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Author Contribution Statement

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Abstract

The purpose of this study is to examine middle school mathematics teachers' preferences and performances in using mathematical models in situations involving different fraction schemes and fraction operations. The study, utilizing the case study survey method, involves fifteen mathematics teachers currently working in the Altınordu district of Ordu Province in Turkey. Purposeful sampling methods, including convenience sampling and criterion sampling, were employed in determining the participants of the study. Accordingly, the criteria for selecting teachers in the study were having a minimum of 10 years of professional experience, being stationed in the central district, and volunteering to participate in the study. In this study, the Questionnaire on Model Use Preferences, Open-Ended Questions on Model Use and semi-structured interviews developed by the researchers were used as data collection tools. According to the results of the study, it was observed that the participant teachers generally preferred to use the rectangle model-circle model-number line model and finally the set model when different fraction schemes were considered, and the rectangle model-number line model-circle model and finally the set model when fraction operations were considered. Teachers generally preferred continuous models and did not use discrete models. When the teachers' performances of using models were examined, it was seen that their performance levels were generally adequate except for the cases involving iterative fraction schemes. When the performances for fraction operations were analyzed, it was seen that the teachers generally performed adequately except for multiplication and division operations. In general, teachers used mathematical models not as a tool to support learning, but to complete the tasks assigned to them in the study process. In this context, it can be said that the models used by teachers do not fully include the conceptual meanings and differences related to the current situation in general.

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Research Article

Investigation of Middle School Math Teachers' Pedagogical Preferences towards Using Models in the Context of Different Fraction Schemes *

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Abstract

The purpose of this study is to examine middle school mathematics teachers' preferences and performances in using mathematical models in situations involving different fraction schemes and fraction operations. The study, utilizing the case study survey method, involves fifteen mathematics teachers currently working in the Altınordu district of Ordu Province in Turkey. Purposeful sampling methods, including convenience sampling and criterion sampling, were employed in determining the participants of the study. Accordingly, the criteria for selecting teachers in the study were having a minimum of 10 years of professional experience, being stationed in the central district, and volunteering to participate in the study. In this study, the Questionnaire on Model Use Preferences, Open-Ended Questions on Model Use and semi-structured interviews developed by the researchers were used as data collection tools. According to the results of the study, it was observed that the participant teachers generally preferred to use the rectangle model-circle model-number line model and finally the set model when different fraction schemes were considered, and the rectangle model-number line model-circle model and finally the set model when fraction operations were considered. Teachers generally preferred continuous models and did not use discrete models. When the teachers' performances of using models were examined, it was seen that their performance levels were generally adequate except for the cases involving iterative fraction schemes. When the performances for fraction operations were analyzed, it was seen that the teachers generally performed adequately except for multiplication and division operations. In general, teachers used mathematical models not as a tool to support learning, but to complete the tasks assigned to them in the study process. In this context, it can be said that the models used by teachers do not fully include the conceptual meanings and differences related to the current situation in general.

Keywords: Fraction schemes, mathematical models, middle school mathematics teachers, pedagogical preferences

1. INTRODUCTION

The most abstract concept that students encounter in primary education is the concept of fraction (Newstead & Murray, 1998) and learning fractions is one of the biggest obstacles for students in the process of learning mathematics (Behr et al., 1993). Studies show that students have difficulties in learning the concept of fraction at all levels. These difficulties stem from the mathematical structure of the fraction and the processes related to its teaching (Birgin & Gürbüz, 2009; Soylu & Soylu, 2005; Yazgan, 2007; Yılmaz & Yenilmez, 2008). Another reason is the abstract structure of the concept. Therefore, for meaningful fraction teaching, it should be concretized and supported with different representations and notations. Considering that most of the students, especially at the primary school level, are in the concrete operations period, it is important to use mathematical representations/models to make abstract objects or situations as concrete and understandable as possible.

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According to Yanık (2016), students need to learn conceptual and procedural knowledge in a meaningful and harmonious way in order to fully form the concept of fraction in their minds. However, considering the results of the studies conducted with teachers and pre-service teachers, it was observed that the participants were more successful in procedural knowledge, which is based on rules, than conceptual knowledge (Zembat, 2007). Non-conceptualized knowledge based on rules and memorization will not make sense in the minds of students. For this reason, fractions and operations with fractions should be concretized as much as possible for better understanding. One of the most effective methods that can be used for this is to utilize different representations. In this context, there are many studies revealing the importance of different representations and models in fraction teaching (Cramer & Henry, 2002; Ertem-Akbaş, 2019; Siebert & Gaskin, 2006). Studies (Çiltaş & Işık, 2012; Çiltaş & Yılmaz, 2013; Gümüş et al., 2008) state that the use of models in mathematics teaching gives positive results and support the use of models.

1.1. Mathematical Representation/Model

A review of the literature reveals different uses of the concepts of representation and model in mathematics education. Nemirovsky (1994) refers to mathematical representations as 'symbol systems' and states that representations are used to emphasize a set of similar objects rather than a single object. Similarly, the National Council of Teachers of Mathematics (NCTM) (2000) document uses the term 'representation' for all types of mathematical presentations. The use of the word “model” is broader than the concept of “representation” in mathematics education. A mathematical model can substitute a variety of representational systems, which may include written symbols, spoken languages, pictures or diagrams, concrete manipulatives or metaphors. The distinction between representation and model is that the term model emphasizes the dynamic and interactive properties of the systems being modeled, whereas the term representation draws attention to the objects within these systems. Models tend to refer to the whole system, whereas representations are treated as collections of inertial objects (Lesh & Doerr, 2000).

Niss (1987) defines the concept of model as a system of mathematical concepts and relationships between them to represent real life situations. In the NCTM (2000) document, it is stated that mathematical representations are used in modeling processes. Accordingly, it can be said that representations function as a kind of mathematical model (Yavuz-Mumcu, 2023). When different studies are examined, it is seen that the common emphasis on the use of the concepts of representation and model is that mathematical models are generally used to show mathematical relationships and the properties of systems, while representations are used to draw attention to the properties of mathematical objects; however, depending on the level of education, any type of representation that emphasizes mathematical relationships is called a mathematical model (Van de Walle et al., 2014). When it comes to fraction teaching, different representations used in learning environments on the meaning of fractions are generally called models because they help emphasize the relationships between concepts and conceptual structures. In this context, within the scope of this study, all of the different representations used in teaching fractions were named and used as models.

