http://www.newtheory.org ISSN: 2149-1402

Journal of New Theory

Received: 27.11.2017 *Year*: **2018**, *Number*: **20**, *Pages*: **57-63**
Published: 10.02.2018 *Original Article*

On Nano πgb-Closed Sets

Ilangovan Rajasekaran[1,*] <sekarmelakkal@gmail.com>
Ochanan Nethaji[2] <jionetha@yahoo.com>

[1]Department of Mathematics,Tirunelveli Dakshina Mara Nadar Sangam College
T. Kallikulam-627 113, Tirunelveli District, Tamil Nadu, India
[2]2/71, West Street, Sangampatti - 625 514, Madurai District, Tamil Nadu, India

Abstaract − In this paper we introduce a new class of sets called nano πgb-closed sets and nano πgb-open sets. We study some of its basic properties.

Keywords − *Nano π-closed set, nano πg-closed set, nano πgp-closed set, nano πgs-closed set and nano πgb-closed set*

1 Introduction

Lellis Thivagar et al. [3] introduced a nano topological space with respect to a subset X of an universe which is defined in terms of lower approximation and upper approximation and boundary region. The classical nano topological space is based on an equivalence relation on a set, but in some situation, equivalence relations are nor suitable for coping with granularity, instead the classical nano topology is extend to general binary relation based covering nano topological space

Recently, Rajasekaran et al. [6, 7, 8] initiated the study nano πg-closed sets and new classes of sets called nano πgp-closed sets and nano πgs-closed sets in nano topological spaces is introduced and its properties and studied.

In this paper, a new class of sets called nano πgb-closed sets in nano topological spaces is introduced and its properties are studied and studied of nano πgb-closed sets.

2 Preliminaries

Throughout this paper $(U, \tau_R(X))$ (or X) represent nano topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset H of a space $(U, \tau_R(X))$, $Ncl(H)$ and $Nint(H)$ denote the nano closure of H and the nano

Corresponding Author.

interior of H respectively. We recall the following definitions which are useful in the sequel.

Definition 2.1. *[5] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.*

1. *The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \bigcup_{x\in U}\{R(x) : R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by x.*
2. *The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \bigcup_{x\in U}\{R(x) : R(x) \cap X \neq \phi\}$.*
3. *The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not - X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.*

Property 2.2. *[3] If (U, R) is an approximation space and $X, Y \subseteq U$; then*

1. $L_R(X) \subseteq X \subseteq U_R(X)$;
2. $L_R(\phi) = U_R(\phi) = \phi$ *and* $L_R(U) = U_R(U) = U$;
3. $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$;
4. $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$;
5. $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$;
6. $L_R(X \cap Y) \subseteq L_R(X) \cap L_R(Y)$;
7. $L_R(X) \subseteq L_R(Y)$ *and* $U_R(X) \subseteq U_R(Y)$ *whenever* $X \subseteq Y$;
8. $U_R(X^c) = [L_R(X)]^c$ *and* $L_R(X^c) = [U_R(X)]^c$;
9. $U_RU_R(X) = L_RU_R(X) = U_R(X)$;
10. $L_RL_R(X) = U_RL_R(X) = L_R(X)$.

Definition 2.3. *[3] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then by the Property 2.2, R(X) satisfies the following axioms:*

1. *U and* $\phi \in \tau_R(X)$,
2. *The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$,*
3. *The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.*

That is, $\tau_R(X)$ is a topology on U called the nano topology on U with respect to X. We call $(U, \tau_R(X))$ as the nano topological space. The elements of $\tau_R(X)$ are called as nano open sets and $[\tau_R(X)]^c$ is called as the dual nano topology of $[\tau_R(X)]$.

Remark 2.4. *[3] If $[\tau_R(X)]$ is the nano topology on U with respect to X, then the set $B = \{U, \phi, L_R(X), B_R(X)\}$ is the basis for $\tau_R(X)$.*

Definition 2.5. *[3] If $(U, \tau_R(X))$ is a nano topological space with respect to X and if $H \subseteq U$, then the nano interior of H is defined as the union of all nano open subsets of H and it is denoted by $Nint(H)$.*

That is, $Nint(H)$ is the largest nano open subset of H. The nano closure of H is defined as the intersection of all nano closed sets containing H and it is denoted by $Ncl(H)$.

