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PRESERVICE MIDDLE SCHOOL MATHEMATICS TEACHERS' KNOWLEDGE ABOUT STUDENTS' MATHEMATICAL THINKING RELATED TO PERIMETER AND AREA

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Abstract: The purpose of the current study is to examine preservice middle school mathematics teachers' knowledge about students' mathematical thinking related to perimeter and area and determine the consistency between this knowledge and students' actual mathematical thinking. Case study, one of the qualitative research designs, was used to gain an in-depth understanding of the situation. The study was conducted with four senior preservice middle school mathematics teachers who enrolled in the program of elementary mathematics education at a public university. The data obtained through video recordings from the process of planning, teaching and reflecting on two lessons towards perimeter and area. The videos from teaching were used to identify students' mathematical thinking, difficulties, mistakes and misconceptions whereas the videos from planning and reflecting were used to describe preservice teachers' knowledge of students' mathematical thinking. The data were analyzed through content analysis method. The findings showed that students had lack of knowledge about the meanings of the concepts of perimeter and area, made mistakes related to calculation and use of measurement units. In addition to this, preservice teachers' predictions and expectations about students' mathematical thinking were very limited. Finally, it was observed that there were important differences between students' thinking ways, difficulties, misconceptions and possible mistakes and preservice teachers' expectations and predictions about these issues.
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***Keywords*:** Knowledge of students' thinking, preservice teachers, perimeter and area

Introduction

Geometric concepts such as perimeter, area and surface area have taken important roles in mathematics curricula because of their practical properties and usage in daily lives. However, students are not able to comprehend these concepts (Martin & Strutchens, 2000). Students have various mistakes, difficulties and misconceptions regarding these concepts (Cavanagh, 2007; Zacharos, 2006) due to inabilities in understanding the meaning of them completely (Zacharos, 2006). Research also mentions teachers' and preservice teachers' misunderstanding related to these concepts and their insufficient understanding of students' thinking regarding them (Reinke, 1997; Simon & Blume, 1994). These gaps in their understanding may cause problems in the future because teachers' knowledge affects teaching and student learning (Ball & McDiarmid, 1989). Similarly, Kellogg (2010) advocates that preservice teachers reflect their such misconceptions to their future students and their students might also have them.

When the confusions, difficulties and mistakes of both students and preservice teachers regarding perimeter and area concepts were considered, the importance of planning and teaching based on students' mathematical thinking and investigating preservice teachers' knowledge of it arose. One of the topics in teacher education that was paid more attention is student thinking (Kellogg, 2010). Teachers should know how students think

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mathematically (e.g. the subjects of perimeter and area), what kind of difficulties and misconceptions they may have regarding the subject and what the possible reasons behind these obstacles are (Simon & Blume, 1994).
Several studies highlight that teachers' knowledge about and abilities to anticipate, interpret and use students' thinking in their practices help them to improve their students' achievement (Carpenter, Fennema & Franke, 1996). However, teachers are neither good at predicting students' reactions, difficulties, misconceptions and what they can do (Goldsmith & Seago, 2011; Nathan & Koedinger, 2000a; Kazemi & Franke, 2004), nor in tendency to attend to, anticipate and make sense of students' thinking (Goldsmith & Seago, 2011; Kazemi & Franke, 2004; Wallach & Even, 2005) and incorporate it into the lesson and use it for making instructional decisions (Baş, 2013). If teachers have difficulties in this issue, it is likely that preservice teachers might have similar problems. From this point of view, the purpose of the current study was to examine preservice middle school mathematics teachers' knowledge about students' mathematical thinking related to perimeter and area and determine the consistency between this knowledge and students' actual mathematical thinking. The following research question guided the study:

What is preservice middle school mathematics teachers' knowledge about students' mathematical thinking related to the subjects of perimeter and area?

Method

Case study, which one of the qualitative research methods, was used in the study. Merriam (1998) states that case study is "an intensive, holistic description and analysis of a single instance, phenomenon or social unit" (p. 21). In this study, it was aimed to present preservice teachers' knowledge of students and to what extent their knowledge reflected students' actual mathematical thinking. Four senior preservice middle school mathematics teachers (1 female, 3 male) who enrolled in the program of elementary mathematics education at a public university participated in this study.

