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RESEARCH PAPER

Economic resilience in the face of pandemic: a holistic mathematical analysis of the pandemic in India

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Abstract

COVID-19 was initiated in 2020 and caused an immediate threat to global countries in terms of both economic and health influences. In this present work, we extend the Susceptible-Infected-Recovered (SIR) model by considering two new variables, gross domestic product or GDP (G) and unemployment (U), to study the impact of this epidemic on the Indian economy during the 2020–2023 period. Since our extended SIR model includes two novel compartments, which are GDP and unemployment rate, we can now explore in more detail the sophisticated relationship between health and economic matters. The framework allows us to investigate the following consequences: how changes in the infection rate affect the economy and how changes in GDP and unemployment translate into the spread of this contagion. These visualizations are based on real-time quarterly data and provide full knowledge of the interaction between health and economic dynamics during the COVID-19 crisis in India. Government initiatives and regulations are also reviewed for their efficiency to contain the virus while taming the economic cost. Real-world results are contrasted with the care to find the strengths and weaknesses of the policies that come out with the underlying assumptions in the model. This paper, in other words, deploys an in-depth analysis of the convoluted links between economics, policy, and public health in the face of a pandemic with a geographic focus in India.

Keywords: GDP; COVID-19; unemployment; epidemic model; stability analysis

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1 Introduction

The economic blow caused by the pandemic was not limited to formal sectors but equally aggravated the informal economy, which employs the majority of the Indian workforce. The lockdown

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measures, though helpful in reducing the spread of the virus, led to a significant loss of income for many informal workers, pushing them towards poverty [1]. The pandemic also highlighted the importance of digitization, as those who could access digital platforms were able to continue working, while others faced severe livelihood challenges [2]. In response, the Indian government implemented various measures, including direct cash transfers, food security initiatives, and credit guarantees, to support the economy. However, the effectiveness of these measures in mitigating the economic downturn remains a subject of ongoing analysis [3].

Epidemiological and economic models have been studied for the multifaceted impacts of pandemics [4–7]. For example, Chakraborty and Maity (2020) [8] investigated the economic implications of lockdowns and highlighted that health and economic variables must be modeled simultaneously in order to formulate effective policy interventions [9]. Mishra et al. (2021) [10] pointed out the importance of nonlinear dynamics in such models by applying advanced mathematical methods to explore the long-term interplay between public health crises and economic stability [11]. Moreover, a recent study by Singh et al. (2023) [12] offered empirical evidence regarding the effectiveness of government measures in reducing economic shocks during pandemics, especially in informal economies [13]. The role of numerical methods such as RK-4 in solving complex epidemiological-economic models has also been emphasized, as it allows for the precise simulation of the nonlinear interdependencies. Based on these fundamentals, the current research contributes to the field by formulating a new SIRUG model, integrating unemployment and GDP dynamics in an epidemiological context, utilizing RK-4 and least-square methods to predict and analyze economic consequences of pandemics more holistically.

In this study, we use the Runge-Kutta 4th order method (RK-4) [14] to solve our equations. This method helps us get more accurate results than basic calculation methods because it looks at multiple points when solving each step. RK-4 is especially good at handling sudden changes, like when a pandemic quickly impacts jobs and the economy, and the algorithm of the considered method directly as compared to the complexity of the algorithm of other methods [15]. By using this method, we can better predict both immediate and long-term changes in employment and economic growth.

The key objective of this research is to develop a mathematically sophisticated model that combines the SIR model with unemployment and GDP components, considering Okun's Law to analyze the relationship between GDP and unemployment [16]. The RK-4 method will be applied to simulate and predict the dynamic behaviors of unemployment and GDP, capturing both short-term fluctuations and long-term consequences [17]. This ambitious undertaking aims to unravel the complex linkages among health, labor markets, and economic performance during a time of unprecedented shocks.

The year-on-year unemployment rate in urban India surged from 8.8% in April to June 2019 to 20.8% in April to June 2020, highlighting the heavy toll on the labor force due to the pandemic [18]. This metric underscores the urgency of understanding and addressing the economic repercussions of public health crises.

The choice of the RK-4 method is based on its efficiency in solving differential equations and its suitability for capturing the nonlinear interactions inherent in economic and epidemiological models [19]. The classical SIR model is modified to include parameters characterizing unemployment and GDP, enhancing the model's ability to capture the complexity of real-world economic systems, especially during crises.

This research fills a striking gap in the literature by integrating health-related and economic variables within a common framework. While previous studies have often focused on health or economic aspects alone, this work combines them into an integrated model. Furthermore, the study extends the scope by incorporating the RK-4 method to predict the potential impact of

future pandemics and adds a least square method to provide a forward-looking dimension to our understanding [20].
This study is crucial in providing actionable insights for policymakers and researchers to make informed decisions amid dynamic economic conditions influenced by public health crises. By filling this gap in the literature, the research contributes to mathematical modeling, epidemiology, and economic forecasting, laying the foundation for future studies to build on this integrated framework [21].
Our SIRUG model presents a unified framework to explore the relationship between disease spread and economic changes during public health crises. By combining traditional disease modeling with economic indicators like unemployment and GDP through Okun's Law and employing the RK-4 method to analyze their interactions, this research provides valuable insights into both immediate and future economic impacts of pandemics. The addition of least square analysis enhances our ability to make accurate predictions, making this work particularly valuable for policymakers and researchers. This study establishes a foundation for future research in this field, offering new ways to understand and address the economic challenges that arise during public health emergencies.

2 Basic results

The following results played a crucial role in the comprehensive analysis and validation of our model, significantly contributing to its efficacy and reliability.

Theorem 1 *[22] The autonomous system $x'(t) = Ax(t), \quad x(0) = x_0$ is asymptotically stable iff $|arg(\lambda(A))| > \frac{\pi}{2}$. Stable if and only if either it is asymptotically stable, or those critical eigenvalues which satisfy $|arg(\lambda(A))| = \frac{\pi}{2}$ have geometric multiplicity one. Here, $arg(\lambda(A))$ denotes the argument of the eigenvalues of the square matrix A.*

Theorem 2 *[23] Let $f(t)$ be a continuous function on $[0, \infty)$ and satisfies*

$$\frac{df}{dt} \leq -\Phi f(t) + \gamma_1, f(t_0) = f_{t_0},$$

where $\Phi, \gamma_1 \in \mathbb{R}$ and $\Phi \neq 0$, then

$$f(t) \leq \left(f_{t_0} - \frac{\gamma_1}{\Phi}\right) e^{-\Phi(t-t_0)} + \frac{\gamma_1}{\Phi}.$$

Definition 1 *The function $f : E \to \mathbb{R}^n$ is said to admit the Lipschitz condition on the open subset E of $\mathbb{R}^n$ if there is a positive constant K such that*

$$|f(x) - f(y)| \leq K|x - y|, \quad \forall x, y \in E.$$

3 Model construction

In this section, we introduce the model that we consider for the present study. The following statement contains a simplified explanation of the SIRUG model. The explanation has been primarily based on assuming there would be no future uncertainties.

