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The Dynamics of Thirring Optical Solitons by Kerr Law Nonlinearity

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ABSTRACT: This article studies the dynamics of thirring optical solitons. Jacobi elliptic functions is considered for exact solutions and nonlinearship is described by Kerr law. As a result, the singular solutions with Dark and Bright optical solitons are obtained.

Key words: Optical soliton, Dark soliton and Bright soliton

Kerr Law Lineer Olmayanlık ile Thirring Optikal Solitonların Dinamikleri

ÖZET: Bu makalede Thirring optikal solitonların dinamiği çalışıldı. Tam çözümler için Jacobi eliptik fonksiyonlar düşünüldü ve lineer olmayanlık Kerr law ile tanımlandı. Sonuçta, Dark ve Bright optikal solitonlar ile tekil çözümler elde edildi

Anahtar Kelimeler: Optikal soliton, Dark soliton ve Bright soliton

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INTRODUCTION

Solitary waves that are nonlinear waves, arise from a balance between nonlinear transmission and scattering terms. The discovery of solitons dates back to 1834 (Kivshar and Agrawal, 2003; Biswas, 2009; Sassaman and Biswas, 2010; Biswas, 2010; Biswas et al., 2013; Mirzazadeh et al., 2015). The discovery of optical solitons that are dates back to 1971. Optical solitons are interesting subject in optical fiber communication because of their capability of propagation over long distance without attenuation and change in shapes. In 1971, Zhakarov and Shabat were the first to show the nonlinear Schrödinger equation (NLSE) which describes propagation in a fiber by the inverse scattering theory (Zakharov and Shabat, 1972; Tang et al., 2014; Agrawal et al., 2015; Krishnan et al., 2015; Sardar et al., 2016). The NLSE that gives the phase shift of the wave used to model wave packets in diverse fields of science such as hydrodynamics, nonlinear optics, nonlinear acoustic, plasma waves and bio-molecular dynamics (Kivshar and Agrawal, 2003). In later years, many scientists interested in Dark and Bright optical solitons (Tang et al., 2014; Agrawal et al., 2015).

In section 2 of this study, we will consider the dynamics of Thirring optical soliton is studied with Jacobi elliptic function.

MATHEMATICAL ANALYSIS

The dynamics of thirring solitons is governed by coupled nonlinear Schrödinger equations (CNLSE) and is given by (Guzman et al., 2015)

$$i\,\psi_t + a_1\,\psi_{xx} + b_1\,\psi_{xt} + c_1|\mu|^2\,\psi = 0 \tag{1}$$

$$i\mu_t + a_2\mu_{xx} + b_2\mu_{xt} + c_2|\,\psi|^2\mu = 0 \tag{2}$$

From Eqs.(1)-(2),

$$\psi(x,t) = \mathbb{P}_1(x,t)e^{i\emptyset_1} \tag{3}$$

$$\mu(x,t) = \mathbb{P}_2(x,t)e^{i\emptyset_2}. \tag{4}$$

This, $\emptyset$ for $i=1,2$ as

$$\emptyset_i(x,t) = -k_i x + w_i t + \theta_i \tag{5}$$

where k is the soliton frequency, w is the soliton wave number and θ is the phase constant. Substituting (3) and (4) into (1) and (2) and equating the real and imaginary parts yields. The real parts

$$a_i\frac{\partial^2\mathbb{P}_i}{\partial x^2} + b_i\frac{\partial^2\mathbb{P}_i}{\partial x\partial t} + (b_i k_i w_i - w_i - a_i k^2)\mathbb{P}_i + c_i\mathbb{P}_i\mathbb{P}_j^2 = 0 \tag{6}$$

where $j=3-i$ and imaginary parts

$$(1 - b_i k_i)\frac{\partial\mathbb{P}_i}{\partial t} + (w_i b_i - 2a_i k_i)\frac{\partial\mathbb{P}_i}{\partial x} = 0. \tag{7}$$

