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BISECTOR CURVES OF COMFORMABLE CURVES IN $\mathbb{R}^2$

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Abstract: In this study, initially, information about the derivative of fractional order was given. Subsequently, one of the fractional derivative types, namely the comformable derivative was discussed in detail. Additionally, the studies conducted on this comformable derivative type were also included. The importance of the bisector structure on the theory of curves was mentioned. In the second part of the study, the materials and methods were demonstrated using the comformable derivative. Finally, in this work, the bisector curves of two regular comformable curves from C^1-regular parametric category is inspected in $\mathbb{R}^2$. Then, multivariable functions which are corresponded to bisector curves of regular comformable curves are calculated. The bisector curves are procured by two similar paths. The methods of finding this function were demonstrated in detail using comformable derivatives. Then, the equations which are corresponded to bisector curves are obtained in $\mathbb{R}^2$.

Keywords: Bisector curve, Comformable Derivative, Frenet Frame

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1. Introduction

The fractional analysis phrase first appears in a letter written by L' Hospital to Leibniz. In this letter, L-Hospital asked Leibniz about a special structure he used in his work for $\frac{d^n y}{dx^n}$, which is the nth order derivative of the linear function $f(x) = x$. L-Hospital wanted to get the number derivative and didn't know how to do, for example if the derivative's order come out to be rational number what would the consequence happen (Nishimoto, 1991). This question created the first glint of fractional analysis. Lately, Khalil at al. defined an unprecedented of the fractional derivative denominated comformable fractional derivative (Khalil et.al., 2014).

Similarly, Gözütok, Çoban and Sağıroğlu (2019) have study the Frenet frame with respect to comformable fractional derivative. Then, the comformable derivative and its properties have been denoted by geometers (Alsaedi, 2015; Atangana, 2015, Baleanu, 2015). In addition, a more precise definition of a comformable derivative is introduced (Anderson and Ulness, 2015). The bisector for two objects is defined as a series of points equality from the two objects. The structure of bisectors plays an important role in many geometric calculation. Curves are one of the areas frequently studied by geometricians. By constructing various frames on a curve, the differential properties of the curve can be obtained. The theory of curves is a very comprehensive subject that finds application in many disciplines such as architecture, engineering, computer technologies, as well as mathematics. Some studies on any regular curve and also some studies on especially comformable and bisector curves are as follows. According to Farouki and Johnstone (1994) the bisector of two geometrical elements (such as points, curves, surfaces ets.) is the path traced by a variable point that remains equidistant from this elements. Elber and Kim (1998) demonstrated that bisector surfaces are rational ruled surfaces. In 3-dimensional Euclidean space, the relationships between N-Bishop and Frenet frames for any regular curve, as well as the relationships between type-1 Bishop and N-Bishop frames, are given (Gür Mazlum, 2024). Additionally, pole vectors associated with these frames are calculated by Gür Mazlum. In addition, Gür Mazlum and Bektaş (2023) studied the involute curves of any non-lightlike curve in 3-dimensional Euclidean space. Many special curves and the Frenet frame, initially defined using classical derivatives, have been redefined through the use of comformable fractional derivatives (Akkurt, 2022; Has, 2022, Yıldırım, 2022, Yılmaz 2022). Bektaş and Gür Mazlum (2022) examined the modified frames with both the non-zero curvature and the torsion of the non-unit speed curves in Euclidean 3-space.

In this study, the bisector curves of two regular plane curves from C^1-regular parametric category is inspected in $\mathbb{R}^2$. Then, multivariable function which is corresponded to bisector curves of regular plane curves is calculated (Dede and Ünlütürk, 2013). The bisector curves are procured by two different paths. As a result, the equations which are corresponded to bisector curves are obtained in $\mathbb{R}^2$.