Doğan-Temur (2011) states that for good fraction teaching, instead of memorizing rules and algorithms, concepts and facts should be used in the context of real-life situations and visualized with the help of different representations. Baykul (2009) emphasizes that especially mathematical models should be used in fraction teaching and states these models as i) area or region model, ii) length (number line) model, iii) set or quantity models. While Alacacı (2014) expresses region, line, cluster and area models as four different ways of representing fractions with concrete models, Van de Walle et al. (2014) express three categories of models for working with fractions. These are i) region/area, ii) length and iii) set/ quantity models. For example, the models used for the fraction number ¾ are given in Figure 1.

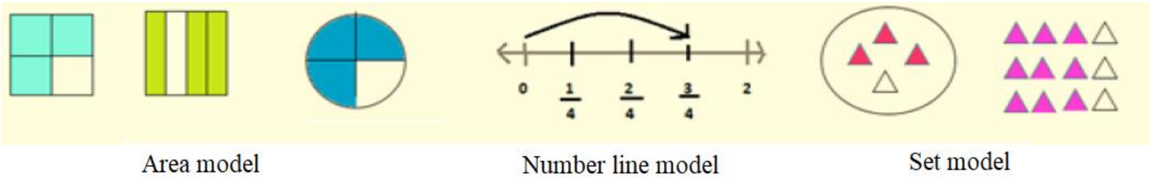


Figure 1. Models used in fraction teaching (Baykul, 2009, p. 239, as cited in Aktaş, 2023)

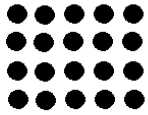
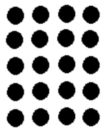
Kieren (1976) categorized fraction models as continuous and discrete. Continuous models include linear, area and volume models in which lengths, two-dimensional regions or three-dimensional objects define reference wholes and parts. For example, long thin rectangles, number lines, circles, rectangles, shapes drawn on a grid or dotted paper, spheres and prisms are continuous models. Discrete models include representations of sets or collections. Concrete manipulatives for teaching and investigating fractions include fraction kits with pre-sectioned circles and squares, fraction bars, geoboards, paper sheets and strips, Cuisenaire bars, pattern blocks, and counting stamps (Cramer et al., 2008). Accordingly, for fractions, the area and length models represent continuous models, while the set model represents discrete models. In fraction representation, a continuous model supports repeated and infinitely varied subdivision of a reference unit, whereas a discrete model supports counting with less emphasis on the reference unit. For example, Kieren (1976) states that the number line model supports the interpretation of rational numbers as a measure, but does not support the multiplication of rational numbers, and that the number line model may cognitively conflict with an area model for generating multiplicative ideas. In this context, it is important to use the models used in fraction teaching in a way that supports conceptual learning in accordance with the characteristics of the subject.

1.2. Fraction Scheme

Steffe and Olive (2010), working on models to be used in fraction teaching, used the concept of ‘scheme’ to analyse the mathematical language and actions used by students and studied various fraction schemes. The concept of fraction schemes is based on Piaget's Constructivism Theory. According to Piaget (1964), the development of knowledge is realised through the formation of a certain structure in the mind as a product of regular interactions in which the individual is involved. In addition, according to Piaget, although the formation of structures is a cognitive process, structures contain more than one scheme. These schemes define the activities that the individual internalises according to different situations based on his/her experiences in the process of structuring the information about a certain concept in the mind (Topcu, 2019). In this context, ‘Fraction Schemes’ can be expressed as a structure that theorises the development of fraction knowledge in terms of conceptual schemes and deals with mental actions such as division, decomposition and iteration. Schemes are investigative structures used to model students' understandings of mathematical situations in order to assimilate and operate within them (von Glasersfeld, 1995). Operations (mental actions) constitute the basic components of schemes. Therefore, Steffe and Olive (2010) made use of these schemes while working on modelling students' fraction understanding.

Steffe and Olive (2010) define three basic operations for fraction schemes. These are *i) partitioning, ii) iterating and iii) disembedding*. Partitioning refers to the mental act of dividing a continuous whole into equal parts (e.g., dividing a candy bar into five equal parts). Iterating refers to the mental act of making related copies of a part (e.g., producing three-fifths of a part given one-fifth). Disembedding refers to the mental act of pulling out a fraction from the whole while keeping the whole intact and unaltered (for example, taking out three-fifths of a stick divided into five equal parts). The fraction schemes related to these operations are given in Table 1.

Table 1. Fraction schemes and mental actions (adapted from Wilkie & Roche, 2023).

Scheme	Related Mental Action	Sample Mathematical Task
Part-Whole Fraction Scheme	Dividing a whole into 'n' parts and separating 'm' of these parts to form 'm/n'	Show 2/5 of the whole below. 
Partitive Unit Fraction Scheme	Determining the size of a unit fraction with respect to a given undivided whole by iterating the unit fraction to produce a continuously divided whole.	What is the length of the short bar below compared to the long bar? (Express as a fraction) 
The Partitive Fraction Scheme	Partitioning a simple fraction to produce a unit fraction and iterating the unit fraction to reproduce the simple fraction and the whole	2/3 of the cake slice below is reserved for you. Draw your slice. 
Reversible Partitive Fraction Scheme	The reproduction of the whole from a given part of the whole by decomposing a given simple fraction to obtain the unit fraction and repeating the resulting unit fraction an appropriate number of times.	The piece below is your friend's chocolate bar and is ¾ of your chocolate bar. Draw your piece of chocolate. 
Iterative Fraction Scheme	Reproducing the whole from a given part of the whole by decomposing the given composite fraction to obtain the unit fraction and repeating the resulting unit fraction an appropriate number of times.	Below is your friend's collection of stickers, which is 4/3 of yours. Draw your own collection. 

Thompson and Saldanha (2003) defined fraction schemes as 'stable ways of thinking that require imagining, connecting, inferring and understanding situations in particular ways' and argued that fractional reasoning is a type of multiplicative reasoning based on a deep understanding of proportionality. Steffe and Olive (2010) explained that schemes are conceptual tools used to analyse students' mathematical language and actions. In most of the studies on fraction schemes, the theoretical framework proposed by Steffe and Olive is used (Topcu & Gürefe, 2020). In this study, these schemes were used and the models used in fraction teaching were examined within the framework of the related schemes.

