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A swarm optimized ANN-based numerical treatment of nonlinear SEIR system based on zika virus.

Zika virüsü temelli doğrusal olmayan SEIR sisteminin sürüklenmiş yapay sinir ağı (ANN) tabanlı sayısal tedavisi.

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A Swarm Optimized ANN-based Numerical Treatment of Nonlinear SEIR System based on Zika Virus

Highlights

- *The Mexican Hat-Wavlevt-based ANN is employed to solve the nonlinear system of Zika virus Spread*
- *Error-based fitness function is demonstrated by using the system of differential equations*
- *Hybrid optimization scheme i.e. PSO-SQP is utilized to optimize the parameters of the neural designed neural network*
- *The correctness and stability of the scheme are analyzed through comprehensive statistical analysis*

Graphical Abstract

The computing efficiency of an artificial neural network is utilized to solve the Zika virus-based nonlinear differential system SEIR.

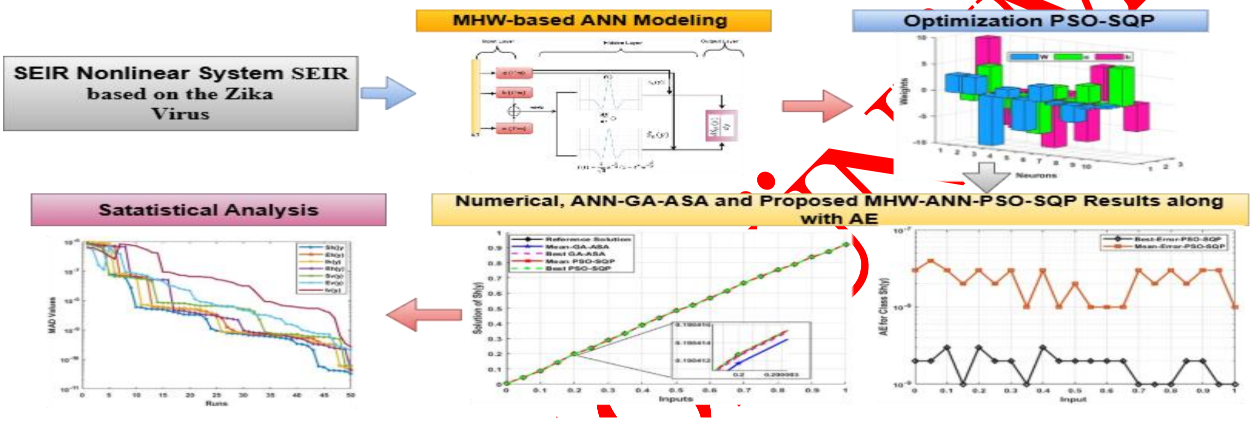

Figure. Flow chart of the Proposed MHW-ANN-PSO-SQP for SEIR based on Zika Virus.

Aim

The study aims to present a novel numerical method for solving a nonlinear SEIR model of the Zika virus using a hybrid computational framework.

Design & Methodology

The dynamics of Zika virus dissemination are examined using this model. The suggested framework solved the SEIR mathematical model by combining the ANN process with the swarm optimization process.

Originality

This study's novelty is a numerical analysis of the SEIR coupled nonlinear mathematical model based on Zika virus spread using the hybrid optimization process of global PSO and local search scheme SQP, as well as the computational efficiency of the Mexican Hat Wavlavt-based activation function

Findings

The suggested scheme's validity is indicated by the overlap solutions, and AE attests to its accuracy. The Min, Mean, and S.T.D. of the statistical operator are likewise within an acceptable range for solving the Zika virus spread-based SEIR nonlinear system.

Conclusion

Although the approximated solution of the proposed scheme is accurate, precise, and stable, the proposed scheme is reliable, however, due to the hybrid optimization procedure, the proposed scheme is computationally expensive.

Declaration of Ethical Standards

The author(s) of this article declares that the materials and methods used in this study do not require ethical committee permission and/or legal-special permission.

A Swarm Optimized ANN-based Numerical Treatment of Nonlinear SEIR System based on Zika Virus

Araştırma Makalesi / Research Article

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ABSTRACT

The purpose of the current study is to present the numerical treatment of a nonlinear mathematical SEIR model based on the Zika virus using the Mexican Hat Wavelet-based feed-forward artificial neural network (MHW-ANN) together with the optimization scheme of global search, Particle Swarm Optimization (PSO) and local search Sequential Quadratic Programming (SQP), i.e. MHW-ANN-PSO-SQP. The Zika virus is an epidemic disease that can spread through the transmission of the virus known as Aedes, its model is based on susceptible-exposed-infected-recovered, i.e. SEIR that investigated the dynamics of virus spread. To solve the model an error-based fitness function is optimized through a hybrid computing scheme of MHW-ANN-PSO-SQP. To validate the precision, accuracy, stability, reliability, and computational complexity of the designed framework various cases have been taken for the virus. The results obtained from the MHW-ANN-PSO-SQP are compared to the well-known RK numerical solver and ANN-based (GA-ASA) to confirm the accuracy. At the same time, the absolute error validated the precision of the designed scheme. Additionally, the statistical analysis through different statistical operators is performed to validate the stability, convergence, and reliability of the MHW-ANN-PSO-SQP. Furthermore, the complexity of the presented scheme is analyzed through the Mean Execution Time (MET).

Keywords: SEIR mathematical model, Artificial neural network computation, swarming techniques, SQP, Mexican hat wavelet

Zika Virüsü Temelli Doğrusal Olmayan SEIR Sisteminin Sürüklenmiş Yapay Sinir Ağı (ANN) Tabanlı Sayısal Tedavisi

ÖZ

Mevcut çalışmanın amacı, Zika virüsü temelli doğrusal olmayan bir SEIR matematiksel modelinin sayısal çözümünü, Meksika Şapkası Dalga Dönüşümü (MHW) tabanlı ileri beslemeli yapay sinir ağı (ANN) ile birlikte, küresel arama optimizasyon şeması olan Parçacık Sürüsü Optimizasyonu (PSO) ve yerel arama olan Ardışık Kuadratik Programlama (SQP) kullanarak sunmaktır. Zika virüsü, Aedes adı verilen virüsün taşınması yoluyla yayılan bir salgın hastalıktır ve bu model, virüsün yayılma dinamiklerini inceleyen Susceptible-Exposed-Infected-Recovered yani SEIR temellidir. Modeli çözmek için, hata tabanlı bir fitness fonksiyonu, MHW-ANN-PSO-SQP hibrit hesaplama şeması ile optimize edilmiştir. Tasarlanan çerçevenin doğruluğunu, güvenilirliğini, stabilitesini, hassasiyetini ve hesaplama karmaşıklığını doğrulamak için, virüsle ilgili çeşitli durumlar incelenmiştir. MHW-ANN-PSO-SQP'den elde edilen sonuçlar, doğruluğu teyit etmek için iyi bilinen RK sayısal çözücü ve ANN tabanlı (GA-ASA) ile karşılaştırılmıştır. Aynı zamanda, mutlak hata, tasarlanan şemanın doğruluğunu doğrulamaktadır. Ayrıca, istatistiksel analiz, MHW-ANN-PSO-SQP'nin stabilitesini, yakınsamasını ve güvenilirliğini doğrulamak için farklı istatistiksel operatörler aracılığıyla yapılmıştır. Dahası, sunulan şemanın karmaşıklığı, Ortalama Çalışma Süresi (MET) ile analiz edilmiştir.

Anahtar Kelimeler: SEIR matematiksel modeli, Yapay Sinir Ağı hesaplaması, sürüleme teknikleri, SQP, Meksika Şapkası Dalga Dönüşümü.

1. INTRODUCTION

In recent years, mathematical models have been applied to solve and understand the dynamics of the models used to investigate the dynamics of the Zika virus [1], dengue virus [2], HIV [3], malaria [4], COVID-19 [5], and LD [6].

The Zika virus is an epidemic that can spread through the transmission of the virus known as Aedes. The symptoms of this virus are treacherous and it can spread within 3 to 14 days. The symptoms are low during the first few days which makes the virus dangerous. The virus can be controlled by using the treatment and the early forecasting of the virus. The Zika Virus was exposed in the middle of the last era [7]. The Zika epidemic disease has been exposed in Uganda. The major cause of the virus is the macaque which belongs to the Rhesus category [8].

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In contrast to the dengue virus, the Zika virus is more dangerous due to its prolonged presence in the human body, the duration of this virus is three days to two weeks whereas the dengue virus remains in the human body from 2 days to 1 week. No appropriate virus treatment has been invented to control the vector along with the insecticide spray and stop the larval breeding atmosphere. The major cause of the virus is the female macaque who is expecting. It designated the susceptible infected as per the circumstance. As the mathematical models are promising to deal with epidemic diseases, researchers proposed various mathematical models for the numerical treatment of the Zika virus [1, 7-17].

