily basis, as they contribute to attaining broad educational objectives. Similarly, Ferrarello et al. (2022) emphasize the educational benefits of integrating optimization problems into middle school curricula. Their study highlights how hands-on problem-solving encourages students to apply mathematical concepts through modeling and fosters connections between real-world contexts and abstract mathematical strategies. This experiential, “learning by doing” approach promotes discovery, critical thinking, validation, and argumentation—key elements of meaningful mathematical engagement.

Reinforcing these findings, Greefrath et al. (2022) show that optimization problems—though not formally part of many curricula—offer rich opportunities for learning and engagement. Their research demonstrates that these problems enhance mathematical reflection and motivation by embedding mathematical thinking within real-world contexts, leading to intensive and diverse modeling processes. Particularly within discrete mathematics, optimization tasks support the development of process-related competencies and sustain student interest through challenging and relevant content. These studies highlight the importance of introducing optimization problems early in the educational journey. Doing so not only cultivates students' modeling abilities but also fosters a more engaged and motivated approach to mathematics, aligning with 21st-century educational goals.

Building on this, Lehmann (2024) explored how mathematical modeling can further support the development of students' algorithmic thinking. The study revealed that mathematical modeling activates crucial modeling competencies for fostering algorithmic thinking. Specifically, students dedicated considerable time to analyzing information and creating abstract representations of problems, identifying relevant variables, and making assumptions—critical factors in their achievements. This underscores how mathematical modeling enhances abstraction skills and problem-solving abilities and complements the findings of Greefrath et al. (2022) by demonstrating the practical benefits of modeling in educational settings. These findings align with the aim of this study, which investigates middle school students' perspectives on optimization problems. By focusing on middle school students' opinions and experiences, this study seeks to understand how they engage with optimization concepts, including the abstraction and problem-solving skills emphasized by Lehmann. Furthermore, it explores how these perspectives influence their attitudes toward mathematics and interactions with optimization-related ideas, providing a deeper understanding of the role of modeling and optimization in mathematics education.

The existing literature reveals a substantial gap in research concerning optimization-based educational initiatives for middle and higher secondary school students, as Raffaele and Gobbi (2021) identified. Schuster (2004) investigated the integration of combinatorial optimization into mathematics and computer science education for high school students. However, a comprehensive review of the literature indicates that most studies addressing students' problem-solving processes in optimization are restricted to high school (Colajanni et al., 2023; Taranto et al., 2024) or university contexts (Villegas et al., 2009). This highlights a need for research focused on introducing and teaching optimization concepts at middle school levels, especially given the growing significance of optimization in contemporary life (Sandefur et al., 2022). Recent studies advocate for a more detailed examination of how optimization can be effectively taught from an earlier age, emphasizing that foundational skills for understanding and applying optimization should be established well before students enter higher education (Greefrath et al., 2022; Sandefur et al., 2022).

Method

The study employed a qualitative method to comprehensively explore the research questions and facilitate in-depth data analysis (Creswell, 2013). A case study approach was explicitly chosen to understand middle school students' perspectives on optimization problems, often regarded as mathematical modeling challenges. The case study method is particularly effective in examining complex phenomena (Merriam, 1998; Yin, 2009). By employing this approach, the study aimed to capture the students' experiences and viewpoints on optimization problems in a detailed and contextually grounded manner. Additionally, conducting in-depth case studies of individual students or small groups allowed for a thorough understanding of their interactions with optimization problems and learning trajectories. This method provided researchers with a nuanced and practical understanding of the behaviors of small groups and the dynamics of specific processes, making it a valuable tool for exploring the challenges and perspectives of middle school students as they engage with optimization problems.

Study Group

This study conducted four optimization activities with 16 high-achieving seventh-grade students at a middle school in the Western Black Sea Region in Türkiye. High-achieving students were selected for this study to provide a deeper insight into the potential of optimization problems to challenge and develop advanced mathematical thinking and problem-solving skills. These students, often possessing a stronger foundation in mathematics and critical thinking, are well-positioned to engage with complex tasks requiring abstraction, representation, and iterative reasoning—key optimization components. The study aims to explore their strategies, achievements, and challenges in approaching real-world mathematical modeling problems by focusing on high-achieving students. This focus allows researchers to identify best practices and instructional approaches that can later be adapted to support a broader range of learners in understanding optimization concepts. Furthermore, selecting high-achieving students ensures a rigorous examination of the activities' educational potential, as these students are more likely to engage deeply with the mathematical and conceptual demands of optimization tasks.

These activities were carefully designed using the mathematical modeling process, drawing from the mathematics applications textbook the Ministry of National Education provided. The activities were carefully selected to align with real-world contexts, ensuring they were both engaging and relevant to students' experiences. The design process emphasized problems that required students to utilize critical thinking, reasoning, and representation skills while promoting an understanding of the iterative nature of modeling. Each activity was structured to challenge students' abilities to mathematize real-world scenarios, formulate mathematical models, and validate their solutions effectively. The activities were also chosen to cater to different facets of optimization, ranging from simple parameter adjustments to more complex scenarios, ensuring a comprehensive exploration of optimization concepts. Special attention was given to clarity in problem statements, accessibility of necessary data, and opportunities for collaborative problem-solving.