1.3. Rationale for the Study

The representations used in the teaching of mathematical concepts are very important in teaching the different meanings of the concept to the students and teaching it in relational ways. Different representations and models used in teaching the concept of fraction, which is one of the concepts that students have difficulty in learning due to its abstract structure and which is taught over a long period of time starting from primary school years, are important in terms of ensuring meaningful learning (Alacacı, 2014; Baki, 2014). Researches (Çiltaş & Işık, 2012; Çiltaş & Yılmaz, 2013; Kutluca & Kaya, 2023) state that the use of models in mathematics teaching gives positive results and support the use of models. However, some studies show that teachers and pre-service teachers lack conceptual knowledge about mathematical models and modelling (Akgün et al., 2013; Deniz & Akgün, 2017; Jung et al., 2019). The results of the limited number of studies on mathematics teachers' use of models in fraction teaching are similar. These studies generally reveal that mathematics teachers do not use mathematical models much in fraction teaching and that they do not have sufficient knowledge although they have a positive attitude towards using these models (Akgün et al. 2013; Bayazıt et al., 2011; Çelik & Çiltaş, 2015; Gökkurt et al., 2012; Gökkurt et al., 2015; Tekin-Dede & Bukova-Güzel, 2013). Although scientific studies (Behr et al., 1993; Çiltaş & Işık, 2012; Çiltaş & Yılmaz, 2013) reveal that using models/representations in fraction teaching has a positive effect on learning and emphasise that models/ representations should be used in these processes, the inadequacy of mathematics teachers' knowledge and skills on the subject points to a problem situation that needs to be prevented.

In order to contribute to the solution of this problem, this study aims to examine the pedagogical preferences of mathematics teachers regarding the use of models in fraction teaching. In this context, the 'fraction scheme' theory, which is a special theory put forward for the conceptualisation of fraction in the mind, was used and it was tried to reveal the models used by the teachers in teaching the fraction concept and the level of use of these models. The point that distinguishes this study from the studies in the literature is that it deals with the mathematical models used in fraction teaching in the context of different fraction schemes and fraction operations. Within the scope of the research, there is an analysis process for almost every situation in the teaching of the fraction concept. When the studies in the literature are examined, it is seen that these studies focus on limited situations related to the fraction concept (such as fraction operations or only division). Therefore, it can be said that this study is different from the studies in the literature and will fill a gap. In the light of the results obtained from the study, it was tried to determine the existing problems regarding the use of these models and to put forward suggestions for mathematics education.

1.4. Purpose of the Study

The aim of this study is to investigate the pedagogical preferences and performances of middle school mathematics teachers towards using models in situations involving different fraction schemes and fraction operations. Within the scope of the study, the answers to the following problems were investigated.

- Which mathematical models do middle school mathematics teachers prefer to use in situations involving different fraction schemes and fraction operations?
- At what level do middle school mathematics teachers use mathematical models in situations involving different fraction schemes and fraction operations?

2. METHOD

In this study, case study survey research design (case study survey method) was used. Case study survey research is defined as a research design in which a questionnaire is administered to a small sample or sample group to describe an aspect or characteristic of the individuals in the group. Researchers ask questions of individuals in the population to examine their personal expressions of opinion, behaviour, ability, belief or knowledge. The responses obtained are analysed to identify the tendencies of the group or to test questions or hypotheses (Mills et al., 2009). In this study, this method was preferred and used because the participant teachers' preferences for using mathematical models and their model use processes were analysed with the help of a questionnaire and open-ended questions.

2.1. Participants

The participants of this study consisted of fifteen mathematics teachers working in the central district of Ordu province. In determining the participants of the study, convenience sampling and criterion sampling methods were used together (Patton, 1987). In the convenience sampling method; depending on conditions such as time, money and location, the sample is selected in accordance with the favourable conditions. In the criterion sampling method, people who meet the criteria constitute the sample of the research. The basis of this method is to study all situations that meet a predetermined set of criteria. Since the teachers in the study were selected from the close environment of the researcher based on the principles of accessibility, time, budget and workforce and by considering certain criteria, these methods were used together. The criteria were determined as having at least 10 years of professional experience, working in the central district and volunteering to take part in the study. The real names of the teachers participating in the study were kept confidential and the names of the teachers were coded as T1, T2, ... T15. Information on teachers' gender, professional experience and educational status is given in Table 2.

Table 2. Demographic characteristics of participating teachers

Teacher	*Gender*	*Professional experience*	*Educational Status*
T1	Female	14	Bachelor's degree
T2	Male	14	Bachelor's degree
T3	Male	17	Bachelor's degree
T4	Female	10	Bachelor's degree
T5	Female	12	Master's degree
T6	Female	14	Bachelor's degree
T7	Male	17	Bachelor's degree
T8	Male	22	Bachelor's degree
T9	Female	15	Bachelor's degree
T10	Male	22	Bachelor's degree
T11	Female	13	Bachelor's degree
T12	Male	21	Bachelor's degree
T13	Male	17	Bachelor's degree
T14	Female	17	Bachelor's degree
T15	Female	16	Bachelor's degree

2.2. Data Collection Tools

In this study, a Likert-type questionnaire and open-ended questions developed by the researchers and semi-structured interviews were used as data collection tools.

2.2.1. Questionnaire on model use preferences (QMUP)

The questions in the QMUP used within the scope of the study were developed by the researcher in order to examine the participant teachers' preference for mathematical models for different fraction schemes and fraction operations. There are a total of 13 five-point Likert-type questions in the QMUP. The questions in this questionnaire were developed based on Olive and Steffe (2002) and Wilkie and Roche (2023). In order to determine the validity of the questions, the expert opinions of four people consisting of two faculty members and two teachers were consulted. For the consistency of the related data collection tool, a pilot study was conducted, and the prepared questions were applied to five mathematics teachers outside the participants. After the pilot study, the question wording of some items was revised to make them more comprehensible.

2.2.2. Open-ended questions on model use (OEQs)

The open-ended questions used in the study were designed to examine the participant teachers' use of mathematical models for different fraction schemes and fraction operations. There are a total of 13 open-ended questions in the OEQs. The studies by Olive and Steffe (2002) and Wilkie and Roche (2023) were utilised in the creation of these questions in a similar way to the QMUP, and expert (same experts with the QMUP) opinions were consulted to determine the validity of the questions in the OEQs. For the reliability of the related data collection tool, a pilot study was conducted, and the open-ended questions were applied to five mathematics teachers outside the participants. After the pilot study, adjustments were made in the question wording of some items.

2.2.3. Semi-structured Interviews

Another data collection tool used in this study is semi-structured interviews. These interviews were used to analyze the participant teachers' responses to the open-ended questions in the OEQ in detail. During the interview process, the reasons for the teachers' answers were tried to be determined and voice recordings were taken to store the data obtained.