To understand the system's dynamics, these differential equations must be solved correctly. Several numerical solvers for the Zika virus mathematical model were created by researchers. [1, 7-17], however in classical approaches, the entire tedious process is carried out to obtain intermediate or floating value outcomes. Furthermore, approaches such as the Adomian family and other perturbation techniques diverge as the input domain grows larger, whereas the suggested scheme remains convergent. Classical methods are particularly prone to variable overwriting and overload. Unlike Stochastic Numerical Solvers (SNS), classical numerical and analytical solvers are not generalizable and so cannot be implemented on parallel architectures.

Stochastic numerical solvers with various hybridization optimization strategies aid in the solution of complex real-world applications such as climate modeling, bioinformatics, energy systems, chemistry, and epidemiology, all of which involve uncertainty, unpredictability, and many dynamics. The SNS combines the stochastic technique with hybrid optimization schemes to get consistent and trustworthy results.

The current study aims to develop a numerical solver for the nonlinear SEIR mathematical model of the Zika disease using efficient stochastic computing techniques by utilizing the Mexican hat wavelet feed-forward ANN. The optimization of the designed ANN is executed by utilizing the computing efficiency of PSO aided with efficient local optimizer SQP i.e. MHW-ANN-PSO-SQP. In the recent past various stochastic computing techniques have been applied to solve the nonlinear mathematical system for Zika disease. However, the MHW-ANN-PSO-SQP technique has never been applied to solve this nonlinear system. The mathematical formulation of SEIR Zika disease along with the parameters' details and values are presented in [1] and written in (1).

$$
\begin{aligned}
&\frac{dS_h(y)}{dy} = A_h - S_h(y)\beta_h\left(I_v(y) + \rho I_h(y)\right) - \mu_h S_h(y), && (S_H)_0 = k_1, \\
&\frac{dE_H(y)}{dy} = \beta_H\left(\rho I_H(y) + I_V(y)\right)S_H(y) - (\chi_H + \mu_H)E_H(y), && (E_H)_0 = k_2, \qquad (1) \\
&\frac{dI_H(y)}{dy} = \chi_H E_H(y) - (\eta + \mu_H + \gamma)I_H(y), && (I_H)_0 = k_3, \\
&\frac{dR_H(y)}{dy} = -\mu_H R_H(y) + \gamma I_H(y), && (R_H)_0 \; k_4, \\
&\frac{dS_v(y)}{dy} = A_v - \beta_v I_H(y)S_v(y) - \mu_H S_v(y), && (S_v)_0 = k_5, \\
&\frac{dE_v(y)}{dy} = \beta_v I_H(y)S_v(y) - (\mu_v + \delta_H)E_v(y), && (E_v)_0 = k_6, \\
&\frac{dI_v(y)}{dy} = E_v(y)S\delta_v(y) - \mu_v I_v(y), && (I_v)_0 = k_7,
\end{aligned}
$$

The list of parameters used in the above system along with the values are tabulated in Table 1. The use of the stochastic computing procedure to solve the various nonlinear coupled and non-coupled differential systems is common and promising, however, the Mexican hat wavelet-based ANN-PSO-SQP numerical solver has never been investigated for the numerical treatment nonlinear coupled model.

Currently, some of the applications of the stochastic computing procedure to solve infectious diseases like the COVID pandemic [18-27], hepatitis [28-30], breakbone fever [31-35], and influenza [36-38], and CD4+T HIV system [39]. The other applications for these solvers are the numerical treatment of predator model [40], Emden system [41, 42], film_flow system [43], mass and heat system [44], hydrothermal system [45], large scale system [46], Emden model [47], life Cycle Optimization [48], Flierl equations [49, 50], Singular Periodic [51], Predictive system[40, 52], Smoke nonlinear model [53], nervous stomach system [54], Engineering problems [55], Transport system[56], and the Love story of Layle [57]. stochastic computing procedure is hybridized with the global bio-inspired GA, Swarm PSO, and Ant Colony Optimization (ACO) along with local search scheme, SQP, SQM, ASM, and simulated annealing [46, 48, 50]. The ANN-based solvers used the Log sigmoid, Morlet Wavelet (MW), as an activation function [28, 29, 31, 40, 54, 58, 59].

While the ANN-based numerical solver GA-ASA lacks precision and incurs high computing costs, numerical solvers struggle with generalizability when addressing the Zika virus nonlinear differential model. Numerous applications have demonstrated the effectiveness of an ANN-based numerical solver that leverages the computational efficiency of Mexican hats wavelet and optimized PSO-SQP hybridization techniques, which can be a good candidate to solve the Zika virus more precisely and accurately at a low computing cost.

Some of the novelties of the proposed MHW-ANN-PSO-SQP are listed as follows:

- A swarm-based stochastic computing procedure, MHW-ANN-PSO-SQP, is presented for numerically treating the nonlinear coupled mathematical system of SEIR to model the dynamics of the Zika virus.
- The error-based fitness function is optimized through PSO and SQP i.e. PSO-SQP in a hybrid manner.
- The accuracy of the developed swarming computational procedure based on MHW is validated by comparing the obtained results with the reference solution and the ANN-GA-ASA solutions.
- The precision of the proposed scheme is tested through the small AE for the numerical treatment of a nonlinear SEIR based on the Zika virus.
- To validate the stability and reliability of the proposed MHW-ANN-PSO-SQP, comprehensive statistical analysis has been made through the Mean Square Error (MSE), Mean Absolute Deviation (MAD), Global Mean Square Error (GMSE), and Global Mean Absolute Deviation (GMAD).
- The computational complexity of the proposed scheme is analyzed through Mean Execution Time (MET).

The rest of the paper is summarized as follows: Section 2 discusses the methodology, optimization procedure, and statistical operators. Section 3 presents the simulation and result discussion, while the last section provides the conclusion and the future direction.

2. DESIGN METHODOLOGY

The current section deals with formulating an error-based fitness function, the hybridization procedure using swarm-based PSO along with SQP, and the construction of statistical operators.

2.1. Construction of Fitness Function

The mathematical presentation of the stochastic computing procedure i.e. MHW-ANN for the system presented in (1) can be written as:

$$\begin{aligned}
&\frac{d\tilde{S}_h(y)}{dy},\frac{d\tilde{E}_h(y)}{dy},\frac{d\tilde{I}_h(y)}{dy},\frac{d\tilde{R}_h(y)}{dy},\frac{d\tilde{S}_v(y)}{dy},\frac{d\tilde{E}_v(y)}{dy} \\
&= \Bigg[\sum_{i=1}^{m} \alpha_{s_{h,i}} f\Big(w_{s_{h,i}}(y) \\
&+ b_{s_{h,i}}\Big), \sum_{i=1}^{m} \alpha_{E_{h,i}} f(w_{I_{h,i}}(y) \\
&+ b_{E_{h,i}}), \sum_{i=1}^{m} \alpha_{I_{h,i}} f(w_{R_{h,i}}(y) \\
&+ b_{I_{h,i}}), \sum_{i=1}^{m} \alpha_{R_{h,i}} f(w_{R_{h,i}}(y) \\
&+ b_{R_{h,i}}), \sum_{i=1}^{m} \alpha_{S_{v,i}} f\Big(w_{S_{v,i}}(y) \\
&+ b_{S_{v,i}}\Big), \sum_{i=1}^{m} \alpha_{E_{v,i}} f(w_{E_{v,i}}(y) \\
&+ b_{E_{v,i}}), \sum_{i=1}^{m} \alpha_{I_{v,i}} f(w_{I_{r,i}}(y) + b_{I_{v,i}})\Bigg]
\end{aligned} \qquad (2)$$

The study used the Mexican hat wavelet (MHW) as an activation function. The mathematical formulation of the MHW is presented in (3).

$$f(y) = \frac{2}{\sqrt{3}}\pi^{-\frac{1}{4}}(1-y^2)e^{-\frac{y^2}{2}} \qquad (3)$$

