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Research Article

New design for the cold part of heat pipes using functionally graded material in heat sink with variable thickness fins: An analytical approach

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ABSTRACT

This study proposed a new design for the cold part of heat pipes that utilizes a Functionally Graded Material (FGM) in the heat sink with power-law fins. The heat transfer in the heat pipes was evaluated using the heat resistance method, and it was shown that the new design improves the performance of the heat pipe. A sensitivity analysis of geometrical and operational parameters was also conducted to examine their effects on the thermal management system. The results indicated that decreasing the inner radius and increasing the outer radius of the FGM fin with a power-law profile improves the thermal behavior by decreasing the absolute temperature of the fin by up to 1 Kelvin degree. It was also found that increasing the ambient air and the fin's inner temperature had a negative effect on the fin's performance by decreasing the amount of convection heat transfer and cooling. The study also showed that increasing the convection heat transfer coefficient and grading index reduces the absolute temperature by up to 1.5 Kelvin degrees, while decreasing the thickness profile coefficients, and conduction heat transfer coefficients positively affected the dimensionless temperature of the fin. The article concludes that using the new heat sink design improves the heat pipe's performance and allows for the quick evaluation of the heat sink's efficiency using an analytical solution.

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INTRODUCTION

The heat and energy transfer between two mediums are critical in thermal management systems. Thus, many types of research have been performed to find the best energy transfer method with maximum performance and minimum heat loss. Heat pipe technology is one of the best devices that can be used for energy transfer over long distances without any input power and movable physical parts. Heat pipes are two-phase capillary heat transfer devices that consist of evaporator and condenser (hot and cold) sections for absorbing and releasing heat. At the evaporator section, a liquid phase of the working fluid turns into vapor using heat absorbed from the hot section. The vapor flows and turns back into the liquid state in the condenser section, with heat rejection in the cold section. This closed loop continues between the hot and cold sections and causes heat transfer between two different mediums. Thus, the

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cold and hot section's efficiency significantly affects the heat pipe efficiency. In recent years, various research has been performed to analyze the influences of different parameters on heat pipes' operational status. In this section, some of this research has been mentioned.

Elnaggar et al. [1] studied a U-shape heat pipe (used in the PC cooling system) and used experimental and numerical methods for the analysis. Elnaggar et al. [2] considered the Characterization of the operating fluid in a vertically positioned U-shape heat pipe for electric cooling. Peng et al. [3] investigated the thermal performance of a flat plate heat pipe with fins located in the vapor chamber. Samana et al. [4] analyzed the development of a wire fin's performance by heat pipe oscillating considering convection heat transfer. Rahman et al. [5] studied the influence of fins and inserts on the Open Loop Pulsating Heat Pipe features. An experimentally numerical model for analyzing the finned heat pipes in crossflow situations has been presented by Stark et al. [6]. Yu et al. [7] examined the different evaporation regimes in a grooved heat pipe. They experimented with various heat load values and tilt angles. Yue et al. [8] performed a CFD simulation on microchannel heat pipes to obtain heat and flow characteristics under various filling ratios. Gaikwad and Mohite [9] have investigated the performance of the microchannel heat sink with secondary flows. Huang et al. [10] presented an optimized design for the heat sink section of the heat pipes with a different height fin array. Behi et al. [11] offered a new concept for the Li-ion battery using heat pipe in electric vehicles. They examined the effects of many parameters on the operation of the considered system. Fikri et al. [12] used a heat pipe in a multistage evaporative cooler. Thermal characteristics of a condenser heat pipe for heat dissipation of an Insulated-Gate Bipolar Transistor have been investigated by Guowei et al. [13]. Xu et al. [14] studied a U-shape micro heat pipe's heat transfer and fluid flow specifications using pin fins. Zeng et al. [15] showed a thermohydraulic study of a new fin design obtained using topology-optimized heat sink arrangements. A review of pulsating heat pipes has been presented by Dave et al. [16]. Abdelkareem et al. [17] analyzed the applications of waste heat recovery systems based on heat pipes. The development of a micro loop heat pipe has been examined by Ahmed et al. [18]. In this analysis, examinations have been done at the microscale level. Baek and Jung [19] studied the thermal performance of a heat pipe for accelerating the operating fluid in start-up and steady-state conditions. Guichet et al. [20] analyzed the impact of heat transfer rate on the performance of a multi-channel heat pipe using experimental/theoretical methods. Guo et al. [21] explored the influences of basic parameters such as filling ratio, geometrical variables, and coolant temperature on the heat transfer of a wraparound heat pipe. Höhne [22] used the homogeneous model for simulating a heat pipe using the Computational Fluid Dynamics Method. Kang et al. [23] examined a single-loop pulsating heat pipe considering a porous wick layer. Kim et al. [24] investigated a flat heat pipe's effective thermal conductivity and thermal resistance. Enhancing the performance of a heat sink using a heat pipe and vapor chamber has been done by Muneeshwaran et al. [25]. Sun et al. [26] examined the heat transfer of a high-temperature heat pipe considering different conditions. Wang et al. [27] studied the dynamic specifications of a cylindrical rotating heat pipe. The impact of different inclination angles on the charging process of a heat storage unit based on a heat pipe has been investigated by Wang et al. [28]. Xu et al. [29] analyzed the influence of different heat input conditions on the heat transfer of a pulsating heat pipe. Xu and Zhang [30] examined the operational characteristics of a cold storage unit based on a heat pipe, considering limited time. Yang et al. [31] studied the start-up performance of a potassium heat pipe by a sensitivity analysis of important parameters.

