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Reflections from the Lesson Study for the Development of Techno-Pedagogical Competencies in Teaching Fractal Geometry

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Abstract: Technological Pedagogical Content Knowledge (TPACK) is a model that explains how teachers use technology more effectively in the context of technological, pedagogical, and content knowledge. Teachers' TPACK competencies play great importance in this regard. Lesson study has also been playing significant roles in the development of teachers' professional trainings. When the researches on TPACK and lesson study have been analyzed, the research is expected to provide significant contributions to the literature. This study aims to present reflections from a lesson study practice that carried out to urge techno-pedagogical competencies of the secondary school mathematics teachers and to reveal the development of teachers' progress. The study used case study method, and it was conducted with three in-service teachers. The research data were collected through semi-structured interviews, voice recorder, and observation notes. To analyze the collected data, descriptive analysis method was used. The results have revealed that teachers have made much more progress in designing, implementing, and problem solving in terms of TPACK competencies. It has also been determined that teachers' development of openness to the innovations was limited. This limitation appeared to emerge as a result of teachers' time anxiety and insufficient knowledge regarding the use of technology.

Keywords: *Lesson study, Techno-pedagogical competencies, Secondary school mathematics teachers, Fractal geometry.*

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Introduction

Technology, pedagogy, and content knowledge have been considered as a whole, teacher competencies have been reassessed, and new criteria have been put forward (Niess, 2005). Among these models, the technological pedagogical content knowledge (TPACK) is a conceptual framework that is necessitated for effective technology integration of prospective teachers and teachers (Bos, 2011). TPACK is a model that explains how teachers incorporate technology into learning-teaching processes and how to use technology more effectively in the context of technological, pedagogical, and content knowledge (Koehler, Mishra & Yahya, 2007; Mishra & Koehler, 2006). In this regard, it is most probably that teachers' development regarding TPACK competencies is important as the quality of education is directly related to the qualifications of teachers (Demirel & Kaya, 2003). Even if schools have technological equipment, they are the teachers who will make education programs actual through using educational technologies (Ay, 2015; Isiksal & Cakiroglu, 2006). Thus, the teachers who put technology into practice should also have the ability in order to organize learning activities through technology together with new teaching techniques (Erbas, Cetinkaya & Ersoy, 2009; Koh, Chai & Tsai, 2013). The question that "can the level of teachers' techno-pedagogical competencies be improved through appropriate professional training?" comes to the mind. Numerous studies have put forward the necessity for providing teachers with receiving education that is practiced and discussed in the class (Doruk, Aktumen & Aytekin, 2013; Fernandez & Yoshida, 2004; Lewis, Perry & Hurd, 2009; Yildiz, 2013). Recently, lesson study has played a significant role in the development of teachers' professional trainings (Back & Joubert, 2011; Fernandez & Yoshida, 2004; Yoshida & Jackson, 2011; Werhoef & Tall, 2011).

Lesson study require steps such as collaborative planning of the lesson, monitoring, and discussing the lesson during the practices, replanning of the lesson (optional), implementation and discussion of a new plan (optional) (Fernandez & Yoshida, 2004). Thus, lesson study has become a system in which participants can express their wishes, gain awareness about a particular subject and develop their abilities together (Senge, Cambron-McCabe, Lucas, Smith, Dutton & Kleiner, 2000). Furthermore, teachers develop a deeper understanding of content and student thinking in this process, and their teaching may become more useful and understandable (Murata, 2011). For instance, Ong, Lim, and Gazali (2010) found that teachers initially asked questions about the steps of the process and the results achieved, but they urge the students to think after lesson study. This can be considered as a difference from in-service courses for lesson study. In

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this context, the reflections from the lesson study on the development of techno-pedagogical competencies of secondary school mathematics teachers were presented and the development of teachers in this process was examined in the current study.

Upon analyzing the researches on TPACK, theoretical studies have mostly concentrated on the conceptualization of TPACK (Angeli & Valanides, 2009; Cox, 2008; Graham, 2011), the determination of TPACK levels (Archambault & Crippen, 2009; Gundogmus, 2013; Tokmak, Konokman & Yelken, 2013), the examination of its development (Akkaya, 2009; Harris & Hofer, 2011; Jaipal & Figg, 2010; Jang & Chen, 2010; Niess, 2005) and the effect of various in-service training programs on teachers' TPACK levels (Chai, Koh & Tsai, 2010; Guzey & Roehring, 2009). Other than that, the researches on lesson study have been identified to introduce different lesson study practices (Sarkar Arani, Keisuke & Lassegard, 2010) and evaluate teachers' impressions by bringing them together in a project (Saito, Harun, Kuboki & Tachibana, 2006). These studies reveal the development of teachers' teaching techniques (Inoue, 2011; Lewis, Perry, Hurd & O'Cannell, 2006). On the other hand, when the studies carried out in our country have been examined, Baki (2012) and Butun (2012) carried out lesson study practices with prospective teachers. Yildiz (2013) also analyzed the effect of lesson study practices conducted with secondary school mathematics teachers on teachers' behaviors towards enhancing their students' metacognition in problem-solving setting. Hence, this research is thought to be original, and it will provide significant contributions to the literature.

