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Characterization of soft sets by $\tilde{\delta}$-soft operations

Esnek kümelerin $\tilde{\delta}$-esnek operasyonlarla tanımlanması

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Characterization of Soft Sets by $\tilde{\delta}$-soft Operations

Araştırma Makalesi / Research Article

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ABSTRACT

The purpose of this paper is to introduce new structures of $\tilde{\delta}$-soft sets in soft ditopological (SDT) spaces and study $\tilde{\delta}$-soft operations such as $\tilde{\delta}$-soft interior, $\tilde{\delta}$-soft closure, $\tilde{\delta}$-soft boundary and $\tilde{\delta}$-soft exterior. This study is therefore organized the analogies between the concepts of $\tilde{\delta}$-soft operations, on the other, are strongly emphasized. Moreover, a result which play a pivotal role in the characterization of $\tilde{\delta}$-soft open sets is found out.

Keywords: Soft sets, $\tilde{\delta}$-soft open(closed)set, $\tilde{\delta}$-soft nbd, $\tilde{\delta}$-soft interior(exterior), $\tilde{\delta}$-soft closure(boundary), soft topology, soft ditopology.

Esnek Kümelerin $\tilde{\delta}$-esnek Operasyonlarla Tanımlanması

ÖZ

Bu araştırmanın amacı, esnek 3-lü topolojik (SDT) uzaylarda esnek kümelerin yeni yapılarını tanıtmak ve $\tilde{\delta}$-esnek iç, $\tilde{\delta}$-esnek kapanış, $\tilde{\delta}$-esnek komşuluk ve $\tilde{\delta}$-esnek dış gibi $\tilde{\delta}$-esnek işlemleri incelemektir. Bu yüzden bu çalışma bir taraftan $\tilde{\delta}$-esnek işlem tanımları arasındaki benzerlikleri düzenlerken, bir taraftan da işlem ilişkilerini güçlü bir şekilde vurgular. Dahası, $\tilde{\delta}$-esnek açık kümelerin tanımlanmasında çok önemli bir rol oynayan bir sonuç da bulunmuştur.

Anahtar Kelimeler: Esnek küme, $\tilde{\delta}$-esnek açık(kapalı)küme, $\tilde{\delta}$-esnek komşuluk, $\tilde{\delta}$-esnek iç(dış), $\tilde{\delta}$-soft kapanış(sınır), esnek topoloji, esnek 3-lü topoloji.

1. INTRODUCTION

The notion of soft sets introduced by Molodtsov [1] in 1999 as a general mathematical tool for dealing with uncertainties while modelling the problems in engineering, physics, computer science, economics, social sciences and medical sciences. Later, he brought out applications of soft set in various areas in [2]. Maji et al. [3] defined and studied several basic notions of soft set theory and developed this theory in [4]. In 2005, Pei and Miao [5] and Chen [6] improved the application of soft sets in various areas.

Recently, few researches (see, for example, [7-10]) introduced and studied the notion of soft topological spaces which are defined over an initial universe with a fixed set of parameters. I tried to account for all the major achievements in soft topology over the last few years. A considerable achievement is introducing product topology and defining soft compactness in [7]. Considering the importance of topological structure in developing soft set theory, more specialized notions from the theory of soft topology will be introduced is needed. This brings about the natural question of wheather or not there is any additional topology on the soft set all possible itineraries. Consequently, soft ditopological (SDT) space on a soft set was introduced by Dizman et al. [11] and Şenel [12] independently. In the notion of soft ditopology [12], the concept of SDT-space on a soft set consists of with two structures on it - a soft topology and a soft subspace topology. The first one is used to describe soft openness properties of a soft topological space while the second one deals with its sub-soft openness properties. This structure enables to study with all soft open sets can be obtained on a soft set. Therefore, we continue investigating the work of [12].

Much of the rest of the paper is devoted to a general study of soft ditopological spaces. In section 2, I lay the foundations for a systematic study of soft sets. In section 3, I proceed with the study of SDT-space. I define and study new structures in soft ditopological spaces with defining soft exterior. I establish several interesting properties of soft interior, soft exterior, soft closure and soft boundary and their relationship which are fundamental for further research on soft ditopology and will strengthen the foundations of the theory of soft ditopological spaces. I intend to obtain an interest in the study of the relationships between $\tilde{\delta}$-soft exterior, $\tilde{\delta}$-soft closure, $\tilde{\delta}$-soft complement, $\tilde{\delta}$-soft interior and $\tilde{\delta}$-soft boundary with respect to $\tilde{\delta}$-soft operations by giving theorems. I study some properties of $\tilde{\delta}$-soft exterior, with the aim of describing $\tilde{\delta}$-soft open and $\tilde{\delta}$-soft close sets directly using $\tilde{\delta}$-soft exterior. The usefulness and interest of this correspondence of $\tilde{\delta}$-soft exterior will of course be enhanced if there is a way of

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returning from the transforms to the $\tilde{\delta}$ -soft sets, that is to say, if there is a formula that characterize $\tilde{\delta}$ -soft set. In closing this paper, all the studies come to fruition and I take up a result which play a pivotal role in the characterization of $\tilde{\delta}$ -soft open and $\tilde{\delta}$ -soft closed sets.

2. PRELIMINARY

Throughout that follows, I shall accept and permanently use elemantary definitions and preliminary results of the works Molodtsov [1], Maji et al. [3, 4], Aktas and Cagman [13] are presented in this section in this paper. Unless otherwise stated, throughout this paper, U refers to an initial universe, E is a set of parameters and $P(U)$ is the power set of U .

Definition 2.1. [1, 14] A soft set f on the universe U is is a set defined by

$f : E \to \mathcal{P}(U)$ such that $f(e) = \varnothing$ if $e \in E \setminus A$ then $f = f_A$ (1)

Here f is also called an approximate function. A soft set over U can be represented by the set of ordered pairs

$$f = \{(e, f(e)) : e \in E\} \tag{2}$$

I will identify any soft set f with the function $f(e)$ and and we shall use that concept as interchangeable. Soft sets are denoted by the letters $f, g, h...$ and the corresponding functions by $f(e), g(e), h(e)$, ...

Throughout this paper, the set of all soft sets over U will be denoted by $\mathbb{S}$. From now on, or all undefined concepts about soft sets, we refer to: [14].

Definition 2.2. [3] Let $f \in \mathbb{S}$. Then,

If $f(e) = \varnothing$ for all $e \in E$, then f is called an empty set, denoted by Φ .

If $f(e) = U$ for all $e \in E$, then f is called universal soft set, denoted by $\tilde{E}$.

Definition 2.3. [3] Let $f, g \in \mathbb{S}$. Then,

f is a soft subset of g , denoted by $f \tilde{\subseteq} g$, if $f \subseteq g$ for all $e \in E$.

f and g are soft equal, denoted by $f = g$, if and only if $f(e) = g(e)$ for all $e \in E$.

