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Hybrid Function Projective Synchronization of Hyperchaotic Financial Systems via Adaptive Control

Vikash [*,1], **Ayub Khan** [α,2] **and Khursheed Alam** [*,3]
[*]School of Basic Sciences and Research, Sharda University, Greater Noida, Uttar Pradesh, 201306, India, [α]Department of Mathematics, Jamia Millia Islamia, New Delhi, India.

ABSTRACT In this manuscript, we establish hybrid function projective synchronization of a new hyperchaotic system using an adaptive control technique with unknown system parameters. In order to prevent either from deriving from participants in the single hyperchaotic financial system, identical master and slave systems are chosen. We design an adaptive controller to achieve global chaos synchronization between these master and slave systems. The synchronization results are based on adaptive control theory and Lyapunov stability theory. Additionally, we outline the basic dynamic characteristics of both hyperchaotic financial systems. Numerical simulations performed in Matlab validate our results excellently.

KEYWORDS
Fundamental dynamical properties
Adaptive control
Synchronization
Chaotic systems
Lyapunov stability theory

INTRODUCTION

In nonlinear science, chaotic dynamics is an interesting area of research that has been a lot of exploration in current decades. Chaotic events have an impact on a wide range of domains, including secure communication, computer science, quantum physics, biological systems, chemical systems, power converters, electrical engineering, psychology, and so on (Chen and G.ed. 1999) .Complex dynamics with unique characteristics, like topological mixing, dense periodic orbits, unusual attractors, broad Fourier transform spectra, limited and fractal motion qualities in phase space, and great sensitivity to beginning circumstances, are characteristics of a hyperchaotic system (Farivar,F. and Teshnehlab 2012). L. M. Pecora and Carroll (1990) established the master-slave idea for synchronization of chaotic systems in 1990.

Given the extensive practical applications of chaotic dynamical systems in the fields mentioned above, numerous theoretical and experimental studies have been conducted on controlling chaos and achieving synchronization (Abd-Elouahab and Wang 2010; Chen, L. and Wu 2011). To be more precise, synchronization of nonlinear dynamical systems allows for a deeper comprehension of collective dynamical behaviour in systems that are physical, chemical, biological, and other. Numerous mathematical, physical, sociological, physiological, and biological systems have been shown to exhibit synchronous behaviour (Koronovskii, A.A. and Hramov 2013).

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[1]vbvikasrohilla@gmail.com
[2]akhan12@jmi.ac.in
[3]khursheed.alam@sharda.ac.in (**Corresponding author**).

Many techniques for controlling and synchronizing, have been designed comparable and non-identical chaotic systems have been developed in an effort to improve ways for chaos management and synchronization. Such methods include backstepping control (Li, S.Y. and Chiu 2012), adaptive control (Khan, A. and Shikha. 2017), linear feedback (Ma, M. and Cai 2012), optimal control (Li, Y. and Li 2013),(Cai, G. and Fang 2013) active control (Kareem, S.O. and Njah 2012), active sliding control (Khan, A. and Prasad 2016), passive control (Motallebzadeh, F. and Cherati 2012), and so on. To derive the controller in these published works, one has to be aware of the values of the system's parameters.

Nevertheless, these factors are frequently unknown in real-world scenarios. Subsequently deriving an adaptive controller, therefore, is an important problem for the control and synchronization of hyperchaotic financial systems with unknown system parameters (Vaidyanathan, S. 2015). For the purpose of synchronizing hyperchaotic financial systems, a number of synchronization techniques have been developed such as complete synchronization (CS) (Chen, H. and Guo 2021), generalized synchronization (GS) (Zheng, Z. and Hu 2000), projective synchronization (PS), and hybrid synchronization (HS) (Wu, X. and Li 2012). Due to its ability to achieve speedier communication with its proportional features There are now two positive Lyapunov exponents (LE) that point to hyperchaotic behaviour. This dissipative hyperchaotic system's mathematical characteristics are shown both theoretically and statistically, including Lyapunov exponents (Al-Azzawi, S and Hasan 2024).

In order to have a thorough understanding of the dynamics of the suggested system, we have examined Hamilton energy and competitive modes at various parameter values. One of the simplest concepts for gaining a deeper knowledge of the dynamics or stability of a chaotic system is Hamilton energy. Research reveals that if one can totally control the energy flow in a chaotic system, one can successfully manage its stability. By handling the Hamilton energy, they can do this. The significance of the Hamilton energy for the formation of nonlinear oscillations has been examined in recent research on chaotic systems (Khan, A. and ALi 2024) projective synchronization has garnered substantial attention and has been thoroughly explored among the numerous types of chaos synchronization (Li., Z. and Xu 2004).