2.3. Data Collection Process

In order to examine which models the teachers used in fraction teaching, they were firstly administered the QMUP. Teachers were given 15 minutes for this application. After that, in order to examine how they used these models, OEQs was applied to the teachers and no time limitation was made in this process. Teachers' responses were taken in written form. After the whole process, teacher responses were analysed by the researchers and semi-structured interviews were conducted with each teacher in sessions planned for a different time. Similarly, no time limit was set in these interviews and the reasons for the teachers' answers to the open-ended questions were tried to be revealed.

2.4. Analysing the Data

In this study, descriptive analysis method was used to analyse the responses of the teachers to the survey questions and different types of responses were examined and reported according to their frequencies. Accordingly, teachers' preferences for using models in situations involving different fraction schemes and fraction operations were expressed and interpreted by calculating the frequencies of positive ('always' and 'most of the time') and negative ('rarely' and 'never') statements in the questionnaire form. For example, it was accepted that the teachers who marked the 'always' and 'most of the time' options for the part-whole scheme were teachers whose pedagogical preferences for the related scheme were positive. Similarly, the teachers who marked 'never' and 'rarely' options in the questionnaire form for the same scheme were accepted to be teachers with negative pedagogical preferences. Accordingly, for the number of teachers who had positive preferences for using mathematical models, the frequency values for the total number of teachers who marked the 'always' and 'most of the time' options were summed up and expressed as 'subtotal frequency'. Similar processes were carried out for negative preferences, and teacher preferences for different types of fraction schemes and fraction operations were evaluated in three different categories: 'preferences in the positive direction', 'preferences in the negative direction' and preferences for the 'sometimes' option.

In analyzing the responses of the teachers in this study to the questions in the OEQ, content analysis method and evaluation criteria developed by the researchers were used. In order to create these criteria, firstly, the interview processes conducted with all participants were listened to and transcribed. Then, the relevant processes were coded by both researchers as 'adequate', 'partially adequate' and 'inadequate'. This process was carried out separately with a faculty member who is an expert in mathematics education and all coders worked together to reach a common idea in the process of creating evaluation criteria. The inter-coder reliability coefficient calculated in this process was 0.64 at the beginning of the process and 0.83 at the end of the process. In line with the criteria obtained, the responses of the participant teachers were coded by the researchers and expressed in the findings section with frequency values.

3. FINDINGS

3.1. Findings from the Questionnaire on Model Use Preferences (QMUP)

Some of the findings obtained as a result of the analyses carried out in order to determine the mathematical models preferred by the teachers are given below. Depending on the fact that this study is derived from the first author's master's thesis, Author (20..) study can be examined for the complete findings.

3.1.1. Preferences for part-whole fraction scheme

The findings regarding the mathematical models preferred by the teachers in cases involving the part-whole fraction scheme are given in Table 3.

Table 3. Findings related to the part-whole fraction scheme

Teacher preferences	*Circle model*	*f*	*Rectangle model*	*f*	*Number line model*	*f*	*Set model*	*f*
Always	T2, T14	2	T2, T3, T13	3	T2	1	-	0
Most of the time	T4, T10	2	T1, T4, T5, T 6, T7, T8, T9, T10, T11, T15	10	T13	1	-	0
SF		4		13		2		0
Sometimes	T1, T3, T5, T6, T7, T8, T11, T12, T13, T15	10	T12, T14	2	T3, T5, T6, T11, T12	5	T12	1
Rarely	T9	1	-	0	T1, T8, T9, Ö T10, T14, T15	6	T4, T5, T9, T13	4
Never	-	0	-	0	T4, T7	2	T1, T2, T3, T6, T7, T8, T10, T11, T14, T15	10
SF		1		0		8		14

SF: Subtotal frequency

According to the data in Table 3, it was seen that the highest frequency related to the circle model belongs to the 'sometimes' option, and the expressions with high frequencies after this belong to the positive and negative expressions respectively. Therefore, it can be said that the participant teachers' preferences for the circle model are generally in the sometimes option, in other words, teacher preferences do not have a positive or negative tendency. When the preferences for the rectangle model are analysed, it is seen that the frequency of positive preferences is significantly higher than the other responses. The following responses belong to the 'sometimes' option. When the teacher responses related to the number line model are analysed, it is seen that the frequency of the responses belong to negative expressions is higher than the other responses, and the responses after this belong to 'sometimes' and positive expressions respectively. When the teacher responses related to the set model were analysed, it was determined that the frequency of negative statements was significantly higher than the other options, and the following responses belong to the 'sometimes' option. Therefore, it can be said that teachers generally prefer the rectangle model for the part-whole fraction scheme. The following preferences are circle model, number line model and set model respectively. It can be said that teachers never prefer the set model in situations involving the part-whole fraction scheme.

3.1.2. Preferences for iterative fraction scheme

The findings regarding the mathematical models preferred by the teachers in this study in cases involving iterative fraction scheme are given in Table 4.

Table 4. Findings related to addition with fractions

Teacher preferences	*Circle model*	*f*	*Rectangle model*	*f*	*Number line model*	*f*	*Set model*	*f*
Always	T2	1	T2, T7, T14, T15	4	-	0	-	0
Most of the time	T6, T9, T13	3	T1, T3, T8, T9, T11	5	T3	1	T4, T5, T8, T10, T12, T13	6
SF		4		9		1	-	6
Sometimes	T8	1	T5, T6	2	T5, T8, T11	3	-	0
Rarely	T4, T5, T12, T14	4	T4, T12, T13	3	T2, T6, T12, T13, T15	5	T1, T9, T15	3
Never	T1, T3, T7, T10, T11, T15	6	T10	1	T1, T4, T7, T9, T10, T14	6	T2, T3, T6, T7, T11, T14	6
SF		10		4		11		9

SF: Subtotal frequency

According to the data in Table 4, it is seen that the highest frequency of the circle model belongs to the negative statements, followed by the statements with high frequencies belonging to the positive statements and the 'sometimes' option, respectively. When the preferences for the rectangle model are analyzed, it is seen that the frequency of positive preferences is higher than the other responses. The following responses belong to negative statements and 'sometimes' option, respectively. When the teacher responses for the number line model were analyzed, it was determined that the frequency of the responses belonging to negative expressions was significantly higher than the other responses, and the responses after this belong to 'sometimes' and positive expressions, respectively. When the teacher responses related to the set model are analyzed, it is seen that the frequency of negative statements is higher than the other options, and the following responses belong to positive statements. Therefore, it can be said that teachers generally prefer the rectangle model for iterative fraction schema. The following preferences are the set model, circle model and number line model, respectively. It can be said that teachers rarely or never prefer the number line model in situations involving iterative fraction scheme.