2. Materials and Methods

Let us consider a smooth curve $\gamma(t) = (x(t), y(t)), \gamma$ from a subset $I \subset \mathbb{R}$ to a two-dimensional space $\mathbb{R}^2$, where t is an arbitrary parameter. Then, the ordinary definition of the length $\tilde{s}$ of a curve γ starting at $t=0$ is given by equation 1

$$\tilde{s}=\int_0^t \left\|\frac{d\gamma}{du}\right\| du \tag{1}$$
$$=\int_0^t \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\, dt$$

for $t\in I$. Then, it is well-known that equation 1 is an arc length of curve for the tangent vector:

$$E_1(\tilde{s})=\left(\frac{dx}{d\tilde{s}},\frac{dy}{d\tilde{s}}\right)$$

Now, let us consider the comformable derivative on the curvature of the curve. Basically, since the curvature is given by the change of the tangent vector of the curve, we can define a comformable tangent vector (equation 2):

$$E_1^{(\alpha)}(\tilde{s})=(T_\alpha x(\tilde{s}),T_\alpha y(\tilde{s})) \tag{2}$$
$$=\left(\frac{d^{(\alpha)}x(\tilde{s})}{d\tilde{s}^{(\alpha)}},\frac{d^{(\alpha)}y(\tilde{s})}{d\tilde{s}^{(\alpha)}}\right)$$

For equation 2, $\left\|E_1^{(\alpha)}(\tilde{s})\neq 1\right\|$. Then, we have

$$\frac{d^{(\alpha)}\gamma(t(s))}{ds^{(\alpha)}}=s^{\mathbf{1}-\boldsymbol{\alpha}}\frac{d\gamma(s)}{dt}\frac{dt}{ds} \tag{3}$$

Considering equation 3, let us define the transformation $\tilde{s}\to s$ as follows (equation 4):

$$s=(\alpha\tilde{s})^{\frac{1}{\alpha}} \tag{4}$$

where α is the degree of the comformable derivative and $0<\alpha\leq 1$. For this parameter s, if the derivative of both sides with respect to t is taken, we can write as follows (equation 5).

$$\frac{ds}{dt}=s^{1-\alpha}\frac{d\tilde{s}}{dt}\,. \tag{5}$$

Then, let us describe the tangent vector of the curve using the parameter s and comformable derivative. In other words (equation 6),

$$e_1{}^{(\alpha)}(s)=T_\alpha\gamma(s)$$
$$=\left(T_\alpha\big(x(s)\big),T_\alpha\big(y(s)\big)\right) \tag{6}$$
$$=\left(\frac{d^\alpha x(s)}{ds^\alpha},\frac{d^\alpha y(s)}{ds^\alpha}\right)$$

The norm of the tangent vector from equation 3 is (equation 7)

$$\left\|e_1{}^{(\alpha)}(s)\right\|=\|T_\alpha\gamma(s)\|=\left\|s^{1-\alpha}\frac{d\gamma}{dt}\frac{dt}{ds}\right\|$$
$$=s^{1-\alpha}\left\|\frac{d\gamma}{dt}\right\|\frac{dt}{ds}$$
$$=s^{1-\alpha}\frac{d\tilde{s}}{dt}s^{\alpha-1}\frac{dt}{d\tilde{s}}$$
$$=1\,. \tag{7}$$

Moreover, the unit vector of the curve orthonormal to $e_1^{(\alpha)}(s)$ is defined by using equation 3 as follows (equation 8):

$$e_2^{(\alpha)}(s)=(-T_\alpha y(s),T_\alpha x(s)). \tag{8}$$

For the γ curve with parameter s, $e_1^{(\alpha)}(s)$ and $e_2^{(\alpha)}(s)$ are, respectively, the unit tangent vector and the normal vector of this curve. Also, the parameter s is the arc length. As a result of this, we have the following definition: Definition 1; Let's take a function $f:[0,\infty]\to\mathbb{R}$. The comformable derivative of the function f of order α is indicated by:

$$T_\alpha f(x)=\lim_{h\to 0}\frac{f(x+hx^{1-\alpha})-f(x)}{h}=x^{1-\alpha}f'(x)$$

for all $x>0,\alpha\in(0,1)$ (Dede, 2013; Ekici, 2013, Ünlütürk, 2013).