The study was conducted under a cyclical process that included phases of planning, teaching and reflecting.

- In the planning phase, preservice teachers endeavored to determine the activities and questions that would be included in the lesson plan, discussed on how students think mathematically, made suggestions on how to respond to them and concluded which materials to use collaboratively. They focused on what kind of solution approaches, mistakes and difficulties might be exhibited by the students.
- In the teaching phase, one of the preservice teachers taught the lesson in a real classroom whereas the other preservice teachers observed the lesson without interacting with the students in the classroom. Preservice teachers took notes on the copy of the lesson plan about the lesson to discuss after implementation together.
- In the reflecting phase, preservice teachers shared their ideas about the effectiveness of the lesson design and discussed on the teaching phase. They evaluated what worked well or not and what need to be changed. They emphasized unexpected responses and made suggestions to revise the lesson plan considering the points that they dwelled on.

This process was repeated for two lessons related to perimeter and area during 4 weeks. Preservice teachers prepared two lesson plans (the former was regarding perimeter and the later was regarding area), conducted two lessons and reflections. The data obtained through video recordings from the process of planning, teaching and reflecting on two lessons towards perimeter and area. The videos from teaching were used to learn students' mathematical thinking, difficulties, mistakes and misconceptions in order to determine the accuracy of preservice teachers' predictions and expectations about students' mathematical thinking. Besides, the videos from planning and reflecting were used to describe preservice teachers' knowledge of students' mathematical thinking. The data were analyzed through content analysis method.

Findings

In terms of students' mathematical thinking, it was found that students confused area and perimeter with each other; they had difficulty in measuring perimeter of a figure with the rope and rule; they did not find the unknown edges to find perimeter; they focused only images without considering the lengths; they confused the properties of rectangle and square; they did not know the meaning of some geometric concepts; they made some calculation mistakes and they used inappropriate measurement units.
Regarding preservice teachers' knowledge on students' mathematical thinking, it was found that there were consistent and inconsistent predictions and lack of knowledge about this issue. Three themes "expected and happened", "expected but not happened" and "not expected but happened" were observed. Preservice teachers

expected some ways of mathematical thinking to come out and they happened as they expected. One of the preservice teachers' comments on the theme of *expected and happened* were as the following:
For example, we prepared exercise sheets including some figures and we wrote numerical values on the edges but we left some edges blank for the students to find themselves. We thought that they might not find them and sum only the numbers that were written. In teaching, I observed that as we expected, students had difficulty in finding the lengths of edges. They could say that opposing edges were the same in a rectangle but they missed out finding and adding the unknown edges to find perimeter. They substantially summed the known edges as calculating the perimeter (see Figure 1). Some students noticed the missing point and emphasized that the unknown edges should be found at first in order to help their friends.

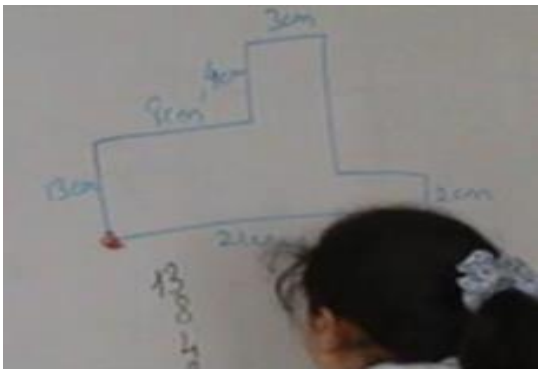

Figure 1. The geometric figure including the unknown edges

Another preservice teacher's explanations about students' mathematical thinking were as follows:
One of the members said that students counted points instead of distance while finding the length of an edge while planning the lesson. Therefore, we decided to use isometric papers and drew figures on them. We asked students to find the lengths of the edges and calculate perimeter of the figures. In teaching, I noticed that the students counted the points instead of the distance to find the lengths of edges on isometric paper and our predictions were correct (see Figure 2). For example, the length of one edge was 7 but the student said that it was 8 because he considered the points and found it as one more.