3.1.3. Preferences for subtraction with fractions

The findings regarding the mathematical models preferred by the teachers in situations involving subtraction with fractions are given in Table 5. According to the data in Table 5, it was seen that the highest frequency related to the circle model belongs to the “sometimes” option, while the frequencies of positive and negative statements were equal. Therefore, it can be said that participant teachers' preferences for the circle model are generally in the sometimes option, in other words, teacher preferences do not have a positive or negative tendency.

Table 5. Findings related to subtraction with fractions

Teacher preferences	*Circle model*	*f*	*Rectangle model*	*f*	*Number line model*	*f*	*Set model*	*f*
Always	T2	1	T1, T2, T3, T5, T9, T14	6	T2, T4, T5, T8	4	-	0
Most of the time	T4, T6, T12	3	T4, T6, T7, T8, T10, T11, T13, T15	8	T3, T6, T9, T10, T13	5	-	0
SF		4		14		9	-	0
Sometimes	T1, T5, T7, T8, T9, T13, T15	7	T12	1	T11, T14, T15	3	-	0
Rarely	T3, T10, T14	3	-	0	T1, T12	2	T5, T15	2
Never	T11	1	-	0	T7	1	T1, T2, T3, T4, T6, T7, T8, T9, T10, T11, T12, T13, T14	13
SF		4		0		3		15

SF: Subtotal frequency

When the teacher responses regarding the rectangle model are analyzed, it is seen that almost all of the teachers have positive preferences. When the preferences for the number line model were analyzed, it was seen that the frequency of positive preferences was higher than the other responses, while the frequencies of 'sometimes' and negative statements were equal. When the teacher responses related to the set model were analyzed, it was determined that all of the preferences belong to negative expressions. Therefore, it can be said that teachers generally prefer the rectangle and number line models to show subtraction with fractions. The next preference is the circle model. It can be said that teachers generally don’t prefer the set model in situations involving subtraction with fractions.

3.1.4. General findings obtained from teachers' preferences for using models

In this section of the study, the general findings obtained from the QMUP are presented. Table 6 presents the participant teachers' preferences for different fraction schemes and fraction operations respectively.

Table 6. General findings from the QMUP

	Different situations	*1st choice*	*2nd choice*	*3rd choice*	*4th choice*
Preferences for Different Fraction Schemes	Part-Whole Fraction Scheme	*Rectangle model*	*Circle model*	*Number line model*	*Set model*
	Partitive Unit Fraction Scheme	*Rectangle model*	*Circle model*	*Number line model*	*Set model*
	The Partitive Fraction Scheme	*Rectangle model*	*Circle model*	*Number line model*	*Set model*
	Reversible Partitive Fraction Scheme	*Rectangle model*	*Circle model*	*Set model*	*Number line model*
	Iterative Fraction Scheme	*Rectangle model*	*Set model*	*Circle model*	*Number line model*
Preferences for Fraction Operations	Comparing Fractions	*Rectangle model*	*Number line model*	*Circle model*	*Set model*
	Showing Equivalent Fractions	*Rectangle model*	*Circle model*	*Number line model*	*Set model*
	Showing Compound Fractions	*Rectangle model*	*Circle model*	*Number line model*	*Set model*
	Showing the Sum of Two Fractions	*Rectangle model*	*Number line model*	*Circle model*	*Set model*
	Addition with Fractions	*Rectangle model*	*Number line model*	*Circle model*	*Set model*
	Subtraction with Fractions	*Rectangle model*	*Number line model*	*Circle model*	*Set model*
	Multiplication with Fractions	*Rectangle model*	*Number line model*	*Circle model*	*Set model*
	Division by Fractions	*Rectangle model*	*Number line model*	*Circle model*	*Set model*

When Table 6 is examined, it is seen that the teachers in the study made similar preferences for all other situations except for the reversible partitive and iterative fraction schemes. Accordingly, all teachers preferred the rectangle model-circle model-number line model and finally the set model, respectively. In the cases involving reversible partitive fraction schemes, the first two preferences of the teachers did not change and were rectangle and circle models, respectively; however, the third preference was the set model instead of the number line model, and the last preference was the number line model. In cases involving iterative fraction schemes, teachers' first preference was the rectangle model, their second preference was the object model, their third preference was the circle model, and their last preference was the number line model.

When the preferences for fraction operations are analyzed, it is seen that teachers preferred the rectangle-number line-circle and set models in all fraction operations, except for showing equivalent fractions and compound fractions, respectively. Only in the two different cases mentioned here, teachers preferred the circle model instead of the number line as the second choice.

3.2. Findings from Open-Ended Questions on Model Use (OEQs)

Some of the findings obtained from the open-ended questions in this study are given below.

3.2.1. Findings from Task 1 for the part-whole scheme

The first of the open-ended questions used in this study involved the part-whole scheme and was named Task-1. In this task, teachers were given a mathematical model of a whole and asked to model two-fifths of the same whole. Teachers completed this task by using the mathematical model that they primarily preferred according to their responses to the QMUP. Accordingly, the findings obtained from the participant teachers' responses for Task 1 are given in Table 7.

Table 7. Findings from Task 1

Teacher	*Model used*	*Level of model use*
T1	Rectangle	Adequate
T2	Circle- Rectangle -Number line	Adequate- Adequate - Adequate
T3	Rectangle	Adequate
T4	Circle- Rectangle	Adequate - Adequate
T5	Rectangle	Adequate
T6	Rectangle	Adequate
T7	Rectangle	Adequate
T8	Rectangle	Adequate
T9	Rectangle	Adequate
T10	Circle - Rectangle	Adequate - Adequate
T11	Rectangle	Adequate
T12	Rectangle	Adequate
T13	Rectangle	Adequate
T14	Circle	Adequate
T15	Rectangle	Adequate

When Table 7 is analysed, it is seen that the level of using models in line with the preferences made by all teachers in the use of part-whole scheme is at an adequate level. There were no teachers who showed partially adequate or inadequate performance for this task.

3.2.1.1. Task 1-Sample case of adequate model use

The answers given by the teachers for Task 1 and coded as 'adequate' are exemplified below.

