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A COINTEGRATION RELATIONSHIP BETWEEN CRYPTOCURRENCIES AND FINANCIAL INSTRUMENTS UNDER STRUCTURAL BREAKS[1]

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Abstract

The aim of this research is to investigate the long-term relationships among the dollar exchange rate (TRY/USD), gold (GAU/USD), the Borsa Istanbul 100 Index (BIST 100) and the prices of Bitcoin (BTC/USD), Ethereum (ETH/USD), and Binance Coin (BNB/USD). Since the series contain structural breaks, Fourier unit root tests were used to model the structural breaks. As the method of this study, the relationships between variables in the long term were examined by using Fourier Shin (FSHIN) and Shin (1994) (SHIN) cointegration tests. The findings of this study showed that cryptocurrencies are cointegrated among themselves under structural breaks in the long term; investment instruments are cointegrated among themselves. In addition, as a result of this study, it was determined financial instruments and cryptocurrencies do not move in along over time under structural breaks.

Keywords: *Cryptocurrency, Dollar Exchange Rate, Gold, BIST 100, Fourier Cointegration Test.*
JEL Classification: *G15, C22, C58*

YAPISAL KIRILMALAR ALTINDA KRİPTO PARALAR İLE FİNANSAL ENSTRÜMANLAR ARASINDA BİR EŞBÜTÜNLEŞME İLİŞKİSİ

Öz

Bu çalışmanın amacı Bitcoin (BTC/USD), Ethereum (ETH/USD) ve Binance Coin (BNB/USD) ile dolar kuru (TRY/USD), altın (GAU/USD) ve Borsa İstanbul 100 Endeksi (BİST 100) arasındaki uzun dönemli ilişkilerin araştırılmasıdır. Çalışmada kullanılan serilerde yapısal kırılmalar tespit edildiği için durağanlık sınamasında yapısal kırılmaları modelleyen Fouirer fonksiyonlu birim kök testleri kullanılmıştır. Bu çalışmanın yöntemi olarak Fourier Shin (FSHIN) ve Shin (1994) (SHIN) eşbütünleşme testleri kullanılarak uzun dönemde değişkenler arası ilişkiler incelenmiştir. Bu çalışmanın bulguları, uzun dönemde yapısal kırılmalar altında kripto para birimlerinin kendi aralarında; yatırım araçlarının da kendi aralarında eşbütünleşik olduğunu göstermiştir. Ek olarak, bu çalışmanın sonucunda, yapısal kırılmalar altında kripto paralar ile yatırım araçlarının uzun dönemde birlikte hareket etmediği belirlenmiştir.

Anahtar Kelimeler: *Kripto Para, Dolar Kuru, Altın, BİST 100, Fourier Eşbütünleşme Testi.*
JEL Sınıflandırması: *G15, C22, C58*

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1. Introduction

Along with rapidly increasing technological developments, significant changes and transformations have been experienced in financial systems. One of these important innovations is cryptocurrencies developed with blockchain technology. Blockchain is a technology that carries records of all past cryptocurrency transactions, including the creation of new cryptocurrencies, and enables cryptocurrency transactions in circulation (Berentsen and Schär, 2018, p.5). With blockchain technology-based digital assets, payments can be made faster and less costly, while alternative investment opportunities to traditional financial products have emerged (Ammous, 2018, p.187).

Bitcoin, a product of blockchain technology and the pioneer of cryptocurrencies, emerged with the work of Nakamoto (2008), who explained the application of this digital currency. Since Bitcoin was first traded, the number of cryptocurrencies and cryptocurrency exchanges has increased significantly. As of March 2024, there are 730 cryptocurrency exchanges. Furthermore, the total market capitalization of the current cryptocurrency market is approximately $2.4 trillion (https://coinmarketcap.com/).

Table 1: **Top 10 Cryptocurrencies by Market Capitalization**

Cryptocurrency	Abbreviation	Market Capitalization ($)
Bitcoin	BTC	1,119,003,469,093
Ethereum	ETH	491,474,197,483
Binance Coin	BNB	93,637,363,247
Tether	USDT	73,800,607,152
Solana	SOL	64,594,733,039
Cardano	ADA	60,598,287,256
Ripple	XRP	50,638,141,259
Polkadot	DOT	39,030,998,997
USD Coin	USDC	34,423,649,601
Dogecoin	DOGE	31,059,527,562

References: (Coinmarket, 2023).

Table 1 shows the ranking of the 10 cryptocurrencies with the highest market capitalization on 15.11.2021. According to the data in the table, Bitcoin is the cryptocurrency with the highest market capitalization. Ethereum is the 2nd cryptocurrency with the highest market capitalization after Bitcoin since 2013 when it was traded.

As a result of the cryptocurrency usage studies conducted in many countries including Turkey in 2023, it was determined that Turkey ranked 12th in cryptocurrency usage (https://www.chainalysis.com/). In addition, cryptocurrencies are among the top 10 investment instruments preferred by investors in Turkey (FODER, 2023). These statistics reveal that cryptocurrencies have an important position as an investment instrument in Turkey and that the level of usage is high.