Key components of the model

i. **Susceptible (S)**: This group represents individuals who are not infected but are susceptible to the disease. Over time, some of them may become infected if they come into contact with infectious individuals.

ii. **Infected (I)**: This group includes individuals who are currently infected and capable of spreading the disease to susceptible individuals. The number of infected individuals typically increases initially.

iii. **Recovered (R)**: The "Recovered" category represents individuals who have recovered from the disease and are no longer infectious. These individuals may have developed immunity to the disease, depending on the disease in question.

iv. **Unemployed Population (U)**: This represents the number of individuals who are currently unemployed and seeking employment.

v. **Gross Domestic Product (G)**: It represents the total value of goods and services produced within a country, serving as a measure of its economic performance.

The SIR model

SIR [24] is a system of ordinary differential equations showing the dynamics of infectious spread. The following equations outline how, over time, the number of people in each category varies:

$$\begin{aligned}\frac{dS}{dt} &= -\beta SI,\\ \frac{dI}{dt} &= \beta SI - \gamma I,\\ \frac{dR}{dt} &= \gamma I.\end{aligned} \tag{1}$$

Uses and practicality in real life:

The SIR model has several important uses and practical applications in real life:

(i) **Epidemic modeling:** SIR is one of the most used models for studying and predicting the dynamics of infectious diseases. It is often used in the simulation of different scenarios by adjusting various parameters like transmission rate and recovery rate. It can assess how interventions like vaccination or social distancing affect the outcome.

(ii) **Public health planning [25]**: Health authorities and policymakers use the SIR model to aid in making decisions concerning disease control strategies, planning resource allocation during an outbreak, and healthcare system readiness. These models aid in the calculation of projections of cases that may occur and identify the critical times in an outbreak and health system needs.

(iii) **Parameter estimation:** Through the SIR model, parameters for the disease can be estimated, including, but not limited to, the basic reproduction number, R_0, which can be described as the average number of secondary infections generated by one infectious individual in an entirely susceptible population. The calibration of these parameters is essential for the design of suitable public health policies.

(iv) **Vaccination campaigns [26]:** The SIR model is utilized in the layout and analysis of vaccination policies. They calculate ideal vaccine coverage levels to achieve the idea of herd immunity, which refers to the idea of a high enough percentage of the population becoming immune to prevent large-scale outbreaks.

(v) **Early warning systems [27]:** Continuous monitoring of the data and the SIR model allows for the development of early warning systems that might help limit the spread of the disease while it is still at its beginning stages.

Thus, SIR models are very helpful in understanding infectious disease transmission dynamics and are an important tool for epidemiologists, public health experts, and policymakers. These models help Inform decisions that can eventually save lives and reduce the impact of epidemics on society.

The SIRUG model

The traditional SIR model categorizes all people into three classes: Susceptible, Infected, and Recovered. This model has been very useful in understanding the basic trends by which infectious diseases spread through a population. It mostly overlooks the bidirectional and nuanced interaction between health and economic stability. Motivated by this important gap in our understanding, we introduce a new holistic modified SIR model. Added to this adaptation are two more compartments, such as the Unemployed and GDP, which will allow for a critical look into the multi-dimensional reality of disease spread and further-reaching implications in society.
This is a modified version of the SIR model, which accounted for an epidemic process and was used to describe and predict the dynamics of infectious diseases within a population. This modified model adds extra compartments defined to include the economic factors influencing the dynamics of the epidemic as follows:

$$\begin{cases} \dfrac{dS}{dt} = -\beta_1 SI + \alpha S + \omega G, \\ \dfrac{dI}{dt} = \beta_2 SI - \gamma_1 I + \eta I, \\ \dfrac{dR}{dt} = \gamma_2 I - \delta R, \\ \dfrac{dU}{dt} = \lambda S - \mu U, \\ \dfrac{dG}{dt} = \phi G - \kappa U G. \end{cases} \tag{2}$$

Explanation of each compartment:

In this adapted SIR model, various factors are taken into consideration that might have an influence on the dynamics of the epidemic and the economy. Following is a detailed explanation of the modifications and what each compartment stands for:

I. **Susceptible dynamics** ($\frac{dS}{dt}$)**:**

i) (-β_1SI): This term is the rate at which the susceptible S entered the infected I state. It thus depends upon the infection rate, β_1 and on the product of the number of people susceptible, S, and infectious, I.

ii) (αS): This term is the birth rate, and it provides the number of people that are added to the susceptible population at any given time. It, therefore, increases the susceptible population.

iii) (ωG): This term is how the Gross Domestic Product, GDP, affects the susceptible population. This shows the way in which the economic characteristics influence the birth rate and, consequently, cause an increase or a decline in the susceptible population.

II. **Infected dynamics** ($\frac{dI}{dt}$)**:**

i) (β_2SI): The expression gives the rate of conversion of Susceptible, S, into Infectious, I, due to COVID-19. In that, it is affected by the conversion rate, β_2, with the product of the number of Susceptible, S and the Infectious, I.

ii) (-γ_1I): This term reflects the rate at which the number of infections is reduced. It accounts for factors like recovery or medical interventions that reduce the number of infectious individuals.

iii) (ηI): This term denotes the disease-induced death rate among the infected population. It represents the mortality associated with the disease.

III. **Recovered dynamics** ($\frac{dR}{dt}$)**:**

i) (γ_2I): This term signifies the rate at which individuals move from the infectious (I) to the recovered (R) compartment. It represents recovery from the disease.

ii) (-δR): This term denotes the natural death rate among the recovered population. It reflects the mortality rate of individuals who have recovered from the disease.

IV. **Unemployed dynamics** ($\frac{dU}{dt}$)**:**

i) (λS): This term represents the increase in unemployment due to the pandemic. It reflects how the susceptible population contributes to the rise in unemployment.

ii) (-μU): This term represents the re-employment rate, indicating the rate at which individuals move from unemployment to employment. It reflects the recovery of the job market.

V. **GDP dynamics** ($\frac{dG}{dt}$)**:**

i) (ϕG): This term represents the GDP growth rate. It indicates the natural growth or expansion of the economy.

ii) (-κUG): This term represents the GDP decay rate due to unemployment. It reflects the negative impact of unemployment on GDP, capturing how economic downturns affect the overall economic output.

4 Parameter estimation

Parameter estimation is one of the most important elements in mathematical modeling and data analysis and, therefore, in our attempt to appreciate the intricate inner workings of complex systems, whether in physics, biology, economics, engineering, or generally in scientific and engineering fields. The main aim of the parameter estimation process is to provide an exact numerical value for parameters underlying a given mathematical model and bring clarity to many of the otherwise elusive behaviors manifested by real-world systems.

Accurate parameter estimation plays a crucial role as a bridge between theoretical concepts and tangible empirical reality. This essential bridge guides scientific investigations grounded in evidence-based, data-driven approaches, providing researchers with a roadmap to navigate the complexities of complex systems.

Using the least square method, we got the best-fitted parameter values, which are presented in Table 1.

5 Stability of equilibrium points

The equilibrium points of system (2) for the parameter values as in Table 1 are

i. $E_1 = (1.67 \times 10^9, 40, 155556, 9.61538, 0)$,

ii. $E_2 = (1.48417 \times 10^9, 0, 0, 8.5625, -296.833)$,

iii. $E_3 = (0, 0, 0, 0, 0)$.

