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Parameterized Statistical Compactness of Topological Spaces

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Abstract. In this research, we present the notion of statistical compactness restricted up to order α, where $\alpha \in (0, 1]$. There exist statistically compact spaces that are not compact. So the parameter α became the measurement of non compactness. Additionally, we looked for the continued existence of order α statistical compactness under open continuous surjection and subspace topology. A finite intersection-like characterization for α statistical compactness has also been established.

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1. Introduction

The density of a set of points in a metric space is the degree to which those points are dispersed densely throughout the space; this is frequently determined by the distances between the points. For example, one can calculate the density of a subset A of a metric space X by evaluating how closely packed the points of A are with respect to a metric on X. It is defined as:

$$\delta(A) = \lim_{n\to\infty} \frac{1}{n}|\{k \leq n : k \in A \subseteq \mathbb{N}\}|, \text{ where } \mathbb{N} \text{ is the set of all natural numbers.}$$

Using the idea of asymptotic density, H. Fast [12] and Schoenberg [16] adapted the concept of typical convergence to statistical convergence. A sequence $\{z_n\}$ in a topological space X is said to converge statistically to a point x if, for any open set U containing x, the density of the set $\{n \in \mathbb{N} : z_n \in U\}$ (i.e., the portion of the sequence's elements that fall within U) converges to 1 as n tends to infinity [10, 13]. Other generalization of statistical convergence can be found in [9, 15]. The notion of s-convergence of real sequences was reinforced in 2012 by Bhunia et al. [8] by limiting asymptotic density up to order α, where $0 < \alpha < 1$. Asymptotic density of order α for $A \subseteq \mathbb{N}$ is given by

$$\delta^\alpha(A) = \lim_{n\to\infty} \frac{1}{n^\alpha}|\{k \leq n : k \in A \subseteq \mathbb{N}\}|.$$

The concept of statistical convergence in topology is extended by statistical convergence of order α. It presents a parameter α that has significance in defining the particular convergence behavior that sequences exhibit. In this case, α denotes a parameter that affects the convergence of sub sequences, offering a more sophisticated interpretation of

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convergence than is possible with conventional thinking. In a topological space X, a sequence $\{y_n\}$ is said to converge statistically of order α to a point x if and only if the following limit holds for any open set U containing x:

$$\lim_{n\to\infty} \frac{|\{n \in \mathbb{N} : x_n \notin U\}|}{n^\alpha} = 0.$$

However, this study has introduced covering qualities like compactness. A basic characteristic that effectively sums up what it means to be"finite" in a broad sense is compactness in a topological space. Every open cover of a topological space that has a finite subcover is said to be compact. To put it another way, there is always a finite number of open sets that"cover" the entire space, regardless of how we decide to "cover" it. Analysis, geometry, and topology are just a few of the mathematical fields where compactness has a wide range of applications and ramifications. From metric spaces to more general topological spaces, it offers a natural extension of the concepts of boundedness and finiteness. Numerous writers have examined other forms of compactness and several selective covering properties [1–6, 14]. We want to pursue our research on the asymptotic density of order α in order to determine a topological characteristic that is associated with compactness.

2. Preliminaries

Prior to studying α statistical compactness in depth, it is important to provide some basic definitions and notions. In this section, we briefly discuss the fundamental instruments and mathematical concepts required to comprehend the key findings. No separation axiom has been assumed in this paper unless otherwise stated. For other notions and symbols, we follow [11].

Definition 2.1 ([11]). A topological space (X, τ) is called a countably compact space if every countable open cover of X has a finite subcover.

Every compact space is countably compact. But the space ω_0 of all countable ordinals is countably compact but not compact [11]. The space $\mathbb{N}$ of all natural numbers equipped with discrete topology is Lindelöf, but it is not countably compact.

Definition 2.2 ([11]). A family $\mathcal{F} = \{F_s\}_{s\in S}$ of subsets of a set X is said to have a finite intersection property if $\mathcal{F} \neq \varnothing$ and and $\bigcap_{i=1}^{n} F_{s_k} \neq \varnothing$ for every finite set $\{s_1, s_2, s_3, \ldots, s_n\} \subseteq S$.