That is, $Ncl(H)$ is the smallest nano closed set containing H.

Definition 2.6. *A subset H of a nano topological space $(U, \tau_R(X))$ is called*

1. *nano semi-open [3] if $H \subseteq Ncl(Nint(H))$.*
2. *nano pre-open [3] if $H \subseteq Nint(Ncl(H))$.*
3. *nano regular-open [3] if $H = Nint(Ncl(H))$.*
4. *nano π-open [1] if the finite union of nano regular-open sets.*
5. *nano b-open [4] if $H \subseteq Nint(Ncl(H)) \cup Ncl(Nint(H))$.*

The complements of the above mentioned sets is called their respective closed sets.

Definition 2.7. *[6] The nano π-Kernel of the set H, denoted by $\mathcal{N}\pi$-Ker(H), is the intersection of all nano π-open supersets of H.*

Definition 2.8. *[8] A subset H of a space $(U, \tau_R(X))$ is called a nano strong $\mathcal{B}_Q$-set if $Nint(Ncl(H)) = Ncl(Nint(H))$.*

Definition 2.9. *A subset H of a nano topological space $(U, \tau_R(X))$ is called;*

1. *nano gb-closed set [2] if $Nbcl(H) \subseteq G$ whenever $H \subseteq G$ and G is nano open.*
2. *nano πg-closed [6] if $Ncl(H) \subseteq G$, whenever $H \subseteq G$ and G is nano π-open.*
3. *nano πgp-closed set [7] if $Npcl(H) \subseteq G$, whenever $H \subseteq G$ and G is nano π-open.*
4. *nano πgs-closed set [8] if $Nscl(H) \subseteq G$, whenever $H \subseteq G$ and G is nano π-open.*

The complements of the above mentioned sets is called their respective open sets.

3 On Nano πgb-Closed Sets

Definition 3.1. *A subset H of a space $(U, \tau_R(X))$ is nano πgb-closed if $Nbcl(H) \subseteq G$ whenever $H \subseteq G$ and G is nano π-open.*

The complement of nano πgb-open if $H^c = U - H$ is nano πgb-closed.

Example 3.2. *Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{b, c\}, \{d\}\}$ and $X = \{b, d\}$. Then the nano topology $\tau_R(X) = \{\phi, \{d\}, \{b, c\}, \{b, c, d\}, U\}$.*

1. *then $\{a\}$ is nano πgb-closed set.*
2. *then $\{b, c\}$ is nano πgb-open set.*

Remark 3.3. *For a subset of a space $(U, \tau_R(X))$, we have the following implications:*

$$\begin{array}{ccccc} & & \textit{nano } \pi gp\textit{-closed} & \Leftarrow & \textit{nano } \pi g\textit{-closed} \\ & & \Downarrow & & \Downarrow \\ \textit{nano } b\textit{-closed} & \Rightarrow & \textit{nano } \pi gb\textit{-closed} & \Leftarrow & \textit{nano } \pi gs\textit{-closed} \end{array}$$

None of the above implications are reversible as shown by the following Examples.

Remark 3.4. *A subset H of a space U is nano πgb-closed $\Longleftrightarrow$ $Nbcl(H) \subseteq \mathcal{N}\pi$-$Ker(H)$.*

Remark 3.5. *In a space $(U, \tau_R(X))$, every nano b-closed set is nano πgb-closed.*

Example 3.6. *Let $U = \{a, b, c\}$ with $U/R = \{\{a, c\}, \{b\}\}$ and $X = \{c\}$. Then the nano topology $\tau_R(X) = \{\phi, \{a, c\}, U\}$. Then $\{a, c\}$ is nano πgb-closed but not nano b-closed.*

Proposition 3.7. *In a space $(U, \tau_R(X))$, every nano πgp-closed set is nano πgb-closed.*

Proof. Let H be nano πgp-closed subset of U and G be nano π-open such that $H \subseteq G$. Then $Npcl(H) \subseteq G$. Since every nano pre-closed set is nano b-closed. Therefore $Nbcl(H) \subseteq Npcl(H)$. Hence H is nano πgb-closed.