Theorem 3 *System* (2) *is stable at* E_1*, but unstable at* E_2 *and* E_3*.*

Proof Following are the eigenvalues of system (2) at the three equilibrium points:

i. The Eigenvalues corresponding to equilibrium point E_1 can be stated as follows:

$$\lambda_{1,1} = -1.3, \quad \lambda_{1,2} = -0.1688462, \quad \lambda_{1,3} = -0.009, \quad \lambda_{1,4} = 0.001i, \quad \lambda_{1,5} = -0.001i.$$

ii. The Eigenvalues corresponding to equilibrium point E_2 can be stated as follows:

$$\lambda_{2,1} = -1.3, \quad \lambda_{2,2} = -0.1095, \quad \lambda_{2,3} = -0.009, \quad \lambda_{2,4} = -0.0011705, \quad \lambda_{2,5} = 0.00117044.$$

iii. The Eigenvalues corresponding to equilibrium point E_3 can be stated as follows:

$$\lambda_{3,1} = 1.37, \quad \lambda_{3,2} = -1.3, \quad \lambda_{3,3} = -1, \quad \lambda_{3,4} = -0.009, \quad \lambda_{3,5} = 1 \times 10^{-6}.$$

Table 1. Value of parameters associated with system (2)

Parameter	Description	Value
β_1	Infection rate	2.5×10^{-8}
β_2	Conversion rate of Susceptible people into infected by COVID-19	6×10^{-10}
α	Birth Rate	1×10^{-6}
ω	Influence of GDP on susceptible population	5
γ_1	Rate at which number of infections are reduced	3
γ_2	Rate at which people move from I to R	35
η	Disease induced death rate	2
δ	Natural death rate	0.009
λ	Unemployment rate	7.5×10^{-9}
μ	Re-employment rate	1.3
ϕ	GDP growth rate	1.37
κ	GDP decay rate due to unemployment	0.16

Table 2. Comparison of argument with $\frac{\pi}{2}$

Eigenvalue	Argument Value	Comparison with $\pi/2$
$\lambda_{1,1}$	π	$\pi > \frac{\pi}{2}$
$\lambda_{1,2}$	π	$\pi > \frac{\pi}{2}$
$\lambda_{1,3}$	π	$\pi > \frac{\pi}{2}$
$\lambda_{1,4}$	$\frac{\pi}{2}$	$\frac{\pi}{2} = \frac{\pi}{2}$
$\lambda_{1,5}$	$\frac{\pi}{2}$	$\frac{\pi}{2} = \frac{\pi}{2}$
$\lambda_{2,1}$	π	$\pi > \frac{\pi}{2}$
$\lambda_{2,2}$	π	$\pi > \frac{\pi}{2}$
$\lambda_{2,3}$	π	$\pi > \frac{\pi}{2}$
$\lambda_{2,4}$	π	$\pi > \frac{\pi}{2}$
$\lambda_{2,5}$	0	$0 < \frac{\pi}{2}$
$\lambda_{3,1}$	0	$0 < \frac{\pi}{2}$
$\lambda_{3,2}$	π	$\pi > \frac{\pi}{2}$
$\lambda_{3,3}$	π	$\pi > \frac{\pi}{2}$

Table 2. Comparison of argument with $\frac{\pi}{2}$ - continued

Eigenvalue	Argument Value	Comparison with $\pi/2$
$\lambda_{3,4}$	π	$\pi > \frac{\pi}{2}$
$\lambda_{3,5}$	0	$0 < \frac{\pi}{2}$

The arguments of the above eigenvalues are presented in Table 2. As it can be seen from those values,

- The equilibrium point E_1 is stable.
- The equilibrium point E_2 is unstable as $\lambda_{2,5} < \frac{\pi}{2}$.
- The equilibrium point E_3 exhibits instability, since $\lambda_{3,1}$, $\lambda_{3,5} < \frac{\pi}{2}$.

6 Existence and uniqueness of solutions

Theorem 4 *The kernels F_1, F_2, F_3, F_4, F_5 admit the Lipschitz condition and contraction when*

$$0 < K_1, K_2, K_3, K_4, K_5 \leq 1,$$

where $K_1 = \beta_1\epsilon_2 + \alpha$, $K_2 = \beta_2\epsilon_1 - \gamma_1 + \eta$, $K_3 = \delta$, $K_4 = \mu$, and $K_5 = \phi + \kappa\epsilon_4$.

Proof We assume that $||S|| \leq \lambda_1$, $||I|| \leq \lambda_2$, $||R|| \leq \lambda_3$, $||U|| \leq \lambda_4$, $||G|| \leq \lambda_5$ and

$$\frac{dS}{dt} = F_1(t, S, I, R, U, G), \tag{3}$$
$$\frac{dI}{dt} = F_2(t, S, I, R, U, G), \tag{4}$$
$$\frac{dR}{dt} = F_3(t, S, I, R, U, G), \tag{5}$$
$$\frac{dU}{dt} = F_4(t, S, I, R, U, G), \tag{6}$$
$$\frac{dG}{dt} = F_5(t, S, I, R, U, G). \tag{7}$$

For Eq. (3), we show

$$||F_1(t, S, I, R, U, G) - F_1(t, S^*, I, R, U, G)|| \leq K_1||S - S^*||,$$

where $K_1 \in [0, 1)$. Now,

$$\begin{aligned} ||-\beta_1 SI + \alpha S + \omega G + \beta_1 S^* I - \alpha S^* - \omega G|| &= || - \beta_1 I(S - S^*) + \alpha(S - S^*)|| \\ &\leq ||\beta_1 I(S - S^*)|| + ||\alpha(S - S^*)|| \\ &\leq |\beta_1| ||I|| ||S - S^*|| + |\alpha| ||S - S^*|| \\ &\leq \beta_1\epsilon_2 ||S - S^*|| + \alpha||S - S^*|| \\ &\leq (\beta_1\epsilon_2 + \alpha)||S - S^*||. \end{aligned}$$

Therefore $K_1 = (\beta_1\epsilon_2 + \alpha)$. For Eq. (4), we need to provide

$$||F_2(t, S, I, R, U, G) - F_2(t, S, I^*, R, U, G)|| \leq K_2||I - I^*||,$$

where $K_2 \in [0,1)$. Now,

$$\begin{aligned}
\|F_2(t,S,I,R,U,G) - F_2(t,S,I^*,R,U,G)\| &= \|\beta_2 SI - \gamma_1 I + \eta I - \beta_2 SI^* + \gamma_1 I^* - \eta I^*\| \\
&= \|\beta_2 S(I - I^*) - \gamma_1(I - I^*) + \eta(I - I^*)\| \\
&\leq |\beta_2|\|S\|\|I - I^*\| - |\gamma_1|\|I - I^*\| + |\eta|\|I - I^*\| \\
&\leq (|\beta_2|\|S\| - |\gamma_1| + |\eta|)\|I - I^*\| \\
&\leq (\beta_2\epsilon_1 - \gamma_1 + \eta)\|I - I^*\|.
\end{aligned}$$