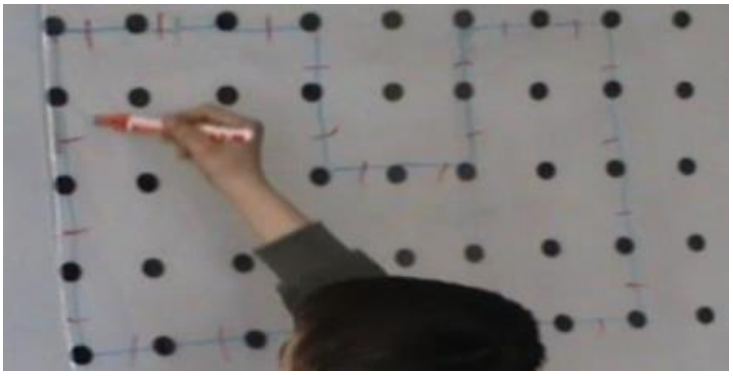
Figure 2. The figure related to calculation of perimeter

Another example addressing expectations and happenings was related to the confusion of perimeter and area. A preservice teacher's reflections on this issue were as below:
When it was asked how to find area of a square, some students said that ax4 or by multiplying one edge by four, namely, perimeter instead of area. I think that it was expected to hear ax4 because they confused perimeter and area and they wanted to make an operation with 4 because all edges of a square were equal. For example, one student found the perimeter of a square by multiplying 7 by 4 as 28 instead of calculating the area multiplying 7 by 7 as 49 (see Figure 3). However, it did not make me surprised because we expected this confusion.

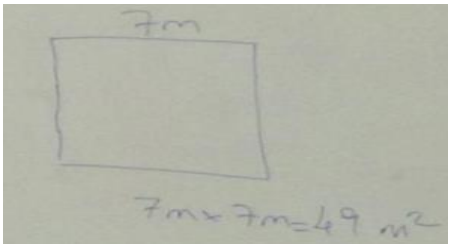

Figure 3. The question regarding area of a square

On the other hand, preservice teachers' predictions were not always consistent with actual mathematical thinking of students. There were situations that preservice teachers expected to occur while planning, however, they did not happen in teaching. One of the preservice teachers' comments on the theme of *expected but not happened* were as the following:

In one activity, we drew four different rectangles which had same perimeter and wanted students to predict perimeter of which figure was bigger. We thought that student might think that they needed to know lengths of all edges to say something about perimeter of the figures. However, there were no students who made comments on this point. It was not as we expected and students gave answers considering only images of the figures such as "perimeter of B is bigger because it is thicker" or "it is A, because it is longer" (see Figure 4).

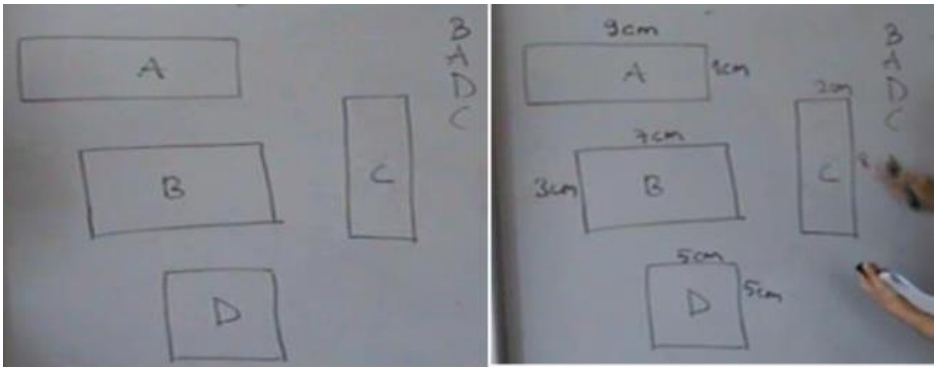

Figure 4. The question regarding different figures with the same perimeter

Another preservice teacher's explanations about students' mathematical thinking were as follows:

In one activity, we wanted the students to cover the same of two rectangles with different two squares to show the need for use of standard measurement units. We thought that students may have difficulty there and they may not understand the reason on why the numbers of squares to cover the rectangles were different. However, students explained the difference between the needed number of squares with statements like "one identical square was bigger than the other", "one square was 1x1 and the other was 4x4" and "their size is not the same", contrary to what we thought (see Figure 5).