The current paper presents a new heat sink design for the cold section of heat pipes. The proposed design utilizes a functionally graded material (FGM) and fins with power-law profiles to improve the thermal efficiency of the heat sink. The study begins by analyzing the heat transfer in the heat pipes using the thermal resistance method, which clearly shows the effects of the heat sink on the performance and heat duty of the heat pipe. Due to the dependence of the heat pipe's performance on the specifications of the heat sink and fins, a differential heat transfer equation is developed. The novelty of this research is the use of FGM to manufacture the fins, which results in fins with thermophysical properties that are dependent on location and temperature. Therefore, considering the power-law profile of the fins, the governing differential equation has a high degree of nonlinearity. Conventional methods are not suitable for solving this equation, so the authors propose using one of the efficient analytical methods known as DTM. The analytical results are then validated using the numerical method (Finite Element solution), showing that the proposed analytical solution can quickly evaluate the new heat sink temperature profile and efficiency in different conditions without the need for a numerical solution. The study investigates the influences of important characteristics on the heat sink's temperature profile and thermal performance using the analytical solution. The proper parameters are determined, and the effects of using the new fins on the heat pipe's performance are evaluated. Finally, the necessary suggestions for using the new heat sink design are presented.

MODELING

Description of Problem

This article applied the heat pipe's thermal investigation with a power-law profiled fins and variable thermal conductivity. The schematic view of the heat pipe and proposed fins is shown in Figure 1. The proposed fins are used for the heat pipe's cooling section (condenser).

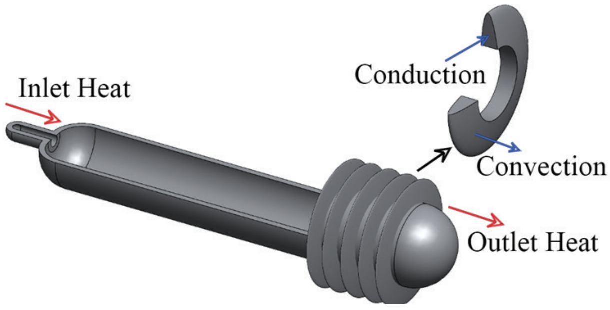


Figure 1. Schematic view of the heat pipe with FGM fins.

The condenser and evaporator sections are physically identical. So, the sufficient length of the heat pipe is [32]:

$$L_{eff} = \frac{L_E}{2} + L_A + \frac{L_C}{2} \tag{1}$$

Where L_A, L_C, and L_E are the length of adiabatic, condenser, and evaporator sections, respectively. The effective thermal conductivity of the heat pipe is [32]:

$$k_{eff} = \frac{k_w\{(k_w + k_c) - (1-\varepsilon)(k_w - k_c)\}}{(k_w + k_c) + (1-\varepsilon)(k_w - k_c)} \tag{2}$$

Where k_w and k_c are the water and copper thermal conductivity. The total heat duty of the heat pipe is calculated by evaluating its resistances. The schematic view of thermal resistances in the heat pipe system is displayed in Figure 2 [32].

Thermal resistances in Figure 2 can be introduced as follows [32]:

$R_{p,c}$ Conduction resistance/condenser section
$R_{w,c}$ Liquid-wick combination resistance/condenser section
$R_{i,c}$ Liquid-vapor interface resistance/condenser section
$R_{ext,c}$ (Heat sink + condenser) contact resistance
$R_{p,e}$ Conduction resistance/evaporator section
$R_{w,e}$ Liquid-wick combination resistance/evaporator section
$R_{i,e}$ Liquid-vapor interface resistance/evaporator section
$R_{ext,e}$ (Heat source + evaporator) contact resistance
$R_{v,a}$ Vapor resistance / adiabatic section
$R_{w,a}$ Liquid-wick combination resistance / adiabatic section
$R_{p,a}$ Conduction resistance / adiabatic region

The total heat duty of the heat pipe using the thermal resistance definition can be calculated below [32]:

$$q_{Heat\ pipe} = \frac{\Delta T_{Heat\ pipe}}{R_{Heat\ pipe}} \tag{3}$$

$$R_{Heat\ pipe} = R_{internal} + R_{heat\ sink} + R_{contact} \tag{4}$$

$\Delta T_{Heat\ pipe}$ is the maximum temperature difference of the heat pipe sections and $R_{Heat\ pipe}$ is the total thermal resistance of the heat pipe. Also, $R_{internal}$ and $R_{contact}$ can be calculated as below [32]:

$$R_{internal} = 2 \times \left(R_{p,c} + R_{w,c}\right) \tag{5}$$

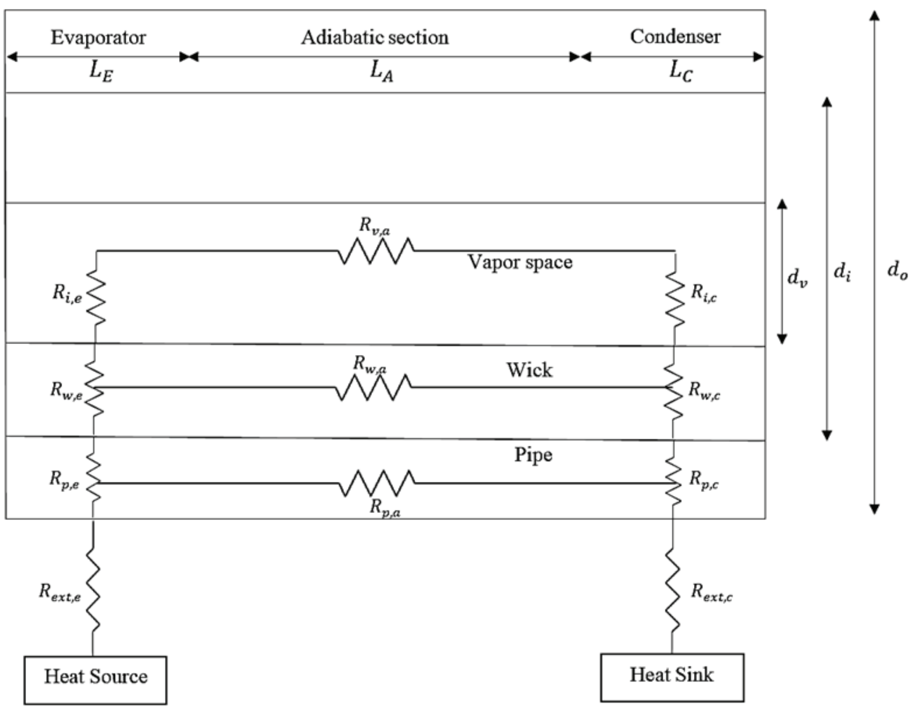


Figure 2. The thermal resistances of the heat pipe.