Therefore $K_2 = (\beta_2\epsilon_1 - \gamma_1 + \eta)$. For Eq. (5), we need to show

$$\|F_3(t,S,I,R,U,G) - F_3(t,S,I,R^*,U,G)\| \leq K_3\|R - R^*\|,$$

where $K_3 \in [0,1)$. Then

$$\begin{aligned}
\|F_3(t,S,I,R,U,G) - F_3(t,S,I,R^*,U,G)\| &= \|\gamma_2 I - \delta R - \gamma_2 I + \delta R^*\| \\
&= \| - \delta(R - R^*)\| \leq |\delta|\|R - R^*\| \\
&\leq \delta\|R - R^*\|.
\end{aligned}$$

Therefore $K_3 = \delta$. For Eq. (6), we need to obtain

$$\|F_4(t,S,I,R,U,G) - F_4(t,S,I,R,U^*,G)\| \leq K_4\|U - U^*\|,$$

where $K_4 \in [0,1)$. Now, we have

$$\begin{aligned}
\|F_4(t,S,I,R,U,G) - F_4(t,S,I,R,U^*,G)\| &= \|\alpha G - \mu U - \alpha S + \mu U^*\| \\
&= \| - \mu(U - U^*)\| \leq |\mu|\|U - U^*\| \\
&\leq \mu\|U - U^*\|.
\end{aligned}$$

Therefore $K_4 = \mu$. For Eq. (7), we need to show

$$\|F_5(t,S,I,R,U,G) - F_5(t,S,I,R,U,G^*)\| \leq K_5\|G - G^*\|,$$

where $K_5 \in [0,1)$. Then

$$\begin{aligned}
\|\phi G - \kappa UG - \phi G^* + \kappa UG^*\| &\leq \|\phi(G - G^*) - \kappa U(G - G^*)\| \\
&\leq |\phi|\|G - G^*\| + \kappa\epsilon_4\|G - G^*\| \\
&\leq (\phi + \kappa\epsilon_4)\|G - G^*\|.
\end{aligned}$$

Therefore $K_5 = (\phi + \kappa\epsilon_4)$.
Here, K_1, K_2, K_3, K_4, K_5 are the Lipschitz constants for the functions F_1, F_2, F_3, F_4 and F_5, respectively.
Now that we have proved the existence of Lipschitz constants K_1, K_2, K_3, K_4, K_5, the existence of a unique solution to system (2) is also ensured following the methodology shown in [28].

7 Boundedness

Theorem 5 *The proposed S-I-R-U-G model Eq. (2) is bounded by Theorem 2.*

Proof Let $X(t) = S(t) + I(t) + R(t) + U(t) + G(t)$.
On differentiating $X(t)$, we have

$$\frac{dX}{dt} + \Phi X = \frac{d(S + I + R + U + G)}{dt} + \Phi(S + I + R + U + G).$$

Simplifying, we have

$$\begin{aligned}\frac{dX}{dt} + \Phi X = &-\beta_1 SI + \alpha S + \omega G + \beta_2 SI - \gamma_1 I + \eta I + \gamma_2 I - \delta R \\ &+ \lambda S - \mu U + \phi G - \kappa UG + \Phi S + \Phi I + \Phi R + \Phi U + \Phi G.\end{aligned}$$

Removing the negative terms, we get

$$\frac{dX}{dt} + \Phi X \leq \alpha S + \omega G + \beta_2 SI + \eta I + \gamma_2 I + \lambda S + \phi G + \Phi S + \Phi I + \Phi R + \Phi U + \Phi G.$$

Now, the solution of system (2) exists uniquely in

$$\{(S, I, R, U, G) : max(|S|, |I|, |R|, |U|, |G|) \leq M\},$$

where M is a positive constant. Therefore, we can write

$$\frac{dX}{dt} + \Phi X \leq (\alpha + \omega + \beta_2 M + \eta + \gamma_2 + \lambda + \phi)M + 5\Phi M = \gamma_1.$$

Using Theorem 1, we get

$$X(t) \leq \left(X_{t_0} - \frac{\gamma_1}{\Phi}\right) e^{-\Phi(t-t_0)} + \frac{\gamma_1}{\Phi}.$$

Therefore, system (2) is bounded.

8 Numerical method

The Runge-Kutta 4th order (RK4) method occupies a pivotal position in the arsenal of numerical techniques applied to SIR (Susceptible-Infectious-Recovered) modeling within epidemiology. Renowned for its adept balance between accuracy and computational efficiency, RK4 is widely embraced for its reliability and ease of implementation. Its enduring popularity stems from its ability to deliver precise solutions while remaining relatively straightforward to employ, robust in the face of diverse scenarios, and stable across a range of conditions. As a consequence, RK4 has emerged as a cornerstone in epidemiological simulations, serving as a linchpin for researchers seeking to unravel the complexities of disease dynamics.

At its core, RK4 functions by breaking down the differential equations governing infectious disease transmission into discrete steps, allowing for the meticulous exploration of various epidemiological scenarios. By leveraging RK4, researchers can simulate and analyze the impact of different interventions, ranging from vaccination campaigns to social distancing measures, thereby informing evidence-based public health strategies and policy decisions.

In practical terms, RK4 enables epidemiologists to simulate disease outbreaks with a high degree of fidelity, providing invaluable insights into the progression and containment of infectious diseases. Its versatility extends beyond simple SIR models, as RK4 can be adapted to explore

more complex dynamics, such as spatial spread, heterogeneous populations, and the interplay of multiple infectious agents. Moreover, RK4's computational efficiency makes it well-suited for real-time epidemic forecasting and scenario planning, empowering public health officials to anticipate and respond effectively to emerging threats.
To solve system (2) using the classical Runge-Kutta method of 4th order, we define the system as

$$\begin{aligned}
f_1(S,I,R,U,G) &= -\beta_1 SI + \alpha S + \omega G,\\
f_2(S,I,R,U,G) &= \beta_2 SI - \gamma_1 I + \eta I,\\
f_3(S,I,R,U,G) &= \gamma_2 I - \delta R,\\
f_4(S,I,R,U,G) &= \lambda S - \mu U,\\
f_5(S,I,R,U,G) &= \phi G - \kappa UG.
\end{aligned} \tag{8}$$

Using the RK4 method, we compute the intermediate values as follows

$$\begin{aligned}
k_1^{(i)} &= hf_i(S_n, I_n, R_n, U_n, G_n),\\
k_2^{(i)} &= hf_i\left(S_n + \frac{k_1^{(1)}}{2}, I_n + \frac{k_1^{(2)}}{2}, R_n + \frac{k_1^{(3)}}{2}, U_n + \frac{k_1^{(4)}}{2}, G_n + \frac{k_1^{(5)}}{2}\right),\\
k_3^{(i)} &= hf_i\left(S_n + \frac{k_2^{(1)}}{2}, I_n + \frac{k_2^{(2)}}{2}, R_n + \frac{k_2^{(3)}}{2}, U_n + \frac{k_2^{(4)}}{2}, G_n + \frac{k_2^{(5)}}{2}\right),\\
k_4^{(i)} &= hf_i\left(S_n + k_3^{(1)}, I_n + k_3^{(2)}, R_n + k_3^{(3)}, U_n + k_3^{(4)}, G_n + k_3^{(5)}\right).
\end{aligned} \tag{9}$$