The Aim of Research

This research aims to present reflections from lesson study practice carried out to ensure techno-pedagogical competency to secondary school mathematics teachers, and reveal the development of teachers in this process. This process has been limited to the fractal geometry. Fractal geometry is much more dominated compared to Euclidean geometry in terms of modeling nature through mathematics (Baki, 2001). Lonell and Westerberg (1999) describe fractal geometry as the geometry of nature. Thus, fractal geometry has become one of the most popular topics that have been studied in mathematics and mathematics education as a new field of mathematics in recent years (Karakus, 2010). Several studies also explore the fact that fractals help students see mathematics as a whole (Lornell & Westerberg, 1999; Karakus, 2007; Fraboni & Moller, 2008).

Methodology

This section covers research model, participants of the research, implementation process, data collection and analysis.

Research Method

In this research, we have used the method of case study as it enables us to examine a particular group in depth and to assess the data obtained through data collection tools without being concerned about generalization. According to Yin (2003) a case study design should be considered when: (a) the focus of the study is to answer "how" and "why" questions; (b) it is impossible to manipulate the behavior of those involved in the study.

Participants

The research was carried out with three secondary school mathematics teachers who work at the same school and who were selected among those capable of utilizing dynamic software voluntarily. Both participants were male, one of whom has been working for 10 years while the other is 7 years. While presenting the data, we coded each in-service teacher. For example, T1 stands for the first teacher and R stand for the researcher.

Data Collection Tools

The research data were collected through interviews with the teachers during the lesson study, the voice recorder by which teachers' dialogues were recorded in the planning, and observation notes that the teachers held during the practice. Before conducting the research, we determined possible questions for the interview. Therefore, the interview questions were designed to present the process of teaching fractals and to answer how this process could be planned better.

Implementation Process

The guideline, which is considered to be used during lesson study, was formed with relevant literature and interviews with experts. Hence, the guideline includes learning theories, teaching methods / techniques / strategies, design of effective learning setting, software / technological products that are used in mathematics education, measurement and evaluation application topics, and activities related to these topics.

The implementation process was performed in the second semester of 2015-2016 academic years. Some parts of these studies were conducted as a group with three teachers working at the same school including the researcher. The informative and discussion phase of the lesson study were performed in groups. The three teachers except for the researcher carried out the planning and implementation phases. On that point, the other teacher also entered the teacher's course. The implementation was carried out with the 8th grade students.

Hence, teachers were asked to record the planning phase via a voice recorder and to take reflective observation notes in the learning setting. Therefore, the discussion phase of the lesson study was realized within the framework of these notes and teacher observations. Each meeting lasted about 40 minutes in which group members come together out of course hours. Thus, the cycle of three lesson study was completed.

Data Analysis

Having a qualitative data analysis design, this research used descriptive analysis. Thus, interviews with the teachers and teachers' dialogues in the planning phase were written verbatim. Yildirim and Simsek (2005) have suggested that direct citations must be included in the descriptive analysis so that it can conspicuously reflect the interviews. Each interview was written without any correction by taking the order of interviewer-interviewee in to consideration. Then, the data analysis was finalized by bringing the independent analyzes together and discussing them separately by the researchers. Researchers' consensus has been paid great attention. The study used performance indicators developed by Yurdakul, Odabasi, Kilicer, Coklar, Birinci and Kurt (2014) for teachers' techno-pedagogical competencies. Thus, the development in these indicators was revealed by the expressions of the teachers during the lesson study and by the behaviors during the practice. On the other, codes were handed to the researcher and teachers during the presentation of findings.

Findings

The first lesson study cycle begins with the teachers' following thoughts.

T1: It is difficult to understand abstract expressions, especially within fractal geometry.
T2: Yes. We are incompetent in fractals while drawing the next steps.
T1: It is Ok. Then, let's create an alternative plan for the current lesson plan. Let's plan an activity for drawing the next steps of the fractal, which our students are forced to do.
T3: I think that the activities that we will do or the abstract concepts may be best understood with the GeoGebra program as it will be more effective for students to learn by doing. We have already observed this situation within some implementations or in our conversation with R.

In general, following the dialogues mentioned above, teachers formed patterns and fractal examples through GeoGebra software. Teachers were determined to experience several difficulties in performing the activities by means of a slider object. T1 who will implement this practice presented these examples to the students and directed the students to the discovery process. These examples are as regards:

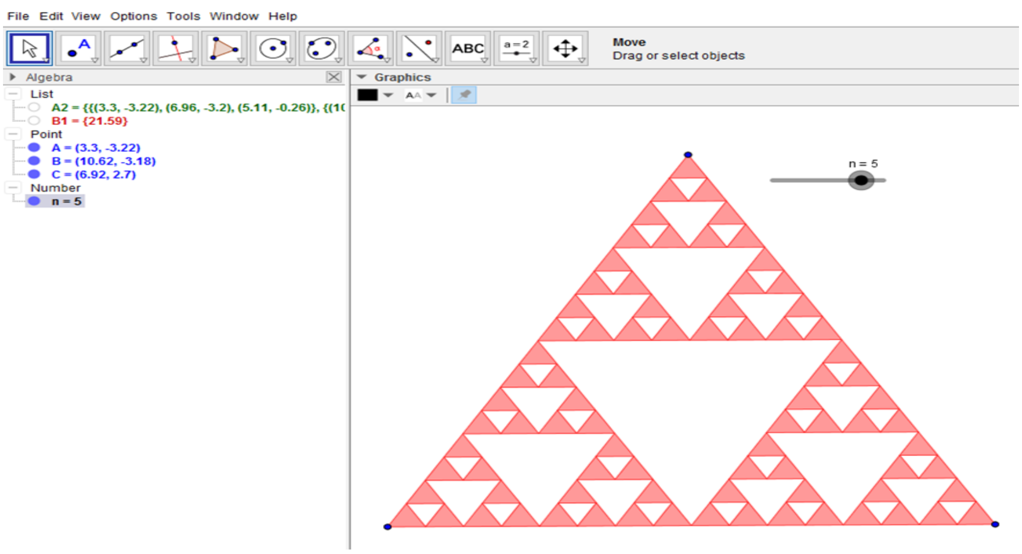

Figure 1. Sierpinski triangle activity created through GeoGebra software.

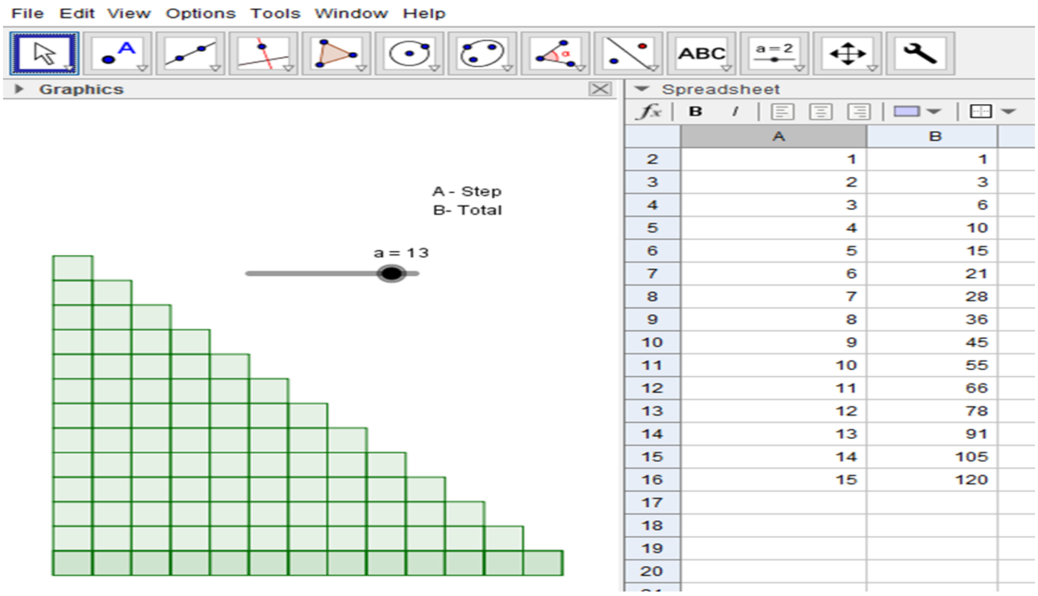

Figure 2. The number of bricks in the ladder steps created through GeoGebra software.

As seen in the above figures, teachers preferred Sierpinski triangle for fractals while ladder steps for pattern. Students were expected to make observations regarding fractal for different steps by moving GeoGebra's slider object, and thus finding the pattern rule.

In the implementation process, T1 enabled students to make relations with daily life through the use of examples from nature and technology. Then, the same teacher presented examples to the students in accordance with the purpose mentioned above, and he requested to find the difference between the pattern and the fractal. Finally, he finished the lesson with the Antropi Teach program during the evaluation stage. However, it was identified that T1 could not solve some of the technological problems that students encountered in this process.

Time is the mostly dominant topic during the discussion phase for both teachers. Teachers' views are as follows.

T1: I think such activities actually take a long time. We may get behind in the curriculum in the future.
R: Our first goal is to provide students with learning the acquisitions by constructing them. If technology will help this process, we must use it. Yet, this does not require such a situation in which technology-aided training is used for learning each acquisition. We must use technology without hesitation the moment we feel that there is a need for learning. As we talked earlier, we are required to design and manage the learning environment.
T2: In fact, as we prefer to present much more real-life events with more slides, we do not have difficulty in using technology. But we cannot even do it due to time constraint.

Since both teachers share the same ideas, teachers are unable to demonstrate their ability to *use technology in conducting in-class and out-of-class educational activities (homework, projects, internships, etc.)* and *produce solutions to the technological problems encountered while preparing activities related to real life.* This may derive from teachers' time anxiety and insufficient knowledge regarding the use of technology.
Teachers were reminded of the potential for group work in the informational phase of the second lesson study cycle, and some of the deficiencies in using GeoGebra software were illustrated on some examples. In the planning stage, the dialogue between the teachers is summarized as follows.