The MHW-based ANN formulation of the nonlinear coupled SEIR system based on Zika virus $\hat{S}_h(y), \hat{E}_h(y), \hat{I}_h(y), \hat{R}_h(y), \hat{S}_v(y), \hat{E}_v(y), \hat{I}_v(y)$ along with the derivative is:

$$
\begin{aligned}
&\left[\frac{d\tilde{S}_h'(y)}{dy},\frac{d\tilde{E}_h'(y)}{dy},\frac{d\tilde{I}_h'(y)}{dy},\frac{d\tilde{R}_h'(y)}{dy},\frac{d\tilde{S}_v'(y)}{dy},\frac{d\tilde{E}_v'(y)}{dy}\right.\\
&=\left[\sum_{k=1}^{m}\alpha\tilde{S}_h\left[\frac{2}{\sqrt{3}}\pi^{-0.25}\left(1-(w\tilde{S}_h(y)+b\tilde{S}_h)^2\right)e^{-0.5(w\tilde{S}_h+b\tilde{S}_h)^2}\right],\sum_{k=1}^{m}\alpha\tilde{E}_h\left[\frac{2}{\sqrt{3}}\pi^{-0.25}\left(1-(w\tilde{E}_h(y)+b\tilde{E}_h)^2\right)e^{-0.5(w\tilde{I}_h+b\tilde{I}_h)^2}\right],\right.\\
&\sum_{k=1}^{m}\alpha\tilde{I}_h\left[\frac{2}{\sqrt{3}}\pi^{-0.25}\left(1-(w\tilde{I}_h(y)+b\tilde{I}_h)^2\right)e^{-0.5(w\tilde{I}_h+b\tilde{I}_h)^2}\right],\sum_{k=1}^{m}\alpha\tilde{R}_h\left[\frac{2}{\sqrt{3}}\pi^{-0.25}\left(1-(w\tilde{R}_h(y)+b\tilde{R}_h)^2\right)e^{-0.5(w\tilde{R}_h+b\tilde{R}_h)^2}\right],\\
&\sum_{k=1}^{m}\alpha\tilde{S}_v\left[\frac{2}{\sqrt{3}}\pi^{-0.25}\left(1-(w\tilde{S}_v(y)+b\tilde{S}_v)^2\right)e^{-0.5(w\tilde{S}_v+b\tilde{S}_v)^2}\right],\sum_{k=1}^{m}\alpha\tilde{E}_h\left[\frac{2}{\sqrt{3}}\pi^{-0.25}\left(1-(\tilde{E}_h(y)+b\tilde{E}_h)^2\right)e^{-0.5(w\tilde{E}_h+b\tilde{E}_h)^2}\right],\\
&\left.\sum_{k=1}^{m}\alpha\tilde{I}_v\left[\frac{2}{\sqrt{3}}\pi^{-0.25}\left(1-(w\tilde{I}_h(y)+b\tilde{I}_v)^2\right)e^{-0.5(w\tilde{I}_v+b\tilde{I}_v)^2}\right]\right] \qquad (4)
\end{aligned}
$$

The unknown parameters of the proposed scheme can be presented as:

$$
P=\begin{bmatrix}P_{\tilde{S}_h}\\P_{\tilde{E}_h}\\P_{\tilde{I}_h}\\P_{\tilde{R}_h}\\P_{\tilde{S}_v}\\P_{\tilde{E}_v}\\P_{\tilde{I}_v}\end{bmatrix}\text{ for }P_{\tilde{S}_h}=\begin{bmatrix}w_{\tilde{S}_h}\\\alpha_{\tilde{S}_h}\\b_{\tilde{S}_h}\end{bmatrix},P_{\tilde{E}_h}=\begin{bmatrix}w_{\tilde{E}_h}\\\alpha_{\tilde{E}_h}\\b_{\tilde{E}_h}\end{bmatrix},P_{\tilde{I}_h}=\begin{bmatrix}P_{\tilde{I}_h}\\\alpha_{\tilde{I}_h}\\b_{\tilde{I}_h}\end{bmatrix},
$$

$$
P_{\tilde{R}_h}=\begin{bmatrix}w_{\tilde{R}_h}\\\alpha_{\tilde{R}_h}\\b_{\tilde{R}_h}\end{bmatrix},P_{\tilde{S}_v}=\begin{bmatrix}w_{\tilde{S}_v}\\\alpha_{\tilde{S}_v}\\b_{\tilde{S}_v}\end{bmatrix},P_{\tilde{E}_v}=\begin{bmatrix}w_{\tilde{E}_v}\\\alpha_{\tilde{E}_v}\\b_{\tilde{E}_v}\end{bmatrix},P_{\tilde{I}_v}=\begin{bmatrix}w_{\tilde{I}_v}\\\alpha_{\tilde{I}_v}\\b_{\tilde{I}_v}\end{bmatrix}
$$

The error based fitness function is as follow:

$$
\epsilon_F=\epsilon_{\tilde{S}_h}+\epsilon_{\tilde{E}_h}+\epsilon_{\tilde{I}_h}+\epsilon_{\tilde{R}_h}+\epsilon_{\tilde{S}_v}+\epsilon_{\tilde{E}_v}+\epsilon_{\tilde{I}_v}+\epsilon
$$

Error error-based fitness function of ϵ_F can be derived by the expressions given below:

$$
\begin{aligned}
\epsilon_{\tilde{S}_h}&=\frac{1}{N}\sum_{i=1}^{N}\left[\frac{d\check{S}_H(y)}{dy}-A_H+\check{S}_H(y)\beta_H\left(\check{I}_v(y)-\rho\check{I}_H(y)\right)+\mu_H\check{S}_H(y)\right]^2\\
\epsilon_{\tilde{E}_h}&=\frac{1}{N}\sum_{k=1}^{N}\left[\frac{d\check{E}_H(y)}{dy}-\beta_H\left(\rho\check{I}_H(y)-\check{I}_V(y)\right)\check{S}_H(y)+(\chi_H+\mu_H)\check{E}_H(y)\right]^2\\
\epsilon_{\tilde{I}_h}&=\frac{1}{N}\sum_{k=1}^{N}\left[\frac{d\check{I}_H(y)}{dy}-\chi_H\check{E}_H(y)+(\eta+\mu_H+\gamma)\check{I}_H(y)\right]^2\\
\epsilon_{\tilde{R}_h}&=\frac{1}{N}\sum_{k=1}^{N}\left[\frac{d\check{R}_H(y)}{dy}+\mu_H\check{R}_H(y)-\gamma\check{I}_H(y)\right]^2 \qquad (5)\\
\epsilon_{\tilde{S}_v}&=\frac{1}{N}\sum_{k=1}^{N}\left[\frac{d\check{S}_v(y)}{dy}-A_v+\beta_v\check{I}_H(y)\check{S}_v(y)+\mu_H\check{S}_v(y)\right]^2\\
\epsilon_{\tilde{E}_v}&=\frac{1}{N}\sum_{k=1}^{N}\left[\frac{d\check{E}_v(y)}{dy}-\beta_v\check{I}_H(y)\check{S}_v(y)+(\mu_v+\delta_H)\check{E}_v(y)\right]^2\\
\epsilon_{\tilde{I}_v}&=\frac{1}{N}\sum_{k=1}^{N}\left[\frac{d\check{I}_v(y)}{dy}-\check{E}_v(y)\check{S}\delta_v(y)+\mu_v\check{I}_v(y)\right]^2
\end{aligned}
$$

$$
\begin{aligned}
\epsilon=\frac{1}{5}\Big[&\left(\tilde{S}_h(0)-k_1\right)^2+\left(\tilde{E}_h(0)-k_2\right)^2+\left(\tilde{I}_h(0)-k_3\right)^2+\left(\tilde{R}_h(0)-k_4\right)^2\\
&+\left(\tilde{S}_v(0)-k_5\right)^2+\left(\tilde{E}_v(0)-k_6\right)^2,\left(\tilde{I}_v(0)-k_7\right)^2\Big] \qquad (05)
\end{aligned}
$$

$\check{S}_h(y),\check{E}_h(y),\check{I}_h(y),\check{R}_h(y),\check{S}_v(y),\check{E}_v(y),\check{I}_v(y)$ are the ANN-based outcomes for each class whereas the $\epsilon_{\tilde{S}_h},\epsilon_{\tilde{E}_h},\epsilon_{\tilde{I}_h},\epsilon_{\tilde{R}_h},\epsilon_{\tilde{S}_v},\epsilon_{\tilde{E}_v},\epsilon_{\tilde{I}_v}$ *and* ϵ are the error-based fitness functions. The fitness function ϵ is calculated through the initial conditions. Fig. 1 Shows the MHW-based ANN structure through global swarming (PSO) and local search (SQP) procedure to solve the coupled nonlinear mathematical model of SEIR based on the Zika virus.

2.2. Optimization Procedure: PSO-SQP

This section provides the optimization technique for the numerical treatment of a nonlinear coupled system presented in (1) through MHW-ANN-PSO-SQP.

PSO is a swarm-based search algorithm that was presented during the last decade to solve global optimization problems. It is used to solve the constrained and unconstrained optimization problems using the global best approach. It works with the iterative process through local best and global best particles. It is the alternative global search technique of genetic algorithms. It is more efficient and has less computational cost. Some of the applications of the PSO are found in [18, 22, 38, 56, 60-64]. The PSO is upgraded by adding an effective local optimization scheme. The study used the local search SQP to get fast and accurate results by using the hybridization procedure with PSO. During the recent century, the SQP has been used in various applications [46, 48, 50, 65-67]. The Steps of the proposed PSO-SQP optimization algorithm is given in Algorithm 1. Parameters used in the proposed hybrid PSO-SQP along with the values can be found in [68].