$$R_{contact} = \frac{R''_{t,c}}{\pi d_o L_C} \tag{6}$$

$R''_{t,c}$ is the thermal contact resistance with a constant value for each solid/solid interface. The radial conduction resistance of the heat pipe wall and the liquid-wick combination's thermal resistance at the condenser section can be calculated with Equations (7) and (8) [32].

$$R_{p,c} = \frac{\ln(\frac{d_o}{d_i})}{2\pi L_C k_c} \tag{7}$$

$$R_{w,c} = \frac{\ln(\frac{d_i}{d_v})}{2\pi L_C k_{eff}} \tag{8}$$

Where d_i/d_o, d_v, and L_C are the inner/outer diameter of the heat pipe, the diameter of vapor space, and the length of the condenser section. It is assumed that the length of the condenser, adiabatic, and evaporator sections are equal ($L_C = L_A = L_E$).

The total resistance of the heat sink section depends a lot on its configuration, which can be calculated below [32]:

$$R_{heat\ sink} = \frac{\Delta T_{heat\ sink}}{q_{heat\ sink}} \tag{9}$$

Where $\Delta T_{heat\ sink}$ is the maximum temperature difference of the heat sink section and $q_{heat\ sink}$ is the heat duty of the heat sink. Also, the total heat transfer of the heat sink section and single fin can be evaluated as [32]:

$$q_{heat\ sink} = (Number\ of\ fins) \times q_{fin} \tag{10}$$

$$q_{fin} = -k_0 A_c \frac{dT}{dr} : (r = r_i) \tag{11}$$

Therefore, the heat transfer of the heat pipe indirectly depends on the heat transfer through the fin (in the heat sink section) and its temperature distribution. For this reason, the temperature distribution of the proposed new FGM fin with a power-law profile is examined and evaluated in the next section.

Energy Balance Equation

In this analysis, two heat transfer phenomena were considered in the heat sink section of the heat pipe: Conduction and Convection heat transfer. The functionally graded material has been used for fins manufacturing; thus, the thermal conductivity is presented as a temperature and location function introduced in equation (17). Thermal conductivity is a linear function of temperature and a polynomial function of location. This function has an appropriate form; because the final thermal differential equation can be solved using the DTM method, and a comprehensive model for analysis can be considered. The primary energy balance equation for the proposed fin can be presented below [33]:

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = \dot{E}_{st} = 0 \tag{12}$$

Considering input/output conduction heat transfer term and output convective heat transfer from the specific element, the input and output energy can be presented below [33]:

$$\dot{E}_{in} = dq_{conduction} = \frac{d}{dr}\left(KA_c\frac{dT}{dr}\right)dr \tag{13}$$

$$\dot{E}_{out} = dq_{convection} = h\, dA_s(T - T_a) \tag{14}$$

$$\dot{E}_{gen} = 0 \tag{15}$$

So, the general thermal differential equation is derived as follows:

$$\frac{d}{dr}\left(KA_c\frac{dT}{dr}\right) - h\, dA_s(T - T_a) = 0 \tag{16}$$

K is the functionally graded material thermal conductivity and can be presented as the function of location and temperature as below [34, 35]:

$$K = k_0 r^n (1 + \beta\theta) \tag{17}$$

Where r, r_i, and r_o are the fin's radius at an arbitrary location, the inner and the outer radius of the fin, respectively. Also, T, T_a, and T_b are the fin temperature, the ambient air, and the base temperature of the fin, respectively.

$f(r)$ is the thickness profile function of the fin. y and Ω are constant parameters. Also, A_c and dA_s are the conduction and convective heat transfer area of the power-law fin. These parameters can be calculated as a function of location as below:

$$f(r) = yr^{-\Omega} \tag{18}$$

$$A_c = (2\pi r)(2f(r)) \tag{19}$$

$$dA_s = 2(2\pi r dr) \tag{20}$$

Also, the volume of the fin element can be presented as:

$$dV = (2\pi r)(2f(r))dr \tag{21}$$

For the simplification of the governed thermal differential equation, the dimensionless parameters are defined below:

$$\theta = \left(\frac{T - T_b}{T_a - T_b}\right) \tag{22}$$

$$\eta = \left(\frac{r - r_i}{r_o - r_i}\right) \tag{23}$$

By replacing the above expressions in the general thermal differential equation (Equation (16)), the final thermal differential equation can be presented:

$$\begin{aligned}&\left\{\frac{(T_a - T_b)}{(r_o - r_i)^2}\frac{d}{d\eta}\left((\eta(r_o - r_i) + r_i)^{n+1}(1+\beta\theta)\, y(\eta(r_o - r_i) + r_i)^{-\Omega}\frac{d\theta}{d\eta}\right)\right\}\\ &\quad - \left\{\left(\frac{h_0}{k_0}\right)(T_a - T_b)(\theta - 1)(\eta(r_o - r_i) + r_i)\right\} = 0\end{aligned} \tag{24}$$

The boundary conditions of the problem according to the fin specifications and the changes in the variables can be stated as follows:

$$r = r_i : \eta = 0 \rightarrow \theta = 0 \tag{25}$$

$$r = r_o : \eta = 1 \rightarrow \frac{d\theta}{d\eta} = 0 \tag{26}$$

The internal temperature of the fin is equal to T_b. Also, due to the small thickness of the outer radius, the outer part of the fin can be considered insulated.