Definition 2.4. [14] $f, g \in \mathbb{S}$. Then, the

intersection of f and g , denoted $f \tilde{\cap} g$, is defined by and the union of f and g , denoted $f \tilde{\cup} g$, is defined by

$$(f \tilde{\cup} g)(e) = f(e) \cup g(e) \text{ for all } e \in E \tag{3}$$

Definition 2.5. [14] $f \in \mathbb{S}$. Then, the soft complement of f , denoted $f^{\tilde{c}}$, is defined by

$$f^{\tilde{c}}(e) = U \setminus f(e), \text{for all } e \in E \tag{4}$$

Definition 2.6. [14] Let $f \in \mathbb{S}$. The power soft set of f is defined by

$$\mathcal{P}(f) = \{ f_i \tilde{\subseteq} f : i \in I\} \tag{5}$$

and its cardinality is defined by

$$|\mathcal{P}(f)| = 2^{\sum_{e \in E} |f(e)|} \tag{6}$$

where $|f(e)|$ is the cardinality of $f(e)$.

Example 2.7. Let $U = \{u_1, u_2, u_3\}$ and $E = \{e_1, e_2\}$. $f \in \mathbb{S}$ and

$$f = \{(e_1, \{u_1, u_2\}), (e_2, \{u_2, u_3\})\} \tag{7}$$

Then,

$$\begin{aligned}
f_1 &= \{(e_1, \{u_1\})\}, \\
f_2 &= \{(e_1, \{u_2\})\}, \\
f_3 &= \{(e_1, \{u_1, u_2\})\}, \\
f_4 &= \{(e_2, \{u_2\})\}, \\
f_5 &= \{(e_2, \{u_3\})\}, \\
f_6 &= \{(e_2, \{u_2, u_3\})\}, \\
f_7 &= \{(e_1, \{u_1\}), (e_2, \{u_2\})\}, \\
f_8 &= \{(e_1, \{u_1\}), (e_2, \{u_3\})\}, \\
f_9 &= \{(e_1, \{u_1\}), (e_2, \{u_2, u_3\})\}, \\
f_{10} &= \{(e_1, \{u_2\}), (e_2, \{u_2\})\}, \\
f_{11} &= \{(e_1, \{u_2\}), (e_2, \{u_3\})\}, \\
f_{12} &= \{(e_1, \{u_2\}), (e_2, \{u_2, u_3\})\}, \\
f_{13} &= \{(e_1, \{u_1, u_2\}), (e_2, \{u_2\})\}, \\
f_{14} &= \{(e_1, \{u_1, u_2\}), (e_2, \{u_3\})\}, \\
f_{15} &= f, \\
f_{16} &= \Phi
\end{aligned} \tag{8}$$

are all soft subsets of f . So $|\tilde{P}(f)| = 2^4 = 16$.

Definition 2.8. [15] The soft set f is called a soft point in $\mathbb{S}$, if for the parameter $e_i \in E$ such that $f(e_i) \neq \varnothing$ and $f(e_j) = \varnothing$, for all $e_j \in E \setminus \{e_i\}$ is denoted by $(e_{i_f})_j$ for all $i, j \in \mathbb{N}^+$.

Note that the set of all soft points of f will be denoted by $SP(f)$.

Example 2.9. [15] Let $U = \{u_1, u_2, u_3, u_4, u_5\}$ and $E = \{e_1, e_2, e_3\}$. $f \in \mathbb{S}$ and

$$f = \{(e_1, \{u_1, u_3\}), (e_2, \{u_2, u_3\}), (e_3, \{u_1, u_2, u_3\})\}$$

Then the soft points for the parameter e_1 are;

$$\begin{aligned}
(e_{1_f})_1 &= (e_1, \{u_1\}) \\
(e_{1_f})_2 &= (e_1, \{u_3\}) \\
(e_{1_f})_3 &= (e_1, \{u_1, u_3\})
\end{aligned} \tag{9}$$

one of them can be chosen as soft point.

For the the parameter e_2 one of three occasions can be chosen as soft point likewise;

$$\begin{aligned} (e_{2_f})_1 &= (e_2,\{u_2\}) \\ (e_{2_f})_2 &= (e_2,\{u_3\}) \\ (e_{2_f})_3 &= (e_2,\{u_2,u_3\}) \end{aligned} \quad (10)$$

The soft points for the parameter e_3 are;

$$\begin{aligned} (e_{3_f})_1 &= (e_3,\{u_1\}) \\ (e_{3_f})_2 &= (e_3,\{u_2\}) \\ (e_{3_f})_3 &= (e_3,\{u_3\}) \\ (e_{3_f})_4 &= (e_3,\{u_1,u_2\}) \\ (e_{3_f})_5 &= (e_3,\{u_1,u_3\}) \\ (e_{3_f})_6 &= (e_3,\{u_2,u_3\}) \\ (e_{3_f})_7 &= (e_3,\{u_1,u_2,u_3\}) \end{aligned} \quad (11)$$

Definition 2.10. [7] Let $f \in \mathbb{S}$. A soft topology on f, denoted by $\tilde{\tau}$, is a collection of soft subsets of f having following properties:

i. $f, \Phi \in \tilde{\tau}$,

ii. $\{g_i\}_{i\in I} \subseteq \tilde{\tau} \Rightarrow \bigcup_{i\in I} g_i \in \tilde{\tau}$,

iii. $\{g_i\}_{i=1}^{n} \subseteq \tilde{\tau} \Rightarrow \bigcap_{i=1}^{n} g_i \in \tilde{\tau}$.

The pair $(f,\tilde{\tau})$ is called a soft topological space.

Example 2.11. Refer example Example 2.7., $\tilde{\tau}^1 = \mathcal{P}(f), \tilde{\tau}^0 = \{\Phi, f\}$ and $\tilde{\tau} = \{\Phi, f, f_2, f_{11}, f_{13}\}$ are soft topologies on f.

Definition 2.12. [7] Let $(f,\tilde{\tau})$ be a soft topological space. Then, every element of $\tilde{\tau}$ is called soft open set. Clearly, Φ and f are soft open sets.

Definition 2.13. [12] Let f be a nonempty soft set over the universe U, $g \tilde{\subseteq} f$, $\tilde{\tau}$ be a soft topology on f and $\tilde{\tau}_g$ be a soft subspace topology on g. Then, $(f,\tilde{\tau},\tilde{\tau}_g)$ is called a soft ditopological space which is abbreviated as SDT-space.

A pair $\tilde{\delta} = (\tilde{\tau},\tilde{\tau}_g)$ is called a soft ditopology over f and the members of $\tilde{\delta}$ are said to be $\tilde{\delta}$ -soft open in f.

The complement of $\tilde{\delta}$ -soft open set is called $\tilde{\delta}$ -soft closed soft set.