Example Case 1:

Below is the task and response of the teacher coded T12 who used the rectangle model.

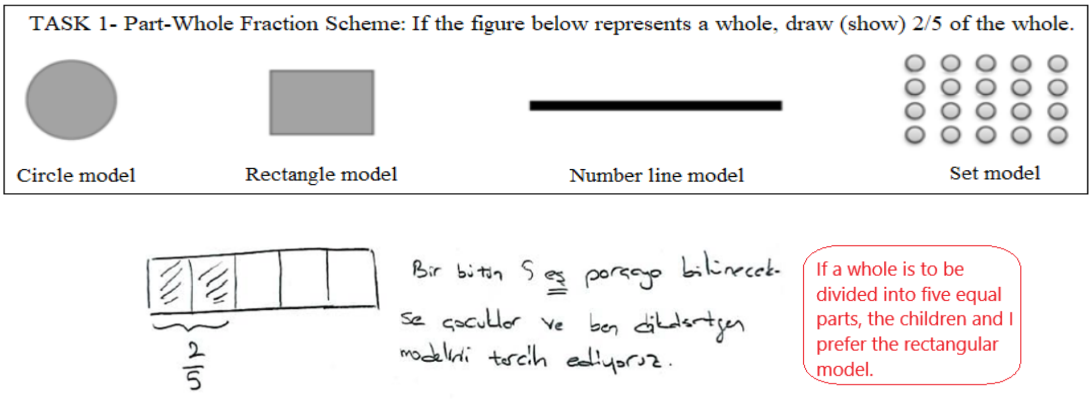


Figure 2. Teacher T12's response to the first question

When Figure 2 is analysed, it is seen that the teacher used mathematical models correctly. Therefore, the related response was coded as adequate.

3.2.2. Findings from Task 5 for the iterative fraction scheme

The fifth of the open-ended questions used within the scope of this study included the iterative fraction scheme and was named as Task-5. In this task, the whole of a fraction modelled as 4/3 is required. The teachers completed this task by using the mathematical model they preferred according to their responses to the QMUP. Accordingly, the findings obtained from the participant teachers' responses for Task-5 are given in Table 8.

Table 8. Findings from Task 5

Teacher	*Model used*	*Level of model use*
T1	Rectangle	Partially adequate
T2	Circle- Rectangle	Partially adequate- Partially adequate
T3	Rectangle	Adequate
T4	Set	Adequate
T5	Set	Adequate
T6	Circle	Adequate
T7	Rectangle	Partially adequate
T8	Rectangle	Partially adequate
T9	Circle - Rectangle	Partially adequate - Adequate
T10	Set	Adequate
T11	Rectangle	Adequate
T12	Rectangle	Partially adequate
T13	Circle	Partially adequate
T14	Rectangle	Partially adequate
T15	Rectangle	Partially adequate

When Table 8 is analysed, it is seen that the level of using models in line with the preferences of all teachers in the use of iterative fraction schema is at adequate and partially adequate levels. Here, it is seen that seven of the 17 different performances are at adequate level and 10 of them are at partially adequate level.

3.2.2.1. Task 5-Sample case of partially adequate model use

Example Case 12:

Below is the task and response of the teacher coded T5 who preferred the rectangle model.

> TASK 5-Iterative Fraction Scheme: Using the model you chose in survey question 5, show the whole fraction given as 4/3 (model 4/3 of the fraction and the whole fraction separately).

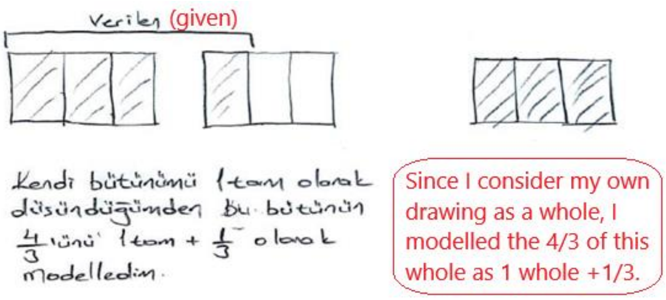


Figure 3. Teacher T15's response to the fifth question

The answer given by the teacher coded T15 to the fifth question is given above. It is seen that the teacher preferred the rectangle model. T15 chose the rectangle model for this task, and instead of determining a whole and dividing it into four equal parts to show the whole of a given fraction, he drew two wholes, divided them into three equal parts and coloured a total of four of them. He completed the task by showing the whole formed by three parts. It was observed that the teacher could partially associate the model she chose with the given operation and could not use it completely correctly. Therefore, her response was coded as partially adequate.

3.2.3. Findings from Task 11 for subtraction with fractions

The eleventh of the open-ended questions used within the scope of this study included subtraction with fractions and was named as Task-11. In this task, teachers were asked to perform the operation ½ -1/3. Teachers completed this task by using the mathematical model they preferred according to their responses to the QMUP. Accordingly, the findings obtained from the participant teachers' responses for Task-11 are given in Table 9.

Table 9. Findings from Task 11

Teacher	*Model used*	*Level of model use*
T1	Rectangle	Adequate
T2	Circle- Rectangle	Adequate- Adequate
T3	Rectangle	Adequate
T4	Number line	Inadequate
T5	Rectangle - Number line	Adequate -Partially Adequate
T6	Circle- Rectangle -Number line	Inadequate- Adequate - Inadequate
T7	Rectangle	Adequate
T8	Number line	Inadequate
T9	Rectangle	Adequate
T10	Rectangle - Number line	Adequate – Partially adequate
T11	Rectangle	Adequate
T12	Circle	Adequate
T13	Rectangle - Number line	Adequate-Adequate
T14	Rectangle	Inadequate
T15	Rectangle	Adequate

When Table 9 is analysed, it is seen that while showing the subtraction of two different fractions with unequal denominators, adequate, partially adequate and inadequate level performance examples were found in line with the preferences of all teachers. Here, it is seen that 16 of 23 different performances are at adequate level, five of them are at inadequate level and two of them are at partially adequate level.

3.2.3.1. Task 11-Sample case of inadequate model use

<u>Example Case 24:</u>

Below is the task and response of the teacher who preferred the rectangle model.