Ᵽ is follows

$$Ᵽ(x,t) = g(x-vt) \tag{8}$$

where *v* is the velocity, *g* is the functional form of the wave profile. Thus from real and imaginary parts obtained *v*. Thus (7) lead to

$$v = \frac{w_i b_i - 2a_i k_i}{1-k_i b_i}. \tag{9}$$

From equation (9),

$$k_i b_i \neq 1 \tag{10}$$

and

$$(1-b_2k_2)(w_1b_1-2a_1k_1) = (1-b_1k_1)(w_2b_2-2a_2k_2). \tag{11}$$

Hence, the cNLSE will be considered for the following cases of Kerr law.

Dark Optical Solitons

We suppose that $Ᵽ_1$ and $Ᵽ_2$ are as

$$Ᵽ_1(x,t) = A_1 sn^{p_1}(\delta,l) \tag{12}$$

$$Ᵽ_2(x,t) = A_2 sn^{p_2}(\delta,l) \tag{13}$$

with

$$\delta = B(x-vt) \tag{14}$$

where *A* is the amplitude, *B* is the inverse width of the soliton and *l* is the modulus of jacobi elliptic function. Substituting (12) and (13) equations into (6) yields

$$\begin{aligned}&(a_ip_i(p_i-1)-b_ip_i(p_i-1)v)A_iB^2sn^{p_i-2}+(-a_ip_i(p_i+l-l^2+p_il^2)B^2\\ &\quad +b_ivp_i{}^2(1+l^2)B^2+b_ik_iw_i-a_ik_i{}^2-w_i)A_isn^{p_i}+(a_ip_il(1+p_il)\\ &\quad -b_ivp_il^2(1+p_i))A_iB^2sn^{p_i+2}+c_iA_iA_j^2sn^{p_i+2p_j}=0.\end{aligned} \tag{15}$$

Now, from (15) equating exponent $(p_i + 2, p_i + 2p_j)$ lead to

$$p_1 = p_2 = 1. \tag{16}$$

In Eq.(15) setting the coefficients of terms to zero, from (15) we obtain

$$w_i = \frac{1}{1-b_i k_i}\{-a_i k_i^2 + B^2(-a_i(1+l) + b_i v(1+l^2))\} \tag{17}$$

and

$$B = \left(\frac{-c_i A_j^2}{a_i l(1+l) - 2b_i v l^2}\right)^{1/2}. \tag{18}$$

As a result, solutions of the equation system (1-2) are given as

$$\psi(x,t) = A_1 sn[B(x - vt), l]e^{i\emptyset_1}, \tag{19}$$

$$\mu(x,t) = A_2 sn[B(x - vt), l]e^{i\emptyset_2}, \tag{20}$$

where the relation between the amplitude *A* and the inverse width *B* is given in (18), the wave number *w* is given by (17). When the modulus $l \to 1$ in (19) and (20), occurs dark optical soliton solutions as

$$\psi(x,t) = A_1 tanh[B(x - vt), l]e^{i\emptyset_1}, \tag{21}$$

$$\mu(x,t) = A_2 tanh[B(x - vt), l]e^{i\emptyset_2}, \tag{22}$$

when $l \to 1$, the wave numbers of the solitons and width are given by

$$w_i = \frac{1}{1-b_i k_i}\{-a_i k_i^2 + 2B^2(-a_i + b_i v)\} \tag{23}$$

and

$$B = \left(\frac{-c_i A_j^2}{2a_i - 2b_i v}\right)^{1/2} \tag{24}$$

respectively. Also, from (24) Eq. are connected to the following limitations

$$c_i A_j^2(2a_i - 2b_i v) < 0 \tag{25}$$

and

$$\frac{A_1}{A_2} = \left(\frac{c_1(a_2 - b_2 v)}{c_2(a_1 - b_1 v)}\right)^{1/2}. \tag{26}$$