The synchronization of the master and slave systems up to a scaling factor is the defining feature of projective synchronization. Chen and Li have considered the function projective synchronization (FPS), a unique synchronization approach (Chen, Y. and Li 2007) . As opposed to projective synchronization, function projective synchronization (FPS) allows synchronization between the response and drive systems up to a certain scaling function rather than a constant. Projective synchronization (PS) or complete synchronization (CS), respectively, can be achieved by choosing the scaling function to be either a constant or unity. Thus, a more inclusive definition of projective synchronization is function projective synchronization. Function projective synchronization is very helpful for safe communications because of the unpredictable nature of the scaling function, which can further improve communication security.

In more general terms, though, not every element in the vector can synchronize to the required scaling function. All of the vector's scaling functions differ in hybrid function projective synchronization (HFPS) (Ojo, K.S.. and Omeike 2014) which increase complexity and fortifies secure communication even more. The primary benefit of employing the adaptive control technique is that it enables controllers to accomplish drive and response system synchronization without requiring knowledge of parameter values. This method effectively synchronizes the systems with less information needed. The active control technique is used to create synchronization and anti-synchronization between the drive and response systems. Controller design requires parameter values. These days, secure communication is a major concern. Hybrid function projective synchronization (HFPS), as previously mentioned, increases controller complexity and makes it more difficult for hackers to interpret communications. This combination enhances secure communication.

Additionally, most reported research on hybrid function projective synchronization achieve synchronization between two hyperchaotic financial systems that are both part of the unified hyperchaotic financial system. Inspired by the above discussion, in this work, we address the HFPS via adaptive control. Complete synchronization (CS), projective synchronization (PS), anti-synchronization (AS), and hybrid projective synchronization (HPS) are the subcases of hybrid function projective synchronization.

This manuscript organized as: The problem of statements for the hyperchaotic financial system's synchronization are covered in Section 2. In section 3 A description of the hyperchaotic financial system's basic dynamical features is given. Section 4 is succeeded by the hyperchaotic financial system's hybrid function projective synchronization (HFPS) via adaptive control. A numerical simulations and discussions Section 5. Finally, conclusion is delivered in Section 6.

PROBLEM STATEMENT FOR SYNCHRONIZATION OF CHAOTIC SYSTEM

Assume that a hyperchaotic financial system with a state vector is a driving system.
$X_m \in R^n$ and $P \in R^{n\times n}$ is system matrix given by

$$\dot{X_m} = PX_m + f(X_m) \tag{1}$$

Where $f(X_m) : R^n \to R^n$ is the system's nonlinear part. Another highly hyperchaotic financial system can be thought of as a slave system with a state vector. The system matrix $Y_s \in R^n$ and $Q \in R^{n\times n}$ with controller is provided by P

$$\dot{Y_s} = QY_s + g(Y_s) + \sigma(X_m, Y_s) \tag{2}$$

Where $g(Y_s) : R^n \to R^n$ is nonlinear part of the slave system and σ is the adaptive controller added in slave system for synchronization of the systems (1) and (2).

For hybrid function projective synchronization, the error $e \in R^n$ between states X_m and Y_s is defined as:

$$e = Y_s - A(t)X_m \tag{3}$$

Where $A(t) = \text{diag}\,(\eta_1(t), \eta_2(t), \ldots\ldots., \eta_n(t))$ is the diagonal matrix and $\eta_i(t) : R^n \to R(i = 1, 2, \ldots\ldots, n)$ are functions that are bounded and continuously differentiable, $\eta_i(t) \neq 0 \forall t$.

From (1) and (3) error dynamics as:

$$\dot{e} = QY_s + g(Y_s) + \sigma(X_m, Y_s) - PX_m - f(X_m) \tag{4}$$

Therefore, for hybrid function projective synchronization, the goal is to determine the controller $\sigma(X_m, Y_s)$, so that $\lim_{t\to\infty} \|e(t)\| = 0, \forall e \in R^n$.

Remark 1: If $A(t) = \text{diag}\,(\eta_1(t), \eta_2(t), \ldots\ldots\ldots, \eta_n(t))$ where $\eta_i(t) \in R$ are constants, then hybrid function projective synchronization simplifies to hybrid projective synchronization. Furthermore, when all $\eta_i(t)$ are identical, the problem reduces to projective synchronization.