The values at the next time step are computed as

$$\begin{aligned}
S_{n+1} &= S_n + \frac{1}{6}\left(k_1^{(1)} + 2k_2^{(1)} + 2k_3^{(1)} + k_4^{(1)}\right),\\
I_{n+1} &= I_n + \frac{1}{6}\left(k_1^{(2)} + 2k_2^{(2)} + 2k_3^{(2)} + k_4^{(2)}\right),\\
R_{n+1} &= R_n + \frac{1}{6}\left(k_1^{(3)} + 2k_2^{(3)} + 2k_3^{(3)} + k_4^{(3)}\right),\\
U_{n+1} &= U_n + \frac{1}{6}\left(k_1^{(4)} + 2k_2^{(4)} + 2k_3^{(4)} + k_4^{(4)}\right),\\
G_{n+1} &= G_n + \frac{1}{6}\left(k_1^{(5)} + 2k_2^{(5)} + 2k_3^{(5)} + k_4^{(5)}\right).
\end{aligned} \tag{10}$$

To generate the simulation results presented in this study, the RK4 method was implemented using Python. Python's rich ecosystem of libraries, including NumPy and Matplotlib, was utilized to ensure precision in numerical computations and clarity in visualizing the results. The implementation in Python further underscores the accessibility and reproducibility of the simulation process, enabling researchers to replicate and extend the findings with ease.

Model simulations

The following are the graphs obtained using the Runge-Kutta 4th-order method. The red points showcase the real data value points, and the blue line showcases our model.

Inferences on the different compartments based on the numerical simulations for system (2):

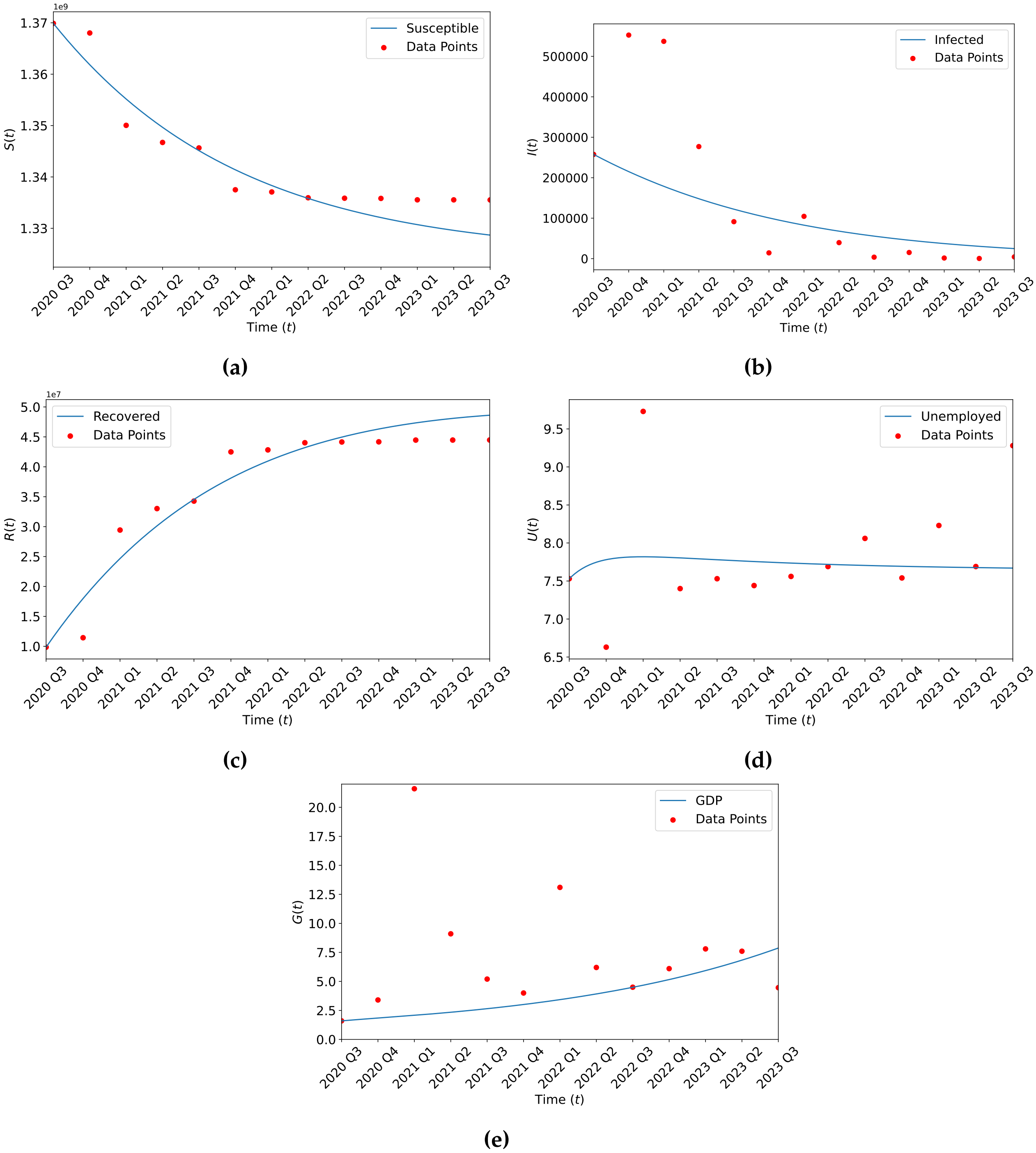

Figure 1. Subplot showing numerical results for (a) Susceptible, (b) Infection, (c) Recovered, (d) Unemployment and (e) GDP at parameters given in Table 1

Susceptible population: In Figure 1a, the susceptible population ($S(t)$) exhibits a decreasing trend over time, indicating potential exposure and infection in the population. The red data points, representing observed values, align closely with the simulated results, underscoring the accuracy of the model.

Infected population: In Figure 1b, the infected population ($I(t)$) displays fluctuations over time, possibly reflecting the impact of interventions or variations in disease spread. The close alignment of the data points with the model's predictions suggests that the model effectively captures observed infection trends.

Recovered population: In Figure 1c, the recovered population ($R(t)$) demonstrates a consistent increase over time, illustrating the cumulative number of individuals who have successfully overcome the infection. The model's trajectory closely follows the provided data points, affirming its reliability in simulating recovery dynamics.

Unemployed population: In Figure 1d, the unemployed population ($U(t)$) experiences fluctuations, possibly influenced by economic factors or external events. The observed data points exhibit variations, and the model successfully captures the general trend, indicating its ability to simulate the dynamics of unemployment in response to changing conditions.

GDP: In Figure 1e, GDP, denoted as ($G(t)$), displays a consistent increase over time, suggesting economic growth. The observed data points align well with the model's predictions, indicating that the simulated economic dynamics accurately represent the growth trends in GDP.

9 Results and discussion

The following sections demonstrate the influence of various parameters like ϕ and κ on GDP and of λ and μ on the unemployment rate.