Although the popularity of cryptocurrencies as an investment instrument is increasing day by day in Turkey, the most preferred investment instruments for investors to utilize their savings are foreign currency, gold and stocks (https://www.tasarrufegilimleri.com/). This study investigates the long-run relationships between the most preferred investment instruments of Turkish investors, namely foreign exchange, gold and stock markets, and the top 3 digital currencies that have the largest share in the cryptocurrency market. The variables used in this study and the method used constitute the originality of the study. Studies that include the variables used in this study together and studies that use Fourier structural break tests as research methodology are very few.

This study consists of five sections. The first section is introduction of the study. The second section summarizes the studies examining the relationship between various investment instruments and cryptocurrencies. The third section provides information about the data and methodology of the study. The fourth section presents the findings of the analysis. In the Evaluation and Conclusion section, the findings of the study are discussed and recommendations are made.

2. Literature

The following summarizes first the studies investigating the relationship between cryptocurrencies and various investment instruments in the world, and then the studies conducted in Turkey on the subject.

Yermack (2013) examined the relationship between Bitcoin and selected macroeconomic factors. The study concluded that Bitcoin has a very low correlation with selected exchange rates and gold price. Baek and Elbeck (2015) investigated the impact of Bitcoin prices on the Standard and Poor's 500 index (S&P 500). As a result of the regression analysis, it is concluded that Bitcoin prices do not have any effect on the S&P 500 index. Alana et al. (2020) investigated the pairwise cointegration relationships between selected cryptocurrencies and six stock market indices (S&P 500 Composite, S&P GSCI Commodity Total Return, VIX, S&P Bond Index, S&P GSCI Gold Total Return and US Nominal Dollar Broad Index) and found no cointegration relationship between cryptocurrencies and selected stock market indices. Sam et al. (2020) investigated the long-run relationship between Bitcoin prices and the Indian stock index, exchange rate and inflation rate. As a result of the analysis, a co-movement was found between the relevant variables in the long run. Chang et al. (2021) investigated the cointegration relationship between Bitcoin prices and Google search volume index (SVI), VIX and S&P 500 Index. As a result of the VAR model, a cointegration relationship was found between Bitcoin prices and VIX in the long run. Gul et al. (2023) examined the relationship between inflation, exchange rate, gold and Gross Domestic Product (GDP) and Bitcoin. As a result of Johansen cointegration analysis, no co-movement was found between the relevant variables in the long run. Krakower (2023) investigated the relationship between Bitcoin and inflation expectations. The Johansen cointegration test revealed that inflation expectations and Bitcoin returns move together in the long run.

Dirican and Canöz (2017) investigated the long-run co-movement between Bitcoin and selected stock market indices using the Autoregressive Distributed Lag (ARDL) method. As a result of the study, while there is a relationship between Bitcoin and US stock markets, there is no relationship between Financial Times Stock Exchange 100 (FTSE100), Nikkei 225 (N225) and BIST100 index. İçellioğlu and Öztürk (2017) examined the relationship between Bitcoin and selected exchange rates using Johansen cointegration analysis. According to the results of the study, it was not found that the relevant variables act in common. Çütçü and Kılıç (2018) analyzed the relationship between the dollar exchange rate and Bitcoin using the Maki test. As a result of the study, it was concluded that the relevant variables are common between the dollar exchange rate and Bitcoin prices. Kılıç and Tütçü (2018) investigated the relationship between Bitcoin and the BIST100 index. Engle-Granger and Gregory-Hansen cointegration test findings indicate that the relevant variables do not move together in the long run. Yıldırım (2018) investigated the relationship between Bitcoin and gold prices with Johansen cointegration analysis. As a result of the study, it is observed that there is no relationship between Bitcoin prices and gold prices in the short run. Kuzucu (2019) analyzed the relationship between the dollar exchange rate and Bitcoin returns using the ARDL method. As a result of the study, a positive relationship was found between the variables. İşcan (2020) examined the long-term relationship between Bitcoin and the dollar exchange rate, BIST 100 index and interest rate with the Johansen cointegration test. As a result of the analysis, it was observed that Bitcoin and selected variables do not move together in the long run. Telek and Şit (2020) examined the long-run relationship between Bitcoin and gold and foreign exchange using the ARDL Frontier test. As a result of the analysis, it was determined that there is a cointegration relationship between Bitcoin and selected variables. Gürbüz and Zeren (2021) examined the relationship between Bitcoin and the Dollar and Euro exchange rates using the Maki method. As a result of the study, it was determined that there is a long-term positive relationship between Bitcoin and Dollar and Euro exchange rate returns. Koç and Çaykara (2021) examined the relationship between selected cryptocurrencies, Euro and Dollar exchange rate returns with the Engle-Granger cointegration test. The results of the analysis showed that the variables move together in the long run. Yiğit and Yiğit (2021) investigated the relationship between Bitcoin price and BIST100 index, gold prices and US dollar exchange rate with Johansen cointegration test. As a result of the study, no long-run relationship was found in the variables. Toprak and Kubar (2023) investigated the relationship between Bitcoin and Ethereum with selected country stock markets (Mexico, Indonesia and South Korea). With the Fourier cointegration analysis, the existence of co-movement of variables in the long run was determined.