Example 3.8. *In Example 3.2, then $\{b, c\}$ is nano πgb-closed but not nano πgp-closed.*

Proposition 3.9. *If H is nano π-open and nano πgb-closed, then H is nano b-closed and hence nano gb-closed.*

Proof. Since H is nano π-open and nano πgb-closed. So $Nbcl(H) \subseteq H$. But $H \subseteq Nbcl(H)$. So $H = Nbcl(H)$. Hence H is nano b-closed. Hence nano gb-closed.

Theorem 3.10. *Let H be a nano πgb-closed. Then $Nbcl(H) - H$ does not contain any nonempty nano π-closed set.*

Proof. Let K be a nano π-closed set such that $K \subseteq Nbcl(H) - H$, so $K \subseteq U - H$. Hence $H \subseteq U - K$. Since H is nano πgb-closed and $U - K$ is nano π-open. So $Nbcl(H) \subseteq U - K$. That is $K \subseteq U - Nbcl(H)$. Therefore $K \subseteq Nbcl(H) \cap (U - Nbcl(H)) = \phi$. Thus $K = \phi$.

Corollary 3.11. *Let H be nano πgb-closed set. Then H is nano b-closed $\Longleftrightarrow$ $Nbcl(H) - H$ is nano π-closed.*

Proof. Let H be nano πgb-closed. By hypothesis $Nbcl(H) = H$ and so $Nbcl(H) - H = \phi$, which is nano π-closed.

Conversely, suppose that $Nbcl(H) - H$ is nano π-closed. Then by Theorem 3.10, $Nbcl(H) - H = \phi$, that is $Nbcl(H) = H$. Hence H is nano b-closed.

Theorem 3.12. *If H is nano πgb-closed and $H \subseteq P \subseteq Nbcl(H)$. Then P is nano πgb-closed.*

Proof. Let $P \subseteq G$, where G is nano π-open. Then $H \subseteq P$ implies $H \subseteq G$. Since H is nano πgb-closed, so $Nbcl(H) \subseteq G$ and since $P \subseteq Nbcl(H)$, then $Nbcl(P) \subseteq Nbcl(H) = Nbcl(H)$. Therefore $Nbcl(P) \subseteq G$. Hence P is nano πgb-closed.

Remark 3.13. *In a space $(U, \tau_R(X))$, every nano πgs-closed set is nano πgb-closed.*

Example 3.14. *In Example 3.2, then $\{c, d\}$ is nano πgb-closed set but not nano πgs-closed.*

Theorem 3.15. *For a subset H of U, the following statements are equivalent:*

1. *H is nano π-open and nano πgb-closed.*
2. *H is nano regular-open.*

Proof. (1) $\Rightarrow$ (2) Let H be a nano π-open and nano πgb-closed subset of U. Then $Nbcl(H) \subseteq H$ and so $Nint(Ncl(H)) \subseteq H$ holds. Since H is nano open then H is nano pre-open and thus $H \subseteq Nint(Ncl(H))$. Therefore, we have $Nint(Ncl(H)) = H$, which shows that H is nano regular-open.

(2) $\Rightarrow$ (1) Since every nano regular-open set is nano π-open then $Nbcl(H) = H$ and $Nbcl(H) \subseteq H$. Hence H is nano πgb-closed.

Theorem 3.16. *For a subset H of U, the following statements are equivalent:*

1. *H is nano π-clopen.*
2. *H is nano π-open, nano strong $\mathcal{B}_Q$-set and nano πgb-closed.*

Proof. (1) $\Rightarrow$ (2) Let H be a nano π-clopen subset of U. Then H is nano π-closed and nano π-open. Thus H is nano closed and nano open.

Therefore, H is nano strong $\mathcal{B}_Q$-set. Since every nano π-closed is nano πgb-closed then H is nano πgb-closed.

(2) $\Rightarrow$ (1) By Theorem 3.15, H is nano regular-open. Since H is nano strong $\mathcal{B}_Q$-set, $H = Nint(Ncl(H)) = Ncl(Nint(H))$.Therefore, H is nano regular-closed. Then H is nano π-closed. Hence H is nano π-clopen.