T3: We may teach this subject far more effective with the fractal card activity. (This is planned in advance, which aims to expose the fractal example by cutting a paper into frames of certain rules in the event.) In this way, the students are able to visualize the next steps in their minds through their own practice.
T2: GeoGebra and EBA data will increase the visually of the subject.

Figure 3 depicts the final stage of the fractal card activity that teachers mentioned in the dialogues above. T2, who implemented this practice, firstly had students perform this activity.

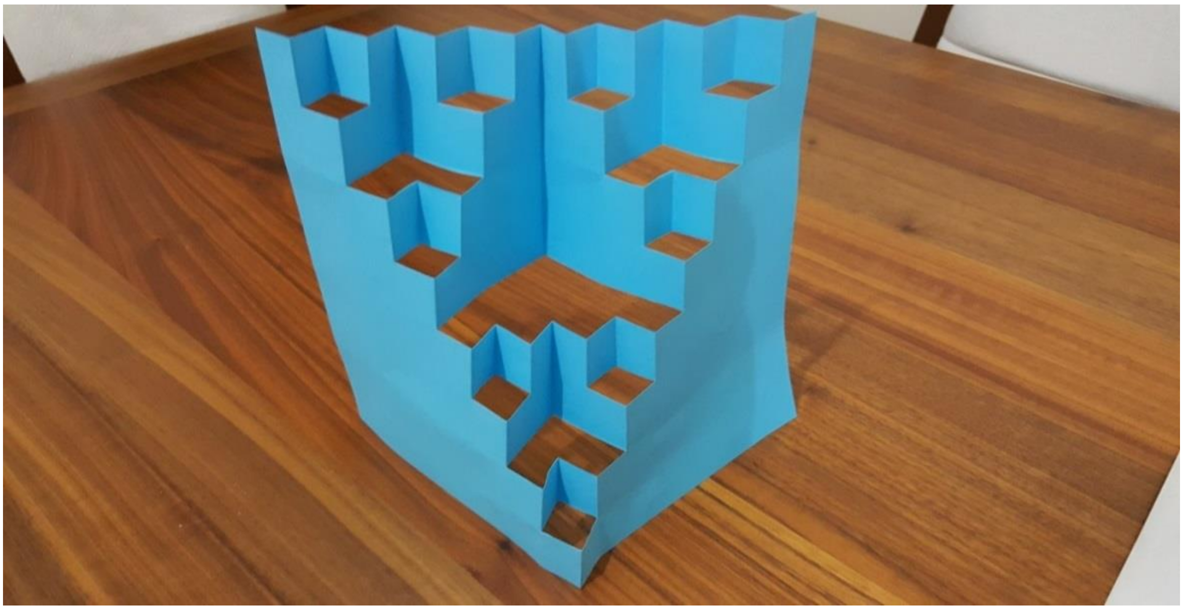

Figure 3. Fractal card activity.

Afterwards, students were asked to do the activity below (Figure 4) that was created by both teachers via GeoGebra software during the planning stage in order that students can have the opportunity for exploring repetition and self-similarity concepts that are the features of fractals and that distinguish them from the shapes of Euclidean geometry.

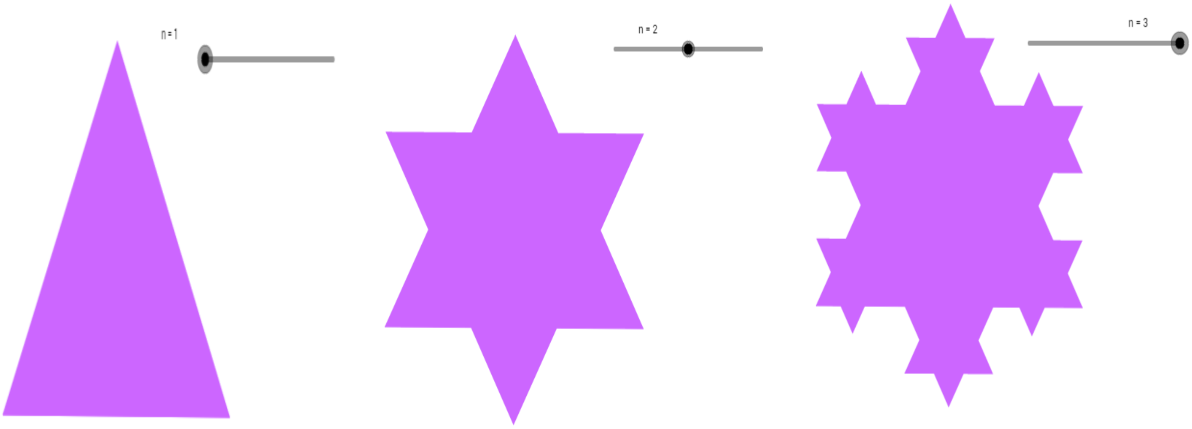

Figure 4. Snowflake created in GeoGebra software.