Method of Solution

Analytical method: Differential transformation method (DTM)

The differential transformation method (DTM) was developed for the fractional differential equation's analytical solution. The DTM of the n-th derivative of the $f(x)$ function can be presented as follows:

$$F_\alpha(k) = \frac{1}{\Gamma(\alpha k + 1)}\left\{(D_{x_0}^{\alpha})^k f(x)\right\}_{x=x_0} \tag{27}$$

The differential inverse transform of $F_\alpha(k)$ is determined as:

$$f(x) = \sum_{k=0}^{\infty} F_\alpha(k)(x - x_0)^{\alpha k} \tag{28}$$

The necessary functions used in DTM are presented in Table 1 [36, 37].

Using the above operations, the differential transformation of the final thermal differential equation (Equation (24)) can be presented as follows:

$$\begin{aligned}&\left(\sum_{z=0}^{k}\sum_{v=0}^{z} H_1[v](\delta(z-v) + \beta\theta[z-v])\big((k-z+1)\theta[k-z+1]\big)\right)\\ &+\left(\sum_{z=0}^{k}\sum_{v=0}^{z} H_2[v]\big((z-v+1)\theta[z-v+1]\big)\big((k-z+1)\theta[k-z+1]\big)\right)\\ &-\left(\sum_{z=0}^{k}\sum_{v=0}^{z} H_3[v](\delta(z-v) + \beta\theta[z-v])\big((k-z+1)\theta[k-z+1]\big)\right)\\ &+\left(\sum_{z=0}^{k}\sum_{v=0}^{z} H_4[v](\delta(z-v)\right.\\ &\left.+\beta\theta[z-v])\big((k-z+1)(k-z+2)\theta[k-z+2]\big)\right)\\ &-\left(\sum_{z=0}^{k} H_5[z]\big(\theta[k-z] - \delta(k-z)\big)\right) = 0\end{aligned} \tag{29}$$

Where:

Table 1. The necessary functions of the differential transformation method

Original function	Transformed function
$b(p) = m(p) \pm q(p)$	$B(w) = M(w) \pm Q(w)$
$b(p) = \beta\, m(p)$	$B(w) = \beta\, M(w)$
$b(p) = \frac{d^i\, m(p)}{d\, p^i}$	$B(w) = (w+1)\,(w+2)\ldots\,(w+i)\,M(w+i)$
$b(p) = m(p)\, q(p)$	$B(w) = \sum_{r=0}^{w} Q(r)\, M(w-r)$
$b(p) = p^i$	$B(w) = \delta(w-i)\,;\ \delta(w-i) = \begin{cases}1\,; & w = i\\ 0\,; & w \neq i\end{cases}$
$b(p) = exp(\eta p)$	$B(w) = \frac{\eta^w}{w!}$
$b(p) = (1+p)^i$	$B(w) = \frac{i\,(i-1)\ldots\,(i-w+1)}{w!}$
$b(p) = \sin(\Omega p + \beta)$	$B(w) = \frac{\Omega^w}{w!}\sin\left(\frac{\pi w}{2} + \beta\right)$
$b(p) = \cos(\Omega p + \beta)$	$B(w) = \frac{\Omega^w}{w!}\cos\left(\frac{\pi w}{2} + \beta\right)$
$b(p) = \int_{p_0}^{p} m(t)dt$	$B(w) = \frac{M(w-1)}{w}\ , w \geq 1\,, B(0) = 0$

$$H_1[t] = \frac{(T_a - T_b)}{(r_o - r_i)}(n+1)\, y(t(r_o - r_i) + r_i)^n (t(r_o - r_i) + r_i)^{-\Omega}$$
$$H_2[t] = \frac{(T_a - T_b)}{(r_o - r_i)^2}\beta\, y(t(r_o - r_i) + r_i)^{n+1} (t(r_o - r_i) + r_i)^{-\Omega}$$
$$H_3[t] = \frac{(T_a - T_b)}{(r_o - r_i)}\Omega\, y(t(r_o - r_i) + r_i)^n (t(r_o - r_i) + r_i)^{-\Omega} \quad (30)$$
$$H_4[t] = \frac{(T_a - T_b)}{(r_o - r_i)^2}\, y(t(r_o - r_i) + r_i)^{n+1} (t(r_o - r_i) + r_i)^{-\Omega}$$
$$H_5[t] = \left(\frac{h_0}{k_0}\right)(T_a - T_b)(t(r_o - r_i) + r_i)$$

k, z, and v are the sigma boundaries used in the DTM method, and the unknown function $\delta(x)$ is defined as below:

$$\delta(x) = \begin{cases} 1, & x = 0 \\ 0, & x \neq 0 \end{cases} \quad (31)$$

Also, the transformed boundary conditions are presented as follows:

$$\theta_0 = 0$$
$$\theta_1 = \mu \quad (32)$$

The following coefficients can be calculated step-by-step using the previous coefficients. For example:

$$\theta_2 = \left(\frac{-1}{2yr_i^{n+1}}\right)\left(\beta y r_i^{n+1}\mu^2 + \Omega y \mu r_i^{n+1} - \Omega y \mu r_i^n r_o - ny\mu r_i^{n+1} + ny\mu r_i^n r_o - y\mu r_i^{n+1} + y\mu r_i^n r_o + \left(\frac{h_0}{k_0}\right)(r_o - r_i)^2 r_i^{\Omega+1}\right) \quad (33)$$

Finally:

$$\theta(\eta) = \sum_{i=1}^{n} \theta_i \,.\, \eta^i \quad (34)$$

Numerical method: Finite-element solution (using flexpde)

The FlexPDE software was selected for the numerical solution in this study to validate the analytical solution using the finite element method (FEM). FlexPDE is a software designed and developed specifically for solving various partial differential equations using the finite element method. It is a self-contained processing system that is easy to use and presents the results after four simple steps. The first step in using FlexPDE is to input the description of the problem. This includes the initial and boundary conditions and the differential equation to be solved. The software forms the Galerkin finite element derivatives, integrals, and dependencies in the next step. These are used to build a coupling matrix in the third step, which is then solved. The final step is to plot the results in a visually appealing format. FlexPDE also applies an Adaptive Mesh Refinement technique, which is a powerful tool for improving the accuracy of the numerical solution. After the initial mesh is created, FlexPDE calculates the solution error and adapts the mesh as required to reach the target accuracy. This is done by adding or removing cells from the mesh, and cells generated by this method can be remerged later. Another advantage of FlexPDE is its ability to handle non-linear differential equations. When the differential equation is non-linear, the FlexPDE software automatically changes the procedure accordingly and can still provide accurate and reliable results. FlexPDE is a powerful and versatile software widely used to solve partial differential equations using the finite element method. It offers a user-friendly interface and efficient tools that are easy to apply, making it a valuable tool for validating analytical solutions and solving complex problems in thermal management.