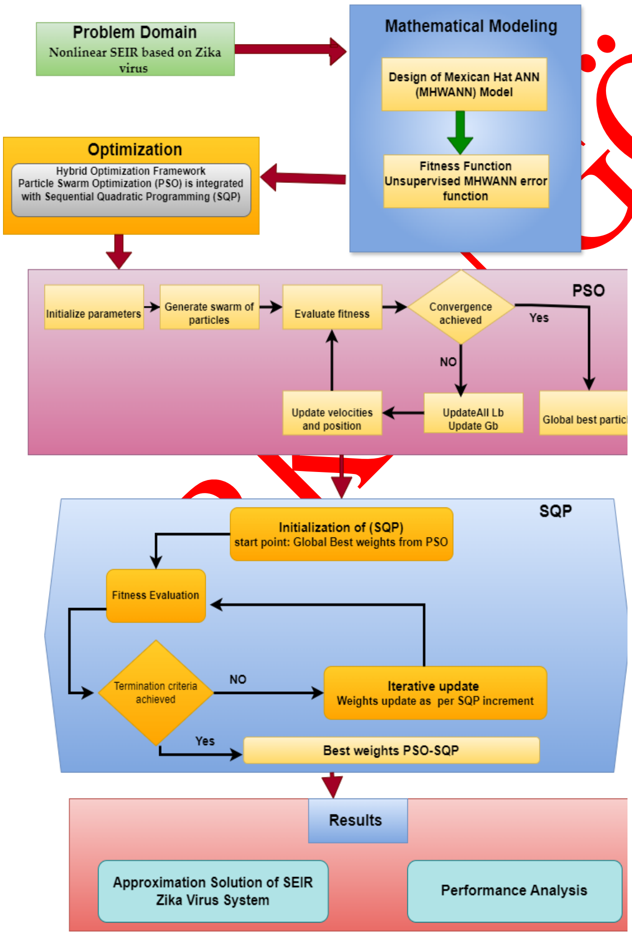


Figure 1. Workflow of the Proposed MHW-ANN-PSO-SQP for the Numerical Treatment of SEIR Coupled System Based on Zika Disease

---

Algorithm 1: Proposed Hybrid PSO-SQP

Optimization of PSO Technique

[Input]: The particles of the network are indicated in vector P.

[Population]: The illustration of the particle set is provided as $W = \begin{bmatrix} P_{\tilde{S}_h} \\ P_{\tilde{E}_h} \\ P_{\tilde{I}_h} \\ P_{\tilde{R}_h} \\ P_{\tilde{S}_v} \\ P_{\tilde{E}_v} \\ P_{\tilde{I}_v} \end{bmatrix}$

Where $P_{\tilde{S}_h} = \begin{bmatrix} w_{\tilde{S}_h} \\ \alpha_{\tilde{S}_h} \\ b_{\tilde{S}_h} \end{bmatrix}$, $P_{\tilde{E}_h} = \begin{bmatrix} w_{\tilde{E}_h} \\ \alpha_{\tilde{E}_h} \\ b_{\tilde{E}_h} \end{bmatrix}$, $P_{\tilde{I}_h} = \begin{bmatrix} P_{\tilde{I}_h} \\ \alpha_{\tilde{I}_h} \\ b_{\tilde{I}_h} \end{bmatrix}$, $P_{\tilde{R}_h} = \begin{bmatrix} w_{\tilde{R}_h} \\ \alpha_{\tilde{R}_h} \\ b_{\tilde{R}_h} \end{bmatrix}$, $P_{\tilde{S}_v} = \begin{bmatrix} w_{\tilde{S}_v} \\ \alpha_{\tilde{S}_v} \\ b_{\tilde{S}_v} \end{bmatrix}$, $P_{\tilde{E}_v} = \begin{bmatrix} w_{\tilde{E}_v} \\ \alpha_{\tilde{E}_v} \\ b_{\tilde{E}_v} \end{bmatrix}$,

$P_{\tilde{I}_v} = \begin{bmatrix} w_{\tilde{I}_v} \\ \alpha_{\tilde{I}_v} \\ b_{\tilde{I}_v} \end{bmatrix}$

Where W, α, and b are the unknown variables of the network.

[Output]: Global Best vector $W_{\text{PSO-Best}}$

[Initialization]: For the given set of particles Specify the random values $M_{\text{PSO-Best}}$

[Fitness Evaluation]: Updated the fitness values of the variable ϵ_F using Eq. (6)

[Termination]: The algorithm will stop when the following conditions are met then store

1. Meet the fitness criteria. The algorithm converges and optimal fitness is achieved.
2. The algorithm reached the maximum iteration.

[Ranking]: Ranke the $W_{\text{PSO-Best}}$ in vector M. The vector contains the optimal values for the unknown variables.

[Storage]: Store the time, weights, function count, iterations

End of PSO Algorithm

Start of [SQP] procedure

Step a: Starting of SQP: the SQP is initialized using the MATLAB built-in function by using the best values generated by PSO along with the parameter's values. Details can be found in [68]

Step b: Calculation of Fitness of Knowns: Fitness values of Unknowns are calculated.

Step c: Termination of SQP: Terminate the recurrent process of SQP to update the weights of the unknowns of the developed PSO-SQP by using the conditions

The number of iterations reached

Maximum fitness values achieved

Step d: Adjust the weights of unknowns: Optimize the weights for each iteration.

End of SQP Algorithm

Generate WPSO-SQP

2.3. Performance Operators

The accuracy of the designed scheme i.e. MHW-ANN-PSO-SQP is validated in various performance indicators. The mathematical formulation of the performance operators is written as follows.

$$absoulte\ error = \sum_{i=1}^{n}|S_{h_i}-\check{S}_{h_i}|,|E_{h_i}-\check{E}_{h_i}|,|I_{h_i}-\check{I}_{h_i}|,|R_{h_i}-\check{R}_{h_i}|,|S_{v_i}-\check{S}_{v_i}|,|E_{v_i}-\check{E}_{v_i}|,|I_{v_i}-\check{I}_{v_i}| \tag{6}$$

$$mean\ squre\ error = \frac{1}{n}\sum_{i=1}^{n}(S_{h_i}-\check{S}_{h_i})^2,(E_{h_i}-\check{E}_{h_i})^2,(I_{h_i}-\check{I}_{h_i})^2,(R_{h_i}-\check{R}_{h_i})^2,(S_{v_i}-\check{S}_{v_i})^2,(E_{v_i}-\check{E}_{v_i})^2,(I_{v_i}-\check{I}_{v_i})^2 \tag{7}$$

$$global\ mean\ squre\ error = \sum_{i=1}^{100}\frac{1}{n}\sum_{i=1}^{n}(S_{h_i}-\check{S}_{h_i})^2,(E_{h_i}-\check{E}_{h_i})^2,(I_{h_i}-\check{I}_{h_i})^2,(R_{h_i}-\check{R}_{h_i})^2,(S_{v_i}-\check{S}_{v_i})^2,(E_{v_i}-\check{E}_{v_i})^2,(I_{v_i}-\check{I}_{v_i})^2 \tag{8}$$

$$mean\ absoulte\ deviation = \frac{1}{n}\sum_{i=1}^{n}|S_{h_i}-\check{S}_{h_i}|,|E_{h_i}-\check{E}_{h_i}|,|I_{h_i}-\check{I}_{h_i}|,|R_{h_i}-\check{R}_{h_i}|,|S_{v_i}-\check{S}_{v_i}|,|E_{v_i}-\check{E}_{v_i}|,|I_{v_i}-\check{I}_{v_i}| \tag{9}$$

$$globale\ mean\ absoulte\ deviation = \sum_{i=1}^{100}\frac{1}{n}\sum_{i=1}^{n}|S_{h_i}-\check{S}_{h_i}|,|E_{h_i}-\check{E}_{h_i}|,|I_{h_i}-\check{I}_{h_i}|,|R_{h_i}-\check{R}_{h_i}|,|S_{v_i}-\check{S}_{v_i}|,|E_{v_i}-\check{E}_{v_i}|,|I_{v_i}-\check{I}_{v_i}| \tag{10}$$

$\check{S}_h(y),\check{E}_h(y),\check{I}_h(y),\check{R}_h(y),\check{S}_v(y),\ \check{E}_v(y,)$ and $\check{I}_v(y)$ are the ANN-based solutions of SEIR mathematical model based on Zika virus. The study also analyzed the complexity of the proposed scheme through mean execution time.

3. NUMERICAL RESULTS

This section deals with the simulation and results discussion for the nonlinear mathematical SEIR system based on the Zika virus through the stochastic computing procedure along with the swarming approach which is aided by the local search scheme SQP. The best weights obtained by using the hybrid optimization procedure, evaluation of the solution, absolute error, statistical operators along complexity analysis have also been presented in this section.