FUNDAMENTAL DYNAMICAL PROPERTIES OF THE SYSTEM

Consider the novel financial system:

$$\begin{aligned}
\dot{x_1} &= {x_3}^2 + (x_2 - a)\,x_1 + x_4 \\
\dot{x_2} &= 1 - bx_2 - {x_1}^2 \\
\dot{x_3} &= -x_1x_2 - cx_3 \\
\dot{x_4} &= -0.05x_1{x_3}^2 - dx_4
\end{aligned} \tag{5}$$

Where the interest rate (x_1), investment demand (x_2), price index (x_3), and average profit margins (x_4) are the four state variables for which the system specifies the temporal evolution. Differentiation with respect to time t is indicated by the dot and $a \geq 0$ the saving amount, $b \geq 0$ the cost per investment, $c \geq 0$ is the elasticity, and $d \geq 0$ is positive systems parameter.

The values of the parameters $a = 0.9, b = 0.2, c = 1.5$ and $d = 0.17$ the Lyapunov exponents are $\lambda_1 = 1.1605, \lambda_2 = 0.6589, \lambda_3 = -0.7145$ and $\lambda_4 = -2.0642$ as shown in Fig 1.
$\sum_{\lambda=1}^{4} \lambda_i = -0.9593 \leq 0$. The considered system is hyperchaotic based on our calculation of the Lyapunov exponent for the system witnessing the two positive Lyapunov exponents. Here two Lyapunov exponents are positive and two are negative, positive Lyapunov exponents shows that system 5 is hyperchaotic.

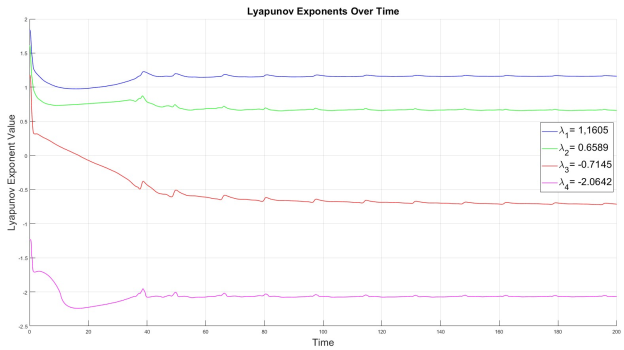

Figure 1 Lyapunov exponents of the 5 hyperchaotic financial system

Lyapunov exponent spectrum

In a dynamical system, the rate at which infinitesimally close paths separate is quantified by the Lyapunov exponent spectrum. The average exponential rate of divergence or convergence in particular directions inside the system's phase space is represented by each of the exponents that make up this representation. When neighbouring trajectories diverge, a positive Lyapunov exponent suggests chaotic activity; conversely, a negative one suggests convergence to a stable point or periodic orbit. Zero exponents frequently imply neutral stability, as in the case of a conservative system's trajectory. The complete range of Lyapunov exponents sheds light on the general stability of the system as well as the characteristics of its attractors.

Bifurcation analysis

Bifurcation analysis as parameter a increases from 0.5 to 2, the system transitions from stable behavior to hyperchaotic oscillations in the interest rate x. For values of parameter b ranging from 0.1 to 0.5, the system exhibits very large fluctuations in the steady state of x reaching magnitudes on the order 10^{12} suggesting instability in the system. As parameter c increases from 1 to 2, the system shows hperchaotic behavior initially, but the fluctuations in x reduce as c increases. Parameter d, ranging from 0.1 to 0.3, leads to hyperchaotic behaviour for the most of its range , with multiple steady-state value of x. The systems remains hyperchaotic.

Dissipation

The divergence of the system 5 is

$$\nabla V = \left(\frac{\partial \dot{x}}{\partial x}\right) + \left(\frac{\partial \dot{y}}{\partial y}\right) + \left(\frac{\partial \dot{z}}{\partial z}\right) + \left(\frac{\partial \dot{w}}{\partial w}\right)$$

$$= -a - b - c - d = -(a + b + c + d) < 0.$$

Since $a, b, c, d \geq 0$, the dynamical system (3.1) is a dissipative system, and

$$\dot{V}(t) = e^{-(a+b+c+d)}.$$

This indicates that as t increases, each volume carrying the trajectory of this dynamical system (3.1) shrinks to zero at an exponential rate of $-(a + b + c + d)$. Hence, the asymptotic motion settles onto an attractor of the new dynamical system (3.1), thereby limiting all of the orbits of the system to a certain subset with zero volume. (Wu, X. and Li 2012).