The implementation occurred during after-school hours during four-day modeling sessions, allowing for an uninterrupted and focused learning environment. Voluntary student participation ensured high engagement and intrinsic motivation. Students worked in groups of three or four, while all groups operated simultaneously in the same environment. This setup facilitated the exchange of ideas and diverse approaches to problem-solving. The problems were presented on paper and through PowerPoint slides on a smartboard to ensure clarity and accessibility. Researchers closely observed the students during the modeling sessions, taking detailed field notes to capture their interactions, problem-solving strategies, and challenges encountered. These observations provided valuable insights into the students' thought processes and engagement with the modeling activities, contributing to the study's data analysis and findings.

During the activities, the students were supervised by two prospective mathematics teachers, who adhered to the principle of minimal assistance. These prospective teachers were in the final semester of their four-year teacher education program, equipping them with theoretical and practical mathematics education expertise. Their involvement as official researchers in a project supported by a national funding source, titled *Optimization from the Perspective of Middle School Students*, further underscored their role in ensuring the study's academic rigor and alignment with research objectives.

To encourage independent modeling activities, no additional instructions were given to the students. Each modeling activity lasted approximately one hour. After the activities, individual interviews were conducted to gather the students' feedback. These interviews lasted 10-15 minutes for each student and took place in a quiet and comfortable environment, with questions designed to be quickly answered by the students. The responses were recorded using audio recording devices, ensuring accuracy and enabling detailed analysis for the research study. The interviews were conducted by three researchers involved in the project.

The research was conducted by the regulations recommended by the ethics committee and was based on the principle of voluntary participation. Ethics committee permissions were obtained. Informed consent forms were obtained from the parents of the participating students. In the research analysis, the students' names were not used; instead, they were assigned codes such as S1, S2, ..., and S16 to ensure confidentiality.

Data Collection Tools

In this study, four optimization activities were implemented and carefully designed to align with the principles of the mathematical modeling process. These activities were thoughtfully structured to engage students in problem-solving and critical thinking, promoting the application of mathematical concepts in real-world contexts. A detailed activity plan with a pedagogical approach and instructional strategies was employed to facilitate meaningful engagement with the optimization problems was implemented (see Appendix).

Four carefully designed optimization problems were administered to students. These problems were carefully crafted by the researchers within the scope of the research project, incorporating principles of mathematical modeling and the distinctive features of optimization problems. The design process involved ensuring that each problem adhered to the key phases of the mathematical modeling cycle—understanding the problem, idealizing and mathematizing, solving, validating, and interpreting results.

To reflect the essential characteristics of optimization problems, the activities were framed around real-world scenarios that required students to determine the best possible solution under specific constraints. These constraints were integral to the problem design, as they aimed to challenge students to explore heuristic strategies, apply approximation methods, and justify their solutions. The problems were designed to develop students' mathematical reasoning and problem-solving skills and encourage critical thinking and collaborative efforts, which are vital in tackling complex optimization tasks.

The researchers drew on existing literature, such as Borromeo Ferri (2018), and adapted modeling problems to align with the project's educational objectives. Each problem was structured to promote engagement. The design process involved iterative refinement based on pilot testing to ensure the problems were appropriately challenging and accessible for seventh-grade high-achieving students. Two of these problems are illustrated in Figure 2.

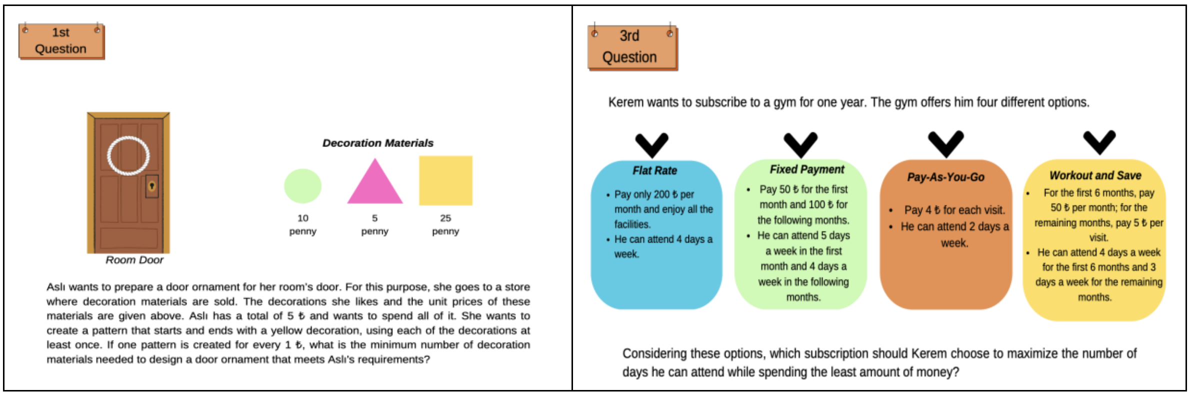

Figure 2. Two Optimization Problems Administered to Students

After completing all optimization activities, individual interviews were conducted with each student. These interviews were conducted in a quiet environment conducive to student comfort, ensuring they felt at ease. During interviews, students responded to questıons such as "Can you share your thoughts on the problems you worked on during this study?, Have you encountered similar problems in your previous experiences?, How do these problems differ from the ones you have solved before?, Were there any particular aspects of the problems that you found challenging? Could you explain those parts in more detail?, What aspects of these problems did you enjoy, and what were those you didn't?" Audio recordings were made during the interviews to capture the richness of their responses. The questions were posed conversationally, allowing students to express their thoughts openly and without pressure. After the interviews, the audio recordings were carefully transcribed into written documents for detailed analysis.