The ANN-based numerical solution of the nonlinear coupled mathematical model based on the Zika virus is presented by using the suitable parameter values tabulated in Table 1 as follows.

$$\begin{aligned}
&\frac{d\check{S}_H(y)}{dy} = (0.1)-\check{S}_H(y)(0.12)\left(\check{I}_v(y)+(0.14)\check{I}_H(y)\right)-(0.1)\check{S}_H(y), \qquad (S_H)_0 = (0.10),\\
&\frac{d\check{E}_H(y)}{dy} = (0.12)\left((0.14)\check{I}_H(y)+\check{I}_V(y)\right)\check{S}_H(y)-\left((0.13)+(0.1)\right)\check{E}_H(y), \quad (E_H)_0 = (0.12),
\end{aligned} \tag{11}$$

$$\frac{d\check{I}_H(y)}{dy} = (0.13)\check{E}_H(y) - \big((0.15) + (0.1) + (0.17)\big)\check{I}_H(y), \quad (I_H)_0 = (0.14),$$

$$\frac{d\check{R}_H(y)}{dy} = -(0.1)\check{R}_H(y) + (0.17)\check{I}_H(y), \quad (0.16),$$

$$\frac{d\check{S}_v(y)}{dy} = A_v - (0.2)\check{I}_H(y)\check{S}_v(y) - (0.1)\check{S}_v(y), \quad (S_v)_0 = (0.18),$$

$$\frac{d\check{E}_v(y)}{dy} = (0.2)\check{I}_H(y)\check{S}_v(y) - (0.3) + (0.22))\check{E}_v(y), \quad (0.20),$$

$$\frac{d\check{I}_v(y)}{dy} = \check{E}_v(y)(0.3)(y) - (0.3)\check{I}_v(y), = (0.22)$$

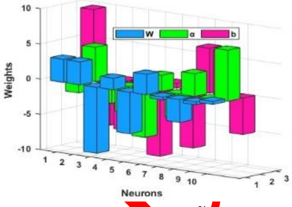
Table 1: List of parameters for SEIR based on Zika along with values

Parameter	Values	Parameter	Values
β_H	0.12	δ_V	0.3
A_H	0.1	μ_V	0.25
μ_H	0.1	k_1	0.10
χ_H	0.13	k_2	0.12
P	0.14	k_3	0.14
η	0.15	k_4	0.16
β_V	0.2	k_5	0.18
γ	0.17	kK_6	0.20
δ_H	0.22	K_7	0.22

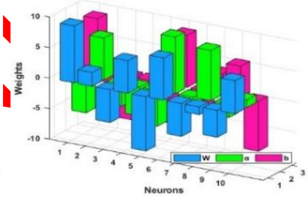
The fitness function ϵ_F become

$$\epsilon_F = \frac{1}{N}\sum_{i=1}^{11}\Bigg[\left[\frac{d\check{S}_H(y)}{dy} - (0.1) - \check{S}_H(y)(0.12)\big(\check{I}_v(y) + (0.14)\check{I}_H(y)\big) - (0.1)\check{S}_H(y)\right]^2 + \left[\frac{d\check{E}_H(y)}{dy} - (0.12)\ \Big(\big((0.14)\check{I}_H(y) + \check{I}_V(y)\big)\check{S}_H(y) - \big((0.13) + (0.1)\big)\check{E}_H(y)\right]^2 + \left[\frac{d\check{I}_H(y)}{dy} - (0.13)\check{I}_H(y) - \big((0.15) + (0.1) + (0.17)\big)\check{I}_H(y)\right]^2 + \left[\frac{d\check{R}_H(y)}{dy} - (0.1)\check{R}_H(y) + (0.17)\check{I}_H(y)\right]^2 + \left[\frac{d\check{S}_v(y)}{dy} - A_v - (0.2)\check{I}_H(y)\check{S}_v(y) - (0.1)\check{S}_v(y)\right]^2 + \left[\frac{d\check{E}_v(y)}{dy} - (0.2)\check{I}_H(y)\check{S}_v(y) - (0.3) + (0.22))\check{E}_v(y)\right]^2 + \left[\frac{d\check{I}_v(y)}{dy} - \check{E}_v(y)(0.3)(y) - (0.3)\check{I}_v(y)\right]^2 \frac{1}{5}\Big[\big(\tilde{S}_h(0) - 0.10\big)^2 + \big(\tilde{E}_h(0) - 0.12\big)^2 + (\tilde{I}_h(0) - 0.14)^2 + \big(\tilde{R}_h(0) - 0.16\big)^2 + \big(\tilde{S}_v(0) - 0.18\big)^2 + \big(\tilde{E}_v(0) - 0.20\big)^2, (\tilde{I}_v(0) - 0.22)^2\Big]\Bigg] \quad (12)$$

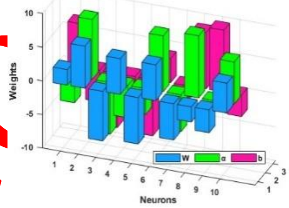
For the numerical solution of the nonlinear SEIR mathematical system for the Zika disease, the fitness function presented in (11) has been optimized through a hybrid computing procedure i.e. PSO-SQP. For the statistical analysis, the 50 independent runs have been performed while 10 neurons i.e. 30 variables for each class. The optimal weights produced by the proposed hybrid PSO-SQP have been demonstrated in Fig. 2 (a-g). These weights are given to the system presented in (14) and used to optimize the fitness function.



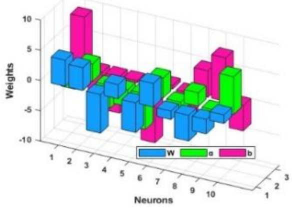
(a): Best Weights $\tilde{S}_h(y)$



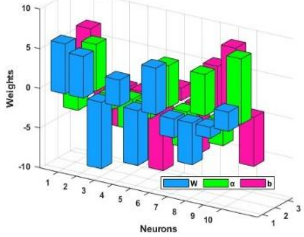
(b): Best Weights $\tilde{E}_h(y)$



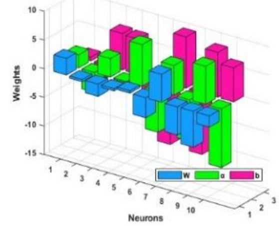
(c): Best Weights $\tilde{E}_h(y)$



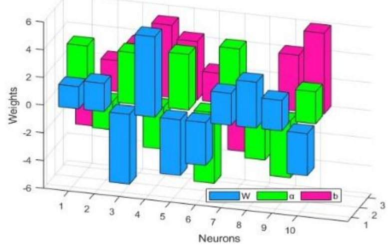
(d): Best Weights $\tilde{R}_h(y)$



(e): Best Weights $\tilde{S}_v(y)$



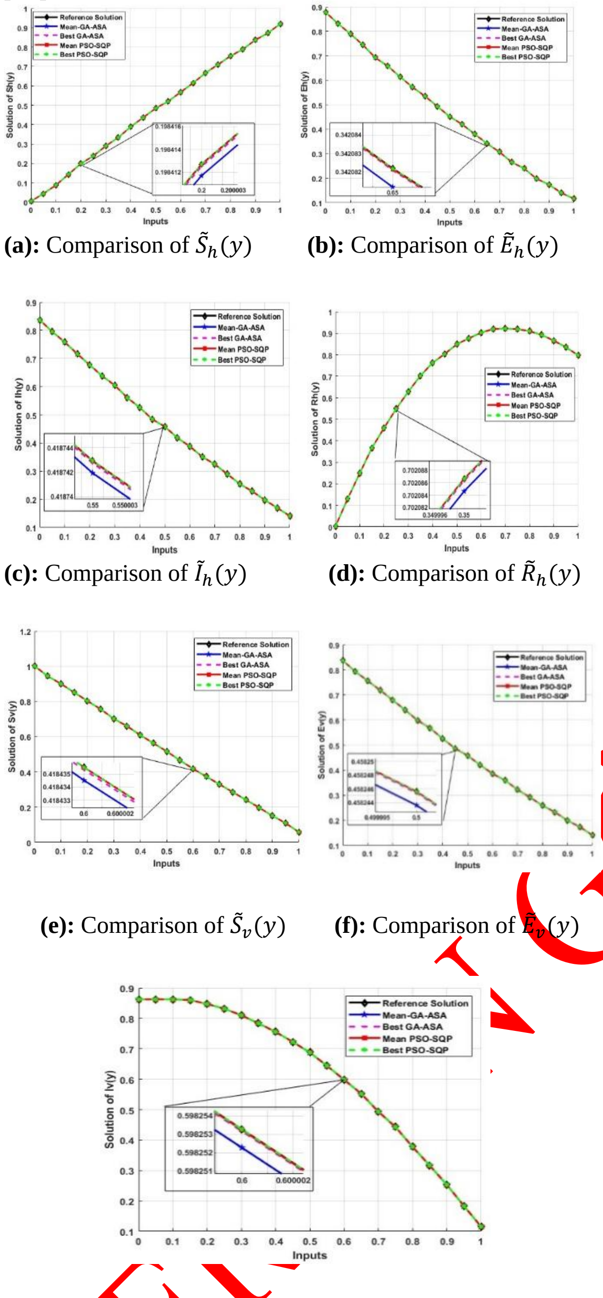
(f): Best Weights $\tilde{E}_v(y)$



(g) : Best Weights $\tilde{I}_v(y)$

Figure 2. Best Weight vector for proposed PSO-NM for the non-linear mathematical SEIR System based on the Zika virüs

The study performs the comparative analysis of the outcome produced by the ANN-GA-ASA, RK numerical solvers, and the proposed stochastic computing procedure MHW-ANN-PSO-SQP through mean and best values. Fig 3 (a-g) shows the outcome for mean and best values. The overlap solution validated the accuracy of the proposed scheme.