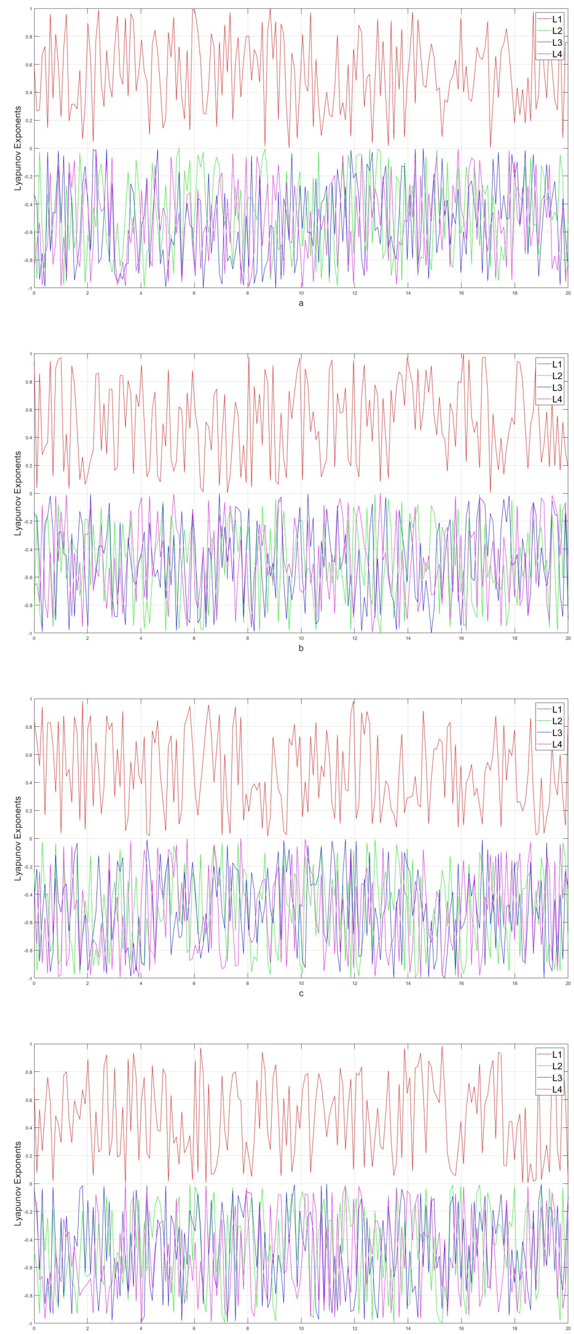

Figure 2 Lyapunov exponent spectrum of the new system 5 and L1 represented largest lyapunov exponent in all cases. (a) represented versus parameter a, (b) represented versus parameter b, (c) represented versus parameter c, (d) represented versus parameter d

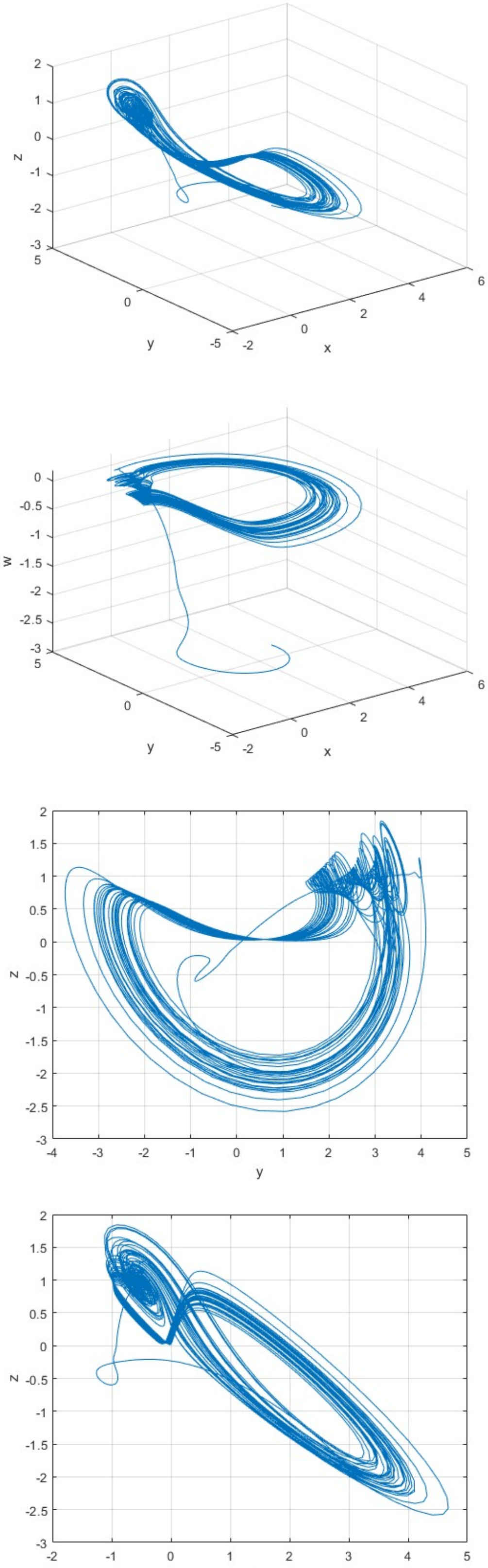

Figure 3 Phase portraits of 5 hyperchaotic financial system (a) in x-y-z space, (b) in the x-y-w space, (c) projection on the y-z plane, and (d) projection on x-z plane