Example 2.14. [12] Let us consider all soft subsets on f in the example Example 2.7.

Let $\tilde{\tau} = \{\Phi, f, f_2, f_{11}, f_{13}\}$ be a soft topology on f. If $g = f_9$, then $\tilde{\tau}_g = \{\Phi, f_5, f_7, f_9\}$, and $(g,\tilde{\tau}_g)$ is a soft topological subspace of $(f,\tilde{\tau})$.

Hence, we get soft ditopology over f as

$$\tilde{\delta} = \{\Phi, f, f_2, f_5, f_7, f_9, f_{11}, f_{13}\} . \quad (12)$$

Definition 2.15. [12] Let $h \tilde{\subseteq} f$. Then, $\tilde{\delta}$ - soft interior of h, denoted by $(g)^{\circ}_{\tilde{\delta}}$, is defined by

$$(h)^{\circ}_{\tilde{\delta}} = \bigcup\{h : k \tilde{\subset} h, k \text{ is } \tilde{\delta}\text{-soft open}\} \quad (13)$$

The $\tilde{\delta}$ - soft closure of h, denoted by $(\bar{h})_{\tilde{\delta}}$, is defined by

$$(\bar{h})_{\tilde{\delta}} = \bigcap\{k : h \tilde{\subset} k, k \text{ is } \tilde{\delta}\text{-soft closed}\} \quad (14)$$

Note that $(h)^{\circ}_{\tilde{\delta}}$ is the biggest $\tilde{\delta}$ -soft open set that contained in h and $(\bar{h})_{\tilde{\delta}}$ is the smallest $\tilde{\delta}$ -soft closed set that containing h.

3. NEW $\tilde{\delta}$ -SOFT STRUCTURES OF SOFT DITOPOLOGICAL SPACE

In this section, the concept of soft ditopological (SDT) space on a soft set is developed with defining soft exterior in SDT. I establish several interesting properties of soft interior, soft exterior, soft closure and soft boundary and their relationship which are fundamental for research on soft ditopology and will strengthen the foundations of the theory of soft ditopological spaces. The results generalize soft topological properties of soft sets by exploiting some general facts seemingly overlooked by the aforementioned authors studied soft topology.

Theorem 3.1. Let $(f,\tilde{\delta})$ be a SDT-space, $h,k \tilde{\subseteq} f$. Then,

i. $(\overline{h \tilde{\cap} k})_{\tilde{\delta}} \tilde{\subseteq} (\bar{h})_{\tilde{\delta}} \tilde{\cap} (\bar{k})_{\tilde{\delta}}$

ii. $(h)^{\circ}_{\tilde{\delta}} \tilde{\cup} (k)^{\circ}_{\tilde{\delta}} \tilde{\subseteq} (h \tilde{\cup} k)^{\circ}_{\tilde{\delta}}$

Proof:

i. Since $h \tilde{\cap} k \tilde{\subset} h$ and $h \tilde{\cap} k \tilde{\subset} k$, then $(\overline{h \tilde{\cap} k})_{\tilde{\delta}} \tilde{\subseteq} (\bar{h})_{\tilde{\delta}}$ and $(\overline{h \tilde{\cap} k})_{\tilde{\delta}} \tilde{\subseteq} (\bar{k})_{\tilde{\delta}}$. So,

$$(\overline{h \tilde{\cap} k})_{\tilde{\delta}} \tilde{\subseteq} (\bar{h})_{\tilde{\delta}} \tilde{\cap} (\bar{k})_{\tilde{\delta}} \quad (15)$$

ii. Since $(h)^{\circ}_{\tilde{\delta}} \tilde{\subset} h$ and $(k)^{\circ}_{\tilde{\delta}} \tilde{\subset} k$, then $(h)^{\circ}_{\tilde{\delta}} \tilde{\cup} (k)^{\circ}_{\tilde{\delta}} \tilde{\subset} h \tilde{\cup} k$ and

$$((h)^{\circ}_{\tilde{\delta}} \tilde{\cup} (k)^{\circ}_{\tilde{\delta}})^{\circ}_{\tilde{\delta}} \tilde{\subset} (h \tilde{\cup} k)^{\circ}_{\tilde{\delta}} \quad (16)$$

where $(h)^{\circ}_{\tilde{\delta}}$ and $(k)^{\circ}_{\tilde{\delta}}$ $\tilde{\delta}$ -soft open. $(h)^{\circ}_{\tilde{\delta}} = s \tilde{\cup} t$ where $s \in \tilde{\delta}$ and $t \in \tilde{\delta}$. $(k)^{\circ}_{\tilde{\delta}} = m \tilde{\cup} n$ where $m \in \tilde{\delta}$ and $n \in \tilde{\delta}$

$(h)^{\circ}_{\tilde{\delta}} \tilde{\cup} (k)^{\circ}_{\tilde{\delta}} = (s \tilde{\cup} t) \tilde{\cup} (m \tilde{\cup} n) = (s \tilde{\cup} m) \tilde{\cup} (t \tilde{\cup} n)$ where $s \tilde{\cup} m \in \tilde{\delta}$ and $t \tilde{\cup} n \in \tilde{\delta}$. So, $(h)^{\circ}_{\tilde{\delta}} \tilde{\cup} (k)^{\circ}_{\tilde{\delta}}$ be a $\tilde{\delta}$ -soft open set. Hence we get

$$((h)^{\circ}_{\tilde{\delta}} \tilde{\cup} (k)^{\circ}_{\tilde{\delta}})^{\circ}_{\tilde{\delta}} = (h)^{\circ}_{\tilde{\delta}} \tilde{\cup} (k)^{\circ}_{\tilde{\delta}} \quad (17)$$

If we apply this equation to the Equation 16 then we get

$$(h)^{\circ}_{\tilde{\delta}} \tilde{\cup} (k)^{\circ}_{\tilde{\delta}} \tilde{\subset} (h \tilde{\cup} k)^{\circ}_{\tilde{\delta}} \quad (18)$$

This proves the theorem (ii). □

I now state a definition of $\tilde{\delta}$ -soft boundary of soft set that will be of use later with $\tilde{\delta}$ -soft openness and $\tilde{\delta}$ -soft exterior:

Definition 3.2. Let $(f,\tilde{\delta})$ be a SDT-space and $h \tilde{\subseteq} f$. Then, $\tilde{\delta}$ -soft boundary of soft set h is denoted by $(h)^{\tilde{b}}_{\tilde{\delta}}$ and is defined as

$$(h)^{\tilde{b}}_{\tilde{\delta}} = (\bar{h})_{\tilde{\delta}} \tilde{\cap} \overline{(h)^{\tilde{c}}_{\tilde{\delta}}} \quad (19)$$