Theorem 2.3 ([11]). *A topological space X is compact if and only if every family of closed subsets of X having the finite intersection property has a non-empty intersection.*

Definition 2.4 ([7]). A topological space (X, τ) will be called a statistical compact (in short, s-compact) space if for every countable open cover $\mathcal{U} = \{U_n : n \in \mathbb{N}\}$ of X, there exists a subcover $\mathcal{V} = \{U_{m_k} : k \in \mathbb{N}\}$ such that $\delta(\{m_k : U_{m_k} \in \mathcal{V}\}) = 0$.

Definition 2.5 ([7]). A family $\mathbb{F} = \{F_n\}_{n\in\mathbb{N}}$ of subsets of a countable set X is said to have δ_r-intersection property if $\mathbb{F} \neq \varnothing$ and $\bigcap_{n\in S} F_n \neq \varnothing$ for every subset $S \subseteq \mathbb{N}$ with $\delta(S) = r$.

Theorem 2.6 ([7]). *A topological space X is s-compact if and only if every family of countable closed subsets of X that has the δ_0-intersection property has a non empty intersection.*

3. s^α-Compact Spaces

In this section, we introduce a restricted and controlled version of statistical compactness and study some of its topological features.

Definition 3.1. A topological space (X, τ) is called a statistical compact space of order α (in short s^α- compact) if for every countable open cover $\mathcal{U} = \{U_n : n \in \mathbb{N}\}$ of X, there exists a sub cover $\mathcal{V} = \{U_{m_k} : k \in \mathbb{N}\}$ such that $\delta^\alpha(\{m_k : U_{m_k} \in \mathcal{V}\}) = 0$.

Theorem 3.2. *Every countably compact space is an s^α- compact space.*

Proof. Let (X, τ) be a countably compact space. Then, every countable open cover $\mathcal{U} = \{U_n : n \in \mathbb{N}\}$ of X has a finite sub cover $\{U_{m_1}, U_{m_2}, \cdots, U_{m_k}\}$. Since $\{m_1, m_2, \cdots, m_k\}$ is a finite subset of $\mathbb{N}$, it has a natural density of order α, which becomes 0. Hence (X, τ) is an s^α-compact space. □

Example 3.3. There exists an s^α-compact space X that is not countably compact.

Let (X, τ) be a topological space, where $X = B_1(0)$ and $\tau = \{B_r(0) : 0 \leq r \leq 1\}$, where $B_r(a)$ represents an open ball of radius r with center a. Consider $\mathcal{A} = \{A_n : n \in \mathbb{N}\}$ to be an open cover of X. For a trivial open cover, $A_k = X$ for some $k \in \mathbb{N}$. Then, we take a subcover $\mathcal{V} = \{A_k\}$ of X. Clearly, $\delta^\alpha(\{k\}) = 0$. Then, X is an s^α-compact space.

Now, we consider $\mathcal{A}$ to be a non-trivial open cover of X. After that, we choose a subsequence $\mathcal{A}' = \{A_{n_k} : k \in \mathbb{N}\}$ of $\mathcal{A}$ such that $A_{n_1} = A_1$ and $A_{n_{k+1}} \supseteq A_{n_k}$ for $k \in \mathbb{N}$. If A_{n_p} is such that for all $m > n_p$ with $A_m \nsubseteq A_{n_p}$, $\cup\mathcal{A} = A_{n_p} \neq X$, a contradiction. Based on the construction of topology, $\mathcal{A}'$ exists, and it is an open cover of X. We now select a subsequence $\mathcal{V}$ of $\mathcal{A}'$ such that $\mathcal{V} = \{A_{n_{k^\beta}} : k \in \mathbb{N}\}$, where $\beta \in \mathbb{N}$ and $\beta > \frac{1}{\alpha}$, it is obvious that $\mathcal{V}$ is a cover of X and $\delta^\alpha\{n_{k^\beta} : k \in \mathbb{N} \text{ and } A_{n_{k^\beta}} \in \mathcal{A}'\} = lim_{n\to\infty} \frac{|n_{k^\beta} : k \in \mathbb{N} : A_{n_{k^\beta}} \in \mathcal{A}'|}{k} = lim_{n\to\infty} \frac{k}{k^\alpha \beta} = lim_{n\to\infty} \frac{1}{k^{\alpha\beta-1}} = 0$ [Since $\alpha\beta > 1$]. Further, $\delta^\alpha\{n_k \in \mathbb{N} \text{ and } A_{n_k} \in \mathcal{A}\} \leq \delta^\alpha\{n_{k^\beta} : k \in \mathbb{N} \text{ and } A_{n_{k^\beta}} \in \mathcal{A}'\} = 0$. Thus, (X, τ) is an s^α-compact space.