The activity presented above includes a triangle first, followed by a marking of the points at one-third of each side of the triangle, followed by a snowflake fractal example. In this context, students made observations by changing the slider object as it is to explore the concepts of repetition and self-similarity.
Then, we met the teachers for the discussion phase. At this time, teachers expressed the favourableness using them effectively in their technology courses over time as, they usually did not use it preliminarily. In this process, teachers' desire to use technology has been determined to increase even more, and that they need to deepen their teaching with their technological products.

T1: Actually, students are happy to see things on the screen and try to interpret them. For instance, I do not show the similarity of shapes in the graphical display of GeoGebra, but I think you have contributed more to their learning. So, I long for utilizing the software more in my classes over time.
T2: ... Yes, I did not use it much in my prior lessons. I now want to use technology efficiently in time.

The planning phase of the third lesson was spent preparing the measuring and assessment practices to assess the students' learning level of the fractals. This is the stage of the application and then the stage of discussion. At this stage, teachers generally discussed the benefits of the activities they have done to students. In this discussion process, the general judgment was that GeoGebra activities increased the students' requests for fractal and they had a more enjoyable course process.

T3: When visual examples are given in this way, students are more interested in the lesson. I have already observed that students are more enthusiastic with GeoGebra. Actually, this is how our class became more enjoyable.
The research results reveal that teachers have made much more progress in designing the teaching process, implementing the teaching process and problem solving in terms of TPCK competencies. To illustrate, the teachers' thoughts are as follows.

T1: Actually, it's better than in-service teacher education that we do not get together like this. We can be more beneficial to our students by discussing what we do and telling each other. For example, using our technology to start planning lessons will improve, students' problem-solving skills. In addition, thanks to various technologies, it is rewarding for students to discuss the subject for good learning and to intervene when a problem arises. In this way, we may construct and discuss something in our minds.
T2: We can improve our knowledge with different studies by keeping it alive at all times. We have already begun to put more of our knowledge into practice in the fields of designing and implementing, which we both discuss in the first place.

However, the development in the performances such as *utilizing technology while preparing appropriate measurement tools for the subject area* in terms of designing, and *using technology effectively in conducting in-class and out-of-class educational activities (homework, project, internship etc.) and in the process of measuring and evaluating student performances* regarding implementation, and *using technology in the development of the ability to produce solutions to technological problems encountered while preparing activities related to real life* in terms of problem solving has been limited due to teachers' beliefs. The expressions for this situation are as follows.

T2: Our biggest concern is actually curriculum development anxiety. We want to give examples and train our curriculum as soon as possible. We have often this in our minds.
T1: Yes, this is significant. And we have never used technology in this way before. We have not encountered with such a technology with in university education. After that, perhaps, we did not learn exactly because of our laziness.

As it is observed, the limited development in these behaviors is due to the teachers' time concerns about the curriculum and the inadequate knowledge of how to use the technology. On the other hand, it has been determined that teachers' development of being open to the innovations of the TPCK competencies is limited. Competences in this area; *Keeping up-to-date information about the technology, keeping up-to-date information about the technology, keeping up to date information about the teaching process, and integrating innovations into real life with the teaching process*. The expressions are as following.

T1: Actually, we do not follow up-to-date training. So, in the education process we do not follow what is new, what is better now. Maybe I do not have time for that.
T2: There was no such dynamic software when I thought of it before, but look how good it is right now. But of course you have to follow up new information to use it. For example, while we were learning GeoGebra, there were some features. Now I see that new features have been included.

Discussion and Conclusion

Teachers have been found to experience how techno-pedagogical competencies can be demonstrated in a classroom environment both in their own and in the practice of group friends during lesson study. As in the findings by Crawford, Chamblee and Rowlett (1988), this may derive from the fact teachers could see effective directions provided these competencies were used. When considered in general, teachers have developed themselves regarding fractals in the context of techno-pedagogical competencies. Of course, the shift in qualifications has been great for some, and small for others. However, this research has revealed that the lesson study has been largely altered in comparison to the previous one in most cases. Similarly, Groth, Spickler, Bergner, and Bardzell (2009) have emphasized that there may be changes in the habit of teachers due to lesson study.
The limitation of improvement has been determined in competencies such as using technology while preparing the appropriate measurement tool for the subject area in terms of designing and using technology in conducting in-class and out-of-class educational activities (homework, project, internship, etc.) related to implementation. Still, at the end of the lesson study, teachers began to express that it would be useful to try to use them effectively in technology courses over time. In fact, the limited improvement in teachers can be enhanced over time for all teachers. Altan (1998) emphasizes the importance of providing technical support for teachers who use technology in teaching curriculum when they do not provide technical support and assistance to solve problems arising in the process of using technology in teaching. In this study, this was achieved with a few weeks of lesson study, and a limited improvement in teachers was ensured. On the other hand, it was postulated that the teachers' ability to produce solutions to the technological problems while preparing activities related to real life problems in problem solving was limited. Likewise, a similar

finding has been put forward by Simsek, Demir, Bagceci, and Kinay (2013) emphasizing that faculty members found themselves less competent in solving the problems that might arise when using the technology in the teaching-learning process.