Equilibrium and Stability

The equilibrium of system 5 can be obtained by solving equations:

$$
\begin{aligned}
x_3^2 + (x_2 - a)x_1 + x_4 &= 0 \\
1 - bx_2 - x_1^2 &= 0 \\
-x_1x_2 - cx_3 &= 0 \\
-0.05x_1x_3^2 - dx_4 &= 0
\end{aligned}
$$

system 5 has a trivial equilibrium points $E_0 = (0, 0, 0, 0)$ and two nontrivial equilibrium points $E_1 = (0.9, 0.7, -0.4, -0.05)$, and $E_2 = (2.6, 5, -9.06, -3.53)$. Therefore, E_0 is stable and E_1 and E_2 are unstable equilibrium points.

HYBRID FUNCTION PROJECTIVE SYNCHRONIZATION OF HYPERCHAOTIC FINANCIAL SYSTEM VIA ADAPTIVE CONTROL

Our aim is to achieve hybrid function projective synchronization between master and slave hyperchaotic systems using the method of adaptive control. Is that regard, we consider the master and slave system, follows:

$$
\begin{aligned}
\dot{x_1} &= {x_3}^2 + (x_2 - a)\, x_1 + x_4 \\
\dot{x_2} &= 1 - bx_2 - {x_1}^2 \\
\dot{x_3} &= -x_1x_2 - cx_3 \\
\dot{x_4} &= -0.05x_1{x_3}^2 - dx_4
\end{aligned}
\tag{6}
$$

Where x_1, x_2, x_3 and x_4 are typical profit margins, price index, investment demand, and interest rate. and a, b, c and d are positive parameters. The above system 6 has already been seen as hyperchaotic for the specified values of parameterise.

The slave system is described as:

$$
\begin{aligned}
\dot{y_1} &= y_3^2 + (y_2 - a)\, y_1 + y_4 + u_1 \\
\dot{y_2} &= 1 - by_2 - y_1^2 + u_2 \\
\dot{y_3} &= -y_1y_2 - cy_3 + u_3 \\
\dot{y_4} &= -0.05y_1y_3^2 - dy_4 + u_4
\end{aligned}
\tag{7}
$$

Where y_1, y_2, y_3 and y_4 are typical profit margins, price index, investment demand, and interest rate. and a, b, c and d are positive parameters and u_1, u_2, u_3 and u_4 , continuously differentiable, non-zero scaling functions. The error dynamics is expressed as the derivative of 8 is

$$
e_i = y_i - \eta_i x_i, \quad \text{where } i = 1, 2, 3, 4 \tag{8}
$$

and $\eta'_i s (i = 1, 2, 3, 4)$ are bounded, continuously differentiable, non-zero scaling functions. The error states' time derivative of 8 is

$$
\dot{e_l} = \dot{y_i} - \dot{\eta_l}(t)x_i - \eta_i(t)\dot{x_l} \tag{9}
$$

Using 6 ,7 and 9 we obtain

$$
\begin{aligned}
\dot{e_1} &= y_3^2 + (y_2 - a)\, y_1 + y_4 + u_1 - \dot{\eta_1}(t)x_1 - \eta_1(t)\left(x_3^2 + (x_2 - a)\, x_1 + x_4\right) \\
\dot{e_2} &= 1 - by_2 - y_1^2 + u_2 - \dot{\eta_2}(t)x_2 - \eta_2(t)\left(1 - bx_2 - x_1^2\right) \\
\dot{e_3} &= -y_1y_2 - cy_3 + u_3 - \dot{\eta_3}(t)x_3 - \eta_3(t)\left(-x_1x_2 - cx_3\right) \\
\dot{e_4} &= -0.05y_1y_3^2 - dy_4 + u_4 - \dot{\eta_4}(t)x_4 - \eta_4(t)\left(-0.05x_1x_3^2 - dx_4\right)
\end{aligned}
\tag{10}
$$