The same set of questions was provided in written form to address the possibility that students might have omitted or forgotten specific details during the interviews. Students were asked to reflect on and respond to these questions in writing, offering additional insight. This approach helped triangulate the data and ensured no important information was overlooked. This comprehensive method marked the conclusion of the data collection process. The study's data consisted of multiple sources, including the students' worksheets, the transcribed information gathered from the interviews, and their written responses to the post-activity questions. By collecting data from diverse formats, the study ensured a robust and multidimensional understanding of the students' experiences and problem-solving processes throughout the optimization activities.

Data Analysis

The qualitative data collected during the study underwent a rigorous descriptive analysis guided by the principles of the mathematical modeling process as outlined by Borromeo Ferri (2018). This analysis involved a detailed examination of the data in relation to the key stages of mathematical modeling, including problem formulation, model development, solution strategies, and interpretation of results, as conceptualized within Borromeo Ferri’s framework. The process ensured that the students' responses were evaluated within established modeling practices. This allowed a deeper understanding of their engagement with the mathematical tasks and problem-solving approaches (Table 1).

Table 1.
Framework for Mathematical Modeling Competencies (+ excellent; * doubtful; − misconceptions/incorrect) adopted from Borromeo Ferri (2018, p. 105).

Task 1					
	Understanding the Problem	Making Assumptions	Mathematizing	Working Mathematically	Validation/Critical Thinking
Student 1	+	*	-	*	*
Student 2	*	*	-	*	*
Student 3	+	+	+	+	+
Student 4	*	+	*	*	+
…					
…					

This framework facilitated the observation of variations in students' modeling competencies over time. It provided valuable insights into individual performance within group settings and offered an alternative assessment method to traditional testing by evaluating students' modeling abilities.

The content analysis method was employed in this study to evaluate students’ responses to interview questions systematically. Content analysis involves identifying codes, categories, and themes from qualitative data. Significant patterns and points within the data were initially extracted as codes, which were then organized into broader categories to develop overarching themes. This method aligns with Creswell’s (2013) approach to qualitative analysis, allowing for a structured examination of the students’ problem-solving strategies, challenges, and perceptions of optimization problems.

Several strategies were implemented to ensure the validity and reliability of the analysis. Validity was ensured through peer debriefing, where other researchers reviewed the coding process to confirm that the themes accurately represented the students’ experiences. Reliability was maintained through inter-coder reliability, where two researchers coded the data independently and then compared the results for consistency (Merriam, 1998). The data analysis process followed a straightforward, step-by-step procedure. First, the interview transcripts were thoroughly read to understand the students' responses and identify recurring patterns. Next, initial codes were generated inductively, based on the data, and deductively, using existing mathematical modeling and optimization literature. As the coding progressed, these codes were refined and grouped into categories, such as “Prior Experience with Similar Problems” and “Discomforts with Problems’ Characteristics.” Finally, overarching themes were developed from these categories to reflect key patterns across the data, such as " Difficulty with Problem Comprehension." This structured coding and theme development approach ensured the content analysis was systematic and comprehensive.

Findings

This section presents the study's findings under two main headings: Students' Views on Optimization Problems and Students’ Achievements and Challenges in the Problem-Solving Process.

Students Views on Optimization Problems

This subsection summarizes students' perspectives on optimization problems, including their prior experience with similar problems, perceived difficulty level, and overall reflections. Table 2 presents students' thoughts on optimization problems, categorized into two main areas: their prior experience with similar issues and the perceived difficulty level.

Table 2.
Prior Experience with Optimization Problems and Difficulty Level

Categories	Common Responses	Students	Frequency
Prior Experience with Similar Problems	I have solved similar problems before.	S-1, S-2, S-7, S-10, S-11	n=5
	I have not solved similar problems before.	S-3, S-4, S-5, S-6, S-8, S-9, S-12, S-13, S-14, S-15, S-16	n=11
Difficulty Level Experienced During Problem-Solving	I struggled while solving the problems.	S-1, S-2, S-4, S-6, S-8, S-9, S-12, S-14, S-15, S-16	n=10
	I did not struggle while solving the problems.	S-3, S-5, S-7, S-10, S-11, S-13	n=6

The findings indicate that most students (n=11) had never encountered optimization problems before, while a smaller group (n=5) had prior experience with such problems. In terms of difficulty, a substantial number of students (n=10) reported struggling while solving the problems, regardless of previous experience. On the other hand, six students (n=6) found the problems manageable and did not struggle during the solving process.

> S-15: I haven't solved them before, but I think these problems exist. I struggled while solving the questions, but I enjoyed solving these problems.

> S-2: These are good questions that make children think more. They are questions that train you to solve complex problems and make them more accessible. I had solved similar problems before.