Theorem 3.17. *Let H be a nano πgb-closed set such that $Ncl(H) = U$. Then H is nano πgp-closed.*

Proof. Suppose that H be nano πgb-closed set such that $Ncl(H) = U$. Let G be an nano π-open set containing H. Since $Nbcl(H) = H \cup (Nint(Ncl(H)) \cap Ncl(Nint(H)))$ and $Ncl(H) = U$, we obtain $Nbcl(H) = H \cup Ncl(Nint(H)) = Npcl(H) \subseteq G$. Therefore, H is nano πgp-closed.

Lemma 3.18. *In a space $(U, \tau_R(X))$,*

1. *every nano open set is nano πgb-closed.*

2. *every nano closed set is nano πgb-closed.*

Remark 3.19. *The converses of statements in Lemma 3.18 are not necessarily true as seen from the following Examples.*

Example 3.20. *In Example 3.2,*

1. *then $\{a, b\}$ is nano πgb-closed set but not nano open.*

2. *then $\{a, c\}$ is nano πgb-closed set but not nano closed.*

Theorem 3.21. *In a space $(U, \tau_R(X))$, the union of two nano πgb-closed sets is nano πgb-closed.*

Proof. Let $H \cup Q \subseteq G$, then $H \subseteq G$ and $Q \subseteq G$ where G is nano π-open. As H and Q are nano πgb-closed, $Ncl(H) \subseteq G$ and $Ncl(Q) \subseteq G$. Hence $Ncl(H \cup Q) = Ncl(H) \cup Ncl(Q) \subseteq G$.

Example 3.22. *In Example 3.2, then $H = \{a\}$ and $Q = \{b, c\}$ is nano πgb-closed sets. Clearly $H \cup Q = \{a, b, c\}$ is nano πgb-closed.*

Theorem 3.23. *In a space $(U, \tau_R(X))$, the intersection of two nano πgb-open sets are nano πgb-open.*

Proof. Obvious by Theorem 3.21.

Example 3.24. *In Example 3.2, then $H = \{a, c\}$ and $Q = \{b, c\}$ is nano πgb-open. Clearly $H \cap Q = \{c\}$ is nano πgb-open.*

Remark 3.25. *In a space $(U, \tau_R(X))$, the union of two nano πgb-closed sets but not nano πgb-closed.*

Example 3.26. *In Example 3.2, then $H = \{b\}$ and $Q = \{d\}$ is nano πgb-closed sets. Clearly $H \cup Q = \{b, d\}$ is but not nano πgb-closed.*

Remark 3.27. *In a space $(U, \tau_R(X))$, the intersection of two nano πgb-open sets but not nano πgb-open.*

Example 3.28. *In Example 3.2, then $H = \{a, b\}$ and $Q = \{a, d\}$ is nano πgb-open sets. Clearly $H \cap Q = \{a\}$ is but not nano πgb-open.*

Acknowledgement

The authors express sincere thanks to ***Prof. P. Sundaram****, Former Principal, NGM College, Pollachi, Tamilnadu for his splendid support.*

References

[1] A. C. Upadhya, *On quasi nano p-normal spaces*, International Journal of Recent Scientific Research, 8(6)2017, 17748-17751.

[2] A. D. A. Mary and I. Arockiarani, *Properties of nano GB-closed maps*, IOSR Journal of Mathematics, 11 (2) (II) (2015) 21-24.

[3] M. L. Thivagar and C. Richard, *On Nano forms of weakly open sets*, International Journal of Mathematics and Statistics Invention, 1 (1) 2013, 31-37.

[4] M. Parimala, C. Indirani and S. Jafari, *On nano b-open sets in nano topological spaces*, Jordan Journal of Mathematics and Statistics, 9 (3) (2016) 173-184.

[5] Z. Pawlak, *Rough sets*, International journal of computer and Information Sciences, 11(5) (1982) 341-356.

[6] I. Rajasekaran and O. Nethaji, *On some new subsets of nano topological spaces*, Journal of New Theory, 16 (2017) 52-58.

[7] I. Rajasekaran, T. S. S. Raja and O. Nethaji, *On nano πgp-closed sets*, Journal of New Theory, 19 (2017) 56-62.

[8] I. Rajasekaran and O. Nethaji, *On nano πgs-closed sets*, Journal of New Theory, 19 (2017) 20-26.