As seen in the literature, there is a general view that there is a relationship between cryptocurrencies and various financial instruments (Yermack, 2013; Çütçü and Kılıç, 2018; Kuzucu, 2019; Sam et al., 2020; Telek and Şit, 2020; Chang et al., 2021; Koç and Çaykara, 2021; Krakower, 2023). However, most of these studies have used traditional methods that do not model breaks. The number of studies using methods that test structural breaks is very limited (Gürbüz and Zeren, 2021; Toprak and Kubar, 2023). In case of structural breaks in the series, using tests that do not include breaks in the analysis reduces the reliability of the results. There are many tests that model structural breaks. However, the Fourier function was chosen in this study because it better models both sharp and smooth breaks. Therefore, by using this method, it was aimed to increase the reliability of the analysis findings.

The cryptocurrencies selected in this study are the cryptocurrencies with the highest market and transaction volume during the analysis period. The financial instruments in which the relationship between the selected cryptocurrencies is analyzed are the financial instruments most commonly included in the portfolios of investors in Turkey. Therefore, the results of the study are expected to be comprehensive and representative of the whole. The results of the study may guide investors in Turkey who try to provide real returns and therefore evaluate their savings in financial instruments. Additionally, the results obtained from this study may provide insight into portfolio diversification strategies for investors investing in the cryptocurrency market, whose market value is increasing. It is expected that the study will contribute to the literature with these aspects.

3. Data and Methodology

3.1. Data

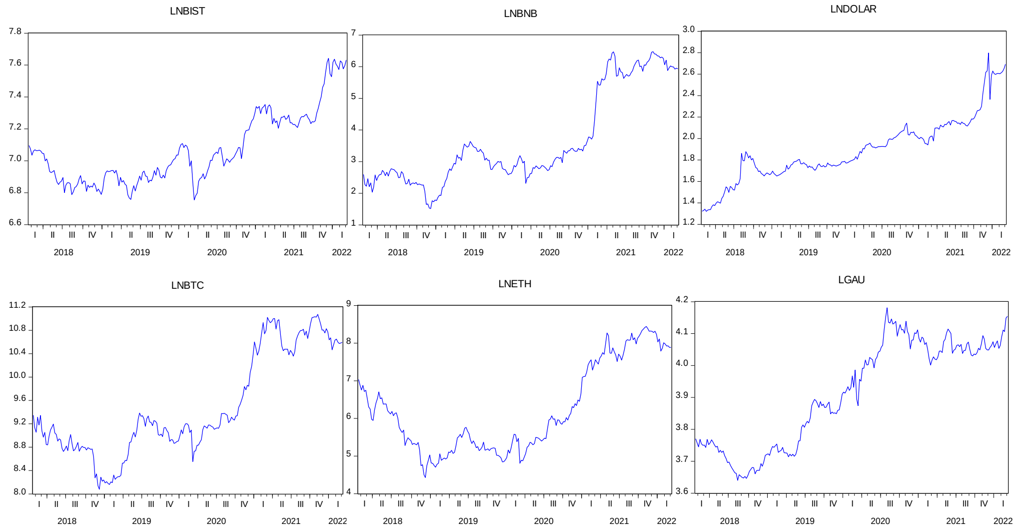
The study investigates the long-run relationship between selected cryptocurrencies and the dollar exchange rate, gold and the BIST 100 index. In this context, the cryptocurrencies used in the study are Bitcoin, Ethereum and Binance Coin. Variables and their abbreviations are shown in Table 2.

Table 2: **Variables**

Variable	Symbol/Unit	Abbreviation
BIST 100	BIST100	LNBIST
Binance coin	BNB/USD	LNBNB
Bitcoin	BTC/USD	LNBTC
Dollar	TRY/USD	LNDOLAR
Ethereum	ETH/USD	LNETH
Gold	GAU/USD	LNGAU