> S-1: It challenged me a bit. Since there wasn't a clear answer, I kept thinking. But some were easy. In the rabbit question, I visualized everything in my mind. Question 3 seemed to have a single answer, but it was clear that the others had different answers. I solved similar questions in primary school, and I am solving them now as well.

Student comments further illustrate these findings. For example, S-15 acknowledged the existence of optimization problems in daily life and enjoyed solving them despite struggling. That implies that S-15 recognized the prevalence of optimization problems in real-life situations and expressed a positive attitude toward solving them despite encountering difficulties during the problem-solving process. This suggests that the student knows the practical relevance of optimization problems and is engaged in the intellectual challenge they present. The struggle during the problem-solving process further highlights the complexity of optimization tasks and the cognitive effort required, which may provide insight into the areas where the student needs additional support or development.

S-2 indicated that these questions promote critical thinking and contribute to developing problem-solving skills, suggesting that engaging with such problems can enhance cognitive abilities related to analytical reasoning and strategic thinking. In contrast, S-1 described experiencing difficulty with the problems, primarily due to the open-ended nature of specific questions. This response highlights the cognitive challenges posed by tasks that lack clear, predefined solutions, which may require higher-order thinking skills.

Table 3 provides an analysis of the aspects students liked and disliked about the optimization activities. The results of the aspects students liked are categorized into four main areas: Problem Language/Writing Style, Enjoyment and Engagement with Problems, Thoughts on Skills Developed by the Problem, and Real-life Related Features of Problems. The results of the aspects students disliked are categorized into three main areas: Discomforts with Problem Characteristics, Preferences in Problem Features, and Absence of Dislikes.

Table 3.
Liked and Disliked Aspects of Problems

	Categories	Common Responses	Frequency
Liked Aspects	Positive Aspects of Problem Presentation (Clarity, Visuals, and Creativity)	I liked that the mathematical concepts were understandable. I liked the visuals in the problem. I liked that it was written in an understandable language. The storytelling in the problem was very nice. I liked that the activity was colorful.	n=10
	Enjoyment and Engagement with Problems	The problems were fun. The problem was easy. There was a variety of questions. The problem was fun and logical.	n=11
	Skills Developed by Problems	I liked that it encouraged thinking differently. I liked that it developed our imagination and ability to notice visuals. I found it fun to reach multiple answers by solving problems through trial and error.	n=3
	Real-life Related Features of Problems	I liked that the questions were based on real-life situations. I liked that the problems involved a lot of steps and were complex. I liked that the element of chance was a bit more prominent.	n=5
Disliked Aspects	Discomforts with Problem Characteristics	I did not like that the questions were long. I found it challenging that the problems had multiple answers instead of a single solution. I did not like that the problems required too many steps.	n=8
	Preferences in Problem Features	I would prefer not to use length measurement units. I did not like that there was no shape in the second question.	n=3
	Absence of Dislikes	There was nothing I didn’t like. The problems were quite nice. There was nothing I didn’t like. Honestly, they were all perfect.	n=6

Students expressed a strong appreciation for several aspects of the optimization activities. Regarding problem language and writing style, they valued clarity and aesthetic appeal, noting that the engaging storytelling, colorful presentation, and understandable language made the problems enjoyable. Positive responses highlighted the pleasure of problem-solving through trial and error and the effective incorporation of visuals and relatable contexts. For instance, S-13 said, “Solving problems through trial and error made reaching multiple answers enjoyable.” S-13 mentioned the enjoyment of solving problems through trial and error and the appealing narrative and visuals.

Regarding problem structure, students favored the inclusion of multiple solutions, which they found logical and reflective of real-life scenarios. The complexity and variety of steps in the problems were praised for maintaining engagement and providing a challenging experience. For instance, S-2 and S-4 said S-2: “I enjoyed that the problems were very challenging, did not have a single answer, included multiple solution paths, and required many operations." and S-4: "I liked the inclusion of geometric shapes in the problems." As noted by S-2, who enjoyed the difficulty and multiple solution paths, and S-4, who appreciated the geometric shapes included in the problems, the complexity and variety of steps in the problems were praised for maintaining engagement and providing a challenging experience.

Students also valued the problems for fostering creative thinking and visual awareness, appreciating the chance to explore multiple answers and develop problem-solving skills. Additionally, the format of the questions, which featured real-life contexts and a higher element of chance, was positively received for making the problems more relatable and engaging. Sample student excerpts are as follows. S-14: "I liked that the parts I enjoyed encouraged different ways of thinking. Overall, it was good." S-1: "One of the aspects I appreciated was that it enhanced our imagination and ability to notice visuals."

On the other hand, many students expressed dissatisfaction with certain aspects of the optimization activities (see sample responses in Table 3), particularly the length and complexity of the problems. Problems that required numerous steps and offered multiple solutions were especially challenging, leading to difficulties for some students. The length of the questions and the complexity of calculating length measurement units were specific sources of discomfort. Additionally, a few students had discrepancies for particular problem features, such as the use of length measurement units and the absence of particular shapes. Despite these issues, a minority of students reported no dislikes, indicating overall satisfaction with the problems.