Now, we consider the open cover $\mathcal{W} = \{W_n = B_{1-\frac{1}{n}}(0) : n \in \mathbb{N}\}$ and assume that $\mathcal{A}$ has finite subcover $\mathcal{W}' = \{W_{n_1}, W_{n_2}, \cdots, W_{n_k}\}$ then a $\{A_{n_p}\}$ exists such that $A_{n_i} \subseteq A_{n_p}$ for all $A_{n_i} \in \mathcal{V}$. Thus, $\cup\mathcal{V} = A_{n_p} = B_{1-\frac{1}{n_p}}(0) \neq X$, a contradiction. Therefore, $\mathcal{A}$ has no finite subcover. Thus, (X, τ) is not countably compact.

Theorem 3.4. *Every s^α-compact space is an s-compact space.*

Proof. Let (X, τ) be an s^α-compact space. Then, every countable open cover $\mathcal{U} = \{U_n : n \in \mathbb{N}\}$ of X has subcover $\mathcal{V} = \{U_{m_k} : k \in \mathbb{N}\}$ such that $\delta^\alpha(\{m_k : U_{m_k} \in \mathcal{V}\}) = 0$. Therefore, $\delta(\{m_k : U_{m_k} \in \mathcal{V}\}) = 0$. □

Open Problem: Does there exist a topological space that is s-compact but not s^α-compact space?

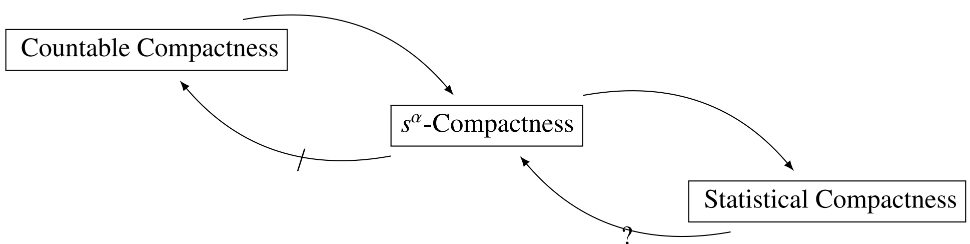

Figure 1. The relation between variations of compactness.

Theorem 3.5. *Every closed subspace of an s^α-compact space is an s^α-compact space.*

Proof. Let (X, τ) be a s^α-compact space and (A, τ_A) be an arbitrary closed subspace of (X, τ). Let $\mathcal{W} = \{W_n : n \in \mathbb{N}\}$ be a countably infinite cover of (A, τ_A)

$$\therefore A = \bigcup_{n\in\mathbb{N}} W_n = \bigcup \mathcal{W}$$

Now, for every $n \in \mathbb{N}$, there exist a $U_n \in \tau$ such that $W_n = A \cap U_n$. $\therefore A = \bigcup_{n\in\mathbb{N}} W_n \subseteq \bigcup_{n\in\mathbb{N}} U_n$. We construct a countably infinite cover $\mathcal{V} = \{V_n : n \in \mathbb{N}\}$ of (X, τ), where

$$V_n = \begin{cases} X \setminus A, & \text{if } n = 1 \\ U_{n-1}, & \text{otherwise.} \end{cases}$$