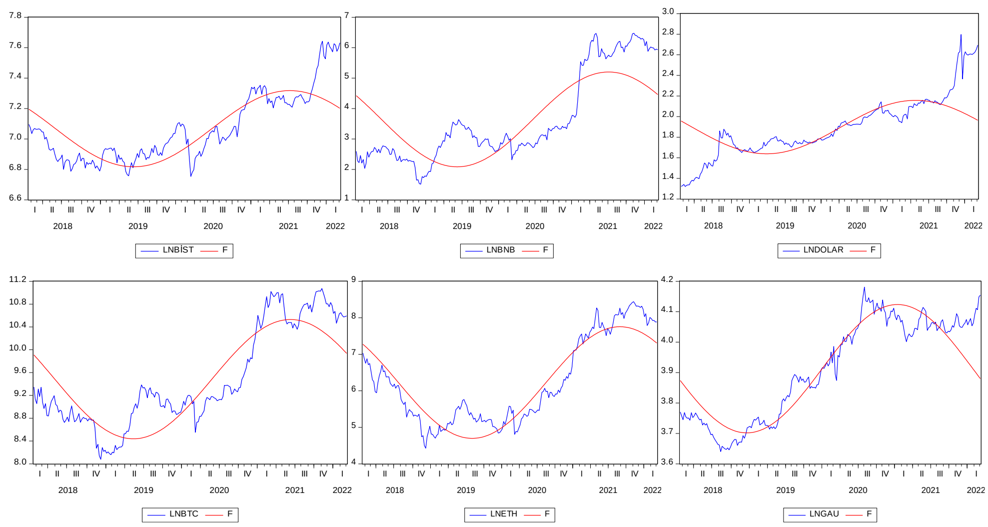
The cryptocurrencies given in Table 2 were selected because they had the highest market value and trading volume on the date of the study. In addition, their data is available without interruption, which is another reason for selection. The study was conducted using weekly data between 19.01.2018 and 11.03.2022. Data on selected cryptocurrencies, dollar exchange rate, gold and BIST 100 index were obtained from Investing official address (https://tr.investing.com/). All series are logarithmically transformed (ln) and the transformed series are denoted as LNBIST, LNBNB, LNBTC, LNDOLAR, LNETH and LNGAU. The graphs of the logarithmically transformed series are shown in Figure 1.

Figure 1: **Natural Logarithmized Series**



Graphs of the series with the Fourier function added after natural logarithmization are shown in Figure 2. The graphs of the logarithmic series (Figure 1) indicate that multiple and gradual structural breaks may be present.

Figure 2: **Series and Fourier Functions**



As seen in Figure 2, it is observed that the series are compatible with the added Fourier functions.

3.2. Method

3.2.1. Traditional Unit Root Tests

3.2.1.1. Augmented Dickey-Fuller Unit Root Test

In this study, both traditional stationarity tests and Fourier function stationarity tests are used. The simplest of the three test equations proposed by Dickey and Fuller (1979, 1981) for the Augmented Dickey-Fuller (ADF) unit root test is the equation:

$$\Delta y_t = \delta y_{t-1} + \sum_{i=1}^{k} a_i \Delta y_{t-i} + e_t \tag{1}$$

It is defined as follows. However, in order to model the series subject to the test, trend (t) and/or constant (μ) can be added when necessary:

$$\Delta y_t = \mu + \delta y_t + \sum_{i=1}^{k} a_i \Delta y_{t-i} + e_t \tag{2}$$

$$\Delta y_t = \mu + \beta t + \delta y_t + \sum_{i=1}^{k} a_i \Delta y_{t-i} + e_t \,. \tag{3}$$

In all three equations, the presence of a stochastic trend is determined by the failure to reject the null hypothesis (usually) against its alternative.

3.2.1.2. Dickey Fuller-General Local Squares Unit Root Test

Dickey Fuller-General Local Squares (DF-GLS) test developed by Elliott, Rothenberg and Stock (1996) is a modified version of the ADF test. The test is performed in two stages. In the first stage, the series is stripped of deterministic components as follows:

$$y_t^d = y_t - \tilde{\beta}' z_t \tag{4}$$

z_t , it is the vector containing all deterministic components. In the second stage, the deterministic components are removed (y_t^d) the classical ADF test is applied to the series using test equation (1).

3.2.1.3. Kwiatkowski-Phillips-Schmidt Shin Unit Root Test

In the method proposed by Kwiatkowski et al. (1992), the series has a deterministic trend (t) and/or constant (μ) whether the series is stationary or not. For this purpose, the following regression is run for the series:

$$y_t = \mu + \beta t + r_t + e_t \tag{5}$$

In the equation r_t is defined as;

$$r_t = r_{t-1} + u_t \tag{6}$$

The purpose of the Kwiatkowski-Phillips-Schmidt Shin (KPSS) test is to determine whether there is a random walk process y_t whether it contains a stochastic trend). For this, the variance of equation (6) is zero ($\sigma_u^2 = 0$) is the null hypothesis $\sigma_u^2 > 0$ is tested against the alternative. If the null hypothesis cannot be rejected r_t is a constant value and y_t does not contain a stochastic trend.

3.2.2. Unit Root Tests with Fourier Function

Traditional stationarity tests do not model the presence of breaks in the series. However, if there is a structural break in the series and the structural breaks are not modeled in the test equation, stationarity tests may lead to the conclusion that there is a stochastic trend in the series when there is no stochastic trend. For this reason, in addition to the ADF, DF-GLS and KPSS tests, it is also necessary to adjust for the possible effects of structural breaks on the test results.

An important feature of the Fourier approach in stationarity tests is that it is not necessary to assume that the number of breaks or the dates of the breaks are known in advance. Therefore, rather than choosing the number, date and structure of breaks, this specification uses the appropriate frequency components ($d(t)$) test equation (Becker et al., 2006, p.383; Enders and Lee, 2012a, Rodrigues and Taylor, 2012):

$$d(t)=\alpha+\sum_{k=1}^{n}\theta_{1k}\sin\left(\frac{2\pi kt}{T}\right)+\sum_{k=1}^{n}\theta_{2k}\cos\left(\frac{2\pi kt}{T}\right),\quad n<\frac{T}{2}\cdot \tag{7}$$

In Equation (7), n is the number of frequencies, k is a specific frequency and T is the number of observations.

