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Research Article / Araştırma Makalesi

Graphical Analysis of Golden Ratio on Selected Works of Western Music*

Batı Müziğinden Seçilmiş Eserlerde Altın Oranın Grafik Analizi

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ABSTRACT
The Golden Ratio is the key to achieving structural, functional and aesthetic balance and harmony in works of art. It is clear from the examples in nature and art that form and function are closely related. Given the lack of analysis on musical works from different periods of western classical music, it is challenging to find studies on the subject. This study fulfils this purpose by presenting the golden ratio on works from various periods with different musical approaches of western classical music and offering a method for analysing the golden ratio in such musical works. The method is a graphical representation of the whole of the music. It overcomes the problem that music cannot be seen as a whole, such as an architectural plan, at a single glance. This is because music writing uses temporally sequential units and often more than one page. The divisions of the golden ratio can be shown on a musical graphic that fits on a standard page width. This allows part-whole relationships to be clearly analysed.

Keywords: Music, composition, golden ratio, musical form, music analysis

ÖZ
Altın Oran, sanat eserlerinin yapısal, işlevsel ve estetik anlamda dengeli ve uyumlu oluşlarında belirleyici bir etkendir. Doğadaki ve sanattaki örnekleri düşünüldüğünde biçim ve işlevin, birbirleri ile sıkı sıkıya ilişkili oldukları görülür. Müzik eserlerindeki varlığı, üzerinde az çalışılmış bir konu olduğundan klasik batı müziğinin farklı dönemlere ait eserler üzerinde yapılmış analizler bulmak güçtür. Bu çalışma, bu amacı yerine getirmek üzere hem klasik batı müziğinin çeşitli dönemlerindeki farklı müzikal yaklaşımlar sergileyen eserler üzerinde altın oranı göstermeye hem de altın oranın klasik batı müziği yapıtlarında ne şekilde incelenebileceğine dair bir yöntem sunmaktadır. Söz konusu yöntem, müzik yazısının zamansal olarak sıralı birimler halinde ve sıklıkla birden fazla sayfa kullanması sebebiyle, müziğin örneğin bir mimari plan gibi bir bütün halinde, tek bakışta görülememesi sorununu aşmak amacıyla geliştirilmiş müziğin tümünü kapsayan grafik bir temsildir. Böylelikle altın orana ait bölmeler standart sayfa genişliğine sığan bir müzikal grafik üzerinde gösterilebilmekte, bu sayede parça bütün ilişkileri net bir şekilde incelenebilmektedir.

Anahtar Kelimeler: Müzik, kompozisyon, altın oran, müzikal form, müzik analizi

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INTRODUCTION

The Golden Ratio is not a recent invention; it is a phenomenon that has been observed throughout history. As Bergil (1992, p. 117) notes, it has been known to mankind since ancient times. As will be seen throughout this study, research carried out in the 20th century has helped us to better understand its causes, results and effects on music, and it has become a topic of increasing interest.

It is not known when the Golden Ratio was first consciously used by composers of western classical music. However, those who make various claims on this subject may not fully grasp the essence of the subject. For example, in an article by Ruth Tatlow on the use of the Golden Ratio in musical compositions, the fact that there is no book on the Golden Ratio in any of the libraries that Bach had access to during his lifetime is presented as evidence that Bach did not know the Golden Ratio. (Tatlow, 2006).

The Golden Ratio (or Golden Section) is a fundamental concept in western music, yet musicians are largely unaware of its significance. This study demonstrates that the Golden Ratio is a fundamental element in the establishment of musical form and formal functions, whether it is a conscious or unconscious process. It is evident that the Golden Ratio can be applied not only to formal sections but also to functional units such as tone and tempo changes, culmination, and unexpected pauses (medial Caesurae), which are the most striking elements of musical expression. The word 'section' which is used alternatively to 'ratio' is important because the so called golden sections refer to the points of exposition, development and recapitulation, which are the basic divisions in all musical works of classical western music.

According to artists such as Goethe, Oscar Wilde, and Delacroix, art is a superior form of expression to that of nature (Kandinsky, 2001, p. 130). This assertion is, to some extent, accurate. There is a consensus that nature itself constitutes a work of art, regardless of whether or not a creator is involved. It can be concluded that artistic forms and expressions imitate a larger and more inclusive art form within which they exist.

In his book, Art, An Introduction, Dale G. Cleaver offers the following insights on the Golden Ratio:

"*Proportion, which concerns the dimensional relationship between parts, is a fundamental aspect of rhythm and balance. The reason why some arrangements of rhythm and balance are more satisfying than others can be partly attributed to the concept of proportion. Throughout history, various theories of proportions have emerged, which are claimed to form the basis of satisfying proportions. The most famous of these ratios is the Golden Ratio, whose mathematical value cannot be expressed as a rational number but can be easily determined by geometric method*" (Bergil, 1992, p.115).

Definition of Golden Ratio

"*The proportion $a : b = \varphi$ is one of the best characterized phenomena in natural geometry, because of the universality of its ratio as $a/b = \varphi$ ($\approx$ 1.6180339...), and is found as general trend, for instance, in the anatomy of animals—including humans; as well as in many plants, especially in a large variety of phanerogams. The golden ratio appears in an enormous diversity of human expressions—in conscious or unconscious usages—through many forms of written language, visual design, painting, sculpture, architecture and music. Most of authors who have studied this topic, including Ghyka (1927, 1931), Huntley (1970), Doczi (1981), Schroeder (1991) and Livio (2002), also explain aspects of structural proportion in terms of characteristic constructivism in organic and inorganic forms, and even as a core issue in crystallography, after one of the first modern treatises on the subject, published by J.D. Bernal (1926)*" (Pareyon, 2011, p. 320).

Golden Ratio is described as ratio of two quantities is the same as the ratio of their sum to the larger of the two quantities. Whether the golden ratio is also related to the Fibonacci and Lucas sequences, which are based on the idea that the sum of the previous two numbers gives the fallowing number, analyses are based on geometric ratios rather than the number sequence relations (Bergil, 1992, p. 60).