Definition 3.3. [12] Let $(f,\tilde{\tau},\tilde{\tau}_g)$ be a SDT-space and $(e_{i_f})_j \tilde{\in} f$. If there is a $\tilde{\delta}$ -soft open set h such that $(e_{i_f})_j \tilde{\in} h$, then h is called $\tilde{\delta}$ -soft open neighborhood (or $\tilde{\delta}$ -soft nbd) of $(e_{i_f})_j$. The set of all $\tilde{\delta}$ -soft neighborhood of $(e_{i_f})_j$, denoted $\mathcal{V}_{\tilde{\delta}}((e_{i_f})_j)$, is called family of $\tilde{\delta}$ -soft neighborhood of $(e_{i_f})_j$, that is

$$\mathcal{V}_{\tilde{\delta}}((e_{i_f})_j) = \{h : h \tilde{\in} \tilde{\delta}, (e_{i_f})_j \tilde{\in} h\} \quad (20)$$

Definition 3.4. Let $(f,\tilde{\delta})$ be a SDT-space, $h \tilde{\subseteq} f$ and $(e_{i_f})_j \tilde{\in} f$. If every $\tilde{\delta}$ -soft nbd of $(e_{i_f})_j$ soft intersects h in some soft points other than $(e_{i_f})_j$ itself, then $(e_{i_f})_j$ is called a $\tilde{\delta}$ -soft limit point of h.

The set of all $\tilde{\delta}$ -soft limit points of h is denoted by $(h')_{\tilde{\delta}}$

I can shortly define it like that:

$$h,k \tilde{\subseteq} f, (e_{i_f})_j \tilde{\in} f, (e_{i_f})_j \tilde{\in} (h')_{\tilde{\delta}} \Leftrightarrow (\forall k \tilde{\in} \mathcal{V}_{\tilde{\delta}}((e_{i_f})_j) : (k \tilde{\cap} (h \tilde{\setminus} \{(e_{i_f})_j\}) \neq \Phi) \quad (21)$$

Example 3.5 Let me consider the soft ditopological space $(f,\tilde{\delta})$ in Example 2.14 and $(e_{i_f})_j = (e_{1_f})_3$ defined in Example 2.7. Then,

$$\mathcal{V}_{\tilde{\delta}}((e_{1_f})_3) = \{ \{(e_1,\{u_1,u_2\}),(e_2,\{u_2,u_3\})\}, \{(e_1,\{u_1,u_2\}),(e_2,\{u_2\})\} \} \quad (22)$$

Proposition 3.6. Let $(f,\tilde{\delta})$ be a SDT-space, $h,k \tilde{\subseteq} f$. Then, the collection of $\tilde{\delta}$ -soft nbd $\mathcal{V}_{\tilde{\delta}}((e_{i_f})_j)$ at $(e_{i_f})_j \tilde{\in} (f,\tilde{\delta})$ has the following properties:

i. If $h \tilde{\in} \mathcal{V}_{\tilde{\delta}}((e_{i_f})_j)$, then $(e_{i_f})_j \tilde{\in} h$.

ii. If $h,k \tilde{\in} \mathcal{V}_{\tilde{\delta}}((e_{i_f})_j)$, then $h \tilde{\cap} k \tilde{\in} \mathcal{V}_{\tilde{\delta}}((e_{i_f})_j)$.

iii. If $h \tilde{\in} \mathcal{V}_{\tilde{\delta}}((e_{i_f})_j)$ and $h \tilde{\subseteq} k$, then $k \tilde{\in} \mathcal{V}_{\tilde{\delta}}((e_{i_f})_j)$.

iv. $h \tilde{\subseteq} f$ is $\tilde{\delta}$ -soft open if and only if h contains a $\tilde{\delta}$ -soft nbd of each of its soft points.

Proof: The others being obvious, only (iv) needs proof.

iv. $(i) \Rightarrow (ii)$: Suppose h is $\tilde{\delta}$ -soft open in f, then $(e_{i_f})_j \tilde{\in} h \tilde{\subseteq} h$ implies that h is a $\tilde{\delta}$ -soft nbd of each $(e_{i_f})_j \tilde{\in} h$.

$(ii) \Rightarrow (i)$: If each $(e_{i_f})_j \tilde{\in} h$ has a $\tilde{\delta}$ -soft nbd $k_{(e_{i_f})_j} \tilde{\subseteq} h$, then

$$h = \{(e_{i_f})_j : (e_{i_f})_j \tilde{\in} h\} \tilde{\subseteq} \tilde{\bigcup}_{(e_{i_f})_j \tilde{\in} h} k_{(e_{i_f})_j} \tilde{\subseteq} h \quad (23)$$

or

$$h = \tilde{\bigcup}_{(e_{i_f})_j \tilde{\in} h} k_{(e_{i_f})_j} \quad (24)$$

This implies that h is $\tilde{\delta}$ -soft open in f. □

The following theorem gives the relationship between $\tilde{\delta}$ -soft limit point and $\tilde{\delta}$ -soft closure of a soft set.

Theorem 3.7. Let $(f,\tilde{\delta})$ be a SDT-space, $h \tilde{\subseteq} f$. Then,

$$h \tilde{\cup} (h')_{\tilde{\delta}} = (\bar{h})_{\tilde{\delta}} \quad (25)$$

Proof: If $(e_{i_f})_j \in h \tilde{\cup} (h')_{\tilde{\delta}}$, then $(e_{i_f})_j \in h$ or $(e_{i_f})_j \in (h')_{\tilde{\delta}}$. In this case, if $(e_{i_f})_j \in h$, then $(e_{i_f})_j \in (\bar{h})_{\tilde{\delta}}$. If $(e_{i_f})_j \in (h')_{\tilde{\delta}}$, then $k \tilde{\cap} (h \tilde{\setminus} \{(e_{i_f})_j\}) \neq \Phi$ for all $k \in \mathcal{V}_{\tilde{\delta}}((e_{i_f})_j)$ and so $k \tilde{\cap} h \neq \Phi$ for all $k \in \mathcal{V}_{\tilde{\delta}}((e_{i_f})_j)$, hence $(e_{i_f})_j \in (\bar{h})_{\tilde{\delta}}$. Conversely, if $(e_{i_f})_j \in (\bar{h})_{\tilde{\delta}}$, then $(e_{i_f})_j \in h$ or $(e_{i_f})_j \notin h$. In this case, if $(e_{i_f})_j \in h$, then, clearly $(e_{i_f})_j \in h \tilde{\cup} (h')_{\tilde{\delta}}$. If $(e_{i_f})_j \notin h$, then $k \tilde{\cap} (h \tilde{\setminus} \{(e_{i_f})_j\}) \neq \Phi$ for all $k \in \mathcal{V}_{\tilde{\delta}}((e_{i_f})_j)$. Therefore, $(e_{i_f})_j \in (h')_{\tilde{\delta}}$ so that $(e_{i_f})_j \in h \tilde{\cup} (h')_{\tilde{\delta}}$.