(a): Comparison of $\tilde{S}_h(y)$ **(b):** Comparison of $\tilde{E}_h(y)$

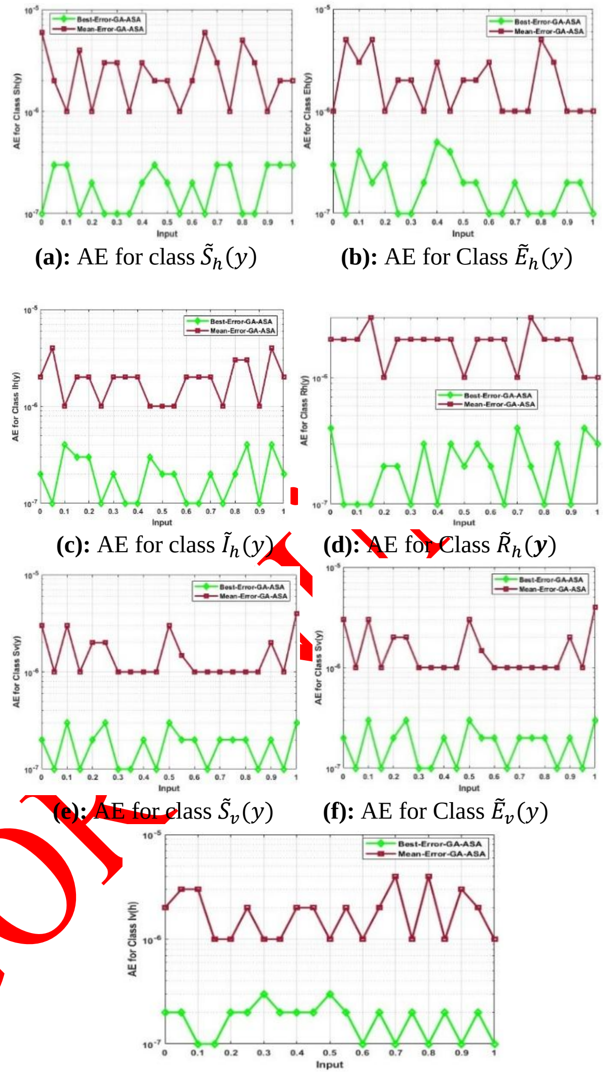
(c): Comparison of $\tilde{I}_h(y)$ **(d):** Comparison of $\tilde{R}_h(y)$

(e): Comparison of $\tilde{S}_v(y)$ **(f):** Comparison of $\tilde{E}_v(y)$

(g): Comparison of $\tilde{I}_v(y)$

Figure 3. (Cont.) Results compression of the Reference, ANN-GA-ASA, and Proposed MHW-ANN-PSO-SQP the non-linear mathematical SEIR System based on the Zika virus

To authenticate the precision of the proposed scheme, absolute error values through the mean and best are also performed. The results are plotted in Fig.4 (a-g) for ANNP-GA-ASA and Fig. 5(a-g) for the proposed MHW-ANN-PSO-SQP. One can observe that the AE of mean and best for ANNP-GA-SA are in the range 10^{-6} to 10^{-7}, while the AE for the developed procedure is between 10^{-8} to 10^{-9} respectively. The low AE measures confirm the precision of the developed scheme.



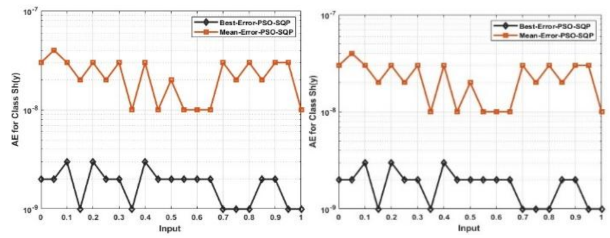
(a): AE for class $\tilde{S}_h(y)$ **(b):** AE for Class $\tilde{E}_h(y)$

(c): AE for class $\tilde{I}_h(y)$ **(d):** AE for Class $\tilde{R}_h(y)$

(e): AE for class $\tilde{S}_v(y)$ **(f):** AE for Class $\tilde{E}_v(y)$

(g): AE for class $\tilde{I}_v(y)$

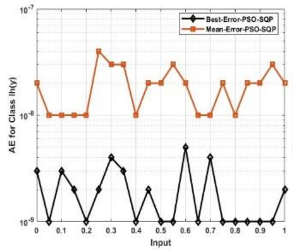
Figure 4. Best/Mean AE of ANN-GA-ASA of the non-linear SEIR System based on the Zika virüs



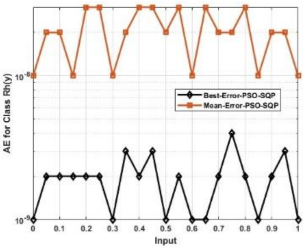
(a): AE for class $\tilde{S}_h(y)$ (b): AE for Class $\tilde{E}_h(y)$

Figure 5. Best/Mean AE of MHW-ANN-PSO-SQP for solving the system

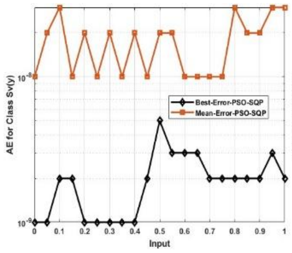
To solve the nonlinear system of SEIR based on the Zika virus, the stability of the proposed scheme is examined through the statistical MSE and MAD operators. Results are demonstrated in Fig. 6(a-b). The MSE measures are in the range 10^{-07} to 10^{-12}, 10^{-07} to 10^{-12}, 10^{-07} to 10^{-12}, 10^{-07} to 10^{-13}, 10^{-07} to 10^{-12}, 10^{-07} to 10^{-13}, 10^{-07} to 10^{-13} respectively.



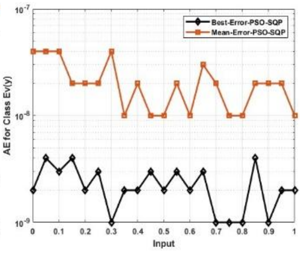
(c): AE for class $\tilde{I}_h(y)$



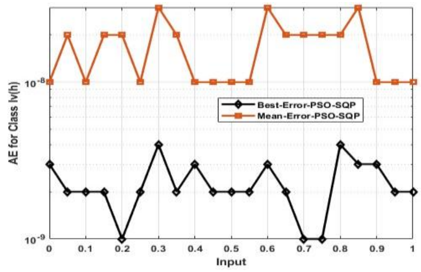
(d): AE for Class $\tilde{R}_h(y)$



(e): AE for class $\tilde{S}_v(y)$



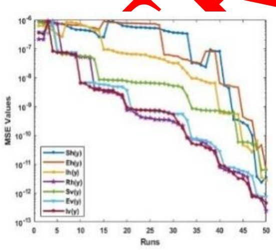
f): AE for Class $\tilde{E}_v(y)$



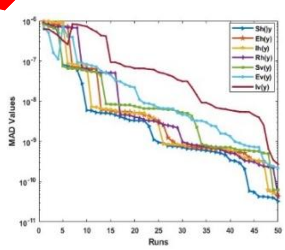
(g): AE for class $\tilde{I}_v(y)$

Figure 5. (Cont.) Best/Mean AE of MHW-ANN-PSO-SQP for solving the system

From the values, it is also observed that the values for MAD are in lie in the range 10^{-07} to 10^{-11}, 10^{-07} to 10^{-11}, 10^{-07} to 10^{-11}, 10^{-07} to 10^{-11}, 10^{-07} to 10^{-11}, 10^{-07} to 10^{-10}, 10^{-7} to 10^{-10}. The data is enlarged for fifty execution runs for the numerical treatment of the SEIR system. One can conclude that the accuracy in terms of MSE and MAD is almost same for the all trials, which confirms the stability of the proposed scheme. The statistical operator's measures for MSE and MAD to solve the SEIR mathematical system are presented in Fig. 7.