In the 3rd-century BC treatise 'Stokheia' (Elements) by Euclid of Alexandria, who is regarded as the most influential mathematician of all time, the division of a line by an extreme and average ratio is discussed. Euclid elucidates this division in the case where the ratio of the whole line to the larger part is equal to the ratio of the larger part to the smaller. Indeed, the history of such a division or ratio can be traced back to 3000 BC, to the Egyptian civilisation. It is asserted that this ratio was introduced to the Greek world by Pythagoras and his followers (Bergil, 1992, p. 3).

This ratio, initially delineated by Luca Pacioli (1445-1515) in his 1509 treatise De Divina Proportione (The Divine Proportion), was subsequently designated "Sectio Aurea." The term 'Golden Ratio' was first used by Leonardo da Vinci, who prepared drawings for the book of the same name and was also the father of the idea of the book. This name has been used ever since (Stakhov, 2007).

If a line AB is divided from any point C in such a way as to give the proportion AB/AC = AC/CB, then C is defined as the 'golden section' of AB. The ratio or value of AB/AC and AC/CB forming this proportion is then defined as the Golden Ratio that can be seen in figure 1 (Bergil, 1992, p. 3).

Figure 1. Golden Section

If AC=x and CB=1 on the line AB, the proportion AB/AC = AC/CB can be written as follows
:x+1/x = x/1
With this equation, the following quadratic equation is obtained:
$x+1 = x^2$, $x^2-x-1 = 0$
Since AC/CB = x/1 = x = Golden Ratio in this equation, the mathematical formula giving the roots of the equation is used to find the numerical value of the Golden Ratio. When the formula is solved for the value x, the following equation and the numerical value of the Golden Ratio are obtained:

$$\frac{\sqrt{5}+1}{2}=1{,}618$$

The inverse or negative value of the Golden Ratio is 0.618. When the value of the Golden Ratio is compared with its inverse value, several unique features emerge. In the whole world of numbers, 1,618 is unique in these unprecedented properties. 1,618 is the only number that turns into its inverse value when 1 is subtracted from it:
1,618-1 = 1/1,618 = 0,618
The Golden Ratio is the only number that gives its square when 1 is added to it:
$1{,}618+1 = 2{,}618 = (1{,}618)^2$
From the early 20th century onwards, the Golden Ratio was represented by the 21st letter of the Greek alphabet, 'ϕ' (phi), the first letter of the name of the famous Greek sculptor Phidias, who lived in the 5th century BC. (Ergil, 1992, p. 6).
The examples so far and the equations expressed in Phi can be summarised by this symbol:
$\phi = 1{,}618$
$1/\phi = 0{,}618$
$\phi^2 = 2{,}618$
$\phi+1 = \phi^2$
$2\phi = 3{,}236$
$2\phi-1 = \sqrt{5} = 2{,}236$

Explanation of the analyses

The objective of this study was to undertake a comprehensive analysis of the western music composers' works which are mainly chosen from baroque to early twentieth century. As can be observed, the Golden Ratio can be defined, in its most basic sense, as the ratio of a part to a whole. However, musical notation is necessary to perceive music as a series of notes, provided that the entire composition is not perceived simultaneously. Furthermore, its temporal structure engages the memory. The most straightforward method for visualising the components of a musical work is to represent it graphically. This approach allows for the illustration of dynamics, a crucial aspect of music, and provides a visual representation of silences and dynamic shifts. To this end, the audio recordings of the selected works were transformed into graphic narratives using software capable of processing sound waves. These graphics were then subjected to analysis.

Another tool is the digital Golden Ratio ruler, designated as the 'Altrise Golden Section'. This ruler has been employed to identify time-related Golden Sections on the sound wave, and to validate analyses based on measurement calculations. Nevertheless, in a small number of cases, the visual compatibility with the result has been minimally affected by the interpretations of the performers or conductors, which, to some extent, are not contingent on the score itself. To illustrate, in a given piece of music, if the performer were to alter the tempo significantly in a manner that is not indicated in the score or were to play a passage with greater intensity than is written, the sound wave would reflect this change, or the relevant musical moment may shift slightly out of alignment with the original intention. Nevertheless, as the result is only marginally affected and the calculations are based on the score, this infrequent occurrence does not impede sound analysis. Additionally, the Golden Sections depicted in the analyses are also displayed in the scores of the corresponding works, employing the same color convention, to enhance the analysis process.

The Altrise Golden Section Ruler is a software program that can be hovered over any graphic or image on the screen. It has a transparent interface with adjustable dimensions. In this manner, the program can be positioned at the edges of the image, allowing the user to identify all the golden sections in a vertical or horizontal orientation. In this study, the

images were utilized in a temporal manner, as they represent the sound waveforms of the related works. The golden sections obtained with this program were represented on the musical notation with the same color as the corresponding measures were represented on the graph.

The works of western music under consideration exhibit multiple significant structural elements, which align with the golden sections observed in both the musical piece and the corresponding sound waveform. Given that the golden section on the horizontal axis is approximately 3/2 of the length, this method can be practically applied as follows: digital music players typically display a bar indicating the duration and progression of the track, along with the total duration at the end. Since 3/2 of the total duration approximates the golden section, it is relatively straightforward to discern the fundamental golden section and the anticipated structural components, if not the complete sections. As will be demonstrated in the analyses, these expected elements are predominantly recapitulations, instrumental cadences and tonic key modulations.

The selected well-known works of classical western music were chosen for analysis based on their established reputations. The role of golden section on the structural elements of music is also investigated through new studies, with a particular focus on contemporary music and composers.

> "*With the spread of serial techniques, the Fibonacci sequence became a common compositional tool. Along with works by Xenakis and Stockhausen, the Fibonacci sequence may also be recognized in the serial compositions of Luigi Nono, Ernst Křenek and many others. Applications of this and similar arithmetic series to the parameters of music represent an easy way to achieve proportionality and a dynamic sort of symmetry, balancing between the traditional strict symmetry and its unpredictable opposite. By the end of the 20th century the Golden Section and the Fibonacci sequence have become legitimate parts of the Western compositional practice and found their place in numerous specialist writings, as well as in a range of composition textbooks*." (Zuvela, 2011, p. 277).