Hence $h \tilde{\cup} (h')_{\tilde{\delta}} = (\bar{h})_{\tilde{\delta}}$. □

A soft subset of a soft ditopological space is $\tilde{\delta}$ -soft closed if it contains all its $\tilde{\delta}$ -soft limit points. The following theorem explains this property:

Theorem 3.8. Let $(f,\tilde{\delta})$ be a SDT-space, $h \tilde{\subseteq} f$. Then, h is a $\tilde{\delta}$ -soft closed if and only if $(h')_{\tilde{\delta}} \tilde{\subseteq} h$.

Proof: $h = (\bar{h})_{\tilde{\delta}} = h \tilde{\cup} (h')_{\tilde{\delta}} \Leftrightarrow (h')_{\tilde{\delta}} \tilde{\subseteq} h$. □

Theorem 3.9. Let $(f,\tilde{\delta})$ be a SDT-space, $k,h \tilde{\subseteq} f$.

Then,

i. $(h')_{\tilde{\delta}} \tilde{\subseteq} (\overline{h})_{\tilde{\delta}}$

ii. $k \tilde{\subseteq} h \Rightarrow (k')_{\tilde{\delta}} \tilde{\subseteq} (h')_{\tilde{\delta}}$

iii. $(k \tilde{\cap} h)'_{\tilde{\delta}} \tilde{\subseteq} (k')_{\tilde{\delta}} \tilde{\cap} (h')_{\tilde{\delta}}$

iv. $(k \tilde{\cup} h)'_{\tilde{\delta}} = (k')_{\tilde{\delta}} \tilde{\cup} (h')_{\tilde{\delta}}$

v. $(\overline{k})_{\tilde{\delta}} = k \iff (k')_{\tilde{\delta}} \tilde{\subseteq} k$

Proof:

i. From the definition of $\tilde{\delta}$-soft closure the proof is trivial.

ii. Let $k \tilde{\subseteq} h$. Because of $k \tilde{\setminus} \{(e_{i_f})_j\} \tilde{\subseteq} h \tilde{\setminus} \{(e_{i_f})_j\}$, then $\overline{(k \tilde{\setminus} \{(e_{i_f})_j\})}_{\tilde{\delta}} \tilde{\subseteq} \overline{(h \tilde{\setminus} \{(e_{i_f})_j\})}_{\tilde{\delta}}$. We obtain $(k')_{\tilde{\delta}} \tilde{\subseteq} (h')_{\tilde{\delta}}$.

iii. $k \tilde{\cap} h \tilde{\subseteq} k$ and $k \tilde{\cap} h \tilde{\subseteq} h$. Then,

$(k \tilde{\cap} h)'_{\tilde{\delta}} \tilde{\subseteq} (k')_{\tilde{\delta}}$ and $(k \tilde{\cap} h)'_{\tilde{\delta}} \tilde{\subseteq} (h')_{\tilde{\delta}}$.

Therefore,

$(k \tilde{\cap} h)'_{\tilde{\delta}} \tilde{\subseteq} (k')_{\tilde{\delta}} \tilde{\cap} (h')_{\tilde{\delta}}$.

iv. $\forall (e_{i_f})_j \tilde{\in} (k \tilde{\cup} h)'_{\tilde{\delta}} \iff ((e_{i_f})_j \in \overline{(k \tilde{\cup} h) \tilde{\setminus} \{(e_{i_f})_j\}})_{\tilde{\delta}}$, therefore

$$\begin{aligned}
\overline{((k \tilde{\cup} h) \tilde{\setminus} \{(e_{i_f})_j\})}_{\tilde{\delta}} &= \overline{(k \tilde{\cup} h) \tilde{\cap} \{(e_{i_f})_j\}^{\tilde{c}}_{\tilde{\delta}}}_{\tilde{\delta}} \\
&= \overline{(k \tilde{\cap} \{(e_{i_f})_j\}^{\tilde{c}}_{\tilde{\delta}}) \tilde{\cup} (h \tilde{\cap} \{(e_{i_f})_j\}^{\tilde{c}}_{\tilde{\delta}})}_{\tilde{\delta}} \\
&= \overline{(k \tilde{\cap} \{(e_{i_f})_j\}^{\tilde{c}}_{\tilde{\delta}})}_{\tilde{\delta}} \tilde{\cup} \overline{(h \tilde{\cap} \{(e_{i_f})_j\}^{\tilde{c}}_{\tilde{\delta}})}_{\tilde{\delta}} \\
&= \overline{(h \tilde{\setminus} \{(e_{i_f})_j\})}_{\tilde{\delta}} \tilde{\cup} \overline{(k \tilde{\setminus} \{(e_{i_f})_j\})}_{\tilde{\delta}} \\
&\iff (e_{i_f})_j \in (k')_{\tilde{\delta}} \tilde{\cup} (h')_{\tilde{\delta}}
\end{aligned}$$

Hence $(k \tilde{\cup} h)'_{\tilde{\delta}} = (k')_{\tilde{\delta}} \tilde{\cup} (h')_{\tilde{\delta}}$.

v. $k = (\overline{k})_{\tilde{\delta}} \iff k = k \tilde{\cup} (k')_{\tilde{\delta}} \iff (k')_{\tilde{\delta}} \tilde{\subseteq} k$ □

Definition 3.10. Let $(f, \tilde{\delta})$ be a SDT-space and $\mathcal{B}_{\tilde{\delta}} \tilde{\subseteq} \tilde{\delta}$. If every element of $\tilde{\delta}$ can be written as a union of element of $\mathcal{B}_{\tilde{\delta}}$, then $\mathcal{B}_{\tilde{\delta}}$ is called $\tilde{\delta}$-soft basis for $(f, \tilde{\delta})$ Each element of $\mathcal{B}_{\tilde{\delta}}$ is called soft ditopological basis element. I can shortly define it like that,

$\mathcal{B}_{\tilde{\delta}} \tilde{\subseteq} \tilde{\delta}, \tilde{\delta}$-soft basis $\iff (\forall h \tilde{\in} \tilde{\delta})(\exists \Psi \tilde{\subset} \mathcal{B}_{\tilde{\delta}}) : (h = \bigcup_{k \in \Psi} k)$

Theorem 3.11. Let $(f, \tilde{\delta})$ be a SDT-space, $h, k \tilde{\subseteq} f$. Then,

i. $((h)^{\circ}_{\tilde{\delta}})^{\tilde{c}}_{\tilde{\delta}} = \overline{(h)^{\tilde{c}}_{\tilde{\delta}}}$

ii. $((h)^{\tilde{c}}_{\tilde{\delta}})^{\circ}_{\tilde{\delta}} = \overline{(h)^{\tilde{c}}_{\tilde{\delta}}}$

iii. $(h)^{\circ}_{\tilde{\delta}} = (\overline{(h)^{\tilde{c}}_{\tilde{\delta}}})^{\tilde{c}}_{\tilde{\delta}}$

iv. $(\overline{h})_{\tilde{\delta}} = (((h)^{\tilde{c}}_{\tilde{\delta}})^{\circ}_{\tilde{\delta}})^{\tilde{c}}_{\tilde{\delta}}$

v. $(h \tilde{\setminus} k)^{\circ}_{\tilde{\delta}} \tilde{\subseteq} (h)^{\circ}_{\tilde{\delta}} \tilde{\setminus} (k)^{\circ}_{\tilde{\delta}}$