(a): MSE Measure



(b): MAD Measure

Figure 6. MSE (a), MAD (b) Measure of the developed MHW-ANN-PSO-SQP

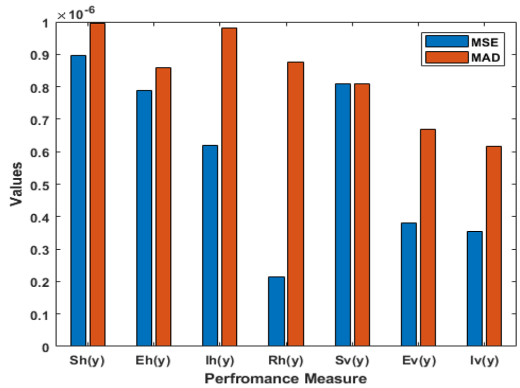


Figure 7. Performance Analysis of MAD and MSE for non-linear mathematical SEIR System based on Zika virus

The accuracy of the developed swarm computing technique is further validated through the statistical operator's minimum (Min), mean, and Standard Deviation (std). The values are tabulated in Table 2-5. Table 2 presents the dynamics of $\tilde{S}_h(y)$ and $\tilde{E}_h(y)$. The min values that represent the best operators lie between 10^{-09} to 10^{-12} and 10^{-08} to 10^{-12} respectively, the values for mean and std are calculated as 10^{-06} to 10^{-07}, and 10^{-06} to 10^{-08} for class $\tilde{S}_h(y)$ and $\tilde{E}_h(y)$. Table 3 demonstrates the min, mean, and std operator measures for classes $\tilde{I}_h(y)$ and $\tilde{R}_h(y)$. The performance measures for the Min, mean and Std are in the range 10^{-09} to 10^{-11}, 10^{-08} to 10^{-12}, 10^{-07} to 10^{-08}, 10^{-07} to 10^{-08}, and 10^{-07} to 10^{-08}, 10^{-07} to 10^{-08} respectively. Table 4 tabulates the min, mean, and std operator measures for classes $\tilde{S}_v(y)$ and $\tilde{E}_v(y)$. The performance measures for the Min, mean and Std are in the range 10^{-07} to 10^{-11}, 10^{-07} to 10^{-08}, 10^{-6} to 10^{-08}, 10^{-08} to 10^{-11}, and 10^{-07} to 10^{-08}, 10^{-07} to 10^{-08} respectively. Moreover, Table 5 illustrates the min, mean, and std operator measures for classes $\tilde{I}_v(y)$. The performance measures for the Min, mean and Std are in the range 10^{-09} to 10^{-12}, 10^{-07} to 10^{-08}, and 10^{-6} to 10^{-8}.

Table 2: Statistical measure for classes $\tilde{S}_h(y)$, $\tilde{E}_h(y)$

y	Min	Mean	Std	Min	Mean	Std
	$\tilde{S}_h(y)$			$\tilde{E}_h(y)$		
0	3.55E-11	3.55E-08	5.55E-06	5.50E-09	5.50E-08	5.35E-07
0.05	4.31E-10	8.35E-08	2.33E-07	2.55E-09	3.55E-08	2.55E-07
0.1	3.52E-09	5.52E-07	5.20E-06	5.30E-10	5.35E-07	5.20E-06
0.15	6.46E-10	6.50E-07	6.25E-06	3.65E-09	2.50E-07	2.50E-06
0.2	4.66E-09	2.22E-08	5.35E-07	5.50E-10	5.20E-07	3.65E-06
0.25	3.83E-09	1.33E-07	9.50E-07	6.30E-09	3.35E-08	5.55E-06
0.3	5.55E-11	6.24E-07	5.20E-07	5.55E-10	2.40E-09	2.25E-06
0.35	3.65E-10	3.62E-07	6.65E-07	6.60E-09	6.75E-08	3.53E-07
0.4	4.64E-09	5.25E-07	3.45E-07	2.50E-09	5.35E-07	5.35E-07
0.45	9.52E-10	3.53E-08	5.50E-07	5.40E-10	2.55E-08	8.64E-06
0.5	3.22E-09	2.66E-07	2.35E-06	8.22E-09	3.22E-06	2.55E-07
0.55	4.56E-11	5.25E-08	4.20E-06	3.55E-10	8.30E-09	5.35E-07
0.6	6.33E-11	6.40E-07	3.55E-07	5.45E-09	2.74E-08	2.55E-08
0.65	2.79E-01	2.65E-07	2.65E-07	2.55E-10	3.55E-09	3.23E-07
0.7	3.64E-10	3.55E-08	5.50E-07	4.35E-09	5.23E-08	5.35E-07
0.75	8.53E-09	5.26E-07	3.35E-06	2.20E-09	6.40E-08	2.50E-07
0.8	5.65E-08	4.54E-07	5.40E-06	5.80E-09	4.55E-07	3.45E-06
0.85	3.22E-09	3.45E-08	5.55E-07	3.50E-08	5.30E-09	8.33E-07
0.9	6.63E-09	2.35E-07	3.25E-07	6.35E-09	3.45E-07	7.25E-07
0.95	2.94E-10	5.53E-08	5.50E-07	5.25E-08	5.55E-06	5.35E-07
1	6.33E-09	3.35E-08	6.30E-07	5.55E-08	6.35E-07	5.42E-07

Table 3: Statistical measure for classes $\tilde{I}_h(y)$, $\tilde{R}_h(y)$

y	Min	Mean	Std	Min	Mean	Std
	$\tilde{S}_v(y$			$\tilde{E}_v(y)$		
0	5.55E-09	5.35E-08	5.58E-07	5.35E-09	5.55E-07	3.25E-07
0.05	2.23E-07	3.55E-08	5.30E-06	3.54E-11	2.34E-08	5.55E-07
0.1	3.40E-11	2.65E-07	2.53E-07	2.65E-10	5.23E-08	4.50E-08
0.15	4.35E-08	4.22E-07	3.55E-07	5.20E-08	5.55E-07	5.35E-07
0.2	2.55E-09	5.53E-07	5.84E-06	2.65E-09	2.45E-07	4.43E-07
0.25	5.20E-10	2.40E-07	4.33E-07	3.55E-07	2.30E-08	2.55E-06
0.3	3.55E-08	3.30E-08	3.25E-07	2.35E-08	3.55E-07	3.55E-07
0.35	2.35E-10	5.20E-07	6.45E-07	5.53E-09	4.25E-08	5.40E-07
0.4	5.55E-09	4.55E-08	5.64E-08	3.50E-11	5.50E-08	9.32E-06
0.45	5.52E-12	3.55E-07	8.33E-06	5.44E-08	2.35E-07	3.55E-06
0.5	3.54E-09	5.65E-07	5.95E-07	2.85E-10	3.25E-07	2.55E-06
0.55	2.35E-09	4.25E-09	2.50E-07	8.50E-05	5.50E-07	4.25E-07
0.6	4.20E-09	3.30E-08	6.76E-07	3.35E-11	2.55E-07	6.55E-07
0.65	5.55E-08	5.55E-07	3.30E-07	4.55E-09	5.35E-07	3.40E-07
0.7	9.33E-10	4.44E-08	4.64E-08	6.53E-10	3.55E-08	4.55E-07
0.75	3.54E-10	6.55E-07	6.56E-07	5.75E-08	5.54E-08	2.30E-06
0.8	5.20E-09	5.33E-07	3.54E-07	4.25E-09	4.45E-07	3.55E-06
0.85	2.55E-11	8.55E-07	9.60E-06	3.57E-10	2.55E-07	5.55E-07
0.9	5.25E-09	5.23E-08	5.35E-08	5.30E-12	3.35E-07	4.25E-07
0.95	3.35E-09	3.50E-08	3.55E-07	2.55E-09	5.52E-07	6.55E-07
1	5.55E-09	2.50E-07	5.55E-07	2.55E-09	5.55E-07	8.55E-07

Table 4: Statistical measure for classes $\tilde{S}_v(y)$, $\tilde{E}_v(y)$

y	Min	Mean	Std	Min	Mean	Std
	$\tilde{\boldsymbol{S}}_{\boldsymbol{v}}(\boldsymbol{y}$			$\widetilde{\boldsymbol{E}}_{\boldsymbol{v}}(\boldsymbol{y})$		
0	5.55E-09	5.35E-08	5.58E-07	5.35E-09	5.55E-07	3.25E-07
0.05	2.23E-07	3.55E-08	5.30E-06	3.54E-11	2.34E-08	5.55E-07
0.1	3.40E-11	2.65E-07	2.53E-07	2.65E-10	5.23E-08	4.50E-08
0.15	4.35E-08	4.22E-07	3.55E-07	5.20E-08	5.55E-07	5.35E-07
0.2	2.55E-09	5.53E-07	5.84E-06	2.65E-09	2.45E-07	4.43E-07
0.25	5.20E-10	2.40E-07	4.33E-07	3.55E-07	2.30E-08	2.55E-06
0.3	3.55E-08	3.30E-08	3.25E-07	2.35E-08	3.55E-07	3.55E-07
0.35	2.35E-10	5.20E-07	6.45E-07	5.53E-09	4.25E-08	5.40E-07
0.4	5.55E-09	4.55E-08	5.64E-08	3.50E-11	5.50E-08	9.32E-06
0.45	5.52E-12	3.55E-07	8.33E-06	5.44E-08	2.35E-07	3.55E-06
0.5	3.54E-09	5.65E-07	5.95E-07	2.85E-10	3.25E-07	2.55E-06
0.55	2.35E-09	4.25E-09	2.50E-07	8.50E-05	5.50E-07	4.25E-07
0.6	4.20E-09	3.30E-08	6.76E-07	3.35E-11	2.55E-07	6.55E-07
0.65	5.55E-08	5.55E-07	3.30E-07	4.55E-09	5.35E-07	3.40E-07
0.7	9.33E-10	4.44E-08	4.64E-08	6.53E-10	3.55E-08	4.55E-07
0.75	3.54E-10	6.55E-07	6.56E-07	5.75E-08	5.54E-08	2.30E-06
0.8	5.20E-09	5.33E-07	3.54E-07	4.25E-09	4.45E-07	3.55E-06
0.85	2.55E-11	8.55E-07	9.60E-06	3.57E-10	2.55E-07	5.55E-07
0.9	5.25E-09	5.23E-08	5.35E-08	5.30E-12	3.35E-07	4.25E-07
0.95	3.35E-09	3.50E-08	3.55E-07	2.55E-09	5.52E-07	6.55E-07
1	5.55E-09	2.50E-07	5.55E-07	2.55E-09	5.55E-07	8.55E-07