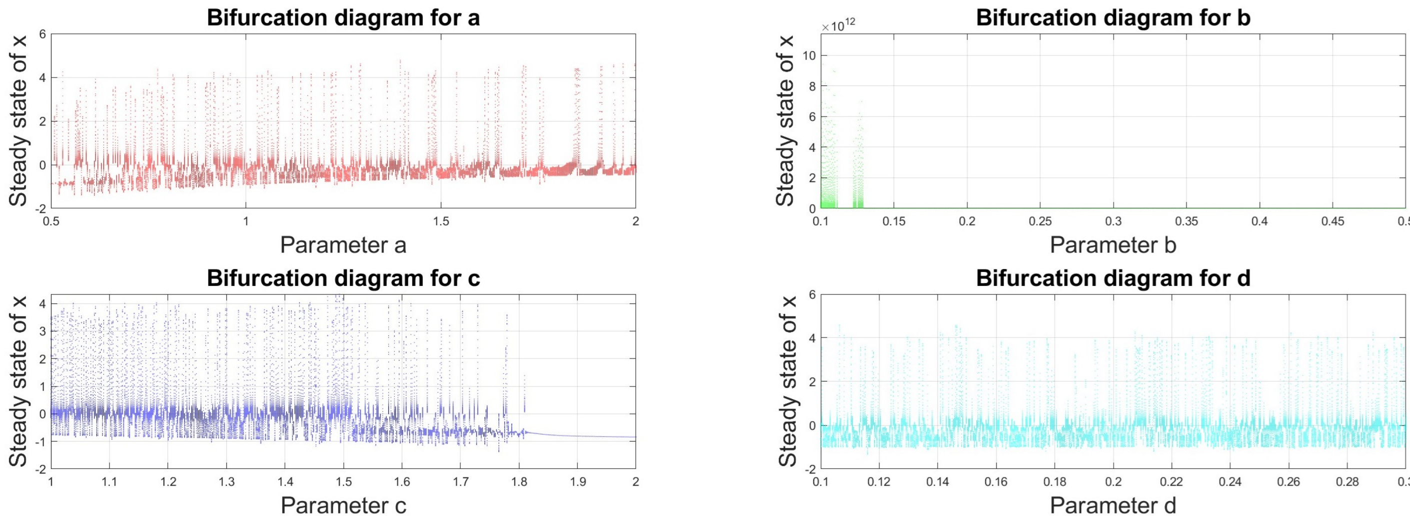

Figure 4 Bifurcation for Parameter a, Chaotic Behaviour with Increasing Parameter a . Bifurcation for Parameter b, Extreme Oscillations and Instability with Parameter b. Bifurcation for Parameter c, Transition to Stability with Increasing Parameter c. Bifurcation for Parameter d, Chaotic Dynamics with Varying Parameter d

To achieve HFPS between master and slave hyperchaotic financial systems with uncertain parameters and arbitrary initial conditions, we need to design appropriate controllers $u_i(t)(i = 1,2,3,4)$ and a parameter update rule. In this way, the unknown parameters will be estimated simultaneously with the assurance that the error dynamical system 10 is asymptotically stable at the origin, and HFPS between the slave system 7 and the master system 6 will be achieved. As a result, for the error dynamical system 10, the synchronization problem is transformed into a stability challenge.
The controller are designed as follows:

$$\begin{aligned} u_1 &= -{y_3}^2 - y_2y_1 + \dot{\eta}_1(t)x_1 + \eta_1(t)x_1 + \eta_1(t)x_2x_1 + \hat{a}e_1 + \eta_1(t)x_4 - k_1e_1 \\ u_2 &= -1 + {y_1}^2 + \dot{\eta}_2(t)x_2 + \eta_2(t) - n_2(t){x_1}^2 + \hat{b}e_2 - k_2e_2 \\ u_3 &= y_1y_2 + \dot{\eta}_3(t)x_3 - \eta_3(t)x_1x_2 + \hat{c}e_3 - k_3e_3 \\ u_4 &= 0.05y_1{y_3}^2 + \dot{\eta}_4(t)x_4 - \eta_4(t)0.05x_1{x_3}^2 + \hat{d}e_4 - k_4e_4 \end{aligned} \tag{11}$$

and the following is how the parameter updating rules are made:

$$\begin{aligned} \dot{\hat{a}} &= -\left(y_1 - \eta_1(t)x_1\right)e_1 - k_5e_a \\ \dot{\hat{b}} &= -\left(y_2 - \eta_2(t)x_2\right)e_2 - k_6e_b \\ \dot{\hat{c}} &= -\left(y_3 - \eta_3(t)x_3\right)e_3 - k_7e_c \\ \dot{\hat{d}} &= -\left(y_4 - \eta_4(t)x_4\right)e_4 - k_8e_d \end{aligned} \tag{12}$$