Proof:

i. Let $(e_{i_f})_j \tilde{\in} h$ such that $(e_{i_f})_j \notin (h)^{\circ}_{\tilde{\delta}}$. Then, for each $\tilde{\delta}$-soft open nbd k of $(e_{i_f})_j$, k $\tilde{\delta}$-soft intersects $(h)^{\tilde{c}}_{\tilde{\delta}}$. Otherwise, for some $\tilde{\delta} - soft\ open\ nbd\ k\ of\ (e_{i_f})_j$, $k \tilde{\cap} (h)^{\tilde{c}}_{\tilde{\delta}} = \Phi$ or $k \tilde{\subseteq} h$. Since $(h)^{\circ}_{\tilde{\delta}}$ is the largest $\tilde{\delta}$-soft open set in h, therefore $(e_{i_f})_j \tilde{\in} k \tilde{\subseteq} (h)^{\circ}_{\tilde{\delta}}$, which is a contradiction. Therefore, $(e_{i_f})_j \tilde{\in} ((h)^{\circ}_{\tilde{\delta}})^{\tilde{c}}_{\tilde{\delta}}$.Hence, $((h)^{\circ}_{\tilde{\delta}})^{\tilde{c}}_{\tilde{\delta}} = \overline{(h)^{\tilde{c}}_{\tilde{\delta}}}$.

Conversely, suppose $(e_{i_f})_j \tilde{\in} \overline{(h)^{\tilde{c}}_{\tilde{\delta}}}$, Then $(e_{i_f})_j \tilde{\in} (h)^{\tilde{c}}_{\tilde{\delta}}$ or $(e_{i_f})_j$ is a $\tilde{\delta}$-soft limit point of $\overline{(h)^{\tilde{c}}_{\tilde{\delta}}}$. If $(e_{i_f})_j \tilde{\in} (h)^{\tilde{c}}_{\tilde{\delta}}$, Then $(e_{i_f})_j \tilde{\in} ((h)^{\circ}_{\tilde{\delta}})^{\tilde{c}}_{\tilde{\delta}}$. In the second case, $(e_{i_f})_j \notin (h)^{\circ}_{\tilde{\delta}}$. Otherwise, by the definition of $\tilde{\delta}$-soft limit point $(h)^{\circ}_{\tilde{\delta}}) \tilde{\cap} (h)^{\tilde{c}}_{\tilde{\delta}} \neq \Phi$, which is false. Therefore, $(e_{i_f})_j \tilde{\in} \overline{(h)^{\tilde{c}}_{\tilde{\delta}}}$. This shows that $\overline{(h)^{\tilde{c}}_{\tilde{\delta}}} \tilde{\subseteq} ((h)^{\circ}_{\tilde{\delta}})^{\tilde{c}}_{\tilde{\delta}}$. Combining, we get the proof.

ii. The proof is trivial.

iii. and iv. are directly obtained by taking the complements of i. and ii., respectively.

v.

$$\begin{aligned}
(h \tilde{\setminus} k)^{\circ}_{\tilde{\delta}} &= (h \tilde{\cap} (k)^{\tilde{c}}_{\tilde{\delta}})^{\circ}_{\tilde{\delta}} \\
&= (h)^{\circ} \tilde{\cap} ((k)^{\tilde{c}}_{\tilde{\delta}})^{\circ}_{\tilde{\delta}} \\
&= (h)^{\circ} \tilde{\cap} ((\overline{k})_{\tilde{\delta}})^{\tilde{c}}_{\tilde{\delta}} \\
&\tilde{\subseteq} (h)^{\circ} \tilde{\cap} ((h)^{\circ}_{\tilde{\delta}})^{\tilde{c}}_{\tilde{\delta}} \\
&= (h)^{\circ}_{\tilde{\delta}} \tilde{\setminus} (k)^{\circ}_{\tilde{\delta}}
\end{aligned}$$

Definition 3.12. Let $(f, \tilde{\delta})$ be a SDT-space, $h \tilde{\subseteq} f$. Then, $\tilde{\delta}$-soft exterior of a soft set h is denoted by $(h)^{\tilde{e}}_{\tilde{\delta}}$ and is defined as $(h)^{\tilde{e}}_{\tilde{\delta}} = ((h)^{\tilde{c}}_{\tilde{\delta}})^{\circ}_{\tilde{\delta}}$

Thus, $(e_{i_f})_j$ is called a $\tilde{\delta}$-soft exterior point of h if there exists a $\tilde{\delta}$-soft open set k such that $(e_{i_f})_j \tilde{\in} k \tilde{\subseteq} (h)^{\tilde{c}}_{\tilde{\delta}}$. We observe that $(h)^{\tilde{e}}_{\tilde{\delta}}$ is the largest $\tilde{\delta}$-soft open set contained in $(h)^{\tilde{c}}_{\tilde{\delta}}$.

Example 3.13. Let $U = \{u_1, u_2, u_3\}$ and $E = \{e_1, e_2, e_3\}$. $f \in \mathbb{S}$ and $f = \{(e_1, U), (e_2, \{u_1, u_2\}), (e_3, U)\}$ $h \tilde{\subseteq} f$ and $h = \{(e_1, U), (e_2, \{u_1, u_2\})\}$ Then, $(h)^{\tilde{c}}_{\tilde{\delta}} = \{(e_2, \{u_3\})\}$, and thus, $(h)^{\tilde{e}}_{\tilde{\delta}} = ((h)^{\tilde{c}}_{\tilde{\delta}})^{\circ}_{\tilde{\delta}} = \Phi$

4. CHARACTERIZATION OF SOFT SETS BY $\tilde{\delta}$-SOFT OPERATIONS

In this section, I study some properties of $\tilde{\delta}$-soft exterior, with the aim of describing $\tilde{\delta}$-soft open and $\tilde{\delta}$-soft close sets directly using $\tilde{\delta}$-soft exterior.

The usefulness and interest of this correspondence of $\tilde{\delta}$-soft exterior will of course be enhanced if there is a way of returning from the transforms to the $\tilde{\delta}$-soft sets, that is to say, if there is a formula that characterize $\tilde{\delta}$-soft set. In closing this section, all the studies come to fruition and we take up a result which play a pivotal role in the characterization of $\tilde{\delta}$-soft open and $\tilde{\delta}$-soft closed sets.