Table 5: Statistical measure for class $\tilde{I}_v(y)$

y	Min	Mean	Std
	$\tilde{\boldsymbol{I}}_{\boldsymbol{v}}(\boldsymbol{y})$		
0	4.55E-09	2.55E-07	4.50E-07
0.05	3.40E-09	5.42E-07	2.35E-06
0.1	2.55E-09	5.35E-08	3.50E-08
0.15	5.35E-08	3.55E-07	5.25E-07
0.2	5.53E-10	5.55E-07	2.50E-06
0.25	4.55E-08	4.23E-07	3.55E-07
0.3	3.40E-09	5.20E-08	5.35E-07
0.35	5.40E-09	2.45E-07	2.23E-07
0.4	6.35E-09	2.55E-08	5.55E-08
0.45	5.25E-12	5.30E-08	4.55E-06
0.5	3.50E-09	3.45E-08	3.24E-06
0.55	2.20E-10	5.50E-08	2.55E-07
0.6	6.50E-09	2.22E-07	5.30E-07
0.65	8.55E-09	4.34E-07	6.26E-07
0.7	5.23E-10	5.85E-07	4.55E-06
0.75	3.55E-09	5.20E-07	3.60E-07
0.8	5.34E-09	3.32E-07	2.55E-07
0.85	2.25E-11	2.55E-07	6.20E-07
0.9	4.52E-09	5.20E-07	2.60E-06
0.95	6.50E-10	3.53E-07	5.46E-07
1	5.25E-10	2.52E-07	5.35E-07

Table 6: Comparative Analysis of the Proposed Scheme with ADM and GA-ASA.

Class	AE			MSE			MAD		
	ADM	GA-ASA	**Proposed**	ADM	GA-ASA	**Proposed**	ADM	GA-ASA	**Proposed**
$\tilde{S}_h(y)$	10^{-4}	10^{-7}	$\mathbf{10^{-8}}$	10^{-5}	10^{-9}	$\mathbf{10^{-11}}$	10^{-4}	10^{-8}	$\mathbf{10^{-11}}$
$\tilde{E}_h(y)$	10^{-4}	10^{-7}	$\mathbf{10^{-8}}$	10^{-5}	10^{-9}	$\mathbf{10^{-11}}$	10^{-4}	10^{-8}	$\mathbf{10^{-11}}$
$\tilde{I}_h(y)$	10^{-3}	10^{-6}	$\mathbf{10^{-8}}$	10^{-5}	10^{-9}	$\mathbf{10^{-12}}$	10^{-4}	10^{-7}	$\mathbf{10^{-10}}$
$\tilde{R}_h(y)$	10^{-4}	10^{-7}	$\mathbf{10^{-9}}$	10^{-6}	10^{-10}	$\mathbf{10^{-13}}$	10^{-4}	10^{-7}	$\mathbf{10^{-10}}$
$\tilde{S}_v(y)$	10^{-3}	10^{-7}	$\mathbf{10^{-8}}$	10^{-5}	10^{-8}	$\mathbf{10^{-12}}$	10^{-5}	10^{-7}	$\mathbf{10^{-10}}$
$\tilde{E}_v(y)$	10^{-4}	10^{-8}	$\mathbf{10^{-9}}$	10^{-5}	10^{-9}	$\mathbf{10^{-12}}$	10^{-4}	10^{-7}	$\mathbf{10^{-11}}$
$\tilde{I}_v(y)$	10^{-4}	10^{-7}	$\mathbf{10^{-9}}$	10^{-5}	10^{-8}	$\mathbf{10^{-12}}$	10^{-4}	10^{-8}	$\mathbf{10^{-10}}$

Table 7: Assessment of the Reliability of the proposed MHW-ANN-PSO-SQP

Class	Method	GMSE		GMAD	
		Values	**STD**	**Values**	**STD**
$\tilde{S}_h(y)$	GA-ASA	3.234E-09	6.432E-09	8.315E-09	8.264E-08
	PSO-SQP	**3.318E-11**	**3.821E-11**	**2.638E-10**	**2.425E-09**
$\tilde{E}_h(y)$	GA-ASA	4.378E-09	2.734E-09	5.942E-08	6.342E-08
	PSO-SQP	**2.321E-12**	**6.354E-10**	**6.248E-09**	**5.916E-09**
$\tilde{I}_h(y)$	GA-ASA	5.348E-09	4.328E-09	3.727E-08	2.538E-08
	PSO-SQP	**6.934E-11**	**7.981E-09**	**5.326E-09**	**6.382E-09**
$\tilde{R}_h(y)$	GA-ASA	8.453E-09	3.642E-09	2.984E-08	3.824E-08
	PSO-SQP	**2.546E-11**	**4.528E-10**	**8.328E-09**	**5.255E-09**
$\tilde{S}_v(y)$	GA-ASA	5.842E-09	5.438E-08	3.237E-08	2.562E-08
	PSO-SQP	**6.438E-11**	**7.327E-10**	**8.547E-10**	**8.623E-09**
$\tilde{E}_v(y)$	GA-ASA	3.328E-09	3.439E-08	2.983E-08	5.177E-08
	PSO-SQP	**7.746E-11**	**9.984E-10**	**6.327E-10**	**5.734E-09**
$\tilde{I}_v(y)$	GA-ASA	3.743E-09	2.324E-09	3.921E-08	3.243E-08
	PSO-SQP	**2.954E-12**	**5.457E-11**	**6.457E-10**	**4.325E-09**

Table 6 shows the comparison of the proposed approach with ADM techniques and the ANN-based solver. The findings show that the proposed approach works better to solve the nonlinear SEIR system based on the propagation of the Zika virus.

The reliability of the developed MHW-ANN-PSO-SQP is testified using the global operators i.e. GMSE and GMAD through 100 runs. The mathematical formulation of the global statistical operations is presented in (9,11). The values tabulated in Table 7 indicate the developed procedure is consistent and stable.

The values for the proposed scheme are in the range 10^{-11} to 10^{-12}, and 10^{-10} the low values are the indication for the precision and accuracy of the scheme. The values for ANN-GA-ASA are in the range 10^{-09} and 10^{-09}.

The execution time of the proposed scheme is further analyzed through mean execution time. It is clear from the results, that the proposed scheme is computationally expensive. This is due to the hybridization of PSO and SQP. However, the designed scheme is accurate and precise in solving the nonlinear mathematical SEIR system based on the Zika virus. The mean execution time of the proposed scheme is 20 minutes while the mean execution of the ANN-GA-ASA [69] is 22.5 minutes. The values for mean execution time are shown in Figure 8. All the simulation is performed on an HP Folio notebook with SSD and 16GB RAM.

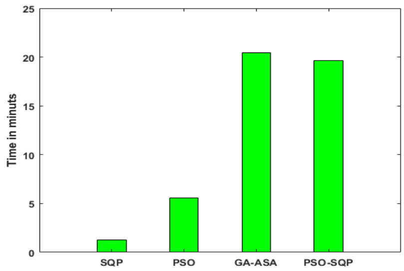


Figure 8. Mean Execution Time (MET) in minutes

4. CONCLUSION

The numerical treatment for the solution of the nonlinear mathematical SEIR model based on the Zika virus has been presented in the current study. The computational efficiency of MHW-based feed-forward artificial neural network, swarming optimization technique based on global particle swarm optimization, and local search sequential quadratic programming have been utilized to solve the SEIR system based on the Zika virus. The mathematical model of the Zika virus is based on the SEIR-coupled nonlinear system. The error-based fitness function based on SEIR differential equations is constructed through MHW-ANN-PSO-SQP in an unsupervised manner. The solution obtained through the proposed MHW-ANN-PSO-SQP is compared with the ANN-GA-ASA and numerical RK method. The overlap solutions designate the correctness of the proposed scheme, and the precision of the proposed scheme is testified through AE. It is found that the AE lies in the range 10^{-8} to 10^{-9}, additionally, the stability, convergence, and reliability are evaluated through detailed statistical analysis. The results indicated that the proposed scheme is stable, convergent, and reliable for solving the SEIR mathematical model based on the Zika virus. The proposed scheme is computationally efficient as compared to the scheme presented in [69].

In forthcoming studies, the proposed MHW-PSO-SQP can be applied to solve the coupled differential system based on a computer virus, COVID-19 system, fluid systems, and higher-order differential equations.

DECLARATION OF ETHICAL STANDARDS

The author(s) of this article declare that the materials and methods used in this study do not require ethical committee permission and/or legal-special permission.

AUTHORS' CONTRIBUTIONS

All authors have equal contributions to performing the experiments analyzing the results, and writing the manuscript.

CONFLICT OF INTEREST

There is no conflict of interest in this study.

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