Influence of ϕ and κ on GDP

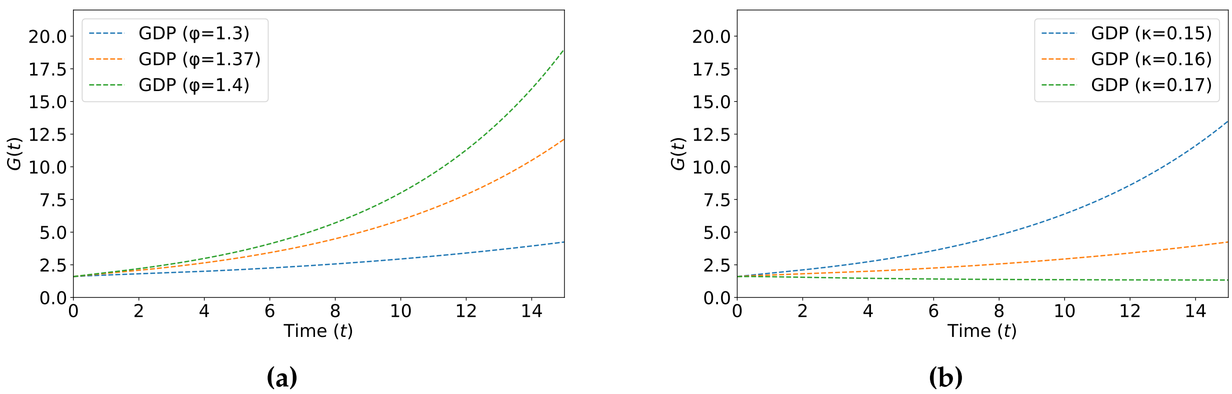

Figure 2. (a) GDP with decreased and increased ϕ value, (b) GDP with decreased and increased κ value at parameters given in Table 1

Influence of ϕ (gross domestic product growth rate) on GDP dynamics:
The parameter ϕ plays a key role in shaping the economy's path. It represents the natural rate at which the Gross Domestic Product grows. In economic terms, ϕ shows how much room an economy has to grow and come up with new ideas. Looking at Figure 2a, we can see that when ϕ drops to 1.3, the GDP growth curve moves to the right. This shift means the economy is growing more. When growth slows down like this, it often leads to other changes. Companies might not want to invest as much money. Workers might not produce as much. And the country might fall behind in developing new tech and building new infrastructure. On the other hand, when ϕ goes up (1.4), it pushes the GDP growth curve to the left, showing faster economic growth. A higher intrinsic growth rate points to a more energetic and ever-changing economy. This means

that a setting that supports new ideas, business creation, and good economic conditions can help overcome the problems caused by disease outbreaks and job losses, leading to a quicker bounce back.

Influence of κ (GDP decay rate due to unemployment) on GDP dynamics:

The κ parameter captures how unemployment hurts GDP, showing the cost to the economy when people can't work. Looking at Figure 2b, we see that a lower κ (0.15) pushes the GDP growth curve to the left, which is good news. This move hints that steps to soften the unemployment's blow can speed up economic growth. A smaller κ points to a tougher job market, less economic decay, and more room for GDP to grow. On the flip side, a higher κ (0.17) pushes the GDP growth curve to the right, meaning unemployment hits economic growth harder. A faster GDP decay rate due to joblessness suggests a job market that's slower to change and respond, which could slow down the whole process of getting the economy back on track.

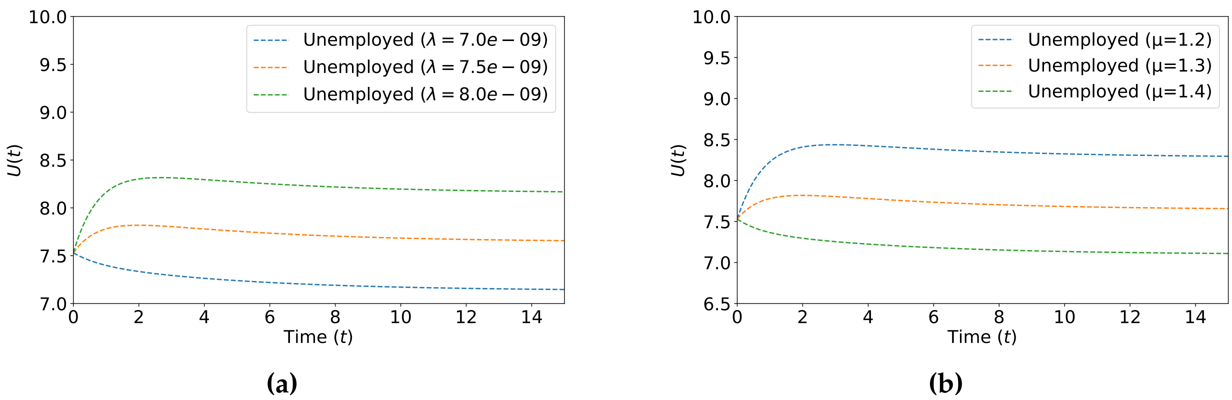

Figure 3. (a) Unemployment with decreased and increased λ value, (b) Unemployment with decreased and increased μ value at parameters given in Table 1

Influence of unemployment rate (λ) on unemployment dynamics:

The unemployment rate, symbolized by λ, plays a key role in shaping how unemployment changes over time. Looking at Figure 3a, we see that when λ goes down (7×10^{-9}), the unemployment curve moves to the right. This shift shows that a lower jobless rate causes unemployment to grow more as time passes. We can link this to things like fewer people quitting their jobs or less job loss in the economy. On the other hand, when λ goes up (8×10^{-9}), it pushes the unemployment curve to the left. This means unemployment grows faster as time passes. This might happen because more people quit their jobs or because jobs disappear quicker in the economy. When unemployment rates go up, it makes the job market tougher. This can put more stress on the economy.

Influence of re-employment rate (μ) on unemployment dynamics:

The re-employment rate, symbolized by μ, plays a key role in the dynamics of unemployment, showing how well the labor market supports job transitions. As seen in Figure 3b, when μ decreases, the unemployment curve shifts to the left. This shift indicates that a lower re-employment rate causes unemployment to rise more quickly over time. This could happen due to a lack of job opportunities or slower job creation in the economy.

In contrast, when μ increases, the unemployment curve moves to the right, indicating a slower rise in unemployment over time. This could be because of more job opportunities or faster job creation in the economy. A higher re-employment rate reflects a more effective labor market, which can help reduce unemployment and promote economic stability.

Influence of infection rate (β_1) on gross domestic product

The influence of infection rates, denoted by β_1, on GDP is important. Figure 4 demonstrates how different infection rates affect GDP growth. Increased speeds will reduce the life of the pandemic, which will mean faster accelerated rates that will foster economic recovery, whereas decelerated rates would only lengthen the downturn. This underlines the delicate balance between public health and economic stability, emphasizing the need for effective strategies to manage infection rates while promoting sustainable growth.

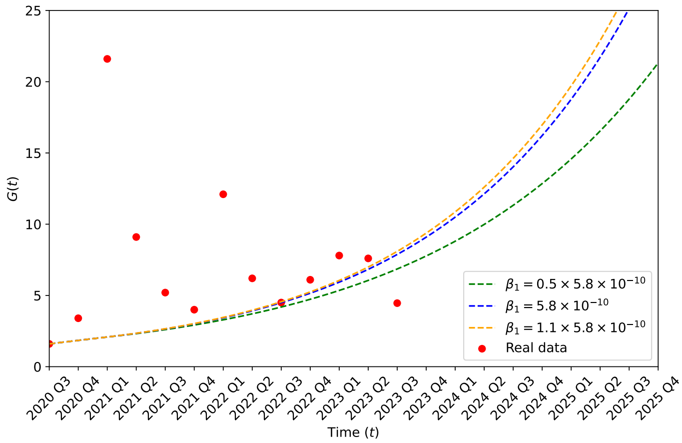

Figure 4. Influence of infection rate on GDP

Accelerated infection rate and economic recovery

An increased infection rate implies a rapid spread of the disease, leading to a quicker rise in the number of infected individuals within a shorter timeframe. A shorter duration of the pandemic may prompt an earlier commencement of the economic recovery phase. The accelerated completion of the pandemic might lead to a more immediate resurgence in economic activities, potentially resulting in a swifter rebound in GDP.