RESULTS AND DISCUSSION

To validate the accuracy of the analytical solution, a numerical solution was performed using the FlexPDE software package. This software package can solve non-linear differential equations, such as equation (24), used in this study's analysis. The numerical solution was performed by inputting the parameters and initial conditions of the system into the FlexPDE software and then solving the non-linear differential equation using a finite element method. The results of the numerical solution were then compared with the results of the analytical solution. The results are compared in Figure 3, demonstrating that the analytical and numerical solutions are in close agreement.

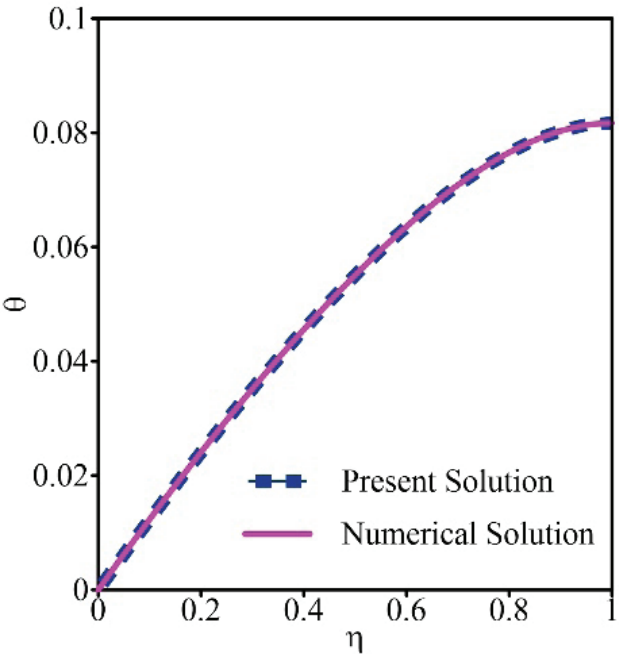


Figure 3. Comparison between analytical and numerical results.

This comparison confirms that the analytical solution is accurate and reliable and can be used to predict the system's thermal behavior. The excellent agreement between analytical and numerical solutions strongly indicates that the proposed analytical solution is accurate and can be used to design and optimize thermal management systems. Additionally, it is important to note that numerical solutions can be computationally expensive and time-consuming, and using an analytical solution can provide a fast and efficient means for evaluating the system's performance. This is especially important when dealing with large and complex systems, where the ability to evaluate different design scenarios quickly is crucial. Therefore, an analytical solution can significantly simplify the design process and save valuable time and resources.

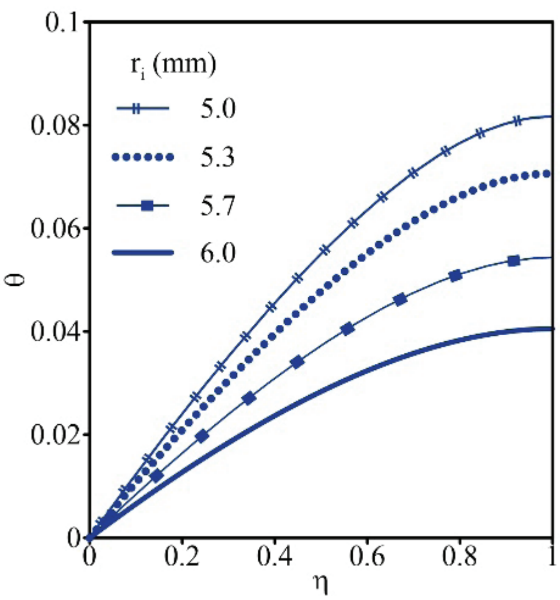
In this section, an in-depth examination of the impact of various parameters on the temperature of the fin is performed. It is worth noting that, as per established relationships, there is an inverse relationship between the dimensionless temperature and the fin's absolute temperature. Furthermore, Table 2 presents the basic parameters of the investigations used in this study.

Table 2. Basic parameters of the analysis

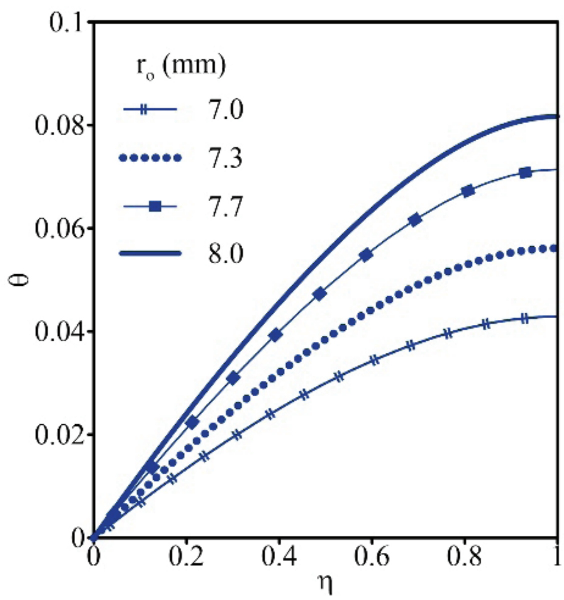
Parameter	Value	Parameter	Value
T_b	303 (K)	h_0	100 (W/m^2K)
T_a	278 (K)	Ω	0.1
r_i	0.005 (m)	y	0.004
r_o	0.008 (m)	n	0.3
k_0	5 (W/mK)	β	-10

The effects of the inner and outer radius of the FGM fin with a power-law profile on the thermal behaviors are presented in Figure 4. The results indicate that the dimensionless temperature will be increased as the inner radius decreases and the outer radius increases. This is because as the inner radius decreases and the outer radius increases, the heat convection from the heat sink's external surface increases. As a result, the fin cools better, and its temperature drops more. This is a significant finding as it highlights the importance of understanding the effect of the inner and outer radius of the FGM fin on the thermal behavior of the heat pipe. Furthermore, the finding can be used to optimize the heat sink's design and improve the heat pipe's performance.