TASK 11: Illustrate the subtraction operation given below using your preferred model in survey question 11.

$$\frac{1}{2}-\frac{1}{3}=$$

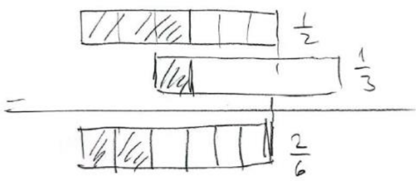


Figure 4. Teacher T14's response to the eleventh question

T14 chose the rectangle model for this task. Although he modelled the fractions correctly, he made an operation error and found the result incorrect because he did not make the part sizes the same. Since it was seen that the teacher could not associate the model he chose with the given operation and could not use it correctly, her answer was coded as inadequate.

3.2.4. General findings obtained from teachers' performances on using models

The general findings obtained from the teachers' performances of using models in situations involving different fraction schemes and fraction operations are given in Table 10.

Table 10. General findings from the OEQs

		Adequate	*Percent*	*Partially adequate*	*Percent*	*İnadequate*	*Percent*
Performances for Different Fraction Schemes	Part-Whole Fraction Scheme	19	100	-	-	-	-
	Partitive Unit Fraction Scheme	14	64	8	36	-	-
	The Partitive Fraction Scheme	15	88	2	12	-	-
	Reversible Partitive Fraction Scheme	16	89	2	11	-	-
	Iterative Fraction Scheme	7	41	10	59	-	-
Performances for Fraction Operations	Comparing Fractions	23	100	-	-	-	-
	Showing Equivalent Fractions	23	100	-	-	-	-
	Showing Compound Fractions	24	100	-	-	-	-
	Showing the Sum of Two Fractions	12	67	1	5	5	28
	Addition with Fractions	15	68	6	27	1	5
	Subtraction with Fractions	16	70	2	9	5	21
	Multiplication with Fractions	4	27	10	67	1	6
	Division by Fractions	-	-	1	7	14	93

When Table 10 is analysed, it is seen that teachers did not show inadequate performance in mathematical situations involving different fraction schemes and they showed at least 60% adequate performance in all situations except for the iterative fraction scheme. However, it is noteworthy that teachers' performances decreased in the cases involving partitive unit fraction scheme. In the cases involving iterative fraction schemes, it was observed that the teachers showed partially adequate performance in general, and their adequate level performance remained at only 41 percent. When different fraction operations are taken into consideration, it is seen that the teachers showed 100% performance in the cases of comparing fractions, showing equivalent and compound fractions. However, when other fraction operations are considered, it is seen that the performance order is subtraction-addition-writing a fraction as the sum of two fractions-multiplication and division. Therefore, it can be said that the lowest performances of the teachers belong to multiplication and division operations. Although the performances related to addition and subtraction operations are close to each other, it is seen that teachers are more successful in using model in addition. When all cases are considered, it is seen that teachers' performances are at the lowest level in division. 93% of the teachers who participated in the study showed inadequate performance in using models in division. There were no teachers who showed adequate performance in this category.

4. DISCUSSION and CONCLUSION

The teachers in this study generally preferred the rectangular model in situations involving the part-whole schema and used it at a completely adequate level. In cases involving the partitive unit fraction, partitive fraction and reversible partitive fraction schemes, teachers generally preferred the rectangular model and performed at a adequate level. In cases involving iterative fraction schemes, teachers generally preferred the rectangular model and used it partially adequately. Therefore, it was observed that the teachers in this study generally preferred the rectangle model and used it at an adequate level in all situations except iterative fraction schema. It was observed that teachers generally did not prefer the set model in all schemes except the iterative fraction scheme. Although the performance of the teachers who preferred the set model in cases involving the iterated fraction schema decreased, no teacher with inadequate performance was found in these cases. When all these results are evaluated, it is seen that the fraction schema in which all of the teachers showed adequate performance is the part-whole schema, and the most unsuccessful performance is the iterative fraction schema. In addition to these findings, during the research process, it was observed that the teachers were generally able to solve the questions about fraction schemes, but they could not express the conceptual differences between these schemes. It is thought that this situation may arise from the fact that in the teaching of fractions, the part-whole meaning of the fraction is focused on from the basic level and other meanings are not sufficiently emphasised. The findings of Acar (2010) also support this conclusion. The results of studies on classroom teachers and prospective classroom teachers also show that when the teachers' knowledge of different meanings of fraction is examined, they generally emphasise the part-whole meaning of fraction and have conceptual problems in explaining other meanings of fraction (Akbaba-Dağ, 2014). Similarly, it is stated that teachers mostly use the part-whole meaning of fractions while explaining fraction subject (Baştürk, 2016). In a different study conducted with secondary school teachers, it was stated that teachers mostly used the area/region model and the number line model in their lectures and taught their lessons based on the part-whole meaning of fractions (Yılmaz, 2016). Şen (2021) examined the teaching knowledge of a secondary school teacher on 5th grade fractions in his study. According to the results of the study, it was revealed that the teacher focused only on the part-whole meaning of fractions within the scope of content knowledge and in parallel to this, she used the area/region model by making representations only for the part-whole meaning and did not use concrete fraction materials in her teaching. One of the reasons for this situation is thought to be the intense focus on the part-whole meaning in teaching (Van de Walle et al., 2014).

In this study, it was observed that teachers generally used the rectangle model for all fraction diagrams and operations and did not prefer the set model in general. Within the scope of the study, the number line model was used in writing a fraction as the sum of two fractions and addition/subtraction operations with fractions. Cramer et al. (2008) stated that the circle model enables students to better comprehend different fraction sizes mentally, so it is the most appropriate model for addition and subtraction of fractions (cited in Van De Walle et al., 2014). However, in this study, it was observed that teachers preferred the circle model only in cases where the whole was divided symmetrically. In other words, teachers preferred to use the circle model when dividing a whole into two, four, six..., but they did not prefer it when dividing it into three, five, seven... on the grounds that it was not useful. Within the scope of the study, it was observed that the set model was used at a partially adequate level by the teachers only in cases involving iterative fraction schema. This situation is thought to be due to the structure of the iterative fraction scheme suitable for the use of discrete models. However, it can be said that teachers have more difficulty in the use of discrete models than continuous models. Similar to this result, in a different study conducted with secondary school mathematics teachers (Can, 2019), when the solutions made by teachers in the conceptual knowledge dimension regarding the subject of operations on fractions were examined, it was stated that mostly the area-region model was preferred, only one teacher used the line model in addition in fractions, while no teacher used the set model. Duran (2017) stated in his study that pre-service teachers mostly preferred the area model in representing fractions. In different studies (Castro, 2008; Parmar, 2003), it is emphasised that the most preferred models among area-region, length and set-object models are area-region models. In a different study, Çelik and Çiltaş (2015) stated that mathematics teachers mostly used area and number line models while teaching fractions and operations with fractions at the 5th grade level and that they had little knowledge about other models. Therefore, it can be said that teachers do not use discrete models for different fraction schemas and generally prefer continuous models. In the study conducted by Topçu and Gürefe (2020) to determine the fraction schemas used by 7th grade students, it was observed that students preferred continuous models and had difficulty with the set model. However, in the interviews conducted during the research process, some teachers stated that they had never used the set model before. However, when the model was explained to them, they stated that it was very useful and that they would use it in their lessons from now on. Therefore, it can be said that some teachers do not have enough knowledge about different representations and models used in fraction teaching and the findings obtained from the research should be interpreted in this direction.