Decelerated infection rate and prolonged economic downturn

A slower infection rate extends the timeline of the pandemic, resulting in a more prolonged period of disease transmission. A longer pandemic timeline might lead to a more prolonged economic downturn.

Economic implications of infection rates on diverse sectors

Healthcare outlays stimulating economic sectors

Elevated infection rates increase healthcare spending, leading to a surge in resource allocation toward healthcare infrastructure, medical supplies, and research. This intensifies during health crises. Heightened healthcare expenditure catalyzes economic activity within specific sectors, fostering favorable growth in GDP. Amid the pandemic, India's public health spending increased from 1.5 percent to 1.8 percent of the GDP [29]. The PM Ayushman Bharat Health Infrastructure Mission scheme intends to enhance infrastructure, funded by the central government [29].

Labor market fluctuations in response to infection rates

Rapid escalations can initially lead to a transient reduction in the labor force. The reintegration of workers into the workforce can contribute to a resurgence in economic productivity.

Innovative resilience and industrial adaptation

Heightened infection rates often spur an incentive for innovation and adaptability within industries. This incentive leads to pivots in production to accommodate the manufacturing of essential goods or services requisite during crises, raising the need for the emergence of novel business models or technological advancements. This adaptive shift can give rise to growth in specific sectors, exerting a positive influence on aggregate GDP.

Dynamic consumer behavioral shifts

Varied infection rates may cause shifts in consumer behavior patterns during pandemics. These shifts lead to alterations in expenditure distributions, with some sectors witnessing a downturn while others experience increased demand. The surge in demand for essential commodities or the accelerated adoption of online services can invigorate specific sectors, thereby bolstering overall GDP growth. There has been a surge of over 100 percent in the demand for essential commodities like rice, wheat, and pulses [30]. Additionally, other food categories such as confectioneries, sweets, organic processed food, and spices have also experienced a notable increase of 15-20 percent [30].

Governmental fiscal interventions and stimuli

Governmental responses to pandemics often include fiscal policies and stimuli aimed at buttressing businesses and individuals impacted by the crisis. Such interventions, spanning financial aid, tax concessions, or infrastructure investment, are designed to stabilize the economy and wield a positive influence on GDP growth trajectories. India's government introduced a COVID-19 social aid package worth INR 1.7 lakh crore (equivalent to 25 billion US dollars) through the Pradhan Mantri Garib Kalyan Yojana (PM-GKY) [31] to offer prompt assistance to those in need.

Research and development investments for long-term economic impacts

Escalating infection rates frequently prompt heightened investments in research and development endeavors, particularly toward vaccines, treatments, or preventive measures. The resultant scientific breakthroughs engendered by such investments manifest long-term positive repercussions across various industries, nurturing innovation and consequent economic growth.

Prospective revival of tourism and hospitality sectors

Subsequent to periods of elevated infection rates and constrained travel, pent-up demand often surfaces for travel and related hospitality services upon the amelioration of the situation. This prospective resurgence in the tourism and hospitality sectors holds the potential to significantly contribute to the resurgence of GDP growth. The recovery of the tourism sector will hinge on enhancing trust in travel and reducing the perceived risks associated with it [32]. The impact of COVID-19 influences consumers' perceptions of tourism products and services [33].

Okun's law

Okun's law originates from the study between unemployment and economic growth by Okun (1962) [34] on the United States economy, where he observed that there was an inverse relationship between the two variables. Okun (1962) observed that a percentage increase in economic growth would result in a 0.3 percent decline in unemployment.

The SIRUG model incorporating Okun's Law provides a comprehensive framework for understanding the complex interplay between epidemiological dynamics and economic factors during the COVID-19 pandemic. It facilitates informed decision-making and policy formulation [35] to mitigate the health and economic impacts of the crisis.

The graph Figure 5 plots the unemployment rate on the y-axis and GDP on the x-axis. The data points show a negative correlation between the two variables, consistent with Okun's Law. In other words, as the unemployment rate increases, GDP decreases, and vice versa. However, the data points also deviate from a straight line, indicating that the relationship between the unemployment rate and GDP is not perfectly linear. The data points in the graph represent the percent change in value with the previous value as the base. A negative value indicates a negative percent change, while a positive value indicates a positive percent change.

Observations

Outliers, such as the sharp decline in GDP accompanied by a high unemployment rate in 2020 Q2, can be understood within the framework of Okun's Law. Such an event could be associated with an economic downturn or recession, where a significant drop in GDP leads to an increase in unemployment. This could be due to factors like reduced consumer spending, investment, and overall economic activity due to the surge of the pandemic.

The period of economic recovery observed in 2021 and 2022, where GDP increases and unemployment decreases, aligns well with Okun's Law. As the economy begins to recover, increased economic output (reflected in rising GDP) typically leads to job creation and a decline in unemployment rates. This can be attributed to factors like increased consumer confidence, government stimulus measures, and business investments.

The increase in unemployment and decrease in GDP observed in 2023 Q4 is again consistent with Okun's Law but in the reverse direction. Such a scenario could signal another economic downturn or slowdown, where a decrease in GDP leads to layoffs and rising unemployment rates. Factors contributing to this could include external shocks, changes in government policies, or shifts in global economic conditions.

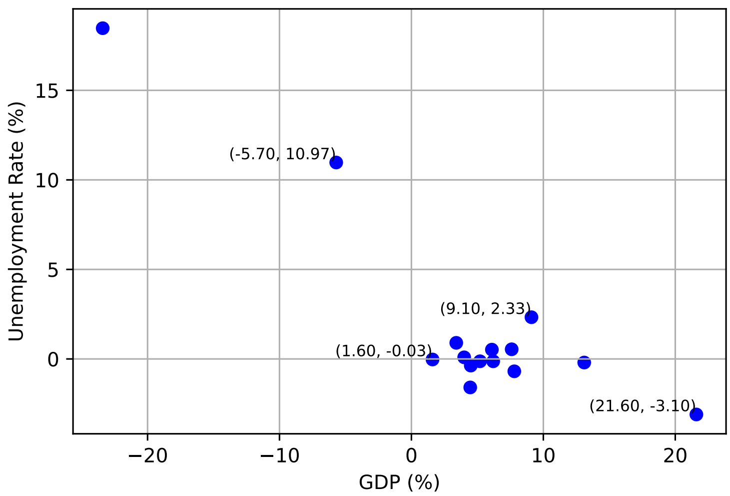

Figure 5. Change in GDP with change in unemployment - demonstration of Okun's Law

Long-term economic implications and policy recommendations

The analysis of the SIRUG model, incorporating epidemiological and economic dynamics, reveals several crucial implications for long-term economic planning and policy formulation. The model's findings demonstrate significant relationships between health metrics, economic indicators, and social outcomes, providing valuable insights for policy development.