But (X, τ) is s^α-compact space. Therefore, there exist a subcover $\mathcal{M} = \{V_{n_k} : k \in \mathbb{N}\}$ with $\delta^\alpha(\{n_k : k \in \mathbb{N}\}) = 0$. Let $\mathcal{M}_A = \{A \cap V_{n_k} : k \in \mathbb{N}\}$. Then, $\mathcal{M}_A$ is a subcover of $\mathcal{W}$. Now, if $V_1 \notin \mathcal{M}$, then $\{n_k : V_{n_k} \in \mathcal{M}\} = \{n_k : A \cap V_{n_k} \in \mathcal{M}_A\}$ So, $\delta^\alpha(\{n_k : V_{n_k} \in \mathcal{M}\}) = \delta^\alpha(\{n_k : A \cap V_{n_k} \in \mathcal{M}_A\}) = 0$. If $V_1 \in \mathcal{M}$, then $\{n_k : V_{n_k} \in \mathcal{M}\} = \{n_k\text{-}1 : A \cap V_{n_k} \in \mathcal{M}_A\} = 0$ So, $\delta^\alpha(\{n_k : V_{n_k} \in \mathcal{M}\}) = \delta^\alpha(\{n_k\text{-}1 : A \cap V_{n_k} \in \mathcal{M}_A\}) = 0$.

Hence, (A, τ_A) is an s^α-compact space. □

Theorem 3.6. *If a subspace (A, τ_A) of a topological space (X, τ) is s^α-compact, then for every family $\{U_n\}_{n\in\mathbb{N}}$ of open subsets of X such that $A \subseteq \bigcup_{n\in\mathbb{N}} U_n$, there exists a subset $S \subseteq \mathbb{N}$ with $\delta^\alpha(S) = 0$ such that $A \subseteq \bigcup_{n\in S} U_n$.*

Proof. Let, $\{U_n\}_{n\in\mathbb{N}}$ be a family of subsets of X such that $A \subseteq \bigcup_{n\in\mathbb{N}} U_n$. Therefore, $A = \bigcup_{n\in\mathbb{N}} (A \cap U_n) \implies \{A \cap U_n : n \in \mathbb{N}\}$ is an open cover of A in (A, τ_A), but (A, τ_A) is s^α-compact. Therefore, there exists $S \subseteq \mathbb{N}$ with $\delta^\alpha(S) = 0$ such that $A = \bigcup_{n\in S} (A \cap U_n)$ implies that $A \subseteq \bigcup_{n\in S} U_n$.
Hence the theorem. □

Theorem 3.7. *Let U be an open subset of a topological space (X, τ). If a family $\{F_n\}_{n\in\mathbb{N}}$ of closed subsets of X contains at least one s^α-compact set such that $\bigcap_{n\in\mathbb{N}} F_n \subseteq U$, then there exists $S \subseteq \mathbb{N}$ with $\delta^\alpha(S) = 0$ and $\bigcap_{n\in S} F_n \subseteq U$.*

Proof. Let, F_{n_0} be the s^α-compact set in the family $\{F_n : n \in \mathbb{N}\}$. Since $U \in \tau, X \setminus U$ is closed. Thus, $(X \setminus U) \bigcap F_{n_0} = F_{n_0} \setminus U$ is an s^α-compact set, being a closed subspace of an s^α-compact space [by Theorem 3.5]. Let $A = F_{n_0} \setminus U$ and consider $\{U_n = X \setminus F_n : n \in \mathbb{N}\}$ as a family of open sets. Now $\bigcup_{n\in\mathbb{N}} U_n = X \setminus \bigcap_{n\in\mathbb{N}} F_n \supseteq X \setminus U \supseteq F_{n_0} \setminus U = A$ implies that $A \subseteq \bigcup_{n\in\mathbb{N}} U_n$. But A is s^α-compact. Therefore, by Theorem 3.6, there exists $S \subseteq \mathbb{N}$ with $\delta^\alpha(S) = 0$ such that $A \subseteq \bigcup_{n\in S} U_n$, which implies that $F_{n_0} \setminus U = X \setminus U \bigcap F_{n_0} \subseteq \bigcup_{n\in S} U_n$. Thus, $X \setminus U \subseteq \bigcup_{n\in S} X \setminus F_n = X \setminus \bigcap_{n\in S} F_n$
Therefore $U \supseteq \bigcap_{n\in S} F_n$. □

Theorem 3.8. *If $\{(X_m, \tau_m) : n = 1, 2, \ldots, p\}$ is a finite collection of s^α-compact spaces of X such that $X = \bigcup_{n=1}^{p} X_p$, then (X, τ) is s^α-compact.*