Bright Optical Solitons

In Eqs.(1)-(2) equations by Kerr law, we suppose that $\mathbb{P}_1$ and $\mathbb{P}_2$ are as follows

$$\mathbb{P}_1(x,t) = A_1 sn^{p_1}(\delta, l) \tag{27}$$

$$\mathbb{P}_2(x,t) = A_2 sn^{p_2}(\delta, l) \tag{28}$$

with

$$\delta = B(x - vt) \tag{29}$$

Substituting (27)-(28) into (6) yields

$$(-a_i p_i(p_i - 1)(l^2 - 1) - b_i v p_i(p_i - 1)(1 - l^2))A_i B^2 cn^{p_i - 2}$$
$$+(a_i p_i(-p_i + l - l^2 + 2p_i l^2)B^2 + bvp_i^2(1 - 2l^2)B^2 + b_i k_i w_i - a_i k_i^2 - w_i)A_i cn^{p_i}$$
$$\left(-a_i p_i l(1 + p_i l) + b_i p_i v l^2(1 + p_i)\right)A_i B^2 cn^{p_i + 2} + c_i A_i A_j^2 cn^{p_i + 2p_j} = 0. \tag{30}$$

In this last equation, $(p_i + 2p_j, p_i + 2)$ lead to

$$p_1 = p_2 = 1. \tag{32}$$

Similarly to the Dark optical soliton solutions, from the coefficients of

$$w_i = \frac{1}{1 - b_i k_i}\{(a_i(l^2 + l - 1) + b_i v(1 - 2l^2))B^2 - a_i k_i^2\} \tag{33}$$

and

$$B = \left(\frac{-c_i A_j^2}{2b_i v l^2 - a_i l(1 + l)}\right)^{1/2}. \tag{34}$$

Thus, solutions of the equation system (1-2) are given by

$$\psi(x,t) = A_1 cn[B(x - vt), l]e^{i\emptyset_1}, \tag{35}$$

$$\mu(x,t) = A_2 cn[B(x-vt),l]e^{i\emptyset_2}, \tag{36}$$

From Eqs. (35)-(36), we obtain following bright optical soliton solutions as the modulus $l \to 1$

$$\psi(x,t) = A_1 sech[B(x-vt),l]e^{i\emptyset_1}, \tag{37}$$

$$\mu(x,t) = A_2 sech[B(x-vt),l]e^{i\emptyset_2}, \tag{38}$$

when $l \to 1$, and B are given by

$$w_i = \frac{1}{1-b_i k_i}\{(a_i - b_i v)B^2 - a_i k_i^2\} \tag{39}$$

and

$$B = \left(\frac{-c_i A_j^2}{2b_i v - 2a_i}\right)^{1/2} \tag{40}$$

respectively. Also, from (40) Eq. are connected to the following limitations

$$c_i A_j^2 (2b_i v - 2a_i) > 0 \tag{41}$$

and

$$\frac{A_1}{A_2} = \left(\frac{c_1(b_2 v - a_2)}{c_2(b_1 v - a_1)}\right)^{1/2}. \tag{42}$$

Singular Solutions

Let, in the form

$$₱_1 = \frac{A_1}{sc^{p_1}(\delta,l)} \tag{43}$$

and

$$₱_2 = \frac{A_2}{sc^{p_2}(\delta,l)}. \tag{44}$$

Substituting (43) and (44) into (6) yield

$$(-a_i + b_i v)p_i(-1-p_i)A_i B^2 sc^{-2-p_i} + ((-a_i + b_i v){p_i}^2(l^2-2)B^2 + b_i k_i w_i - a_i k_i^2 - w_i)A_i sc^{-p_i} + (-a_i + b_i v)p_i(1-p_i)(1-l^2)A_i B^2 sc^{2-p_i} + c_i A_i A_j^2 sc^{-p_i-2p_j} = 0. \tag{45}$$