3. Results

At $\mathbb{R}^2$, we suppose that there is regular comformable curves from C^1-regular parametric category. Furthermore, these curves are parameterized by (equation 9)

$m_{\mathbf{1}}(\varrho)=(v_1(\varrho),w_1(\varrho))$

and

$$m_2(\varsigma)=(v_2(\varsigma),w_2(\varsigma))\,. \tag{9}$$

Now, multivariable function $F(\varrho,\varsigma)=0$ which correspond to the bisector curves of $m_1(\varrho)$ and $m_2(\varsigma)$ is being calculated for $i=1,2$.

3.1. $\boldsymbol{F_1(\varrho,\varsigma)}$ Function

Tangent vector of curves $m_1(\varrho)$ and $m_2(\varsigma)$ are defined by

$t_1(\varrho)=(T_\alpha v_1(\varrho),T_\alpha w_1(\varrho))$

and (equation 10)

$$t_2(\varsigma)=(T_\alpha v_2(\varsigma),T_\alpha w_2(\varsigma))\text{, respectively.} \tag{10}$$

Moreover, the normal vectors of curves are written by

$n_1(\varrho)=(-T_\alpha w_1(\varrho),T_\alpha v_1(\varrho))$

and (equation 11)

$$n_2(\varsigma)=(-T_\alpha w_2(\varsigma),T_\alpha v_2(\varsigma))\text{, respectively.} \tag{11}$$

On the other and, for multivariable functions, which are $\lambda=\lambda(\varrho,\varsigma)$ and $\beta=\beta(\varrho,\varsigma)$, crossing points in accordance with the normals of curves $m_1(\varrho)$ and $m_2(\varsigma)$ denoted by

$$m_1(\varrho)+n_1(\varrho)\lambda=m_2(\varsigma)+n_2(\varsigma)\beta. \tag{12}$$

Later, by use of equation 9, equation 11 in equation (12), we get (equation 13)

$$\big(v_1(\varrho),w_1(\varrho)\big)+\big(-T_\alpha w_1(\varrho),T_\alpha v_1(\varrho)\big)\lambda$$
$$=\big(v_2(\varsigma),w_2(\varsigma)\big)+\big(-T_\alpha w_2(\varsigma),T_\alpha v_2(\varsigma)\big)\lambda$$

or

$$\big[v_1(\varrho)-\big(T_\alpha w_1(\varrho)\big)\lambda+(T_\alpha v_1(\varrho))\lambda\big]$$
$$=[v_2(\varsigma)-(T_\alpha(w_2(\varsigma))\beta,w_2(\varsigma)+(T_\alpha(v_2(\varsigma))\beta]$$

or

$$v_1(\varrho)-\big(T_\alpha w_1(\varrho)\big)\lambda=v_2(\varsigma)-\big(T_\alpha w_2(\varsigma)\big)\beta,$$
$$w_1(\varrho)+(T_\alpha v_1(\varrho))\lambda=w_2(\varsigma)+(T_\alpha v_2(\varsigma))\beta$$

or

$$(-T_\alpha w_1(\varrho))\lambda+(T_\alpha w_2(\varsigma))\beta=v_2(\varsigma)-v_1(\varrho),$$
$$T_\alpha v_1(\varrho)\lambda-T_\alpha v_2(\varsigma)\beta=w_2(\varsigma)-w_1(\varrho). \tag{13}$$

We accept it so (equation 14)

$$\Delta = \begin{vmatrix} (-T_\alpha w_1(\varrho)) & (T_\alpha w_2(\varsigma)) \\ T_\alpha v_1(\varrho) & -T_\alpha v_2(\varsigma) \end{vmatrix} \neq 0,$$

$$\Delta_1 = \begin{vmatrix} v_2(\varsigma) - v_1(\varrho) & (T_\alpha w_2(\varsigma)) \\ w_2(\varsigma) - w_1(\varrho) & -T_\alpha v_2(\varsigma) \end{vmatrix} \tag{14}$$

and

$$\Delta_2 = \begin{vmatrix} (-T_\alpha w_1(\varrho)) & v_2(\varsigma) - v_1(\varrho) \\ T_\alpha v_1(\varrho) & w_2(\varsigma) - w_1(\varrho) \end{vmatrix}$$

in order that equation 13 solution to be calculated Cramer's rule.