3.3. Cointegration Test

3.3.1. Shin Cointegration Test

The data generation process of this test is based on the Kwiatkowski et al. (1992) test equation given in equation (5). The regression model proposed by Shin (1994) for the cointegration test is given below (where necessary, deterministic components such as trend and constant or only constant can be added to the model):

$$y_t=x_t'\beta+\eta_t \qquad t=1,2,\ldots,T \tag{8}$$

This is similar to the processes used in the KPSS test:

$$\eta_t=\omega_t+v_{1t} \tag{9}$$

$$\omega_t=\omega_{t-1}+u_t,\quad \omega_0=0, \tag{10}$$

$$x_t=x_{t-1}+v_{2t} \tag{11}$$

$y_t \sim I(1)$ and x_t, $I(1)$ are an m-dimensional vector of variables. u_t; zero-mean and σ_u^2 variance. Thus, ω_t refers to the random walk process. $v_{1t}\sim i.i.d.(0,\sigma^2)$ and $v_{2t}\sim i.i.d.(0,\Sigma)$ is a vector of m-dimensional errors.

In order to perform this test, regressors are added to the KPSS test equation. As in the KPSS test, the null hypothesis is tested against the alternative hypothesis in order to test the existence of cointegration relationship (Shin, 1994, pp.92-94).

3.3.2. Tsong, Lee, Tsai and Hu Cointegration Test

The regression model established in the FSHIN cointegration test developed from the FKPSS stationarity equation is given below (Tsong et al., 2016, p.1087):

$$y_t=f(t)+x_t'\beta+\eta_t \tag{12}$$

In the model x_t, η_t as defined in equations (11) and (9), respectively. In the model, the deterministic trend $f(t)$ defined as follows (Tsong et al., 2016, p.1088):

$$f(t)=\sum_{i=0}^{m}\delta_i t^i+d(t) \tag{13}$$

$m=0$ constant; $m=1$ constant and trend, and $d(t)$ is the function given in equation (7). In this method, the existence of cointegration relationship $\sigma_u^2=0$ null hypothesis $\sigma_u^2>0$ against the alternative hypothesis. Acceptance of the null hypothesis y_t and x_t indicates the existence of a cointegration relationship between the two countries.

To perform the test, the regression model in equation (12) under the null hypothesis is estimated by the Dynamic Least Squares (DOLS) method (in this case will be; $\eta_t=v_{1t}$). FSHIN cointegration test statistic (CI_f^m) is listed below:

$$CI_f^m=T^{-2}\sigma_v^{-1}\sum_{t=1}^{T}S_t^2 \tag{14}$$

$S_t = \sum_{i=1}^{T} \hat{v}_{1t}$; sum of EKK residuals from equation (12), σ_v^2 ; v_{1t} reflects the estimator of its long-run variance (Tsong et al., 2016, p.1092).

If the null hypothesis is rejected as a result of the test using the FSHIN test statistic obtained from equation (14), it means that the variables do not move together in the long run, that is, there is no cointegration relationship between them.

For the analyses applied in this study, the TSPDLIB library created by Nazlioglu (2021) was used.

4. Empirical Findings

To ensure the reliability of the results obtained from the analyses, breaks in the series were tested before applying the unit root tests. In the first stage of the analysis, the Bai and Perron (1998, 2003) test was conducted following Pascalau, Lee, Nazlioglu and Lu (2022) to determine whether there is a break in the series. Bai-Perron structural breaks test results are reported in Table 3.

Table 3: **Bai Perron Structural Break Test Results**

N.B.		LNBIST	LNBNB	LNBTC	LNDOLAR	LNETH	LNGAU
H_0	H_1	F- stat.	F- stat.	F- stat.	F- stat.	F- stat.	F- stat.
Break at Level							
0	1	651,224***	1937,928***	1226,010***	339,762***	863,944***	1066,167***
1	2	55,832***	203,018***	86,210***	96,192***	132,645***	138,108***
2	3	37,341***	28,733***	80,603***	127,491***	130,537***	77,569***
3	4	14,175***	27,514***	80,603***	32,435***	11,176	7,352
4	5	0,000	0,000	9,817	2,311		
N.B.		**LNBIST**	**LNBNB**	**LNBTC**	**LNDOLAR**	**LNETH**	**LNGAU**
H_0	H_1	F- stat.	F- stat.	F- stat.	F- stat.	F- stat.	F- stat.
Break at Level and Trend							
0	1	133,384***	375,504***	158,245***	105,743***	374,991***	107,433***
1	2	40,661***	46,941***	119,596***	95,639***	68,019***	125,283***
2	3	42,537***	51,384***	27,470***	23,709***	101,068***	45,468***
3	4	34,641***	0,000	6,124	12,526***	0,00	16,489***
4	5	21,233***			3,506		10,295

Note: N.B: Number of breaks, F-stat: F-statistic.
***p<0,01, **p<0,05, *p<0.10

Bai-Perron test results revealed the presence of breaks in all series included in the study. These breaks made it necessary to conduct stationarity analysis with unit root tests that model breaks. Therefore, this study aims to eliminate the possible effects of breaks in the series on the test results by using unit root tests that model structural breaks after conventional unit root tests (ADF, DF-GLS and KPSS).