The graphical analysis method developed provides a rapid and effective means of identifying the fundamental structural divisions present in any musical work, with particular efficacy in the case of works exhibiting conventional formal patterns, such as sonata-allegro and rondo forms. Furthermore, it offers a straightforward approach to tracing the structures of works that deviate from these established forms, providing a clear temporal construct upon which these structures can be easily mapped. In addition to the performers, the composer candidates are afforded the opportunity to construct the musical structure with relative ease during the composition process. They are also able to judge and compare their works with the masterpieces of classical western music.

J. S. Bach BWV 537 Fantasia and Fugue Analysis

The Fantasy is structured in the form of an AB $A^1B^1B^1$, with two intertwined themes. The initial AB, which commences with a pedal on the C, is modulated to G minor, the dominant key, in measure 19 and concludes in a tense adagio in measure 21. A^1 commences in G minor in measure 21, this time on the G pedal, and connects to B^1 in measure 30. However, in contrast to B, B1 is in G minor, and this time it is the beginning of a rising tension. Measure 30 marks the beginning of a continuous tension that will persist until the end of the piece. The fantasy concludes with a brief coda and half cadences in measure 48.

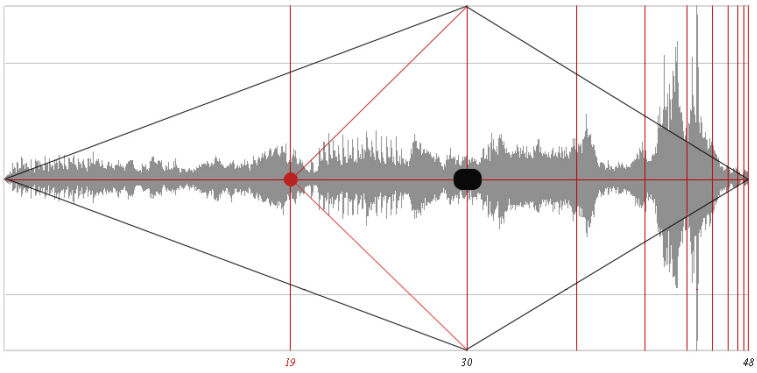

Figure 2. BWV 537 Fantasy Analysis

As illustrated in Figure 2, measures 19 and 30 represent the musical turning points, wherein the attractor tone of C minor is accentuated. It is also noteworthy that measure 30 represents the Golden Section of the Fantasy (48/30 = ϕ), while measure 19 is the reverse Golden Section (48/19 = ~ ϕ^2). As can be observed on the score (fig. 3), these are the points at which the tonal axis-gravitational relations are established, and new melodic divisions emerge. Above mentioned points can be seen through bars on figure 3.

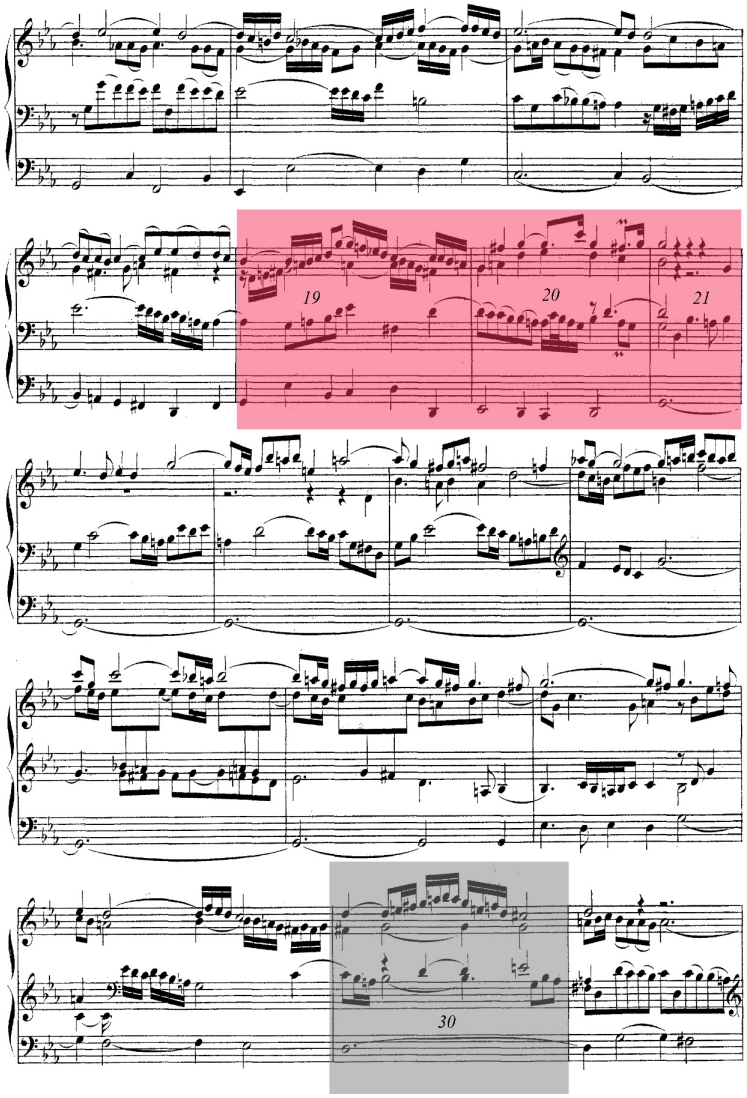

Figure 3. BWV 537 Fantasy Sheet Music

The Fugue is founded upon a dynamic and heroic theme, characterised by a rhythmic motif of repeated notes. Subsequent to the exposition, the subject and countersubject persist in entering continuously in both axis and recessive tones. The development section commences at a later point than anticipated, only in measure 58.

In contrast to the theme, the development section is constructed on a thematic material based on a chromatic ascent in second notes Despite the presence of numerous diatonic elements as a consequence of the harmonisation of the chromatic ascent, the development commences in C minor and subsequently establishes a connection with the dominant tone, G minor, through a cadence in measure 78.