From this equation is as

$$p_1 = p_2 = 1. \tag{46}$$

From coefficients of

$$w_i = \frac{1}{1-b_i k_i}\{((-a_i + b_i v)(l^2 - 2))B^2 - a_i k_i^2\} \tag{47}$$

and

$$B = \left(\frac{c_i A_j^2}{2b_i v - 2a_i}\right)^{1/2}. \tag{48}$$

In the same way from

$$\psi(x,t) = A_1 cs[B(x - vt), l]e^{i\emptyset_1}, \tag{49}$$

$$\mu(x,t) = A_2 cs[B(x - vt), l]e^{i\emptyset_2}. \tag{50}$$

Here, while $l \to 1$ from (45)

$$w_i = \frac{1}{1-b_i k_i}\{(-a_i + b_i v)B^2 - a_i k_i^2\} \tag{51}$$

and singular solutions are given by

$$\psi(x,t) = A_1 csch[B(x - vt), l]e^{i\emptyset_1}, \tag{52}$$

$$\mu(x,t) = A_2 csch[B(x - vt), l]e^{i\emptyset_2}. \tag{53}$$

By using (48) equation as

$$c_i A_j^2(2b_i v - 2a_i) < 0. \tag{54}$$

CONCLUSIONS

In this article, we investigated Jacobi elliptic function solutions of (1)-(2) equations system. The Optical soliton solutions are obtained of Eqs.(1)-(2) with Kerr law nonlinearity of Jacobi elliptic functions. Thus, dark and bright optical solitons and singular solutions are presented when $l \to 1$.

REFERENCES

Agarwal P, Ray A, Chowdhury AR, 2015. Properties of Optical Soliton in a Three Level Medium with Quantic Nonlinearity. International Journal of Physics, 3(2): 45-51.

Biswas A, Kara AH, Bokhari AH, Zaman FD, 2013. Solitons and conservation laws of Klein-Gordon equation with power law and log law nonlinearities. Nonlinear Dynamics, 73(4): 2191-2196.

Biswas A, 2009. Solitary wave solution for KdV equation with power-law nonlinearity and time-dependent coefficients. Nonlinear Dynamics, 58(1-2): 345-348.

Biswas A, 2010. Solitary waves for power-law regularized long-wave equation and R(m,n) equation. Nonlinear Dynamics, 59(3): 423-426.

Guzman JV, Hilal EM, Alshaery AA, Bhrawy AH, Mahmood MF, Moraru L, Biswas A, 2015. Thirring

Kivshar YS, Agrawal GP, 2003. Optical Solitons. Academic Press, USA.

Krishnan EV, Gabshi MA, Mirzazadeh M, Bhrawy A, Biswas A, Belic M, 2015. Optical Solitons for Quadratic Law Nonlinearity with Five Integration Schemes. Journal of Computational and Theoretical Nanoscience, 12(11): 4809 – 4821.

Mirzazadeh M, Arnous AH, Mahmood MF, Zerrad E, Biswas, A, 2015. Soliton solutions to resonant nonlinear Schrödinger's equation with time-dependent coefficients by trial solution approach. Nonlinear Dynamics, 81(1-2): 277-282.

Optical Solitons with Spatio-Temporal Dispersion. Proceedings of The Romanian Academy Series A, 16(1): 41-46.

Sardar A, Ali K, Rizvi STR, Younis M, Zhou Q, Zerrad E, Biswas A, Bahrawy A, 2016. Dispersive Optica

Sassaman R, Biswas A, 2010. Topological and non-topological solitons of the Klein–Gordon equations in 1+2 dimensions. Nonlinear Dynamics, 61(1-2): 23-28.

Solitons in Nanofibers with Schrödinger-Hirota Equation. Journal of Nanoelectronics and Optoelectronics, 11(3): 382-387.

Tang D, Guo J, Song Y, Zhang H, Zhao L, Shen D, 2014. Dark Soliton Fiber Lasers. Optics Express, 22(16): 19831-19837.

Zakharov VE, Shabat AB, 1972. Exact Theory of Two-Dimensional Self-Focusing and One- Dimensional Self-Modulation of Waves in Nonlinear Media. Soviet Physics-Jetp, 34(1): 62-69.