The effects of ambient air and the fin's inner temperature on the fin performance have been studied and presented in Figure 5. The results indicate that an increment in the ambient air and base temperature leads to an increase in the temperature of the fin. This is attributed to the change in thermal potential between the base plate and ambient air caused by these events. In other words, as the ambient air and base temperature increase, the amount of convection heat transfer decreases, leading to less cooling of the fin. It is important to note that this decrease in cooling can significantly impact the overall performance of the heat pipe and thermal management system, and should be considered during the design and optimization process. This highlights the importance of carefully controlling and monitoring the ambient air and base temperature to ensure optimal performance of the heat pipe and thermal management system.

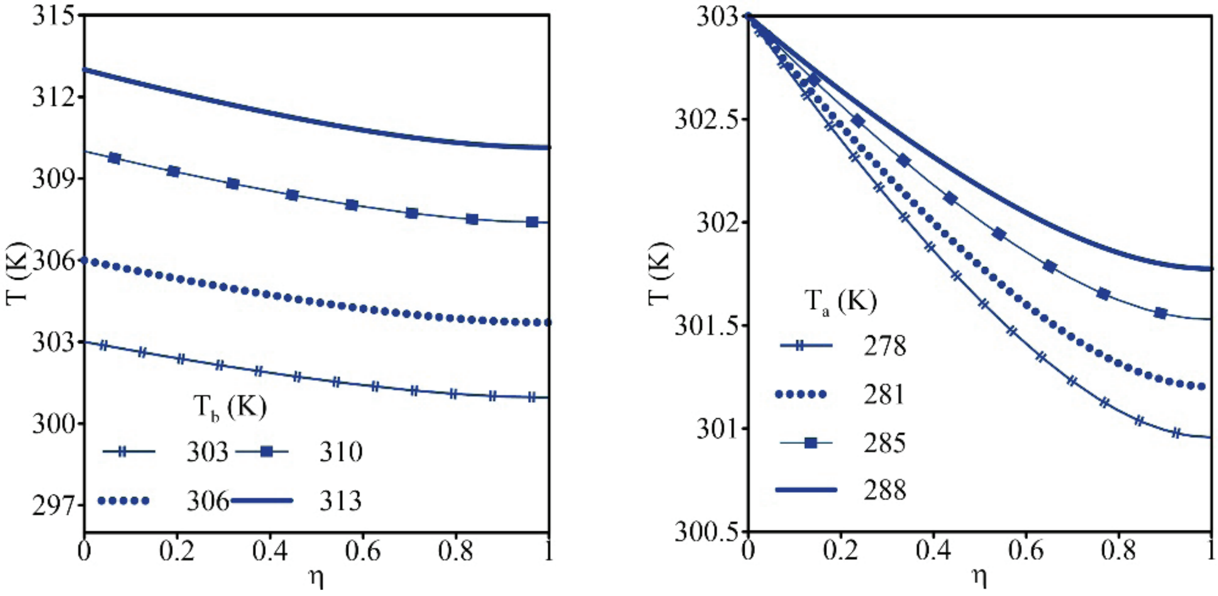


(a)



(b)

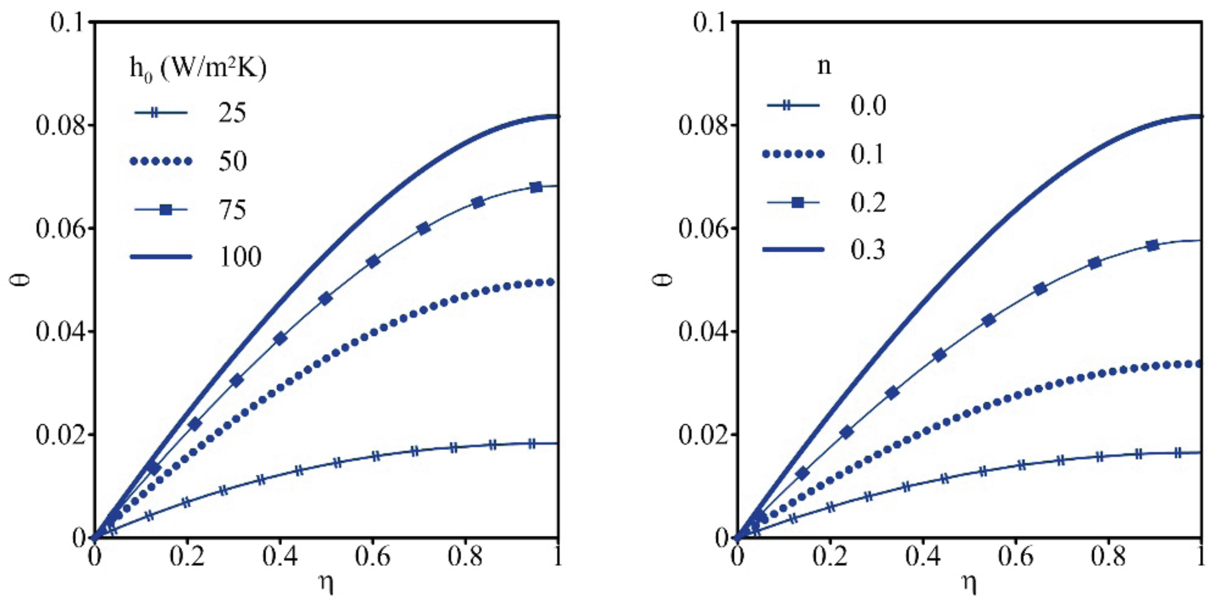
Figure 4. Effect of the radius on temperature distribution of the fin.