It has also been concluded that teachers' developments are limited in terms of being open to innovations in research. Similarly, Hermans, Tondeur, van Braak, and Valcke (2008) and Argon, Ismetoglu, and Celik Yilmaz (2015) found that teachers did not see themselves adequate enough concerning individual innovation. However, teachers are the ones who will bring new ideas, approaches and possibilities and the process of education out of the conventional way. In order to achieve this, teachers need to improve themselves for life. Hence, it is required to take measures that help teachers to improve them and to organize the courses.

Sarkar Arani, Keisuke, and Lassegard (2010) stated that there will not be any positive changes due to lesson study. Carlson and Bloom (2005) and Handal and Herrington (2003) found that teachers are more confident in their beliefs than their contemporary education practices in their courses. In accordance with this claim, this study has determined the impact of teachers' beliefs upon demonstrating some techno-pedagogical competence performances. For instance, the development of the ability to produce solutions to technological problems while preparing activities related to real life has been limited due to teachers' beliefs. Similarly, Anderson and Maninger (2007) and Demir and Bozkurt (2011) reported a strong correlation between the beliefs about technology integration and the use of technology. Besides, Depaepe, Corte, and Verschaffel (2010) noted that one of the factors influencing course content and practice is the timetable and the curriculum that must be trained.

Research results have revealed that, lesson study for the training of the teachers should be extended. It should also be possible for teachers to exchange views with other branch teachers in order to change some of their beliefs on exhibiting techno-pedagogical competence performances.

References

Akkaya, E. (2009). *Matematik ogretmen adaylarinin turev kavramina iliskin teknolojik pedagojik alan bilgilerinin ogrenci zorluklari baglaminda incelenmesi* [Investigating technological pedagogical content knowledge for derivative concept: Knowledge of student difficulties] (Unpublished Doctoral Thesis), Marmara University, Istanbul, Turkey.

Altan, M. Z. (1998). A call for change and pedagogy: A critical analysis of teacher education in Turkey. *European Journal of Education*, *33*(4), 407-417.

Anderson, S. E., & Maninger R. M. (2007). Preservice teachers' abilities, beliefs and intentions regarding technology integration. *Journal of Educational Computing Research*, *37*(2), 151-172.

Angeli, C., & Valanides, N. (2009). Epistemological and methodological issues for the conceptualization, development, and assessment of ICT–TPCK: Advances in technological pedagogical content knowledge (TPCK). *Computers & Education, 52,* 154-168.

Archambault, L., & Crippen, K. (2009). Examining TPACK among K-12 online distance educators in the united states. *Contemporary Issues in Technology and Teacher Education, 9*(1), 71-88.

Argon, T., Ismetoglu, M., & Celik Yilmaz, D. (2015). The opinions of branch teachers about their technopedagogical education competencies and induvial innovativeness levels. *Journal of Research in Education and Teaching*, *4*(2), 319-333.

Ay, Y. (2015). *Ogretmenlerin teknolojik pedagojik alan bilgisi (TPAB) becerilerinin uygulama modeli baglaminda degerlendirilmesi* [Evaluation of teachers' technological pedagogical content knowledge (TPACK) within the framework of practical model] (Unpublished Doctoral Thesis). Eskisehir Osmangazi University, Eskisehir, Turkey.

Back, J., & Joubert, M. (2011, February). *Lesson study as a process for professional development: Working with teachers to effect significant and changes in practice*. Proceedings of 7th Congress of the European Society for Research in Mathematics Education, Rzeszow, Poland.

Baki, A. (2001). Bilisim teknolojisi isigi altinda matematik egitiminin degerlendirilmesi [Evaluation of mathematics education under the light of information technology]. *Milli Egitim Dergisi*, 149, 26-31.

Baki, M. (2012). *Sinif ogretmeni adaylarinin matematigi ogretme bilgilerinin gelisiminin incelenmesi: Bir ders imecesi (lesson study) calismasi* [Investigating development of prospective primary teachers' mathematical pedagogical content knowledge: Lesson study] (Unpublished Doctoral Thesis). Karadeniz Technical University, Trabzon, Turkey.

Butun, M. (2012). *Ilkogretim matematik ogretmeni adaylarinin uygulanan zenginlestirilmis program surecinde matematigi ogretme bilgilerinin gelisimi* [The development of pedagogical content knowledge of preservice mathematics teachers in the process of applied enriched program] (Unpublished Doctoral Thesis), Karadeniz Technical University, Trabzon, Turkey.

Bos, B. (2011). Professional development for elementary teachers using TPACK. *Contemporary Issues in Technology and Teacher Education*, *11*(2), 167-183.

Carlson, M. P., & Bloom, I. (2005). The cyclic nature of problem solving: An emergent multidimensional problem-solving framework. *Educational Studies in Mathematics*, *58*, 45–75.