Proof. Let (X_m, τ_m) for $m = 1, 2, \ldots, p$ are s^α-compact subspaces of (X, τ) such that $X = \bigcup_{m=1}^{p} X_m$. Let $\mathcal{U} = \{U_n : n \in \mathbb{N}\}$ be a countable open cover of (X, τ). Then $\mathcal{U}_m = \{X_m \cap U_n : n \in \mathbb{N}\} \setminus \{\varnothing\}$ are countable open covers of (X_m, τ_m) where $m = 1, 2, \ldots, p$. Therefore, there exists a subcover $\mathcal{V}_m = \{X_m \cap U_{n_k} : k \in \mathbb{N}\}$ for each $\mathcal{U}_m$ of (X_m, τ_m) such that $\delta^\alpha(\{n_k : X_m \cap U_{n_k} \in \mathcal{V}_m\}) = 0$, for $m = 1, 2, \ldots, p$. Now $\delta^\alpha(\bigcup_{m=1}^{p}\{n_k : X_m \cap U_{n_k} \in \mathcal{V}_m\}) \leq \sum_{m=1}^{p} \delta^\alpha(\{n_k : X_m \cap U_{n_k} \in \mathcal{V}_m\}) = 0$.
Moreover $\bigcup_{m=1}^{p} \mathcal{V}_m$ covers of X.
So, $X \subseteq \bigcup_{m=1}^{p} \mathcal{V}_m \subseteq \bigcup_{m=1}^{p} \{U_{n_k} : k \in \mathbb{N}$ and $\{X_m \cap U_{n_k} \in \mathcal{V}_m\}\} = \mathcal{W}$ (say).
Therefore, $\mathcal{W}$ is a sub cover of $\mathcal{U}$ such that $\delta^\alpha(\{n_k : U_{n_k} \in \mathcal{U}\}) = 0$.
Hence, (X, τ) is an s^α-compact space. □

Example 3.9. There exists a non-s^α-compact topological space (X, τ) that can be expressed as the union of countably many s^α-compact subspaces.
Consider a collection of indiscrete topological space (X_i, τ_i) where $X_i = [i-1, i)$, $i \in \mathbb{N}$, which is a countable collection of subsets of $X = [0, \infty)$ and $\tau_i = \{\varnothing, X_i\}$. Thus, $\{(X_i, \tau_i) : i \in \mathbb{N}\}$ is a collection of s^α-compact spaces. Now, the topology τ on X is generated by $\mathcal{B} = \{[i-1, i) : i \in \mathbb{N}\}$ and $X = [0, \infty) = \bigcup_{i\in\mathbb{N}} X_i$.
Let us consider a countable open cover $\mathcal{A} = \{A_i = [i-1, i) : i \in \mathbb{N}\}$ of (X, τ). If possible, let $S \subset \mathbb{N}$ with $\delta(S) = 0$, such that $\bigcup_{i\in S} A_i = X$. So there exist one element, say $p \in \mathbb{N}$, but $p \notin S$ which means that $A_p \notin \{A_i : i \in S\}$, i.e., $\bigcup_{i\in S} A_i \neq X$. which is a contradiction. Therefore, (X, τ) is not s^α-compact.

Theorem 3.10. *An open continuous image of an s^α-compact space is also s^α compact.*

Proof. Let (X, τ) be an s^α-compact space and $f : (X, \tau) \to (Y, \sigma)$ be an open continuous mapping. Let $\{G_n : n \in \mathbb{N}\}$ be a countable open cover for Y. So, $\cup\{G_n : n \in \mathbb{N}\} = Y$, which implies $f^{-1}\{\cup\{G_n : n \in \mathbb{N}\}\} = f^{-1}(Y)$. Thus, $\cup\{f^{-1}(G_n) : n \in \mathbb{N}\} = X$. Also, f is continuous, and G_n is open. Therefore, $\{f^{-1}(G_n) : n \in \mathbb{N}\}$ is an open cover in X. Since X is an s^α-compact space, there exist a countable subcover of X, $\{f^{-1}(G_{n_1}), f^{-1}(G_{n_2}), \ldots\}$ (say), where $\{n_1 < n_2 < \ldots\}$ and $\delta^\alpha(\{n_k : k \in \mathbb{N}\}) = 0$ So, $\cup\left\{f^{-1}(G_{n_k}) : k \in \mathbb{N}\right\} = X$, which gives $f\left[\cup\left\{f^{-1}(G_{n_k}) : k \in \mathbb{N}\right\}\right] = f(X) = Y$ implies that $\cup\{G_{n_k} : k \in \mathbb{N}\} = Y$ for $f\left[f^{-1}(G_{n_k})\right] = G_{n_k}$. Thus, $\{G_{n_k} : n \in \mathbb{N}\}$ is an open sub cover of $\{G_n : n \in \mathbb{N}\}$ for Y and also $\delta^\alpha(\{n_k : k \in \mathbb{N}\}) = 0$.
Therefore, (Y, σ) is also an s^α-compact space.
Now, we will search for a finite intersection type characterization under the influence of s^α-density. □