Although various unit root tests with structural breaks can be used to model fractures, Fourier unit root tests (FADF, FDF and FKPSS) are preferred because they can model sudden and smooth breaks. Considering the structure of the breaks in the series, it was observed that there were both sharp and soft breaks. Therefore, it was determined that Fourier function tests were more suitable for modeling the breaks. In this context, since the Fourier function can model breaks in the series well (Becker et al., 2006), Fourier approach unit root tests are preferred in this study.

ADF, DF-GLS and KPSS tests are applied to determine the degree of integration of the series. The results of the conventional stationarity tests are presented in Table 4.

Table 4: **ADF, DF-GLS and KPSS Unit Root Test Results**

Variable	Model	ADF		DF-GLS		KPSS
		L	T.Stat	L	T.Stat	T.Stat
LNBIST	T&C	0	-2,331	0	-0,839	0,322***
	C	0	0,710	0	0,081	1,441***
LNBNB	T&C	0	-1,814	0	-1,212	0,304***
	C	0	-0,139	0	0,549	1,463***
LNBTC	T&C	0	-2,513	0	-1,194	0,278***
	C	0	-0,465	0	-0,396	1,479***
LNDOLAR	T&C	1	-1,387	1	-1,599	0,177***
	C	0	0,163	1	2,325	1,674***
LNETH	T&C	0	-2,551	0	-0,683	0,413***
	C	0	-0,430	0	-0,582	1,150***
LNGAU	T&C	0	-2,300	0	-1,664	0,225***
	C	0	-0,436	0	0,374	1,68***

Note: T&C: test equation with trend and constant, C: test equation with constant, L: lag length, T.Stat: test statistic.

According to the ADF, DF-GLS and KPSS unit root results in Table 4, the series are not stationary at level. In order to eliminate the possible effects of structural breaks in the series on the test results, the stationarity test is analyzed with Fourier function unit root tests (Fourier ADF, Fourier LGLS and Fourier KPSS unit root test). Accordingly, the t statistic was used as the criterion for selecting lag lengths in the FADF and FLGLS tests. As the optimal frequency, the frequency size that gives the minimum sum of error squares is chosen. The results of the Fourier function tests are presented in Table 5.

According to the FADF and FLGLS test results reported in Table 5, the null hypothesis of unit root cannot be rejected for all series, therefore, the series used in the study are not stationary at level and contain stochastic trend. The results of the FKPSS test are also consistent with the FADF and FLGLS test results. The FKPSS test rejected stationarity at the 1% significance level for each series.

As a result of the stationarity tests, it is determined that the series are not stationary at level. In order to determine the degree of integration of the series, first differences were taken and the stationarity tests were applied to these filtered series again with the structural break test. The abbreviations DLNBIST, DLNBNB, LNBTC, LNDOLAR, LNETH and LNGAU are used for LNBIST, DLNBNB, DLNBTC, DLNDOLAR, DLNETH and DLNGAU, respectively. The Bai Perron test results of the differenced series are given in Table 6.

Table 5: **FADF, FLGLS and FKPSS Unit Root Test Results**

Variable	Model	FADF		FLGLS		FKPSS	
		F	T.Stat	F	T.Stat	F	T.Stat
LNBIST	T&C	1	-3,477	1	-2,873	1	0,415***
	C	1	-1,196	3	-0,308	1	4,412***
LNBNB	T&C	1	-2,848	1	-2,883	1	0,662***
	C	1	-1,888	3	-0,261	1	6,033***
LNBTC	T&C	5	-2,524	1	-2,426	1	0,587***
	C	5	-0,458	5	-0,403	1	4,530***
LNDOLAR	T&C	1	-3,032	1	-3,075	1	0,513***
	C	1	-0,540	4	1,930	1	6,506***
LNETH	T&C	5	-2,562	1	-2,771	1	0,582***
	C	5	-0,191	5	-0,631	1	4,123***
LNGAU	T&C	2	-2,494	1	-2,708	1	0,482***
	C	5	-0,451	2	0,226	1	4,831***

Note: T&C: test equation with trend and constant, C: test equation with constant, F: frequency value, T.Stat: test statistic.

Table 6: **Bai Perron Test Results for Differenced Series**

Number of Breaks H_0	H_1	DLN BIST F-statistic	DLN BNB F- statistic	DLN BTC F- statistic	DLN DOLAR F- statistic	DLN ETH F-statistic	DLN GAU F- statistic
0	1	6,090	3,793	5,715	2,146	11,536***	3,301
1	2					2,742	

Not: ***p<0,01, **p<0,05, *p<0.10

In the basis of the data obtained from Table 6, it is determined that only DLNETH series has 1 break. Therefore, the Fourier function stationarity test was re-applied for the DLNETH series and the stationarity test for the remaining series was performed with conventional unit root tests. The result of the stationarity analysis of DLNETH series is presented in Table 7.