Thus, from the expression $\lambda = \lambda(\varrho, \varsigma) = \frac{\Delta_1}{\Delta}$, we obtain

$$\lambda = \lambda(\varrho, \varsigma) = \frac{\begin{vmatrix} v_2(\varsigma) - v_1(\varrho) & (T_\alpha w_2(\varsigma)) \\ w_2(\varsigma) - w_1(\varrho) & -T_\alpha v_2(\varsigma) \end{vmatrix}}{\begin{vmatrix} (-T_\alpha w_1(\varrho)) & (T_\alpha w_2(\varsigma)) \\ T_\alpha v_1(\varrho) & -T_\alpha v_2(\varsigma) \end{vmatrix}}.$$

Similarly, from the expression $\beta = \beta(\varrho, \varsigma) = \frac{\Delta_2}{\Delta}$, we obtain

$$\beta = \beta(\varrho, \varsigma) = \frac{\begin{vmatrix} (-T_\alpha w_1(\varrho)) & v_2(\varsigma) - v_1(\varrho) \\ T_\alpha v_1(\varrho) & w_2(\varsigma) - w_1(\varrho) \end{vmatrix}}{\begin{vmatrix} (-T_\alpha w_1(\varrho)) & (T_\alpha w_2(\varsigma)) \\ T_\alpha v_1(\varrho) & -T_\alpha v_2(\varsigma) \end{vmatrix}}.$$

Functions of two variables $\lambda(\varrho, \varsigma)$ and $\beta(\varrho, \varsigma)$ can write with function of two variables at vertex point. In other words, we can express the formula as following (equation 15):

$$\begin{aligned} P(\varrho, \varsigma) &= c_1(\varrho) + n_1(\varrho)\lambda(\varrho, \varsigma) \\ &= c_2(\varsigma) + n_2(\varsigma)\beta(\varrho, \varsigma). \end{aligned} \tag{15}$$

Also, function of two variables $P(\varrho, \varsigma)$ has to be equal distance from $m_1(\varrho)$ and $m_2(\varsigma)$ $\|P(\varrho, \varsigma) - m_1(\varrho)\| = \|P(\varrho, \varsigma) - m_2(\varsigma)\|$ can be written like this. Then, by use of (15), in the last equation, we get (equation 16)

$$n_1^2(\varrho, \varsigma)\big(\lambda(\varrho, \varsigma)\big)^2 = n_2^2(\varrho, \varsigma)\big(\beta(\varrho, \varsigma)\big)^2. \tag{16}$$

Later, by use of $\lambda = \lambda(\varrho, \varsigma)$ and $\beta = \beta(\varrho, \varsigma)$ in equation 16, we get (equation 17)

$$n_1^2(\varrho)\left(\frac{\begin{vmatrix} v_2(\varsigma) - v_1(\varrho) & (T_\alpha w_2(\varsigma)) \\ w_2(\varsigma) - w_1(\varrho) & -T_\alpha v_2(\varsigma) \end{vmatrix}}{\begin{vmatrix} (-T_\alpha w_1(\varrho)) & (T_\alpha w_2(\varsigma)) \\ T_\alpha v_1(\varrho) & -T_\alpha v_2(\varsigma) \end{vmatrix}}\right)^2 - n_2^2(\varsigma)\left(\frac{\begin{vmatrix} (-T_\alpha w_1(\varrho)) & v_2(\varsigma) - v_1(\varrho) \\ T_\alpha v_1(\varrho) & w_2(\varsigma) - w_1(\varrho) \end{vmatrix}}{\begin{vmatrix} (-T_\alpha w_1(\varrho)) & (T_\alpha w_2(\varsigma)) \\ T_\alpha v_1(\varrho) & -T_\alpha v_2(\varsigma) \end{vmatrix}}\right)^2 = 0 \tag{17}$$