Students' Achievements and Challenges in the Problem-Solving Process

Table 4 analyzes students' problem-solving skills in different optimization tasks. Although all students (n = 16) grasped the problem well, their performance varied in later stages. Some students excelled in making assumptions and mathematization, while others faced challenges, particularly in mathematical operations and critical thinking, where difficulties in result validation were common.

Table 4.
Evaluation of Students During the First Optimization Activity (+ Excellent; * Suspect; – Conceptual Errors/Mistakes)

	Understanding the Problem				Making Assumptions				Mathematization				Working Mathematically				Validation/Critical Thinking			
Prb# / Std#	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
S-1	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	*	+	+	*
S-2	+	+	+	+	-	-	+	+	+	+	+	+	-	+	+	+	-	*	+	*
S-3	+	+	+	+	+	-	+	-	+	+	+	+	+	+	+	+	+	*	+	*
S-4	+	+	+	-	-	-	+	-	-	-	+	-	-	-	+	-	-	-	+	-
S-5	+	+	+	+	+	-	+	*	+	+	+	+	+	-	+	+	+	-	+	*
S-6	+	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
S-7	+	+	+	+	-	+	+	*	+	+	+	+	+	+	+	+	+	+	+	*
S-8	+	+	+	+	+	+	+	+	+	+	+	+	-	-	+	+	*	+	+	+
S-9	+	-	+	+	-	-	*	*	-	-	+	+	-	-	+	+	*	-	*	*
S-10	+	+	+	+	+	-	+	*	+	+	+	+	+	+	+	+	*	-	+	*
S-11	+	+	+	+	+	-	+	*	+	+	+	+	+	+	+	+	*	-	+	*
S-12	+	+	+	+	-	-	+	+	-	+	+	+	-	+	+	+	*	*	+	*
S-13	+	+	+	+	-	+	+	+	+	+	+	+	+	+	+	+	*	+	+	*
S-14	+	+	+	+	+	-	-	*	+	-	+	*	-	-	+	*	*	-	*	*
S-15	+	+	+	+	+	*	+	*	+	+	+	+	-	+	+	+	*	*	+	*
S-16	+	+	+	+	+	+	+	+	+	+	+	+	-	+	+	+	-	+	+	*

As presented in Table 4, all students (n = 16) demonstrated a comprehensive understanding of the problem, indicating that they effectively grasped the problem's requirements and context. However, a divergence in performance was observed in subsequent stages of the problem-solving process.

Specifically, 9 out of 16 excelled in making assumptions, while 7 out of 16 struggled, suggesting that nearly half of the students faced challenges in formulating accurate or relevant assumptions. In the mathematization phase, 12 out of 16 students effectively translated the problem into mathematical terms, whereas four students encountered difficulties.

When applying mathematical methods, nearly half of the students succeeded in working mathematically, with 9 out of 16 facing challenges, highlighting a significant struggle in executing the required mathematical operations. During the validation and critical thinking stage, 9 out of 16 exhibited uncertainty or potential issues in their results, and four students made significant conceptual errors. Only three students displayed excellence in critical thinking and validation. These findings indicate that while students generally understood the problem well, they experienced substantial difficulties in later stages, particularly in making assumptions, executing mathematical operations, and validating their solutions. The data underscores the need for targeted instructional support to enhance students' mathematical reasoning and critical thinking skills.

As presented in Table 4, the analysis of the solution papers from the second optimization activity revealed that while most students understood the problem, a considerable number faced substantial challenges during the assumption-making stage. This difficulty was particularly evident in students like S-4, S-5, and S-14, who either struggled to establish relevant assumptions or did not engage in this step. The assumption-making phase, critical for simplifying and structuring the problem mathematically, presented a barrier to effective problem-solving for these students. Their difficulty in this area may indicate a need for further instruction or guidance in identifying and formulating key assumptions that can facilitate the progression to subsequent stages in the modeling process.

This difficulty appeared to stem from a misconception that the problem had only one solution approach. For instance, when tasked with designing a garden with a perimeter of 316 meters to maximize the area, 50% of the students mistakenly believed that the garden should exclusively be rectangular, overlooking the fact that a square is also a particular type of rectangle. Additionally, some students incorrectly insisted that the garden be hexagonal. These misconceptions highlighted substantial deficiencies in area calculation. Despite a general understanding of the problem, students faced challenges in making assumptions and the mathematical working stages. The most significant difficulties were observed during the critical thinking phase, where students struggled to interpret their results effectively. It was found that students who excelled in the critical thinking stage provided explanations rooted in their initial assumptions.

Upon examining the students’ responses to the third optimization problem, it was found that most students achieved their highest achievement during the mathematization stage. Remarkably, students demonstrated the most tremendous achievement in clearly presenting their assumptions for solving this problem, with 6th-grade students showing a higher overall achievement level. In contrast, the review of answer papers for the third optimization activity indicated that while students were successful in the mathematization stage and recognized possible solution methods, they encountered prominent issues related to calculation. Specifically, students struggled with ratios and proportions during the calculation process, further underscoring the need for targeted instruction.