Table 7: **Fourier Function Unit Root Test Results**

Variable	FKPSS		FADF		FLGLS	
DLNETH	F	T.Stat	F	T.Stat	F	T.Stat
	1	0,042	1	-7,148***	1	4,586**

Note: C: model with constant, F: frequency value, T.Stat: test statistic. The critical table values are 0.270, 0.172 and 0.132 for the FKPSS test; -4.370, -3.780 and -3.470 for the FADF test; -4.593, -4.041 and -3.749 for the FLGLS test at 1%, 5% and 10% significance levels respectively.

According to the FKPSS and FADF test results, the DLNETH series given in Table 7 is stationary at 1% significance level; according to the FLGLS test results, it is stationary at 5% significance level. Therefore, DLNETH series is integrated at first order (I(1)). The results of stationarity tests of the differenced series using conventional unit root tests are presented in Table 8.

Table 8: **ADF, DF-GLS and KPSS Unit Root Test Results**

Variable	ADF T.Stat	DF-GLS T.Stat	KPSS T.Stat
DLNBIST	-6,278***	-1,785***	0,349
DLNBNB	-3,738***	-1,044***	0,172
DLNBTC	-14695***	-0,669***	0,238
DLNDOLAR	-20,309***	-20,356***	0,140
DLNETH	-13,763***	0,473***	0,542
DLNGAU	-16,500***	-0,838***	0,136

Note: ***p<0,01. T.Stat: test statistic.

The ADF, KPSS and DF-GLS test results in Table 8 indicate that the first differences of all variables are stationary at 1% significance level. This means that all series are integrated at first order.

Since it is concluded through the stationarity tests that all series used in the study are generated by first order integrated processes, the long-term relationships between variables are investigated by cointegration tests. In this direction, FSHIN and SHIN cointegration tests were used to examine the relationships between variables. The FSHIN cointegration test results for the variable groups of cryptocurrencies and investment instruments are presented in Table 9.

Table 9: **Fourier Shin Cointegration Test Results**

Function	F	CI^m_f	CI_m	F-Stat.
LNBTC=*f(LNBNB, LNETH)*	1	0,024	0,111	3,427
LNBIST=*f(LNDOLAR, LNGAU)*	1	0,031	0,103	1,599

Note: F: Frequency value, CI^m_f: Tsong et al. (2016) test statistic, CI_m: Shin (1994) test statistic. The critical value for the Fourier Shin cointegration test at the 5% significance level for the Frequency 1 value is 0.092; the critical value for the Shin cointegration test at the 5% significance level is 0.221.

When the results obtained from Table 9 are evaluated at 5% significance level, it is concluded that LNBNB, LNBTC and LNETH variables move together in the long run. Similarly, LNBIST, LNDOLAR and LNGAU variables are found to move together in the long run. From these results, it is concluded that the cryptocurrencies and investment instruments used in the study are cointegrated among themselves.

The FSHIN cointegration test results, which examine the cointegration relationship between each cryptocurrency variable and investment instruments, are given in Table 10.

Table 10: **Fourier Shin Cointegration Test Results between Cryptocurrencies and Investment Instruments**

Function	F	CI^m_f	CI_m	F-Stat.
LNBNB=*f(LNBIST, LNDOLAR, LNGAU)*	2	0,831	0,172	88,431
LNETH= *f(LNBIST, LNDOLAR, LNGAU)*	1	0,704	0,573	98,124

Note: F: Frequency value, CImf: Tsong et al. (2016) test statistic, CIm: Shin (1994) test statistic. LNBNB and LNETH are dependent variables. LNBIST, LNDOLAR and LNGAU are independent variables and given as f(LNBIST, LNDOLAR and LNGAU) respectively. The critical value for the Fourier Shin cointegration test at 5% significance level is 0.076 for Frequency 1 and 0.132 for Frequency 2. The critical value for Shin cointegration test at 5% significance level is 0.221.

The results reported in Table 10 show that the structural breaks in the test equations for LNBNB and LNETH are statistically significant and that there is no cointegration relationship between these cryptocurrencies and other investment instruments used in the study.

Unlike the other test equations, when LNBTC is expressed as a function of all macroeconomic variables, the component modeled by the Fourier function is statistically insignificant. Therefore, the Shin cointegration test proposed by Shin (1994) was utilized for the cointegration relationship between LNBTC and LNBIST, LNDOLAR and LNGAU variables (Özer, 2019). The results of this test are presented in Table 11.

Table 11: **Shin Cointegration Test Results**

Function	CI_m	Critical Value (%5)
LNBTC=*f(LNBIST, LNDOLAR, LNGAU)*	0,661	0,085

Note: CI_m: Shin (1994) test statistic. LNBTC is the dependent variable. LNBIST, LNDOLAR and LNGAU are independent variables and given as f(LNBIST, LNDOLAR and LNGAU) respectively.

The findings in Table 11 show that there is no co-movement between BTC and BIST100, dollar and gold. Considering this result, it shows that Bitcoin does not act together with investment instruments frequently preferred by investors in Turkey, such as gold, dollars and stocks, in the long term.