The second part of the development places an emphasis on G minor, the tonality of the fugue, and builds up a progressively increasing tension until it reaches its zenith in measure 102 with a trill occurring simultaneously in the soprano, alto and tenor parts. In measure 104, the technique of stretto is initiated with the fugue theme being heard once more in the main tone (dux). The Fugue reaches its conclusion in measure 131 with a brief coda, which follows a four-measure pedal point on G.

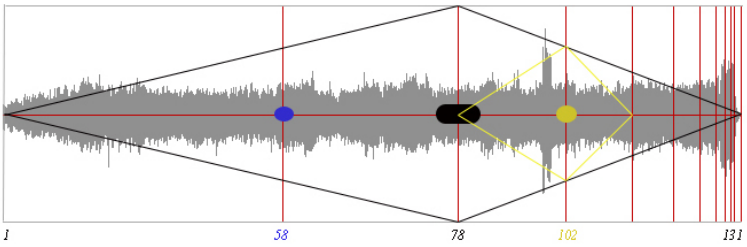

Figure 4. BWV 537 Fugue Analysis

Figure 4 illustrates that measure 78 represents the moment of tension in the second part of the development, preparing the climax to be reached in Measure 102. Additionally, Measure 78 can be identified as the Golden Section of the Fugue, with a ratio of 131:78, which is equivalent to the Golden Ratio (ϕ).

Measure 102, representing the culmination of the fugue, is situated within the Golden Section of the entire fugue, spanning from the development beginning in Measure 58 to the conclusion of the fugue. This placement is determined by an error of one measure (131-58/1,618+58 = 103). Indeed, depending on whether the beginning (102) or the end (103) of the climax is to be taken into account, the one-measure discrepancy can be disregarded. Measure 58 can be considered as approximately reverse Golden Section. These can be seen on score a figure 5 and 6.