The results of the fourth optimization activity, as presented in Table 4, indicate that students achieved their highest achievement at the problem-understanding stage. An analysis of the students' solution papers revealed that most students relied on a trial-and-error approach to solve the problem. While most students demonstrated a strong understanding of the problem, as evidenced by their "+" ratings, their performance in making assumptions varied. Some students showed proficiency in this area, whereas others had mixed or questionable performance. In the mathematization stage, most students were successful, receiving favorable ratings, and many also performed well in the mathematical working stage, though some encountered issues or gaps. The most significant variability was observed in the validation and critical thinking stage, where some students excelled, but others struggled, indicating deficiencies in these areas. These findings suggest that while students generally excelled in understanding the problem and mathematization, they faced challenges in assumption-making and critical thinking, highlighting areas where further instructional support may be necessary.

A comparative analysis of the results across the optimization activities reveals distinct patterns in students' problem-solving abilities at various stages. In the second optimization activity, students generally comprehended the problem well but struggled significantly with making assumptions. This challenge was partly due to a misconception that the problem had only one solution approach. Additionally, there were notable deficiencies in area calculations, particularly when students were required to consider different geometric shapes. In contrast, students in the third optimization activity succeeded in the mathematization stage but encountered difficulties in the calculation process, specifically with ratios and proportions. This indicates a gap in their ability to apply mathematical concepts accurately during problem-solving.

In the fourth optimization activity, students achieved the highest achievement at the problem-understanding stage, like the earlier activities. However, the reliance on a trial-and-error approach was prominent. While many students performed well in the mathematization and mathematical working stages, challenges persisted in the assumption-making and validation stages. The critical thinking stage revealed significant variability, with some students excelling while others struggled, indicating deficiencies in interpreting and validating their results.

While students consistently demonstrated an ability to understand the problem across all activities, their performance varied in subsequent stages. The assumption-making stage emerged as a recurring challenge, particularly in the second and fourth activities, whereas the third activity highlighted difficulties in mathematical calculations. The critical thinking stage consistently presented challenges. Table 5 summarizes students' achievement on optimization problems, divided into positive and negative perspectives.

Table 5.
Students' Achievement on Optimization Problems

Categories	Common Responses	Students	Frequency
Positive Perspectives	Code 3. It strengthened my perspective on mathematics. Code 5. I saw that people can come up with different solutions to problems. Code 1. I learned that not all questions have definite answers. Code 4. It helped me understand the difference between the math taught in school and real-world math problems.	S-1, S-2, S-3, S-7, S-8, S-9, S-10, S-11, S-13, S-14, S-15, S-16	n=12
Neutral Perspectives	Code 2. It didn’t change my perspective.	S-4, S-5, S-6, S-12	n=4

Most students (n=12) expressed positive views, while a smaller group (n=4) held neutral opinions. Among those with positive perspectives, students reported several benefits from engaging with optimization problems. Many students (Code 3) felt that the problems strengthened their understanding of mathematics. Others (Code 5) appreciated seeing that people can develop different solutions to the same problem, which broadened their problem-solving approach. Some students (Code 1) found it valuable to learn that not all questions have definite answers, which added complexity and interest to their mathematical thinking. Additionally, several students (Code 4) recognized that these problems helped them understand the difference between the math taught in school and real-world mathematical applications. Following are sample excerpts from the students.

> S-8: Yes, it positively changed my perspective. Normally, I enjoy solving math problems, but after a while, I get bored. Sometimes, I think about not reading or solving them. However, I liked that there could be more than one answer because I automatically lose interest in math when I can’t find the answer to a problem. But here, there is no such thing as not finding an answer because I can give multiple answers.

> S-13: Yes, my perspective changed a bit. I learned that a situation could have more than one answer. I realized I could solve this by trying different approaches. It also created a different perspective for me in my daily life.

> S-16: I saw that the questions have different solutions, making it fun.

> S-7: Yes. These questions are not like other questions; they are easier, and you don't feel like you could get the wrong answer, so you don't feel pressure, making it easier to reach the correct solution.

S-8 shared that the problems positively changed their perspective by allowing for multiple answers, which made math more engaging and reduced the pressure of finding a single correct solution. S-13 mentioned that understanding a situation could have various answers, and applying different methods to reach a solution contributed to a shift in their daily life perspective. S-16 found the variety of solutions fun, and S-7 appreciated that the problems were less intimidating, making it easier to arrive at the correct answer. On the other hand, four students with neutral perspectives (Code 2) indicated that the problems did not change their view of mathematics.

These excerpts indicate that most students gained valuable insights and a more positive outlook on mathematics through their experiences with optimization problems. However, a small subset of students did not experience a shift in their perspective.

The analysis of challenges students experienced with optimization problems, as summarized in Table 6, reveals two primary categories of difficulty: Difficulty with Problem Comprehension and Complexity of Solution Process.