(a) **(b)**

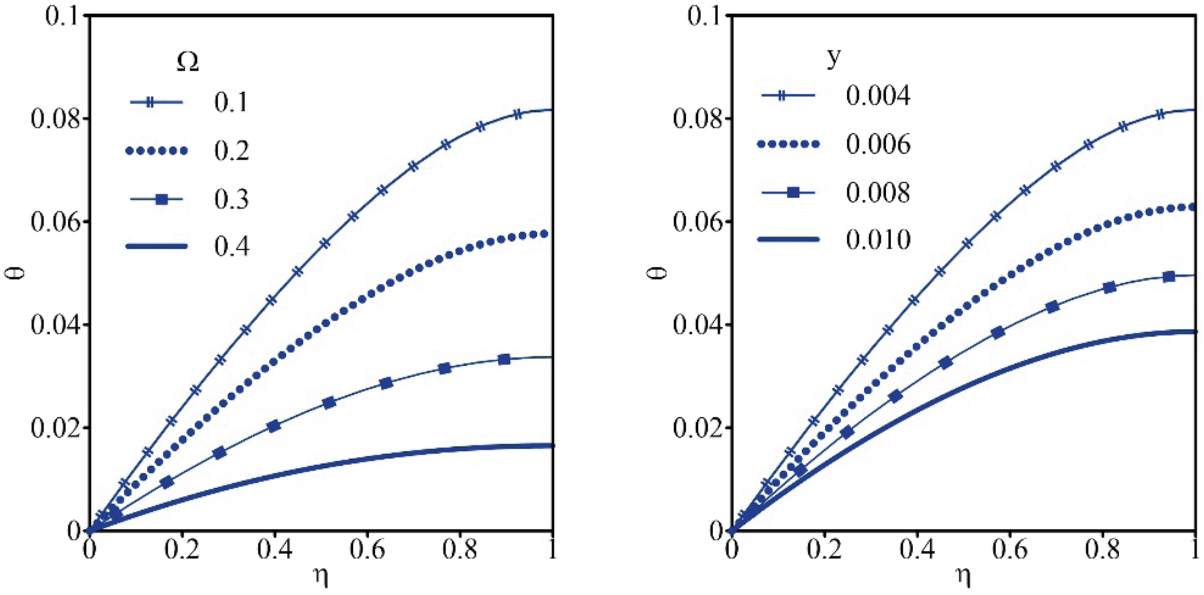
Figure 5. Effect of ambient air and fin base temperature on the temperature profile.

The study examines the effect of the convection heat transfer coefficient and grading index on the dimensionless temperature of the fin in the new design for the cold part of heat pipes. The results, presented in Figure 6, demonstrate that an increment in the values of h_0 and n leads to an increase in the dimensionless temperature. This implies that the absolute temperature of the fin decreases when the convection heat transfer coefficient and grading index are increased. The reason for this is that a higher convection coefficient (h_0) leads to an enhancement in the convection heat transfer, which results in a reduction in the temperature of the fin. Similarly, an increase in the value of n coefficient improves the conduction heat transfer, thus enhancing the cooling process of the fin.



(a) **(b)**

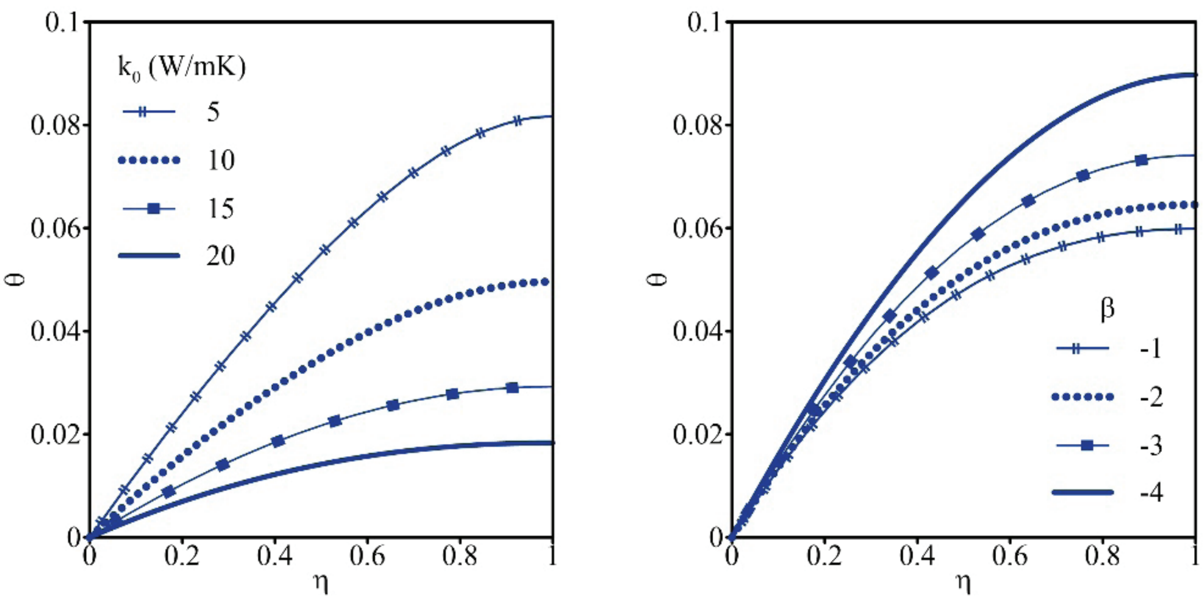
Figure 6. Effect of the convection heat transfer coefficient and grading index on the temperature profile.



(a) **(b)**

Figure 7. Influence of the thickness profile coefficients on the thermal performance of the fin.