Where the control gain $k_i > 0(i = 1,2,\ldots,8), \hat{a}, \hat{b}, \hat{c}$, and $\hat{d}$ are the parameters for the estimated variable that are unknown. $e_a = \hat{a} - a, e_b = \hat{b} - b, e_c = \hat{c} - c$, and $e_d = \hat{d} - d$ are corresponding parameter errors. We select the Lyapunov function that satisfies the requirements of Lyapunov stability theory for the parameter update methods designed above. That will demonstrate the stability of the faulty dynamical system and the achievement of the necessary synchronization. However, we select the subsequent Lyapunov function candidate for the error system 10:

$$V(t) = \frac{1}{2}\left({e_1}^2 + {e_2}^2 + {e_3}^2 + {e_4}^2 + {e_a}^2 + {e_b}^2 + {e_c}^2 + {e_d}^2\right) \tag{13}$$

Undoubtedly, $V(t) > 0$. Along the trajectories of the error system 10, the time derivative of V(t) equals

$$\begin{aligned} \dot{V}(t) &= e_1\dot{e_1} + e_2\dot{e_2} + e_3\dot{e_3} + e_4\dot{e_4} + e_a\dot{e_a} + e_b\dot{e_b} + e_c\dot{e_c} + e_d\dot{e_d} \\ \dot{V}(t) &= e_1\left(y_3^2 + (y_2 - a)\,y_1 + y_4 + u_1 - \dot{\eta}_1(t)x_1 - \eta_1(t)\left(x_3^2 + \right.\right. \\ &\left.\left.(x_2 - a)\,x_1 + x_4\right)\right) + e_2\left(1 - by_2 - {y_1}^2 + u_2 - \dot{\eta}_2(t)x_2 - \right. \\ &\left.\eta_2(t)\left(1 - bx_2 - {x_1}^2\right)\right) + e_3\left(-y_1y_2 - cy_3 + u_3 - \dot{\eta}_3(t)x_3 - \right. \\ &\left.\eta_3(t)\left(-x_1x_2 - cx_3\right)\right) + e_4\left(-0.05y_1{y_3}^2 - dy_4 + u_4 - \dot{\eta}_4(t)x_4 - \right. \\ &\left.\eta_4(t)\left(-0.05x_1{x_3}^2 - dx_4\right)\right) + e_a\dot{e_a} + e_b\dot{e_b} + e_c\dot{e_c} + e_d\dot{e_d} \end{aligned} \tag{14}$$

Using 11, 12 and 14, we obtain

$$\begin{aligned} \dot{V}(t) &= e_1\left(e_a\,\left(y_1 - \eta_1(t)x_1\right)\right) - k_1{e_1}^2 + e_2\left(e_b\left(y_2 - \eta_2(t)x_2\right)\right) - k_2{e_2}^2 \\ &+ e_3\left(e_c\left(y_3 - \eta_3(t)x_3\right)\right) - k_3{e_3}^2 + e_4\left(e_d\left(y_4 - \eta_4(t)x_4\right)\right) - k_4{e_4}^2 \\ &+ e_a\left(-\left(y_1 - \eta_1(t)x_1\right)e_1 - k_5e_a\right) + e_b\left(-\left(y_2 - \eta_2(t)x_2\right)e_2 - k_6e_b\right) \\ &+ e_c\left(-\left(y_3 - \eta_3(t)x_3\right)e_3 - k_7e_c\right) + e_d\left(-\left(y_4 - \eta_4(t)x_4\right)e_4 - k_8e_d\right) \\ &= -k_1{e_1}^2 - k_2{e_2}^2 - k_3{e_3}^2 - k_4{e_4}^2 - k_5{e_a}^2 - k_6{e_b}^2 - k_7{e_c}^2 - k_8{e_d}^2 \\ &= eKe < 0 \end{aligned}$$

Where $e = (e_1, e_2, e_3, e_4, e_a, e_b, e_c, e_d)$ and $K = diag\,(k_1, k_2, k_3, k_4, k_5, k_6, k_7, k_8)$.
Based on the Lyapunov stability theory, the error vector e asymptotically converges to zero, meaning that $\lim_{t\to\infty}\|e(t)\| = 0$. since $\dot{V}(t) < 0$ It also suggests that the unknown parameters are approximated simultaneously the hybrid function projective synchronization of the master and slave systems is both globally and asymptotically synchronized.