When the performances of the teachers in this study for different fraction schemes and fraction operations were analysed, it was seen that teachers' performances decreased in unit fraction and iterative fraction schemes, multiplication and division by fractions, and the lowest performance belonged to division by fractions. In other studies, it is stated that students have difficulties in situations involving partitive unit fraction scheme (Norton & Wilkins, 2009) and iterative fraction scheme (Olive & Steffe, 2002). Norton and McCloskey (2008) state that when students use the part-whole scheme, they reason based on a partitioned whole; therefore, they have difficulty in determining the size of a certain fractional part by iteratively determining the size of a fractional part within an unpartitioned whole in situations involving a partitive unit fraction schema. Olive and Steffe (2002) state that the reason why students have difficulty in situations involving iterative fraction schema is that they have difficulty in performing division in compound unit fractions. Similarly, Yavuz-Mumcu (2018), in his study with pre-service teachers, states that pre-service teachers generally have difficulty in showing the algorithm/mathematical meaning of fraction operations using models, and this situation is more common in multiplication and division operations. This supports the view of Toptaş et al. (2017) that classroom teachers' knowledge of different meanings of fractions and different modelling types is insufficient and that they cannot model different fractions at the desired level. It is thought that teachers' difficulties in modelling while solving problems related to these schemes and fraction operations stem from their lack of conceptual knowledge or the fact that they do not use these models much in their lessons. As a result of the research, it was seen that many teachers used modelling operations with fractions as a purpose rather than a tool. Accordingly, it was observed that some teachers first made a procedural solution in order to model the operation presented to them, and then tried to model the situation in question according to the mathematical result they obtained. The results of Şahin (2019) and Yavuz-Mumcu (2018) studies also support this view. In addition, as a result of this study, it was observed that some teachers were able to model multiplication with fractions but could not explain it conceptually. Some teachers' statements such as "we use transparent fraction cards, when we put one horizontally and one vertically on top of each other, the region hatched with both gives us the result" in modelling the multiplication of two fractions show that teachers cannot explain the multiplication process at the conceptual level. Cluff (2005) states that pre-service teachers do not know fractions and the meaning of multiplication and division with fractions sufficiently, and that they have procedural knowledge rather than conceptual knowledge about the subject. This situation is similar to the results of the study conducted by Gürbüz and Birgin (2008). In the said study, the researchers stated that modelling operations with fractions were taught and learned with a rote memorisation approach and that no conceptual learning took place in this subject. The same view is also supported by the study conducted by Bulgar (2003). The results of different studies also show that teachers do not have sufficient conceptual knowledge about fractions (Can, 2019; Gökkurt et al. 2012; Işık, 2011; Lo & Luo, 2012).

When the model use performances of the teachers in this study were examined, it was seen that the lowest performances belonged to the division by fractions. It was observed that only one of the 15 participant teachers in this study was able to model and conceptually express the division of two fractions correctly. The results of Bayazıt et al. (2011) study can be shown to support this result. Bayazıt et al. (2011) examined elementary mathematics teachers' understanding of mathematical models and model building competencies and found that teachers had difficulties in understanding the models in mathematics textbooks and in building and using appropriate models to explain the mathematical situations given to them. Especially when it comes to multiplication and division, they found that teachers had serious problems in creating models and none of the teachers could create a model suitable for division by fractions. It is thought that the failure of most of the teachers in modelling division with fractions is due to the fact that teachers do not give enough time to these modelling in their lessons. In this context, Van De Walle et al. (2014) argue that the ‘invert multiply’ algorithm may be one of the least understood rules for students. With approaches such as rule memorisation and repetition, students try to memorise formulas and rules instead of understanding them conceptually (Gökalp & Sharma, 2004). The reasons for the rule ‘Take the first fraction exactly, invert the second and multiply it by the first’ are not known by teachers and pre-service teachers, and it is transferred to students with the logic of memorisation (Durmuş, 2005; Gökkurt et al., 2012; Işık, 2011). Kılcan (2006) also found that the mathematics teachers who participated in his study did not have conceptual knowledge about division by fractions and taught division by fractions to their students by giving rules. He also stated that whether teachers have conceptual knowledge about division by fractions is not related to their professional experience. The fact that teachers' conceptual knowledge of division by fractions is not sufficient may cause them to transfer these concepts to their students as rote knowledge. In the MoNE (2018) mathematics teaching programme, it is seen that the emphasis is not on all division operations, but on modelling simple operations such as dividing a natural number by a fraction or a fraction by a natural number or dividing a large fraction by a small fraction and the result is a whole number. In addition, it is seen that only such examples are included in some textbooks. Therefore, it can be said that teachers use modelling very limited in division with fractions.

In this study, it was tried to examine the preferences and performance levels of mathematics teachers towards using mathematical models in situations involving different fraction schemes and fraction operations. Depending on the results obtained, it is recommended that mathematical models should be used more in teaching fraction concept in learning environments. For this, it is thought that teachers should be guided and encouraged to use these models depending on the importance of teaching at conceptual level. Modelling applications can be included more in teaching programmes at undergraduate and graduate levels. Through cooperation programmes between the Ministry of National Education and universities, teachers can be enabled to follow academic developments more closely. Academicians working in the field of mathematical models and modelling can organise seminars etc. for teachers. In addition, it is suggested that teachers should use mathematical models in their lessons to support conceptual learning in the context of student thinking.

In similar studies to be conducted on the subject, teachers' use of discrete models can be studied in more detail. In this context, it is recommended to investigate teachers' attitudes towards using these models, why they do not prefer to use these models in their lessons and the points where they have difficulties in using models. In addition, since there are more activities in the freely distributed textbooks on modelling fractions compared to the auxiliary resources, it is suggested that teachers should be encouraged to use the textbooks.

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