Figure 7 presents the results of the investigation on the effect of varying the thickness profile coefficients on the temperature of the fin. The results indicate that using lower values for Ω and y can lead to an increase in the dimensionless temperature of the fin or a decrease in the absolute temperature. This is because reducing the fin's thickness results in it absorbing less heat from the condenser section. This reduction in heat absorption results in a decrease in the temperature of the fin. It is worth noting that the thickness of the fin plays an important role in the thermal performance of the heat pipe system and should be carefully considered during the design and optimization of these systems. Additionally, it was observed that decreasing the values of Ω and y, leads to a reduction in the fin's thermal resistance, which results in a better performance of the heat pipe system.



(a) **(b)**

Figure 8. Effect of variable conduction coefficients on the temperature profile.

The study has investigated the effects of conduction heat transfer coefficients on the dimensionless temperature of the fin. The results, presented in Figure 8, indicate that the dimensionless temperature of the fin will be enhanced when using lower values for k_0 and β. This translates to a reduction in the absolute temperature of the fin. Decreasing the value of the k_0 and β coefficients can weaken the conduction heat transfer process, and as a result, less heat will be transferred from the condenser section to the middle areas of the fin. Therefore, the fin temperature will be lower. This finding is significant as it highlights the importance of considering conduction heat transfer coefficients in the design of thermal management systems, as it can significantly impact the fin's performance. The study's results suggest that to achieve optimal thermal performance, it is necessary to optimize the values of conduction heat transfer coefficients.

CONCLUSION

In conclusion, this study presents a new design for the cold part of heat pipes, which utilizes a Functionally Graded Material (FGM) in the heat sink with power-law fins. The proposed design aims to improve the performance of the heat pipe by increasing the heat duty. The study evaluates the heat transfer in the heat pipes using the heat resistance method and shows the heat sink's effect on the heat pipe's performance. The authors also perform a sensitivity analysis of geometrical and operational parameters to examine their effects on the thermal management system. The study's results indicate that changing the inner and outer radius of the FGM fin with a power-law profile improves the thermal behavior by decreasing the absolute temperature of the fin by up to 1 Kelvin degree.

Additionally, increasing the ambient air and the fin's inner temperature has a negative effect on the fin's performance by decreasing the amount of convection heat transfer and cooling by up to 1.5 Kelvin degrees. Increasing the convection heat transfer coefficient and grading index improves the dimensionless temperature while decreasing the thickness profile coefficients and conduction heat transfer coefficients has a positive effect on the dimensionless temperature of the fin. Overall, the study concludes that using the new heat sink design significantly improves the heat pipe's performance. The analytical solution used in this study allows for the quick evaluation of the heat sink's efficiency for different configurations without requiring numerical solutions. The results of this study can be used in designing and optimizing thermal management systems in various applications.

NOMENCLATURE

r_i	Inner radius of the fin, m
r_o	Outer radius of the fin, m
T_a	Ambient air temperature, K
T_b	Fin base temperature, K
h_0	Convection heat transfer coefficient, W/m^2 K
k_0	Fin base thermal conductivity, W/mK
y	Fin profile coefficient
$f(r)$	Fin profile function
A_c	Area of conduction heat transfer, m^2
dA_s	Area of convection heat transfer, m^2
n	Grading index
d_o	The outer diameter of the heat pipe, m
d_i	The inner diameter of the heat pipe, m
d_v	The diameter of the vapor space, m
d_w	The wire diameter, m
L_{total}	The total length of the heat pipe, m
L_A	The adiabatic region length, m
L_E	The evaporator section length, m
L_C	The condenser section length, m
L_{eff}	The effective length of the heat pipe, m
$R_{p,c}$	Radial conduction resistance of the heat pipe wall at the condenser section
$R_{w,c}$	Resistance of the liquid-wick combination at the condenser section
$R_{i,c}$	Resistance of the liquid-vapor interface at the condenser section
$R_{ext,c}$	Contact resistance between the heat sink and the condenser section
$R_{p,e}$	Radial conduction resistance of the heat pipe wall at the evaporator section
$R_{w,e}$	Resistance of the liquid-wick combination at the evaporator section
$R_{i,e}$	Resistance of the liquid-vapor interface at the evaporator section
$R_{ext,e}$	Contact resistance between the heat source and the evaporator section
$R_{v,a}$	Resistance of the vapor at the adiabatic section
$R_{w,a}$	Resistance of the liquid-wick combination in the adiabatic region
$R_{p,a}$	Axial conduction resistance of the heat pipe wall in the adiabatic region
k_c	The copper thermal conductivity, W/mK
k_w	The water thermal conductivity, W/mK
k_{eff}	Effective thermal conductivity of the heat pipe, W/mK
$q_{Heat\ pipe}$	The total heat duty of the heat pipe, W
$\Delta T_{Heat\ pipe}$	The maximum temperature difference of the heat pipe sections, K
$R_{Heat\ pipe}$	The total thermal resistance of the heat pipe, K/W
$R_{t,c}''$	The thermal contact resistance, m^2 K/W
$q_{Heat\ sink}$	The heat duty of the heat sink, W
$\Delta T_{Heat\ sink}$	The maximum temperature difference of the heat sink section, K
$R_{Heat\ sink}$	The total thermal resistance of the heat sink, K/W
q_{fin}	The heat duty of a single proposed fin, W
$H\{_t\}$	DTM transformation coefficients

Greek symbols

Ω	Fin profile coefficient
β	Coefficient of conduction heat transfer
θ	Fin dimensionless temperature
η	Fin dimensionless radius
$\delta(x)$	Dirac delta function
ε	Coefficient of effective thermal conductivity of the heat pipe

AUTHORSHIP CONTRIBUTIONS

Authors equally contributed to this work.

DATA AVAILABILITY STATEMENT

The authors confirm that the data that supports the findings of this study are available within the article. Raw data that support the finding of this study are available from the corresponding author, upon reasonable request.

CONFLICT OF INTEREST

The author declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

ETHICS

There are no ethical issues with the publication of this manuscript.

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