Figure 5. The same rectangles with different size squares

Reflections of one of preservice teachers regarding expectations and happenings were as below:

As planning the lesson, we thought that we should make students discover the formula of area from the relationship between column and row, and we also thought that one example was not enough and asked two questions in order to help students generalize the area formula of a rectangle and square. However, they could understand formula at the first example on the contrary what we think and preservice teacher who taught the lesson decided to skip the second examples for both rectangle and square feeling that it would not be necessary. I think he gave the correct decision at this point (see Figure 6).

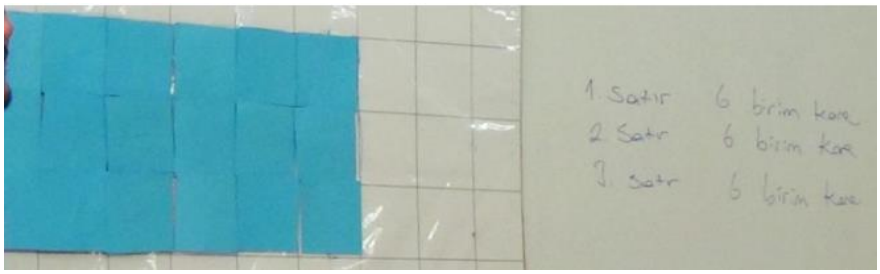

Figure 6. The activity for introducing the area formula of a rectangle and square

Furthermore, preservice teachers did not expect or could not predict correctly students' some mathematical thinking ways while planning the lesson but they arose in teaching. One of the preservice teachers' comments on the theme of *not expected but happened* was as the following:

For example, while students were summing the edges to find perimeter in one question, they confused which edges they wrote or not. Thus, they missed some lengths to include in calculation or they added twice (see Figure 7). We did not expect this confusion and we decided to determining a point on one corner, wanting students to write the edge lengths respectively until they reach this point in order not to be confused while calculating. Moreover, the students could not draw the figure on the isometric paper in their notebooks. It made me surprised because isometric paper and their notebook were similar and I expected them to have no difficulty here.

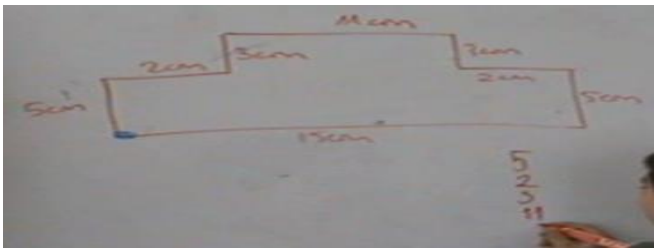

Figure 7. Another figure related to calculation of perimeter

Another preservice teacher's explanations about students' mathematical thinking were as follows:

For example, while drawing a rectangle, if it was similar to a square, students asked whether it was a square. Therefore, we must be careful to draw the figures correctly because they focus on this point. Students should understand long edge is bigger than short edge when they look at it. For example, even though we wrote 3 and 4 on the edges, they perceived it as a square without considering the numerical values and inequality of edges because the lengths of edges look alike in view (see Figure 8). We could not predict students' this approach while planning.