or (equation 18)

$$n_1^2(\varrho)\big[-\big(v_2(\varsigma) - v_1(\varrho)\big)T_\alpha v_2(\varsigma) - (w_2(\varsigma) - w_1(\varrho))T_\alpha w_2(\varsigma)\big]^2 - n_2^2(\varsigma)\big[-\big(w_2(\varsigma) - w_1(\varrho)\big)T_\alpha w_1(\varrho) - (v_2(\varsigma) - v_1(\varrho))T_\alpha v_1(\varrho)\big]^2 = 0. \tag{18}$$

Here, by use of equations 11, we obtain (equation 19)

$$P(\varrho, \varsigma) = \big(T_\alpha^2 w_1(\varrho) + T_\alpha^2 v_1(\varrho)\big)\big[\big(v_2(\varsigma) - v_1(\varrho)\big)T_\alpha v_2(\varsigma) + \big(w_2(\varsigma) - w_1(\varrho)\big)T_\alpha w_2(\varsigma)\big]^2 - \big(T_\alpha^2 w_2(\varsigma) + T_\alpha^2 v_2(\varsigma)\big)\big[\big(w_2(\varsigma) - w_1(\varrho)\big)T_\alpha w_1(\varrho) + \big(v_2(\varsigma) - v_1(\varrho)\big)T_\alpha v_1(\varrho)\big]^2 = 0. \tag{19}$$

The equation 11 is bisector curve of $m_1(\varrho)$ and $m_2(\varsigma)$.

3.2. $F_2(\varrho, \varsigma)$ Function

In this method, we obtained bisector point of $P(\varrho, \varsigma)$. When the bisector of the curves $m_1(\varrho)$ and $m_2(\varsigma)$ are at one point of $P(\varrho, \varsigma)$, the curves $m_1(\varrho)$ and $m_2(\varsigma)$ have two points such as $m_1(\varrho)$ and $m_2(\varsigma)$, which contain P. As a result, the point P provides linear equations as follows (equation 20):

$$\begin{aligned} &L_1(\varrho): \langle P - m_1(\varrho), T_1(\varrho)\rangle = 0, \\ &L_2(\varsigma): \langle P - m_2(\varsigma), T_2(\varsigma)\rangle = 0. \end{aligned} \tag{20}$$

Hence, with direct calculation, we get (equation 21 and 22)

$$\langle P - T_1(\varrho)\rangle = \langle c_1(\varrho), T_1(\varrho)\rangle, \tag{21}$$

$$\langle P - T_2(\varsigma)\rangle = \langle c_2(\varsigma), T_2(\varsigma)\rangle. \tag{22}$$

If we take $P(\varrho, \varsigma) = (v(\varrho, \varsigma), w(\varrho, \varsigma))$ and by use of equation 9 and 10 in equation 21, we get (equation 23)

$$\begin{aligned} \langle\big(v(\varrho, \varsigma), w(\varrho, \varsigma)\big), \big(T_\alpha v_1(\varrho), T_\alpha w_1(\varrho)\big)\rangle &= \langle\big(v_1(\varrho), w_1(\varrho)\big), \big(T_\alpha v_1(\varrho), T_\alpha w_1(\varrho)\big)\rangle \\ &= v(\varrho, \varsigma)T_\alpha v_1(\varrho) + w(\varrho, \varsigma)T_\alpha w_1(\varrho) \\ &= v_1(\varrho)T_\alpha v_1(\varrho) + w_1(\varrho)T_\alpha w_1(\varrho). \end{aligned} \tag{23}$$