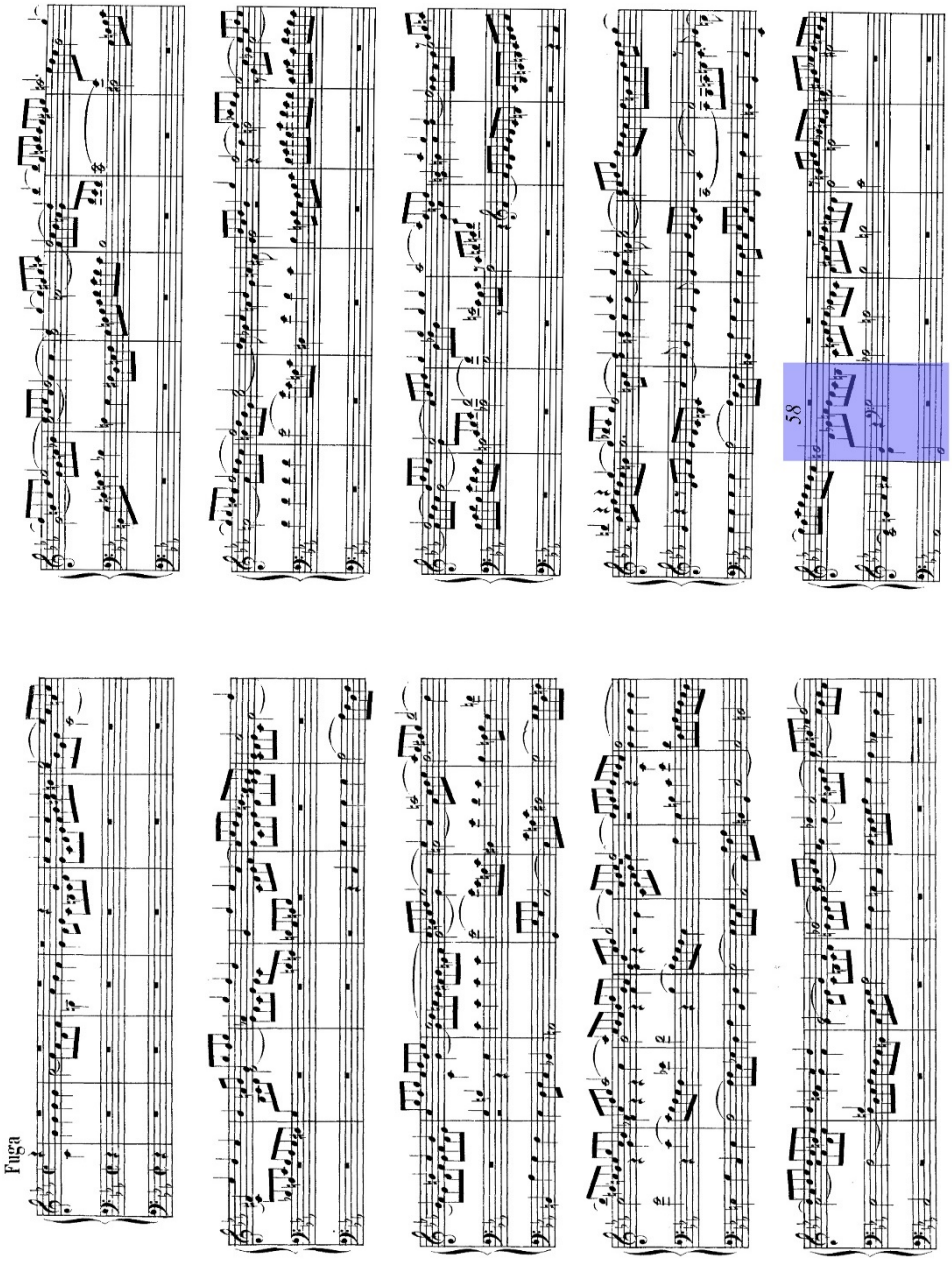

Figure 5. BWV 537 Fugue Sheet Music 1

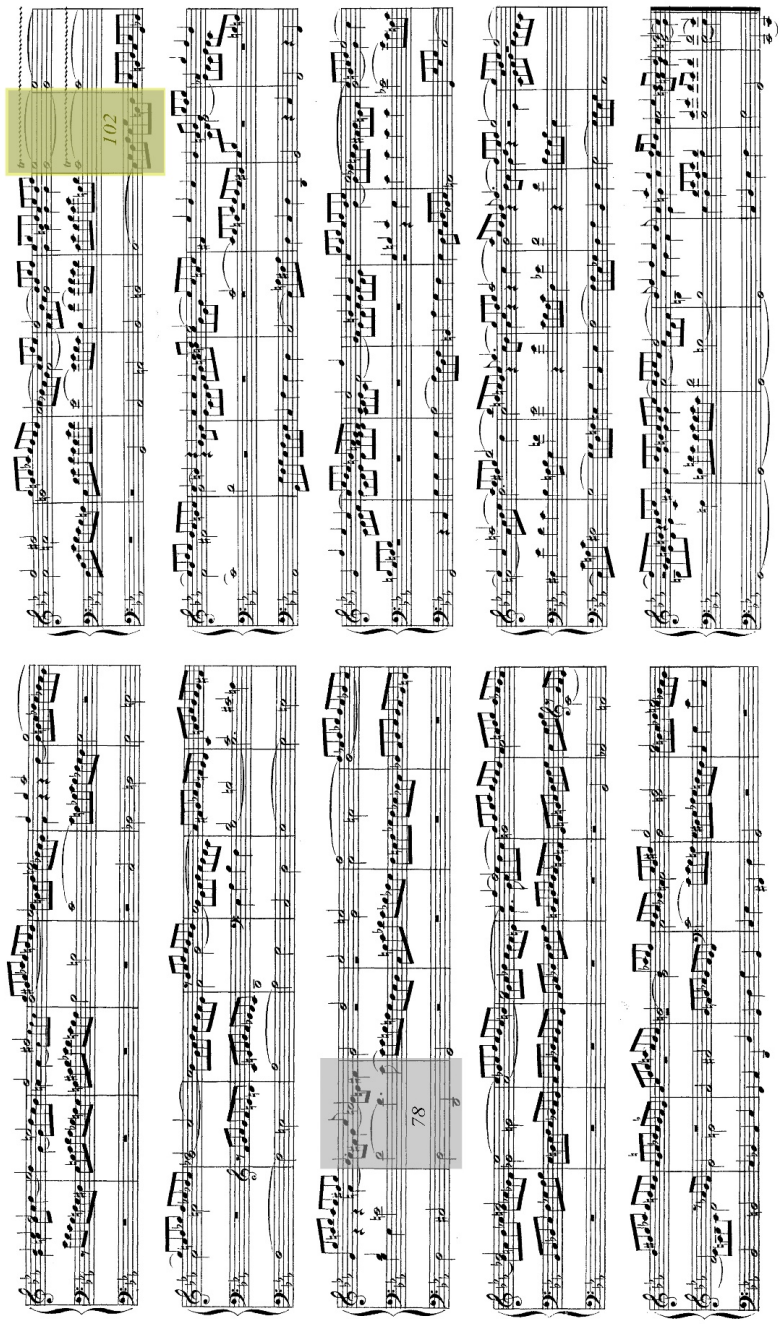

Figure 6. BWV 537 Fugue Sheet Music 2

Mozart Requiem "Requiem aeternam" K 626

The piece commences with a fugato introduction, initially presented by the orchestra and subsequently by the choir. At measure 21, a second homophonic division commences in C major, featuring a soprano solo. In measure 32, the principal theme is reintroduced, and in measure 34, with the addition of the basses and polyphonic writing, the development section commences, accompanied by an increase in tension towards the climax, which is reached in measure 41. The culmination of the piece, occurring in measure 41, is emphasized by the highest A note in the sopranos, which is followed by the completion of the movement with strong chords in measure 43. The tension gradually decreases until measure 46, where it reaches its zenith, before gradually dissipating until the piece reaches its conclusion in measure 48. This is marked by a short coda connecting to the Kyrie on a half-step.

The intertwined Golden Chapters are readily discernible in Figure 7. In measure 32, the moment when the main theme returns in G minor, the tone of the lower register constitutes approximately the Golden Section of the piece (48/32 = 1.5). Measure 21 is in the Golden Section of the whole, from the beginning of the music to the introduction of the basses in measure 34 (34/21 = ϕ).

The main theme and the Golden Section of the final section, which connects to the codas in measure 46, represents the culmination of the piece, reaching its zenith in measure 41 (46-32/1,618+32 = 40.6). Ultimately, the Golden Section between the culmination reached in measure 41 and the conclusion of the piece, measure 48, serves to connect the codas with the half-stay in measure 46 (48-41/1,618+41 = 45.3).

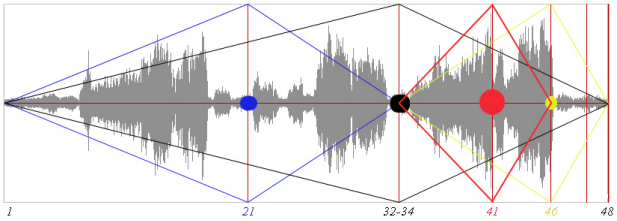

Figure 7. K626 Requiem Aeternam Analysis

All these Golden Ratio relationships can also be followed on the sheet music in Figure 8.