Labor market dynamics and economic growth

The examination of unemployment (λ) and re-employment (μ) rates reveals crucial patterns in labor market behavior. The model demonstrates that decreased re-employment rates shift unemployment curves leftward, indicating accelerated unemployment growth. When the natural growth rate (ϕ) increases to *1.4*, the economy exhibits faster growth patterns, highlighting the importance of supporting innovation and entrepreneurship. Additionally, lower GDP decay rates ($\kappa = 0.15$) correlate with enhanced economic resilience, suggesting that robust unemployment protection mechanisms significantly contribute to economic stability.

Healthcare infrastructure and sectoral adaptations

The study establishes a clear correlation between infection rates (β_1) and economic performance. Analysis reveals that while accelerated infection rates may shorten pandemic duration, they can trigger severe economic shocks. This finding is supported by India's strategic increase in health spending from *1.5%* to *1.8%* of GDP [29]. The model further indicates substantial shifts in consumer behavior during crisis periods, with essential commodities experiencing demand surges exceeding *100%* [36]. These patterns emphasize the necessity for sector-specific adaptation strategies and modernized healthcare infrastructure.

Economic stabilization and future preparedness

The relationship between unemployment and GDP, as demonstrated through the model's application of Okun's Law, provides crucial insights for economic stabilization mechanisms. The implementation of support programs, exemplified by India's PM-GKY scheme providing INR *1.7 lakh crore* in aid [37], demonstrates the effectiveness of timely governmental intervention. The study indicates that anticipatory policy frameworks, encompassing both immediate response capabilities and long-term resilience mechanisms, are essential for future crisis management.

Research investment and policy integration

The model's findings emphasize the critical role of research and development in crisis resilience. Analysis suggests that integrated approaches combining healthcare research, technological advancement, and economic adaptation yield optimal outcomes. This necessitates sustained investment in research infrastructure and the development of flexible policy frameworks capable of responding to evolving challenges. The study demonstrates that successful economic recovery requires coordinated efforts across healthcare, employment, and fiscal policy domains.

The findings support a comprehensive approach to policy development, integrating health infrastructure investment, labor market flexibility, and research advancement. These elements, working in concert, provide the foundation for robust economic recovery and long-term resilience against future crises. The model's insights suggest that policy effectiveness depends on the ability to implement coordinated responses across multiple sectors while maintaining flexibility for rapid adaptation to changing circumstances..

10 Future directions

The SIRUG model opens up several exciting avenues for future research in understanding how diseases affect economies. Future studies could enhance the model by exploring the complex ways that health crises and economic factors influence each other, going beyond the current straightforward relationships. An important area for development would be incorporating the effects of different government policies, such as economic support packages and healthcare initiatives, to better predict their impact on recovery. Additionally, future research could break down the analysis by different economic sectors, as studying how various industries respond uniquely to health crises could provide more targeted policy recommendations.

The model's framework could be further enriched by exploring regional variations and demographic factors, as health crises often affect communities differently. This could lead to more tailored intervention strategies based on local conditions and population characteristics. Furthermore, incorporating international factors such as global supply chains and trade relationships would make the model more comprehensive and applicable to our interconnected world economy. These enhancements would build upon the current SIRUG model's foundation, making it an even more powerful tool for understanding and responding to future health and economic challenges..

11 Conclusion

The paper provides an overall analysis of the complex relationship between health variables and economic variables during the COVID-19 pandemic period in India from 2020 to 2023. In a bid to understand the impact of the pandemic on public health and the economy, this research modified the classic SIR model by adding the components of Gross Domestic Product and rate of unemployment. Anchoring on SIRUG, we have combined Okun's Law aspect with epidemiological dynamics and relevant economic factors in our model. Our results suggest, as expected, that on average, unemployment is negatively correlated with GDP. In keeping with Okun's Law, changes in one variable do seem to influence another. We also showed deviations from a perfect linear relationship, which further proves the multifaceted nature of this relationship of variables.

The trends that came out were the steep fall of GDP followed by a surge in unemployment in 2020 Q2, commensurate with the economic downturns attributed to the pandemic. Similarly, the economic recoveries during 2021 and 2022, accompanied by rising GDP and decreased unemployment, are not only in conformity with Okun's Law but also represent the strong bounce-back ability of the economy after crisis periods. On the other hand, challenges have been found in the research, like increasing reduction in GDP observed in 2023 Q4, indicating the likelihood of economic slowdowns or recessions.

These findings highlight how aggressive policy measures at both ends can dampen the adverse impacts of health shocks on the economy and vice versa. This paper opens the pathway to future and deep research into this complex interaction of epidemics and economic variables for India in several possible ways. First, the SIRUG model could be fine-tuned to enhance policy decisions during pandemics, its parameters can be calibrated with India-specific data, effectuating an equilibrium between GDP growth rate, unemployment level, and disease diffusion. This model could also be extended to incorporate behavior changes in order to speed up the program impact on the transmission and recovery rates, respectively. This will help fight any negative repercussions arising from educating the population on proper health measures that aim at reducing morbidity and mortality in general. Thirdly, incorporating variables like viral mutation patterns and healthcare delivery capacity will future-proof it for other pandemics while capturing regional differences across India. Fourthly, in the pursuit of long-term resilience, strategies such as health sector development or supporting a diversity of industries are necessary to sustain them over time. Further, it will be able to assess government programs for the control of unemployment and various other health issues that they confront today. Lastly, sensitivity analyses, real-time forecasting, and the use of present values data would enhance the model's accuracy and relevance. In summary, the effort put into this research creates an in-depth insight into the interactive dynamics at play during pandemics and becomes very resourceful to policymakers and researchers. We establish a platform for an informed decision– by combining health and economic variables in a unified model, making provision for policy formulation that will help to address the challenges that may be triggered by future health and economic crises.

Declarations

Use of AI tools

The authors declare that they have not used Artificial Intelligence (AI) tools in the creation of this article.

Data availability statement

The data used in this study can be accessed through the following links:
https://dge.gov.in/dge/sites/default/files/2023-05/Employment_and_Unemployment_scenario_of_India_May_2023.pdf
https://www.mospi.gov.in/dataviz-quarterly-gdp-growth-rates
https://www.thehindu.com/business/Economy/gdp-surges-76-per-cent-in-2023-july-to-september-quarter-goes-past-rbi-forecast/article67591434.ece

Ethical approval (optional)

The authors state that this research complies with ethical standards. This research does not involve either human participants or animals.

Consent for publication

Not applicable

Conflicts of interest

The authors declare that they have no conflict of interest.

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Author's contributions

A.T.: Conceptualization, Methodology, Investigation, Formal analysis, Writing – original draft. A.M.S.: Conceptualization, Methodology, Investigation, Formal analysis, Writing – original draft. N.C.: Conceptualization, Methodology, Investigation, Formal analysis, Writing – original draft. A.C.: Supervision, Formal analysis, Writing – Reviewing and editing. P.V.: Investigation, Supervision, Formal analysis, Writing – Reviewing and editing. All authors discussed the results and agreed to publish the manuscript.

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