Similarly, by use of equation 9 and 10 in equation 22, we get

$$\begin{aligned} \langle\big(v(\varrho, \varsigma), w(\varrho, \varsigma)\big), \big(T_\alpha v_2(\varsigma), T_\alpha w_2(\varsigma)\big)\rangle &= \langle\big(v_2(\varsigma), w_2(\varsigma)\big), \big(T_\alpha v_2(\varsigma), T_\alpha w_2(\varsigma)\big)\rangle \\ &= v(\varrho, \varsigma)T_\alpha v_2(\varsigma) + w(\varrho, \varsigma)T_\alpha w_2(\varsigma) \\ &= v_2(\varsigma)T_\alpha v_2(\varsigma) + w_2(\varsigma)T_\alpha w_2(\varsigma). \end{aligned} \tag{24}$$

Thus, from expression equation 23 and equation 24, we get system of equations

$$v(\varrho, \varsigma)T_\alpha v_1(\varrho) + w(\varrho, \varsigma)T_\alpha w_1(\varrho) = v_1(\varrho)T_\alpha v_1(\varrho) + w_1(\varrho)T_\alpha w_1(\varrho)$$

and

$$v(\varrho, \varsigma)T_\alpha v_2(\varsigma) + w(\varrho, \varsigma)T_\alpha w_2(\varsigma) = v_2(\varsigma)T_\alpha v_2(\varsigma) + w_2(\varsigma)T_\alpha w_2(\varsigma). \tag{25}$$

In equation 25, let's take it as

$$\widetilde{\Delta} = \begin{vmatrix} T_\alpha v_1(\varrho) & T_\alpha w_1(\varrho) \\ T_\alpha v_2(\varsigma) & T_\alpha w_2(\varsigma) \end{vmatrix} \neq 0.$$

If this equation 25 tried to be solve according to Cramer's rule, we get

$$\widetilde{\Delta}_1 = \begin{vmatrix} v_1(\varrho)T_\alpha v_1(\varrho) + w_1(\varrho)T_\alpha w_1(\varrho) & T_\alpha w_1(\varrho) \\ v_2(\varsigma)T_\alpha v_2(\varsigma) + w_2(\varsigma)T_\alpha w_2(\varsigma) & T_\alpha w_2(\varsigma) \end{vmatrix}$$

and

$$\widetilde{\Delta}_2 = \begin{vmatrix} T_\alpha v_1(\varrho) & v_1(\varrho)T_\alpha v_1(\varrho) + w_1(\varrho)T_\alpha w_1(\varrho) \\ T_\alpha v_2(\varsigma) & v_2(\varsigma)T_\alpha v_2(\varsigma) + w_2(\varsigma)T_\alpha w_2(\varsigma) \end{vmatrix}.$$

Thereby, from expression $v(\varrho, \varsigma) = \frac{\widetilde{\Delta}_1}{\widetilde{\Delta}}$, we get

$$v(\varrho, \varsigma) = \frac{\begin{vmatrix} v_1(\varrho)T_\alpha v_1(\varrho) + w_1(\varrho)T_\alpha w_1(\varrho) & T_\alpha w_1(\varrho) \\ v_2(\varsigma)T_\alpha v_2(\varsigma) + w_2(\varsigma)T_\alpha w_2(\varsigma) & T_\alpha w_2(\varsigma) \end{vmatrix}}{\begin{vmatrix} T_\alpha v_1(\varrho) & T_\alpha w_1(\varrho) \\ T_\alpha v_2(\varsigma) & T_\alpha w_2(\varsigma) \end{vmatrix}}.$$

Similarly, from expression $w(\varrho, \varsigma) = \frac{\widetilde{\Delta}_2}{\widetilde{\Delta}}$, we get

$$w(\varrho, \varsigma) = \frac{\begin{vmatrix} T_\alpha v_1(\varrho) & v_1(\varrho)T_\alpha v_1(\varrho) + w_1(\varrho)T_\alpha w_1(\varrho) \\ T_\alpha v_2(\varsigma) & v_2(\varsigma)T_\alpha v_2(\varsigma) + w_2(\varsigma)T_\alpha w_2(\varsigma) \end{vmatrix}}{\begin{vmatrix} T_\alpha v_1(\varrho) & T_\alpha w_1(\varrho) \\ T_\alpha v_2(\varsigma) & T_\alpha w_2(\varsigma) \end{vmatrix}}.$$