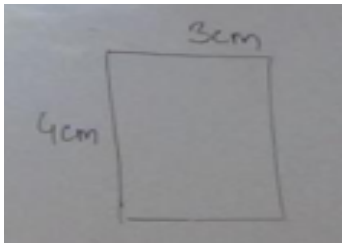

Figure 8. The figure that students perceive as square

Another example addressing expectations and happenings was related to the confusion of representations. A preservice teacher's reflections on this issue were as below:
While giving the formulas of perimeter and area, we did not expect but the letters of l and s confused the students. I think there is no difference between the use of a,b and l,s. On the contrary, l and s was more meaningful because "l" was used to represent the long edge and "s" was used to represent the short edge in line with their first letter. However, students got used to a and b and when they encountered with different representations such as l and s, they could not generalize and transfer knowledge. Therefore, we decided to use a and b to represent edges of rectangle and square (see Figure 9).

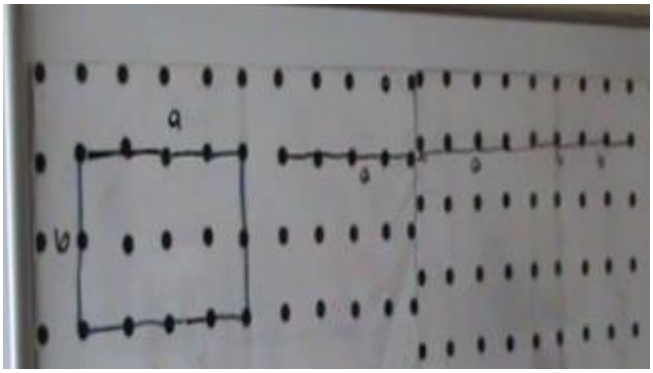

Figure 9. The activity to teach perimeter formula of a rectangle and square

Preservice teachers' some comments also showed their lack of knowledge about students' mathematical thinking and abilities directly. For example, they discussed on whether they should draw half squares in the form of triangle while calculating area of figures. However, they were not sure about the students' ability to complete two half squares to a whole unit square. Therefore, they did not ask this kind of question but they noticed that students could do it in teaching and their questions were easy. Preservice teachers' predictions and expectations about students' mathematical thinking were very low. In teaching phase, they mostly realized that the questions and activities in their lesson plans were very simple. Therefore, in reflecting phase, they discussed on making difficult some of them so that the content of lesson plans would be more convenience in terms of students' levels.

Discussion and Conclusion

The results indicated that there were differences between preservice teachers' expectations and students' actual mathematical thinking. Similarly, many studies also revealed that there were differences between students' mathematical thinking ways, difficulties, previous knowledge, misconceptions and teachers' expectations and predictions about these issues (Baş, Erbaş & Çetinkaya, 2011; Bergqvist, 2005; Hadjidemetriou & Williams, 2002; Nathan & Koedinger, 2000a, 2000b). Some research also showed that teachers and preservice teachers do not understand students' mathematical thinking ways sufficiently and they cannot utilize them during teaching (Kılıç, 2011; Tirosh, 2000).

In planning, it was observed that preservice teachers made incorrect or incomplete predictions about students' thinking (Baş et al., 2011; Bergqvist, 2005; Hadjidemetriou & Williams, 2002; Nathan & Koedinger, 2000a, 2000b). They had difficulty in determining some questions and activities since they were not sure about students' previous knowledge or they could not predict students' reactions. In this case, they underestimated students' potential and it was noticed that some questions were easy or the tasks did not include all necessary concepts. Bergqvist (2005) also indicates that teachers tend to underestimate the students' reasoning levels. The reasons behind preservice teachers' incorrect or incomplete predictions may result from their knowledge or the effects of their own thinking ways (Doerr & Lesh, 2003; Zeytun, Çetinkaya & Erbaş, 2010).

The results showed that students may mathematically think differently from what preservice teachers expected or recognized. It supported the results of the studies in literature (Bergqvist, 2005; Kılıç, 2011; Zeytun et al., 2010; Tirosh, 2000) and revealed that preservice middle school mathematics teachers' knowledge of students' thinking were limited (Didiş, Erbaş, Çetinkaya, Çakıroğlu & Alacacı, 2015). However, it was also found out that the preservice teachers were better in understanding students' mathematical thinking during the process that included planning, teaching-observing and reflecting. Therefore, it is recommended that this kind of process should be integrated into teacher education programs.

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