I intend to obtain an interest in the study of the relationships between $\tilde{\delta}$-soft exterior, $\tilde{\delta}$-soft closure, $\tilde{\delta}$-soft complement, $\tilde{\delta}$-soft interior and $\tilde{\delta}$-soft boundary with respect to $\tilde{\delta}$-soft operations by given theorems below:

Theorem 4.1. Let h,k be $\tilde{\delta}$-soft subsets of a SDT-space $(f,\tilde{\delta})$. Then,

i. $(h)^{\tilde{e}}_{\tilde{\delta}} = ((h)^{\tilde{c}}_{\tilde{\delta}})^{\tilde{e}}_{\tilde{\delta}}$

ii. $(h\tilde{\cup}k)^{\tilde{e}}_{\tilde{\delta}} = (h)^{\tilde{e}}_{\tilde{\delta}}\tilde{\cap}(k)^{\tilde{e}}_{\tilde{\delta}}$

iii. $(h)^{\tilde{e}}_{\tilde{\delta}}\tilde{\cup}(k)^{\tilde{e}}_{\tilde{\delta}}\tilde{\subseteq}(h\tilde{\cap}k)^{\tilde{e}}_{\tilde{\delta}}$

Proof:

i. It is trivial from the definition of $\tilde{\delta}$-soft exterior.

ii.
$$\begin{aligned}(h\tilde{\cup}k)^{\tilde{e}}_{\tilde{\delta}} &= ((h\tilde{\cup}k)^{\tilde{c}}_{\tilde{\delta}})^{\circ}_{\tilde{\delta}}\\ &= ((h)^{\tilde{c}}_{\tilde{\delta}}\tilde{\cap}(k)^{\tilde{c}}_{\tilde{\delta}})^{\circ}_{\tilde{\delta}}\\ &= ((h)^{\tilde{c}}_{\tilde{\delta}})^{\circ}_{\tilde{\delta}}\tilde{\cap}((k)^{\tilde{c}}_{\tilde{\delta}})^{\circ}_{\tilde{\delta}}\\ &= (h)^{\tilde{e}}_{\tilde{\delta}}\tilde{\cap}(k)^{\tilde{e}}_{\tilde{\delta}}\end{aligned}$$

iii.
$$\begin{aligned}(h)^{\tilde{e}}_{\tilde{\delta}}\tilde{\cup}(k)^{\tilde{e}}_{\tilde{\delta}} &= ((h)^{\tilde{c}}_{\tilde{\delta}})^{\circ}_{\tilde{\delta}}\tilde{\cup}((k)^{\tilde{c}}_{\tilde{\delta}})^{\circ}_{\tilde{\delta}}\\ &\tilde{\subseteq} ((h)^{\tilde{c}}_{\tilde{\delta}}\tilde{\cup}(k)^{\tilde{c}}_{\tilde{\delta}})^{\circ}_{\tilde{\delta}}\\ &= ((h\tilde{\cap}k)^{\tilde{c}}_{\tilde{\delta}})^{\circ}_{\tilde{\delta}}\\ &= (h\tilde{\cap}k)^{\tilde{e}}_{\tilde{\delta}}\end{aligned}$$

Theorem 4.2. Let $(f,\tilde{\delta})$ be a SDT-space, $h\tilde{\subseteq}f$. Then,

i. $((h)^{\tilde{b}}_{\tilde{\delta}})^{\tilde{c}}_{\tilde{\delta}} = (h)^{\circ}_{\tilde{\delta}}\tilde{\cup}(h)^{\tilde{e}}_{\tilde{\delta}}$

ii. $(\overline{h})_{\tilde{\delta}} = h\tilde{\cup}(h)^{\tilde{b}}_{\tilde{\delta}}$

iii. $(h)^{\circ}_{\tilde{\delta}} = h\tilde{\setminus}(h)^{\tilde{b}}_{\tilde{\delta}}$

Proof:

i.
$$\begin{aligned}(h)^{\circ}_{\tilde{\delta}}\tilde{\cup}(h)^{\tilde{e}}_{\tilde{\delta}} &= (h)^{\circ}_{\tilde{\delta}}\tilde{\cup}((h)^{\tilde{c}}_{\tilde{\delta}})^{\circ}_{\tilde{\delta}}\\ &= (((h)^{\circ}_{\tilde{\delta}})^{\tilde{c}}_{\tilde{\delta}})^{\tilde{c}}_{\tilde{\delta}}\tilde{\cup}(((((h)^{\tilde{c}}_{\tilde{\delta}})^{\circ}_{\tilde{\delta}})^{\tilde{c}}_{\tilde{\delta}})^{\tilde{c}}_{\tilde{\delta}})\\ &= [((h)^{\circ}_{\tilde{\delta}})^{\tilde{c}}_{\tilde{\delta}}\tilde{\cup}(((h)^{\tilde{c}}_{\tilde{\delta}})^{\circ}_{\tilde{\delta}})^{\tilde{c}}_{\tilde{\delta}}]^{\tilde{c}}_{\tilde{\delta}}\\ &= [(\overline{(h)^{\tilde{c}}_{\tilde{\delta}}})_{\tilde{\delta}}\tilde{\cap}(\overline{h})_{\tilde{\delta}}]^{\tilde{c}}_{\tilde{\delta}}\\ &= ((h)^{\tilde{b}}_{\tilde{\delta}})^{\tilde{c}}_{\tilde{\delta}}\end{aligned}$$

ii.
$$\begin{aligned}h\tilde{\cup}(h)^{\tilde{b}}_{\tilde{\delta}} &= h\tilde{\cup}((\overline{h})_{\tilde{\delta}}\tilde{\cap}(\overline{(h)^{\tilde{c}}_{\tilde{\delta}}})_{\tilde{\delta}})\\ &= [h\tilde{\cup}(\overline{h})_{\tilde{\delta}}]\tilde{\cap}[h\tilde{\cup}(\overline{(h)^{\tilde{c}}_{\tilde{\delta}}})_{\tilde{\delta}}]\\ &= (\overline{h})_{\tilde{\delta}}\tilde{\cap}[h\tilde{\cup}(\overline{(h)^{\tilde{c}}_{\tilde{\delta}}})]\\ &= (\overline{h})_{\tilde{\delta}}\tilde{\cap}f = (\overline{h})_{\tilde{\delta}}\end{aligned}$$

iii.
$$\begin{aligned}h\tilde{\setminus}(h)^{\tilde{b}}_{\tilde{\delta}} &= h\tilde{\cap}((h)^{\tilde{b}}_{\tilde{\delta}})^{\tilde{c}}_{\tilde{\delta}}\\ &= h\tilde{\cap}((h)^{\circ}_{\tilde{\delta}}\tilde{\cup}((h)^{\tilde{c}}_{\tilde{\delta}})^{\circ}_{\tilde{\delta}})\\ &= [h\tilde{\cap}(h)^{\circ}_{\tilde{\delta}}]\tilde{\cup}[(h\tilde{\cap}(h)^{\tilde{c}}_{\tilde{\delta}})^{\circ}_{\tilde{\delta}}]\\ &= (h)^{\circ}_{\tilde{\delta}}\tilde{\cup}\Phi\\ &= (h)^{\circ}_{\tilde{\delta}}\end{aligned}$$