That is to say, from expression $P(\varrho,\varsigma)=(v(\varrho,\varsigma),w(\varrho,\varsigma))$ we get

$$\begin{aligned}P(\varrho,\varsigma)=&(T_\alpha^2 w_1(\varrho)\\&+T_\alpha^2 v_1(\varrho)\big[(v_2(\varsigma)-v_1(\varrho))T_\alpha v_2(\varsigma)\\&+(w_2(\varsigma)-w_1(\varrho))T_\alpha w_2(\varsigma)\big]^2\\&-(T_\alpha^2 w_2(\varsigma)\\&+T_\alpha^2 v_2(\varsigma))\big[(w_2(\varsigma)-w_1(\varrho))T_\alpha w_1(\varrho)\\&+(v_2(\varsigma)-v_1(\varrho))T_\alpha v_1(\varrho)\big]^2=0.\end{aligned}$$

Furthermore, $P(\varrho,\varsigma)$ provides equation as follow

$$\|P(\varrho,\varsigma)-c_1(\varrho)\|=\|P(\varrho,\varsigma)-c_2(\varsigma)\|.$$

Therefore, provides equation as follow

$$\begin{aligned}&\langle P(\varrho,\varsigma)-m_1(\varrho),P(\varrho,\varsigma)-m_1(\varrho)\rangle\\&=\langle P(\varrho,\varsigma)-m_2(\varsigma),P(\varrho,\varsigma)-m_2(\varsigma)\rangle\end{aligned}$$

or

$$\langle P(\varrho,\varsigma),P(\varrho,\varsigma)\rangle-2\langle P(\varrho,\varsigma),m_1(\varrho)\rangle+\langle m_1(\varrho),m_1(\varrho)\rangle=\langle P(\varrho,\varsigma),P(\varrho,\varsigma)\rangle-2\langle P(\varrho,\varsigma),m_2(\varsigma)\rangle+\langle m_2(\varsigma),m_2(\varsigma)\rangle$$

or

$$\begin{aligned}&2(\langle P(\varrho,\varsigma),m_1(\varrho)\rangle-\langle P(\varrho,\varsigma),m_2(\varsigma)\rangle)\\&=\langle m_2(\varsigma),m_2(\varsigma)\rangle-\langle m_1(\varrho),m_1(\varrho)\rangle\end{aligned}$$

or

$$\begin{aligned}&\langle P(\varrho,\varsigma),m_1(\varrho)-m_2(\varsigma)\rangle\\&=\frac{\langle m_2(\varsigma),m_2(\varsigma)\rangle-\langle m_1(\varrho),m_1(\varrho)\rangle}{2}.\end{aligned}$$

4. Conclusion

In differential geometry, the theory of curves has become a widely studied field. However, recently, using the comformable derivative has provided many scientists different perspectives. In this study, methods for finding functions corresponding to angle bisector curvatures of two plane curves in $\mathbb{R}^2$ are provided using comformable derivatives. We present a geometric interpretation of the multivariable function corresponding to the angle bisector curvatures by using different derivatives. For example; Caputo, Rieman-Liouville and non-local derivative and so.

Author Contributions

The percentages of the authors' contributions are presented below. All authors reviewed and approved the final version of the manuscript.

	Ş.Ö.	M.B.
C	40	60
D	40	60
S	40	60
DCP	40	60
DAI	30	70
L	40	60
W	40	60
CR	40	60
SR	40	60
PM	30	70
FA	40	60

C=Concept, D= design, S= supervision, DCP= data collection and/or processing, DAI= data analysis and/or interpretation, L= literature search, W= writing, CR= critical review, SR= submission and revision, PM= project management, FA= funding acquisition.

Conflict of Interest

The authors declared that there is no conflict of interest.

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