Table 6.
Challenges Students Experienced with Optimization Problems

Categories	Common Responses	Students	Frequency
Difficulty with Problem Comprehension	I struggled to understand the problem situation. Using length measurement units made it difficult for me to understand the question.	S-4, S-11, S-10, S-15	n=4
Complexity of Solution Process	The solution steps were too long. I got confused while solving it. The many solution strategies and operations required made it difficult for me.	S-2, S-3, S-5, S-7, S-12, S-13, S-14, S-16	n=8
No Difficulty Experienced	I didn't struggle at all.	S-1, S-8, S-15	n=3

A notable subset of students encountered difficulties in understanding the problem situation, impeding their ability to engage with optimization problems effectively. Specifically, Student S-4 said, "Understanding the problem situation and many solution strategies made it difficult for me." S-4 reported challenges in grasping the problem scenario, highlighting difficulties with the problem context and the required solution strategies. Similarly, S-16 stated that "Mathematical concepts seemed confusing. The visuals in the first question were confusing." S-16 found the mathematical concepts and accompanying visuals confusing, further exacerbating the problem.

Some students mentioned that they experienced difficulties because the mathematical concepts were not understandable. These issues underscore the critical role that clarity and effective presentation of mathematical concepts play in facilitating students' comprehension and problem-solving abilities in optimization tasks. A significant proportion of students (n=8) expressed the challenges related to the complexity of the solution processes in optimization problems. For instance, S-2 said, "Question 3 was the part I struggled with because the operations were lengthy. Performing all four premises one by one was challenging for me." Student S-2 reported that the problem-solving steps were excessively lengthy, leading to confusion and difficulty managing multiple solution strategies.

This complexity, characterized by numerous steps and operations, emerged as a significant barrier to effectively solving the problems. Student S-13 noted, "Question 3 was the part I struggled with because the operations were lengthy. Performing all four premises one by one was challenging for me." Thus, S-13 stated that Question 3 was particularly problematic due to the extensive operations required, making performing all four premises sequentially challenging. These findings highlight that the intricacy of the solution processes significantly impedes students' ability to navigate and solve optimization problems successfully.

A smaller group of students (n=3) reported no difficulties with the optimization problems, suggesting that these individuals either found the problems straightforward or possessed strategies that mitigated the common challenges faced by others. For instance, students S-1, “I didn’t struggle with any part,” and S-8, “I didn’t struggle at all,” indicated that they did not experience any struggle with the problem-solving tasks. These findings highlight that difficulties with optimization problems are primarily associated with the structure and complexity of the problems themselves. Specifically, issues related to problem comprehension and the extensive nature of solution processes are significant challenges for students.

Discussion, Conclusion, and Suggestions

This study aimed to explore middle school students' perspectives on optimization problems in mathematics, focusing on their opinions, perceptions, and the impact on their learning experiences. This study aimed to explore middle school students' perspectives on optimization problems in mathematics, focusing on their opinions, perceptions, and the impact on their learning experiences. The study sought to investigate how students engage with optimization tasks and the strategies they employ. By examining their attitudes and thoughts about optimization, the research also aimed to uncover the underlying factors influencing students' willingness to tackle complex mathematical challenges. Furthermore, the study intended to identify students' challenges and achievements during their problem-solving processes, shedding light on difficulties in applying mathematical concepts to real-world scenarios. An essential research component was understanding how students conceptualize optimization, particularly in making assumptions, mathematization, and validating their solutions. By exploring these aspects, the study hoped to provide insights into the role of optimization in middle school mathematics education, including the potential for enhancing students' problem-solving skills, critical thinking, and overall mathematical understanding. Through these findings, the research offered recommendations for improving the teaching and learning of optimization concepts, ensuring that students not only grasp mathematical principles but also develop a deeper appreciation for their relevance and application in everyday life.

The findings from this study reveal that middle school students' limited prior exposure to optimization problems significantly contributed to the challenges they faced in solving them. This observation is consistent with the literature, which underscores the importance of early exposure and targeted instructional support to help students navigate the complexities of optimization concepts. Ferrarello et al. (2022) highlighted that integrating optimization problems into the middle school curriculum can provide substantial educational value by fostering a hands-on, experiential learning approach. This "learning by doing" method deepens students' understanding of mathematical concepts and enhances their ability to apply them through critical thinking, verification, and argumentation.

Moreover, the recurring challenges observed in the assumption-making and critical-thinking stages suggest a need for enhanced instructional strategies. Lehmann (2024) supports this notion by demonstrating that mathematical modeling effectively activates critical competencies for fostering algorithmic thinking, which is closely related to optimization skills. Students who engage in mathematical modeling spend considerable time analyzing information, creating abstract representations, and making assumptions, which are crucial for achievement in optimization problems. Therefore, integrating mathematical modeling with optimization can further enhance students' abstraction skills and problem-solving abilities.

The study also found that while students generally demonstrated an ability to understand optimization problems, their performance varied significantly in later stages, particularly in assumption-making and mathematical calculations. These findings align with Greefrath et al. (2022), who emphasized that optimization problems offer significant educational benefits despite not being traditionally included in the curriculum. By engaging students in real-world applications and diverse modeling processes, optimization problems stimulate mathematical reflection and motivation. This reinforces the idea that incorporating optimization problems into the curriculum can help students develop essential process-related competencies and maintain their interest in mathematics.

Given that the students in this study are high achieving, the findings provide valuable insights into their problem-solving abilities and highlight areas where even advanced students face challenges. While all participants demonstrated a solid understanding of the problem requirements and context, the study revealed significant performance variability as students progressed through the stages of problem-solving. Despite their strong foundational grasp, approximately 25% of the students struggled during the mathematization phase. This indicates that even high-achieving students can encounter difficulties transitioning from conceptual understanding to formal mathematical representation. This suggests that although students can comprehend the problem context, they may face challenges in mathematical structuring, a critical skill for optimization tasks.