Definition 3.11. A countable family $\mathbb{F} = \{F_n\}_{n\in\mathbb{N}}$ of subsets of X is said to have Δ_r^α-intersection property if $\mathbb{F} \neq \varnothing$ and $\bigcap_{n\in S} F_n \neq \varnothing$ for every subset $S \subseteq \mathbb{N}$ with $\delta^\alpha(S) = r$.

Theorem 3.12. *Every countable family of closed subsets of X having Δ_0^α-intersection property has non-empty intersection if and only if the topological space X is s^α-compact.*

Proof. Let (X,τ) be an s^{α}-compact space and $\{F_n\}_{n\in\mathbb{N}}$ be a arbitrary family of closed subsets of X with Δ_0^{α}-intersection property. If possible, let $\bigcap_{n\in\mathbb{N}} F_n = \emptyset$. Consider $U_n = X \setminus F_n$. Then, $X = X \setminus \bigcap_{n\in\mathbb{N}} F_n = \bigcup_{n\in\mathbb{N}} (X \setminus F_n) = \bigcup_{n\in\mathbb{N}} U_n$. Therefore, $\{U_n\}_{n\in\mathbb{N}}$ is an open cover of X. But (X,τ) is s^{α}-compact. Thus, there exists a subset $S \subseteq \mathbb{N}$ such that $\delta^{\alpha}(S) = 0$ and $\bigcup_{n\in S} U_n = X$. Now, $X = \bigcup_{n\in S} U_n = \bigcup_{n\in S} X \setminus F_n = X \setminus \bigcap_{n\in S} F_n$. Therefore, $\bigcap_{n\in S} F_n = \emptyset$, which is a contradiction to the fact that $\{F_n : n \in \mathbb{N}\}$ has the Δ_0^{α}-intersection property. Thus, every family of closed subsets of X that has the Δ_0^{α}-intersection property has a non-empty intersection.

Conversely, let $\mathcal{U} = \{U_n : n \in \mathbb{N}\}$ be a countable open cover of X. Now, $\mathcal{F} = \{F_n = X \setminus U_n : n \in \mathbb{N}\}$ is a countable family of closed sets. Now, $\bigcap_{n\in\mathbb{N}} F_n = \bigcap_{n\in\mathbb{N}}(X \setminus U_n) = X \setminus \bigcup_{n\in\mathbb{N}} U_n = \emptyset$. By contrapositivity of our assumption, it does not have the Δ_0^{α}-intersection property. Therefore, there exists $S \subseteq \mathbb{N}$ with $\delta^{\alpha}(S) = 0$ and $\bigcap_{n\in S} F_n = \emptyset$. So, $\bigcap_{n\in S} (X \setminus U_n) = \emptyset$. Thus, $X \setminus \bigcup_{n\in S} U_n = \emptyset$. So $X = \bigcup_{n\in S} U_n$, with $\delta^{\alpha}(S) = 0$. Therefore, (X,τ) is an s^{α}-compact. □

4. Conclusion

s^{α} compactness serves as an intermediate between countable compactness and statistical compactness; it is a closed hereditary property that is preserved under continuous surjection. s^{α} compactness can be characterized by the families of closed subsets by means of the Δ_r^{α}-intersection property. Extension of such concepts can also be useful for the analysis of some other covering properties.

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Conflicts of Interest

The authors do not have any relationship that can be interpreted as conflict of interest.

Authors Contribution Statement

This work was carried out in collaboration between all authors. P. Bal formulated the problem, developed the theoretical framework, and conducted the mathematical analysis. T. Datta wrote and edited the manuscript, contributing to the revisions. P. Das performed the calculations and contributed to the revision process. All authors read and approved the final manuscript.

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