Following these theorems, I now state this remark:

Remark 4.3. It is known that $(h)^{\tilde{b}}_{\tilde{\delta}} = ((h)^{\tilde{c}}_{\tilde{\delta}})^{\tilde{b}}_{\tilde{\delta}}$ and from Theorem 4.2. (i), it follows that

$$f = (h)^{\circ}_{\tilde{\delta}}\tilde{\cup}(h)^{\tilde{e}}_{\tilde{\delta}}\tilde{\cup}(h)^{\tilde{b}}_{\tilde{\delta}}$$

I remark at the outset that this formula makes sense, because I can characterize soft set by the combination of $\tilde{\delta}$- soft interior, $\tilde{\delta}$- soft exterior and $\tilde{\delta}$- soft boundary.

Theorem 4.4. Let $(f,\tilde{\delta})$ be a SDT-space, $h\tilde{\subseteq}f$. Then,

i. $(h)^{\tilde{b}}_{\tilde{\delta}}\tilde{\cap}(h)^{\circ}_{\tilde{\delta}} = \Phi$

ii. $(h)^{\tilde{b}}_{\tilde{\delta}}\tilde{\cap}(h)^{\tilde{e}}_{\tilde{\delta}} = \Phi$

Proof:

i.
$$\begin{aligned}(h)^{\circ}_{\tilde{\delta}}\tilde{\cap}(h)^{\tilde{b}}_{\tilde{\delta}} &= (h)^{\circ}_{\tilde{\delta}}\tilde{\cap}((\overline{h})_{\tilde{\delta}}\tilde{\cap}(h)^{\tilde{c}}_{\tilde{\delta}})\\ &= (h)^{\circ}_{\tilde{\delta}}\tilde{\cap}(\overline{h})_{\tilde{\delta}}\tilde{\cap}(h)^{\tilde{c}}_{\tilde{\delta}}\\ &= \Phi\end{aligned}$$

ii.
$$\begin{aligned}(h)^{\tilde{b}}_{\tilde{\delta}}\tilde{\cap}(h)^{\tilde{e}}_{\tilde{\delta}} &= ((h)^{\tilde{c}}_{\tilde{\delta}})^{\circ}_{\tilde{\delta}}\tilde{\cap}((\overline{h})_{\tilde{\delta}}\tilde{\cap}(\overline{(h)^{\tilde{c}}_{\tilde{\delta}}})_{\tilde{\delta}})\\ &= ((h)^{\tilde{c}}_{\tilde{\delta}})^{\circ}_{\tilde{\delta}}\tilde{\cap}(\overline{h})_{\tilde{\delta}}\tilde{\cap}(\overline{(h)^{\tilde{c}}_{\tilde{\delta}}})_{\tilde{\delta}}\\ &= ((\overline{h})_{\tilde{\delta}})^{\tilde{c}}_{\tilde{\delta}}\tilde{\cap}(\overline{h})_{\tilde{\delta}}\tilde{\cap}(\overline{(h)^{\tilde{c}}_{\tilde{\delta}}})_{\tilde{\delta}}\\ &= \Phi\end{aligned}$$

In the next theorem, I state a characterization of $\tilde{\delta}$ -soft open and $\tilde{\delta}$ -soft closed sets which do not seem to have been noticed previously:

Theorem 4.5. Let $(f,\tilde{\delta})$ be a SDT-space, $h\tilde{\subseteq} f$. Then,

i. h is $\tilde{\delta}$ -soft open if and only if $h\tilde{\cap}(h)^{\tilde{b}}_{\tilde{\delta}}=\Phi$.

ii. h is $\tilde{\delta}$ -soft closed if and only if $(h)^{\tilde{b}}_{\tilde{\delta}}\tilde{\subseteq}h$.

Proof:

i. Let h be a $\tilde{\delta}$ -soft open set. Then, $(h)^{\circ}_{\tilde{\delta}}=h$. By the Theorem 4.4.,

$$h\tilde{\cap}(h)^{\tilde{b}}_{\tilde{\delta}}=(h)^{\circ}_{\tilde{\delta}}\tilde{\cap}(h)^{\tilde{b}}_{\tilde{\delta}}=\Phi \quad (25)$$

Conversely, let $h\tilde{\cap}(\bar{h})_{\tilde{\delta}}=\Phi$, or $\overline{(h)^{\tilde{c}}_{\tilde{\delta}}}\tilde{\subseteq}(h)^{\tilde{c}}_{\tilde{\delta}}$, which implies that $(h)^{\tilde{c}}_{\tilde{\delta}}$ is $\tilde{\delta}$ -soft closed.

Hence, h is $\tilde{\delta}$ -soft open.

ii. Let h is $\tilde{\delta}$ -soft closed set. Then, $\overline{(h)^{\tilde{c}}_{\tilde{\delta}}}=(h)^{\tilde{c}}_{\tilde{\delta}}$. On the other hand,

$$(h)^{\tilde{b}}_{\tilde{\delta}}=(\bar{h})_{\tilde{\delta}}\tilde{\cap}\overline{(h)^{\tilde{c}}_{\tilde{\delta}}}\tilde{\cap}(\bar{h})_{\tilde{\delta}} \quad (26)$$

Or $(\bar{h})_{\tilde{\delta}}\tilde{\subseteq}h$, conversely. □

Actually, the proof gives an even more precise conclusion: $\tilde{\delta}$ -soft open and $\tilde{\delta}$ -soft closed sets can be characterized by $\tilde{\delta}$ -soft boundary.

5. CONCLUSION

In this paper, I establish several new structures as soft interior, soft exterior, soft closure and soft boundary and their relationship which are fundamental for research on soft ditopology. I study some properties of $\tilde{\delta}$ -soft operations, with the aim of describing $\tilde{\delta}$ -soft open and $\tilde{\delta}$ -soft close sets directly using $\tilde{\delta}$ -soft operations. The usefulness and interest of this correspondence of $\tilde{\delta}$ -soft operations will of course be enhanced if there is a way of returning from the transforms to the $\tilde{\delta}$ -soft sets, that is to say, if there is a formula that characterize $\tilde{\delta}$ -soft set. In closing of the last section, all the studies come to fruition and I take up a result which play a pivotal role in the characterization of $\tilde{\delta}$ -soft open and $\tilde{\delta}$ -soft closed sets.

These results lead to several illuminating pieces of information about the insufficiently studied the characterization of $\tilde{\delta}$ -soft open and $\tilde{\delta}$ -soft closed properties in SDT-spaces. It is hoped that a deeper understanding of these residues will help establish new results about the distribution of soft set theory.

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