Furthermore, while some students exhibited proficiency in making assumptions and translating the problems into mathematical terms, others struggled with mathematical operations and critical thinking. The difficulties observed in subsequent stages, such as mathematical operations and validation, point to potential gaps in deeper mathematical understanding or insufficient strategies for checking the accuracy of their solutions. These challenges may be compounded by the complex, open-ended nature of optimization problems, which require technical proficiency and reflective and critical thinking. These findings are consistent with Sokolowski's (2015) observations that students' difficulties with optimization problems often stem from challenges with underlying mathematical concepts rather than the mathematization process itself. This aligns with the notion that even high-achieving students may struggle with certain fundamental aspects of mathematical modeling, particularly when faced with the need for validation and critical reflection.

The integration of scientific and mathematical modeling is particularly crucial, given the role of optimization problems in STEM education. Taranto et al. (2024) highlight the importance of optimization in developing formal models for real-world challenges across various fields such as economics, engineering, logistics, and transportation. By engaging students in constructing and solving mathematical models, optimization problems illustrate the practical relevance of mathematics and enhance students' problem-solving and modeling skills. This engagement can foster a more profound interest in mathematics, reinforcing its significance within the broader STEM curriculum.

Based on the findings and discussion from this study, future research could investigate strategies for systematically integrating optimization problems into the middle school curriculum, which could provide insights into how different levels of exposure to these concepts impact students' problem-solving skills and overall mathematical understanding. Additionally, research should focus on integrating mathematical modeling with optimization problems to enhance students' abstraction and problem-solving skills, evaluating how well modeling activities complement optimization tasks and influence students' abilities to make assumptions, construct models, and validate results. Finally, exploring how optimization problems can be more effectively linked to real-world applications within the STEM curriculum could reveal how contextualizing these problems in fields such as engineering affects student engagement and understanding.

In conclusion, this study highlights the importance of incorporating optimization problems into middle school education. The findings underscore the need for targeted interventions and instructional strategies to address the challenges students face with optimization problems, ultimately contributing to more effective and engaging mathematics modeling. While studies like those by Schuster (2004) and Colajanni et al. (2023) have explored optimization in high school and university contexts, research is needed targeting middle school students. Raffaele and Gobbi (2021) and Sandefur et al. (2022) argue that introducing optimization concepts at an earlier age is crucial, given the growing significance of optimization in contemporary life. Establishing foundational skills in optimization before students reach higher education can better prepare them for the complex challenges they will face in their academic and professional futures.

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Ethics statement: In this study, we declare that the rules stated in the "Higher Education Institutions Scientific Research and Publication Ethics Directive" are complied with and that we do not take any of the actions based on "Actions Against Scientific Research and Publication Ethics." At the same time,

we declare that there is no conflict of interest between the authors, which all authors contribute to the study, and that all the responsibility belongs to the article authors in case of all ethical violations.

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Appendix

Optimization Activity Plan (adapted from Borromeo Ferri (2018))

Time	Section	Activity	Materials / Social Interaction
5 min	Introduction	The researchers introduce the optimization concept to the students before the activity begins. Students are briefed about optimization problems through a researcher-led presentation.	Researchers' presentation
5-10 min	Group Formation & Activity Sheet Distribution	The researchers divide the class into groups of 3-4 students, depending on the number of participants. The activity sheet, illustrated below, is distributed to each group. The researchers provide brief information about the activity, explaining that it includes modeling questions and requires the students to approach drawing and calculations differently. Equal participation from every group member is emphasized, and any questions about the activity are addressed.	Activity sheet, PowerPoint presentation
20-25 min	Group work	Students are given a total of 40 minutes to complete the activity. During this phase: The researchers act as a guide but do not provide direct answers to questions aimed at finding the correct solution. Students collaborate within their groups to solve the problem using problem-solving skills and discuss their ideas with group members. Cooperative learning is encouraged, and enough time is allocated to allow students to explore solutions. The researchers monitor the groups, offering motivational feedback, hints, and comments as needed. Strategic questions are asked to encourage students to question their solutions. At the end of the activity, students prepare for the presentation phase, engaging in discussions within their groups to compare solutions and organize their findings. Cooperative learning is emphasized in this preparation stage. Finally, groups present their solutions to the class, facilitating knowledge exchange and comparing approaches.	Activity sheet, group collaboration
20 min	Presentations	Groups are selected to present their solutions, not solely based on having the correct answer but also on innovative or effective problem-solving strategies. During presentations: Other groups can add comments or suggestions to enhance the discussion. The focus is on generating diverse ideas rather than determining right or wrong answers.	Students' presentations
10 min	Validation/reflection	Students reflect on their solution steps on the activity sheet, even if they are not fully completed. The focus is on the problem-solving process rather than correctness. The researchers facilitate a whole group discussion for students to share their thoughts and reflections.	Whole group discussion
5 min	Feedback	After the presentations, the researchers collect students' thoughts about the activity. Feedback forms allow Students to share their discoveries, likes, or challenges during the activity.	Feedback forms