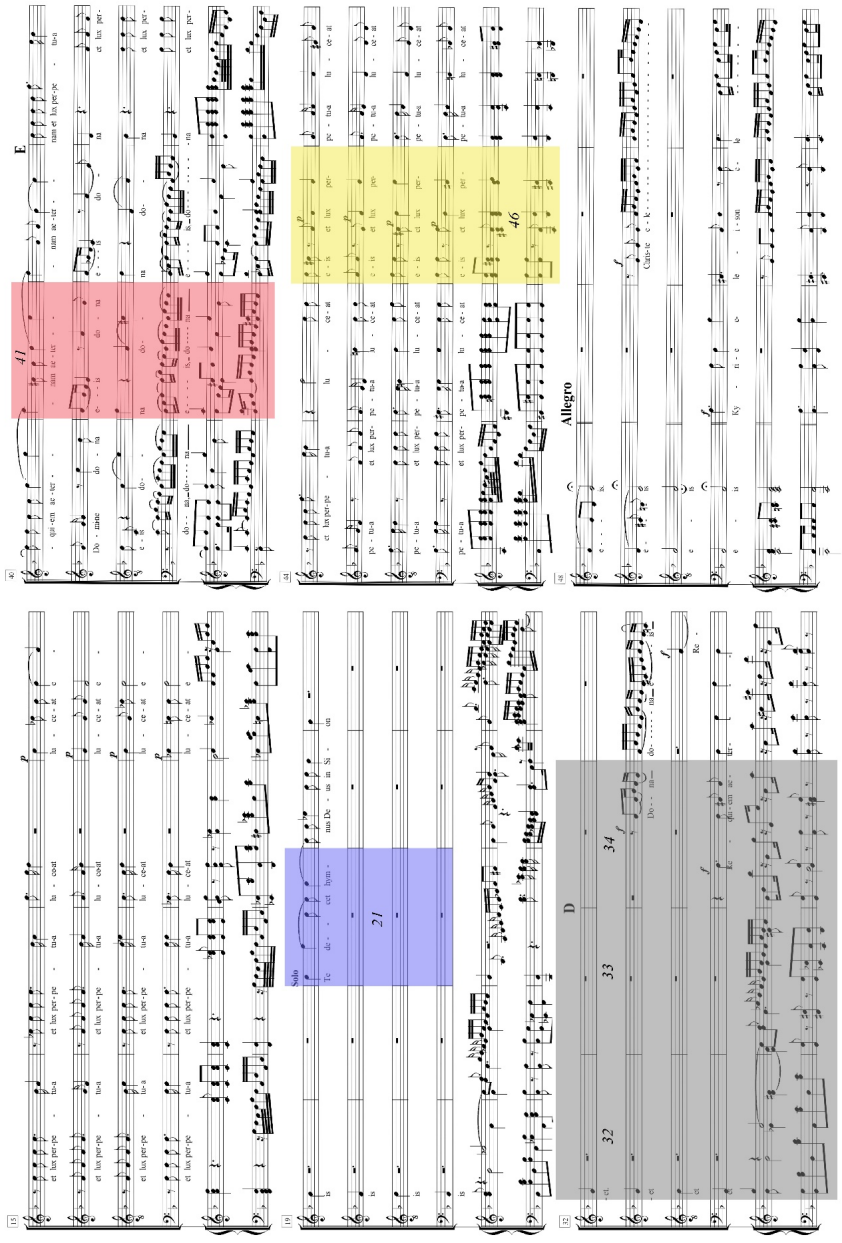

Figure 8. K626 Requiem aeternam Sheet Music

Johannes Bramhs Concerto for Piano and Orchestra No:1 .Op.15

The movement reaches its conclusion in Measure 192, following the initial presentation of the primary theme group and the subsequent introduction of the secondary theme in F Major in Measure 157. The exposition then reaches its end in Measure 226. The subsequent development section is relatively brief but progresses through the skillful use of thematic material, reaching a climax of increasing tension before reconnecting to the re-exposition in Measure 310.

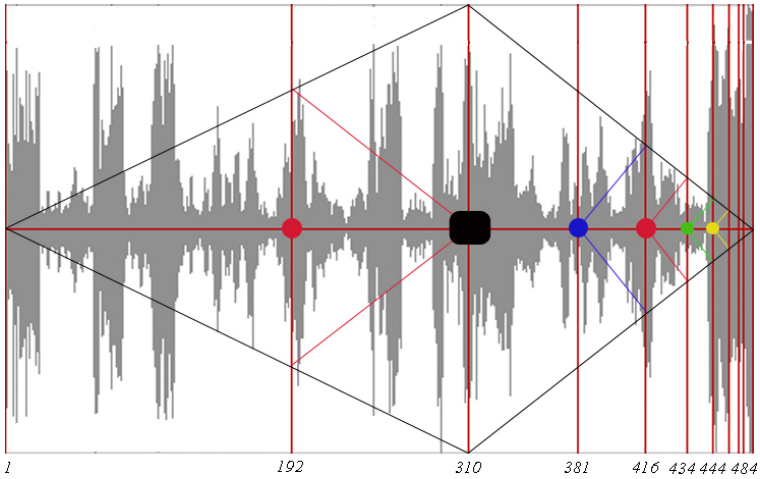

Figure 9. Op.15 1st Episode Analysis

As illustrated in Figure 9 and detailed in the accompanying score, all these measures are intertwined with Golden Sections. 299. The tension and crescendo that begins in measure 299 is concluded in measure 310, after which the exposition recommences in the Golden Section (484/310 = 1.57). Measure 192, which marks the commencement of the closing section of the exposition, is situated within the Golden Section of the exposition (310/192 = 1.6).

The Golden Section of the re-setting in measure 416 serves the same function as measure 192 (484-310/1,618+310 = 417). The re-setting of the second theme in measure 381 is situated approximately in the Golden Section of the re-setting up to the closing section (416-310/1,618+310 = 375). It is evident that other significant points are also situated within the approximate Golden Section. Figure 10-12 illustrates these golden points on score.