Chai, C. S., Koh, J. H. L., & Tsai, C. C. (2010). Facilitating preservice teachers' development of technological, pedagogical, and content knowledge (TPACK). *Educational Technology & Society*, *13*(4), 63–73.

Crawford, A. R., Chamblee, G. E., & Rowlett, R. J.(1998). Assessing concerns of algebra teachers during a curriculum reform: A constructivist approach. *Journal of In-service Education*, *24*(2), 317–327.

Cox, S. (2008). *A conceptual analysis of technological pedagogical content knowledge* [Unpublished Doctoral Dissertation]. Brigham Young University, USA.

Demir, S., & Bozkurt, A. (2011). Ilkogretim matematik ogretmenlerinin teknoloji entegrasyonundaki ogretmen yeterliklerine iliskin gorusleri [Primary mathematics teachers' views about their competencies concerning the integration of technology]. *Elementary Education Online*, *10*(3), 850-860.

Demirel, O., & Kaya, Z. (2003). O*gretmenlik meslegine giris* [Introduction to teaching profession]. Ankara: Pegem Yayincilik.

Depaepe, F., Corte, E., & Verschaffel, L. (2010). Teachers' approaches towards word problem solving: Elaborating or restricting the problem context. *Teaching and Teacher Education*, *26*, 152-160.

Doruk, B. K., Aktumen, M., & Aytekin, C. (2013). Pre-service elementary mathematics teachers' opinions about using GeoGebra in mathematics education with reference to 'teaching practices'. *Teaching Mathematics and Its Aplication. 32*(3), 140-157.

Erbas, A. K., Cetinkaya, B., & Ersoy, Y. (2009). Ogrencilerin basit dogrusal denklemlerin cozumunde karsilastiklari guclukler ve kavram yanilgilari [Student difficulties and misconceptions in solving simple linear equations]. *Education and Science, 34*(152), 44-59.

Fernandez, C., & Yoshida, M. (2004). *Lesson study A Japanese approach to improving mathematics teaching and learning*. New Jersey, Lawrence Erlbaum Associates.

Fraboni, M., & Moller, T. (2008). Fractals in the classroom. *Mathematics Teacher*, *102*(3), 197.

Graham, C. R. (2011). Theoretical considerations for understanding technological pedagogical content knowledge (TPACK). *Computers & Education, 57*, 1953- 1960.

Groth, R., Spickler, D., Bergner, J., & Bardzell, M. (2009). A qualitative approach to assessing technological pedagogical content knowledge. *Contemporary Issues in Technology and Teacher Education, 9*(4), 392-411.

Guzey, S. S., & Roehrig, G.H. (2009). Teaching science with technology: Case studies of science teachers' development of technology, pedagogy, and content knowledge. *Contemporary Issues in Technology and Teacher Education, 9*(1), 25-45.

Gundogmus, N. (2013). *Ogretmen adaylarinin teknolojik pedagojik alan bilgileri ile ogrenme stratejileri arasindaki iliskinin incelenmesi* [Analysis of relationship between technological pedagogical and learning strategies of teacher candidates] (Unpublished Master Thesis), Necmettin Erbakan University, Konya, Turkey.

Handal, B., & Herrington, A. (2003). Mathematics teachers' beliefs and curriculum reform. *Mathematics Education Research Journal*, *15*(1) 59-69.

Harris, J. B., & Hofer, M. J. (2011). Technological pedagogical content knowledge (TPACK) in action: A descriptive study of secondary teachers' curriculum based, technology-related instructional planning. *Journal of Research on Technology in Education, 43*(3), 211-229.

Hermans, R, Tondeur, J., van Braak, J., & Valcke, M. (2008) The impact of primary school teachers' educational beliefs on the classroom use of computers. *Computers & Education*, *51*, 1499-1509.

Inoue, N. (2011). Zen and the art of neriage: Facilitating consensus building in mathematics inquiry lessons through lesson study. *Journal of Mathematical Teacher Education*, 14, 5-23.

Isiksal, M., & Cakiroglu, E. (2006). Ilkogretim matematik ogretmen adaylarinin matematige ve matematik ogretimine yonelik yeterlik algilari [Pre-service mathematics teachers' efficacy beliefs toward mathematics and mathematics teaching]. *Hacettepe Universitesi Egitim Fakultesi Dergisi*, *31*,74-84.

Jaipal, K., & Figg, C. (2010). Unpacking the "Total package": Emergent TPACK chracteristics from a study of preservice teachers teaching with technology. *Journal of Technology and Teacher Education, 18*(3), 415-441.

Jang, S. J., & Chen, K. C. (2010). From PCK to TPACK: Developing a transformative model for pre-service science teachers. *Journal of Science Education and Technology, 19*(6), 553-564.

Kabakci Yurdakul, I., Odabasi, H. F., Kilicer, K., Coklar, A. N., Birinci, G., & Kurt, A. A. (2014). Constructing technopedagogical education based on teacher competencies in terms of national standards. *Elementary Education Online*, *13*(4), 1185-1202.