5. Conclusion

In this study, the long-term relationships between Bitcoin (BTC/USD), Ethereum (ETH/USD), and Binance Coin (BNB/USD) with the exchange rate (TRY/USD), gold (GAU/USD), and the BIST 100 index have been investigated. The study was conducted using weekly data between 19.01.2018 and 11.03.2022. In the FSHIN co-integration test, two separate models were initially created, focusing on cryptocurrencies and investment instruments. The findings obtained from the FSHIN cointegration test have shown that the series are cointegrated with each other in both established models. Therefore, the test results indicate that the BNB, BTC, and ETH cryptocurrencies, similarly to the BIST 100 index, the dollar, and gold, have moved together in the long term.

In the FSHIN cointegration test, the second step involved examining the cointegration relationship between each cryptocurrency and the investment instruments. According to the findings, while there was a common movement in a portfolio that included the BIST 100 index, the dollar, and gold, the addition of relevant cryptocurrencies to the portfolio has eliminated this common movement.

This study is expected to make significant contributions to investors and the finance literature. When evaluated from the perspective of investors, data obtained from studies conducted on how investors in Turkey assess their savings generally indicate that among the financial assets preferred by investors, stocks, the dollar, and gold have maintained their significance over the years. It can be stated that investors in Turkey tend to prefer assets that can preserve the real value of their investments. However, investors should consider portfolio risks while seeking assets with high real returns to evaluate their savings. Because including financial assets that are highly correlated in the same portfolio will increase portfolio risk. In this regard, although research reports indicate that stocks, dollars, and gold are among the investment instruments with the highest real returns in Turkey, this study has found that a portfolio consisting of BIST 100, dollars, and gold moves together under structural breaks. Similarly, among cryptocurrencies, BTC, ETH and BNB have high market value and trading volume compared to other cryptocurrencies. However; Since these cryptocurrencies move together in the long term, it can be stated that a portfolio diversification strategy that includes these cryptocurrencies together carries risks.

In light of the results obtained, it has been determined that including one of the cryptocurrencies BTC, BNB, or ETH in a portfolio containing the BIST 100 index, dollar, and gold eliminates the common movement among the series. Therefore, it is believed that a portfolio diversification created in this way could be a good investment strategy. Because, although the BIST 100 index, the dollar, and gold move together in the long term, adding one of the relevant crypto assets to this portfolio could reduce the portfolio risk. A similar situation applies to cryptocurrencies as well. Although the risk in a portfolio composed of cryptocurrencies cannot be sufficiently reduced, it is believed that adding traditional assets to a cryptocurrency portfolio can significantly lower the portfolio risk.

Results of this study are similar to those obtained in İşcan's (2020) study. However, in İşcan's (2020) study, the relationships between the BTC to dollar exchange rate, the BIST 100 index, and interest rates were examined using the Johansen cointegration test.

In general terms, according to the cointegration analyses conducted in the study, it has been found that cryptocurrencies are cointegrated with each other as investment instruments; however, cryptocurrencies are not cointegrated with investment instruments, meaning that they do not move together in the long term. The findings obtained are consistent with the results from the study by Toprak and Kubar (2023). In the study by Toprak and Kubar (2023), the common movement between variables was examined by the Fourier Shin test, similar to this study. However, in the study, the relationships between the selected country stock exchanges and BTC and ETH have been examined, and the relationships explained differ from those in this study.

On the other hand, the results obtained by this study differ from the findings of Laçin (2019), Telek and Şit (2020), Koç and Çaykara (2021), Gülcü and Kıtkıt (2022), and Akkaya and Tuna (2023). The reasons for this contradictories can be listed as follows: First of all, the variables used in the mentioned studies do not completely resemble the variables used in this study. The specified studies examine the relationships between Bitcoin and the dollar or between Bitcoin and the BIST 100 index. Secondly, the analysis periods of this study differ from those of other studies. Another reason is that when there are breaks in the series, the reliability of the results obtained from applications conducted without modeling these breaks decreases. Therefore, in this study, Fourier function unit root and cointegration tests that model abrupt and smooth breaks have been applied. Therefore, since traditional unit root tests and cointegration tests were used in the studies being compared, the results obtained may differ from those of this study. Additionally, it is thought that the reason for the differing results in the studies by Çütçü and Kılıç (2018) and Gürbüz and Zeren (2021), despite the use of the structural break cointegration test by Maki (2012), may stem from the different ways in which the Fourier function models breaks compared to the cointegration tests used in the studies.

It is expected that the study will also contribute to the financial literature. Firstly, studies that include the variables used in this study together and studies that use Fourier structural break tests as research methodology are very few. Secondly, the number of studies investigating the relationships between cryptocurrencies and various investment instruments using the Fourier method is quite limited. In future studies, the relationships between other cryptocurrencies and traditional investment alternatives can be examined. Additionally, in this study, contributions to the literature can be made by comparing results using break tests that model structural breaks, similar to the Fourier function tests used in the analysis of relationships between variables.

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