NUMERICAL SIMULATIONS

To demonstrate the viability and validity of the proposed synchronization technique, numerical simulations are carried out. The selected parameter of the hyperchaotic financial system as $a = 0.9, b = 0.2, c = 1.5$, and $d = 0.17$. The initial condition of master and slave system are chosen as $x(0) = (3, 1, -2, -3)$ and $y(0) = (5, 3, -6, -3)$. The scaling functions $\eta_1 = \sin(t), \eta_2 = 0.5\cos(t), \eta_3 = 1 + \sin(t)$, and $\eta_4 = \cos(0.1t)$ are chosen at random. It is assumed that the control gains are $k_i = 0.11 \forall i = 1, 2, \ldots, 8$. and Figs. 5 and 6 display the outcomes of the simulation. Figure 4 illustrates how the error dynamics approach zero as t approaches infinity. As seen in Figure3 exhibit that values of the unknown parameters also tend to $\hat{a} \to a, \hat{b} \to b, \hat{c} \to c, \hat{d} \to d$ Consequently, the intended hybrid function projective synchronization between the slave and master systems is achieved.

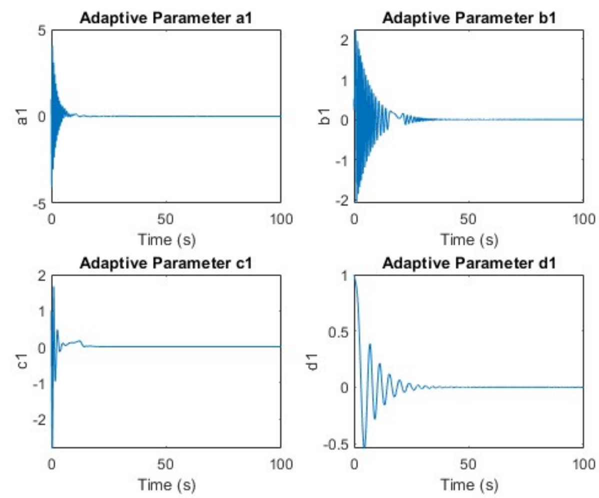

Figure 5 The estimated value of the parameters that are unknown $\hat{a}, \hat{b}, \hat{c}$ and $\hat{d}$ as hybrid function projection synchronization occurs

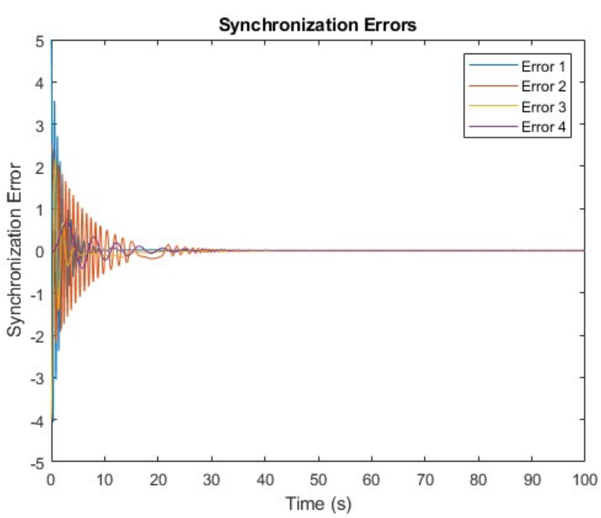

Figure 6 Error in synchronization between the slave and master system states

CONCLUSION

By including more terms and increasing one more variable, average profit margins x_4, based on the chaotic system described in (Wu, X. and Li 2012) is achieved. Both theoretical and numerical analyses are performed on a few fundamental dynamical features, including the Lyapunov exponent spectrum, bifurcations, equilibria, and hyperchaotic dynamical behaviours. This manuscript successfully demonstrates hybrid function projective synchronization (HFPS) of a novel hyperchaotic system utilizing an adaptive control technique, even in the presence of unknown system parameters. The chosen master and slave systems are carefully selected to ensure that they are distinct from any members of the unified chaotic financial system. An adaptive controller is meticulously designed to ensure global chaos synchronization between the master and slave systems. The synchronization is carefully proven using Lyapunov stability theory and adaptive control theory, ensuring theoretical soundness. Additionally, the manuscript provides a detailed analysis of the fundamental dynamical properties of the hyperchaotic financial systems. The effectiveness and accuracy of the proposed synchronization strategy are further confirmed through numerical simulations conducted in MATLAB, validating the theoretical findings and demonstrating practical applicability.

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Availability of data and material

Not applicable.

Conflicts of interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

Ethical standard

The authors have no relevant financial or non-financial interests to disclose.

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