Karakus, F. (2007). Ilkogretim ogrencilerinin fraktal geometri etkinliklerine yonelik gorusleri [Elementary school students' views on fractal geometry activities]. First National Primary Congress, Hacettepe University, Ankara, Turkey.

Karakus, F. (2010). Fraktal kart etkinligiyle fraktal geometriye giris [Introduction to fractal geometry with fractal card activity]. *Elementary Education Online*, *9* (1), 1-6.

Koehler, M. J., Mishra, P., & Yahya, K. (2007). Tracing the development of teacher knowledge in a design seminar: Integrating content, pedagogy and technology. *Computers & Education*, *49*(3), 740-762.

Koh, J. L., Chai, C. S., & Tsai, C. C. (2013). Examining practicing teachers' perceptions of technological pedagogical content knowledge (TPACK) pathways: A structural equation modeling approach. *Instructional Science, 41*(4), 793-809.

Lewis, C., Perry, R., Hurd, J., & O'Cannell, M. P. (2006). *Lesson study comes of age in North America*. 88, 273-281. Phi Delta Kappan.

Lewis, C., Perry, R., & Hurd, J. (2009). Improving mathematics instruction through lesson study: A theoretical model and North American case. *Journal of Mathematics Teacher Education*, *12*, 285-304.

Lornell, R., & Westerberg, J. (1999). Fractals in high school: Exploring a new geometry. *Mathematics Teacher*, *92*(3), 260–269.

Mishra, P., & Koehler, M. (2006). Technological pedagogical content knowledge: A framework for teacher knowledge. *The Teachers College Record*, *108*(6), 1017-1054.

Murata, A. (2011). *Conceptual overview of lesson study: Introduction*. In L. Hart, A. Alston & A. Murata (Eds.), Lesson Study Research and Practice in Mathematics Education: Learning Together, pp. 1-12, NY: Springer.

Niess, M. L. (2005). Preparing teachers to teach science and mathematics with technology: Developing a technology pedagogical content knowledge. *Teaching and Teacher Education*, *21*(5), 509-523.

Ong, E. G., Lim, C. S., & Ghazali, M. (2010). Examining the changes in novice and experienced mathematics teachers' questioning techniques through the lesson study process. *Journal of Science and Mathematics Education in Southeast Asia*, *33*(1), 86-109.

Saito, E., Harun, I., Kuboki, I., & Tachibana, H. (2006). Indonesian lesson study in practice: Case study of Indonesian mathematics and science teacher education project. *Journal of In-service Education*, *32*(2), 171–184.

Sarkar Arani, M. R., Keisuke, F., & Lassegard, J. P. (2010). "Lesson Study" as professional culture in Japanese schools: An historical perspective on teacher classroom practices. *Japan Review*, *22*, 171-200.

Senge, P., Cambron-McCabe, N., Lucas, T., Smith, B., Dutton, J., & Kleiner, A. (2000). *Schools that learn: A fifth discipline field book for educators, parents, and everyone who cares about education*. *NY*: Doubleday / Currency.

Simsek, O., Demir, S. Bagceci, B., & Kinay, I. (2013). Ogretim elemanlarinin teknopedagojik egitim yeterliliklerinin cesitli degiskenler acisindan incelenmesi [Examining technopedagogical knowledge competencies of teacher trainers in terms of some variables]. *Ege Journal of Education, 14*(1), 1-23.

Tokmak, H. S., Konokman, G.Y., & Yelken, T.Y. (2013). Mersin Universitesi Okul oncesi ogretmen adaylarinin teknolojik pedagojik alan bilgisi (TPAB) oz guven algilarinin incelenmesi [An investigation of Mersin University early childhood pre-service teachers' self-confidence about their technological pedagogical content knowledge (TPACK)]. *Journal of Kirsehir Education Faculty, 14*(1), 35–51.

Yildiz, A. (2013). *Ders imecesinin matematik ogretmenlerinin problem cozme ortamlarinda ogrencilerinin ustbilislerini harekete gecirmeye yonelik davranislarina etkisi* [The effect of lesson study to the behaviours of mathematics teachers intended to promote the metacognitions of the students in the problem solving environment] (Unpublished Doctoral Thesis), Karadeniz Technical University, Trabzon, Turkey.

Yildirim, A., & Simsek, H. (2005). *Sosyal bilimlerde nitel arastirma yontemleri* [Qualitative research methods in the social sciences]. Seckin Yayincilik. Ankara.

Yin, R. K. (2003). *Case study research: Design and methods*. Thousand Oaks, CA: Sage.

Yoshida, M., & Jackson, W., C. (2011). *Ideas for developing mathematical pedagogical content knowledge through lesson study*. In L, C, Hart., A. Alston and A. Murata (Eds.), Lesson Study Research and Practice in Mathematics Education (pp, 279- 288), Dordrecht, The Netherlands: Springer.

Werhoef, N. C., & Tall, D. O. (2011). *Lesson study:* The effect on teacher's professional development. In Ubuz, B.(Ed.). Proceedings of the 35th Conference of the International for the Psychology of Mathematics Education, 4, 297-304, Ankara, Turkey: PME.