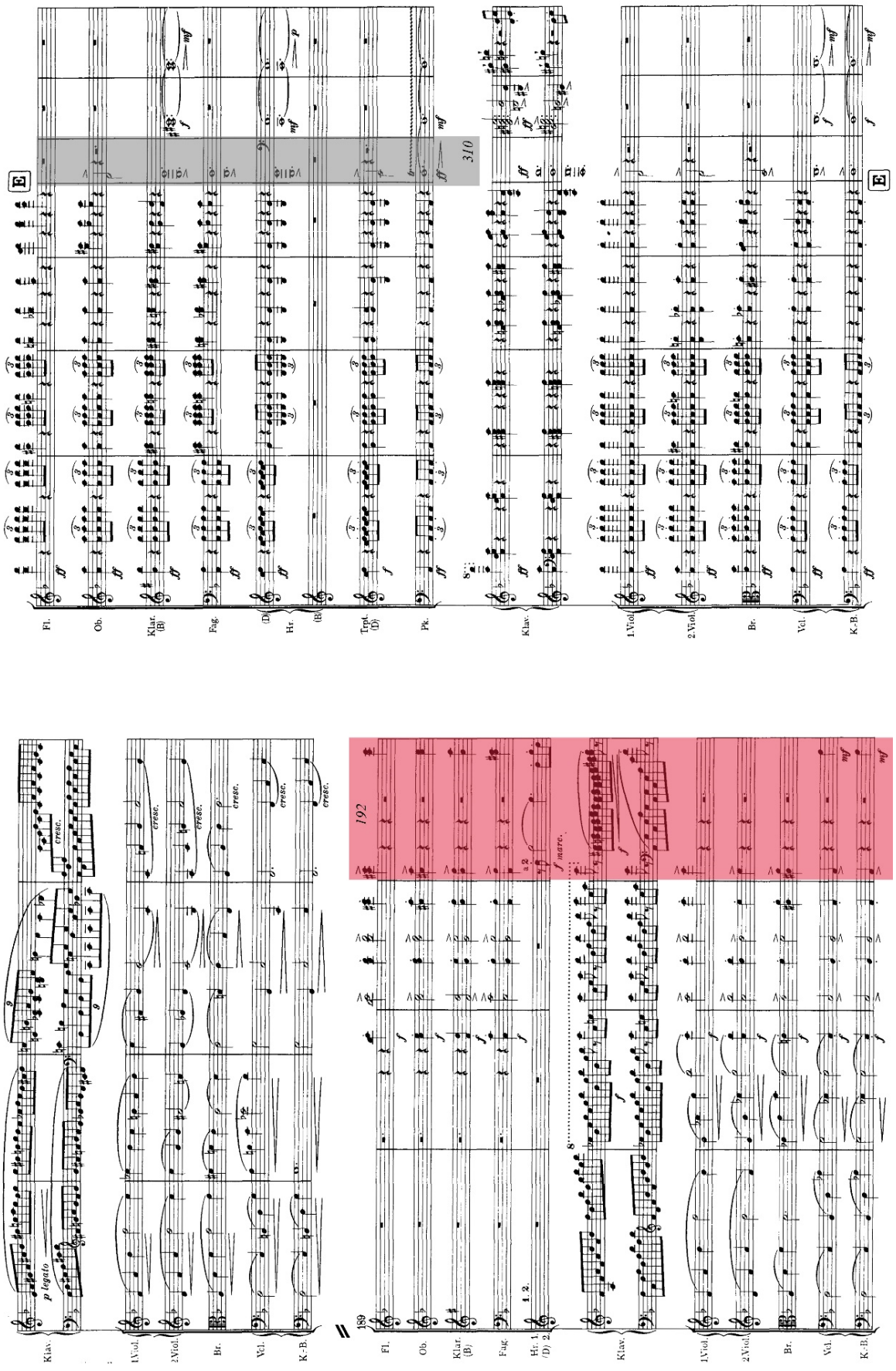

Figure 10. Op.15 1st Episode Sheet Music 1

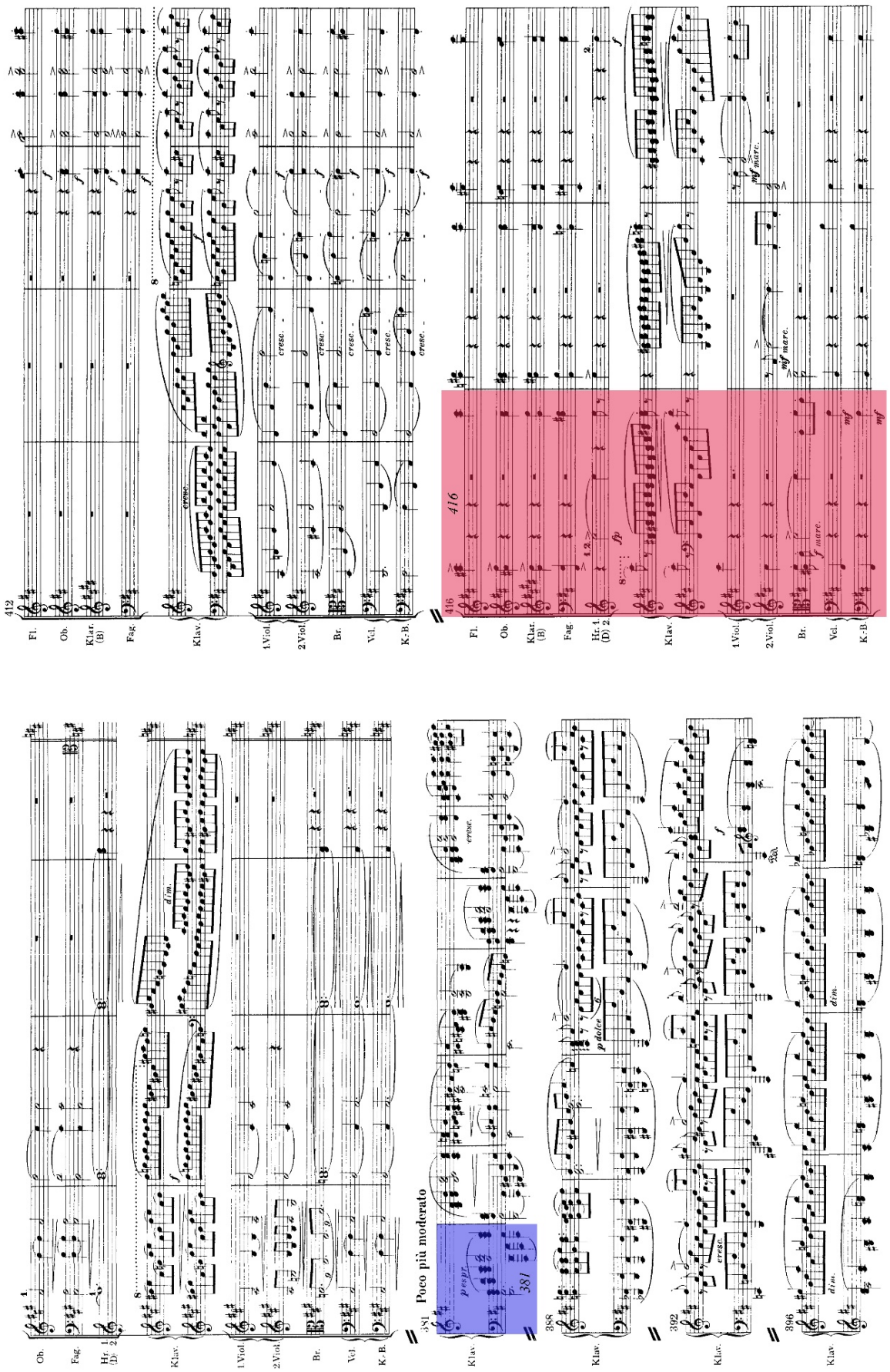

Figure 11. Op.15 1st Episode Sheet Music 2

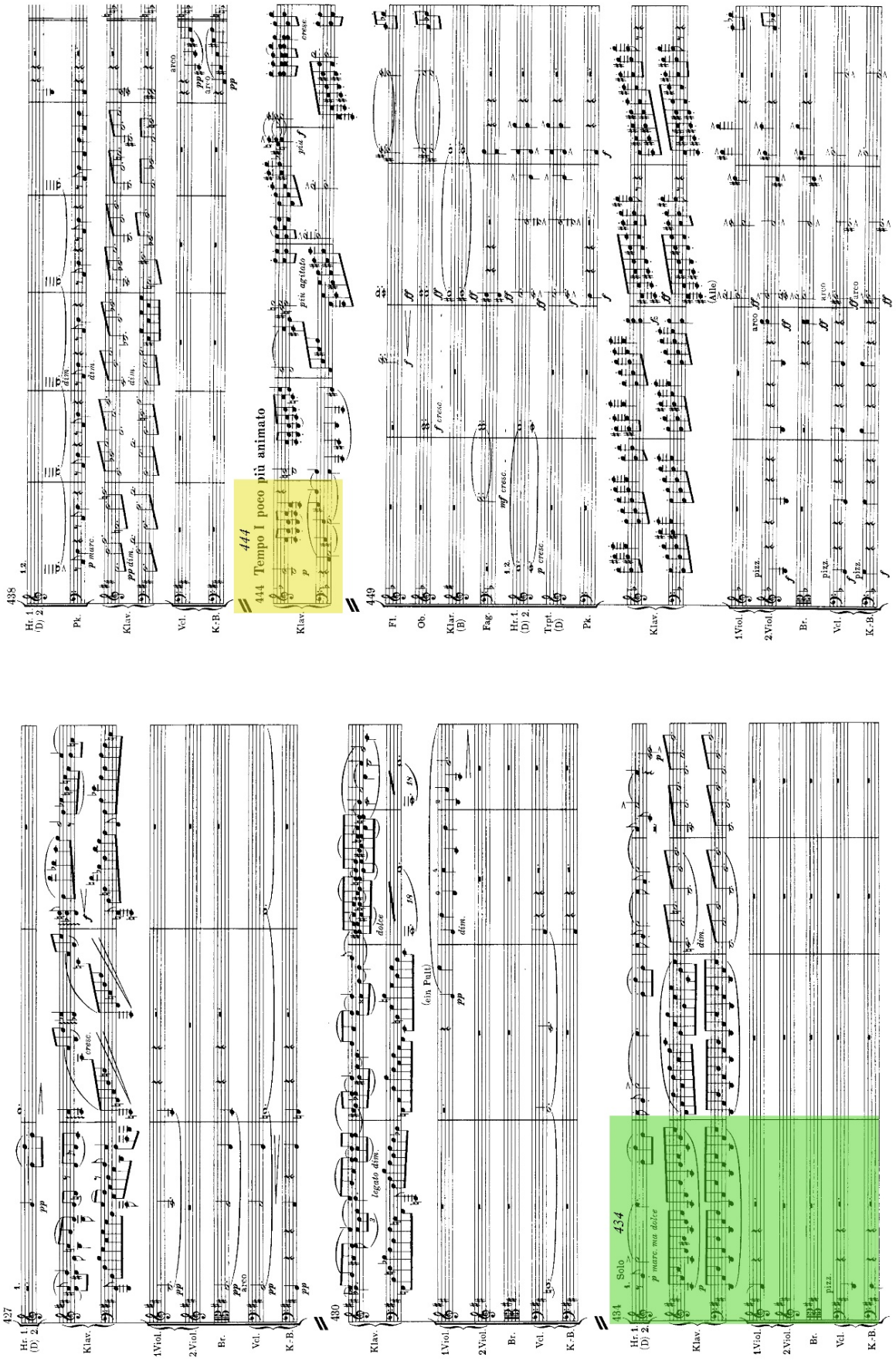

Figure 12. Op.15 1st Episode Sheet Music 3

Frederic Chopin Etude No: 11 Op. 25

This etude, also known as 'The Winter Wind', is one of the most accomplished and sonorous pieces of Chopin's piano music. Figure 13 illustrates a peculiar phenomenon: the analytical approach does not commence at the piece's outset. When the entire work is subjected to analysis, a structural imbalance in accordance with the Golden Ratio becomes evident. However, the addition of the four-bar introductory passage in a slow tempo by Chopin subsequent to the initial composition helps to resolve this issue. Consequently, this introductory passage will be excluded from the analytical process.

The piece is structured around a single theme, with an accompaniment consisting of descending chromatic broken chords in the right hand. It then progresses with an energetic and rhythmic continuity, which is interrupted by an unexpected chromatic march that begins in measure 53. This interruption serves to heighten the suspense and tension, acquiring significance upon the return to the principal tone and theme. It represents the most noteworthy moment in the piece. As illustrated in Figures 13 and 14, this point, specifically measure 53, represents the Golden Section of the 88-measure etude, with a ratio of 88/53 = ϕ.

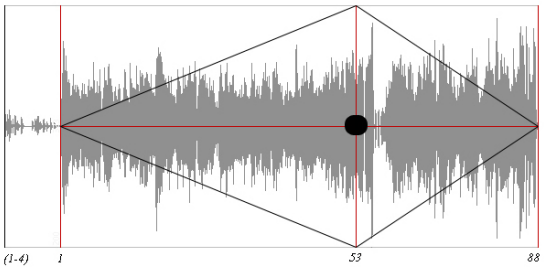

Figure 13. Op.25 no:11 Analysis

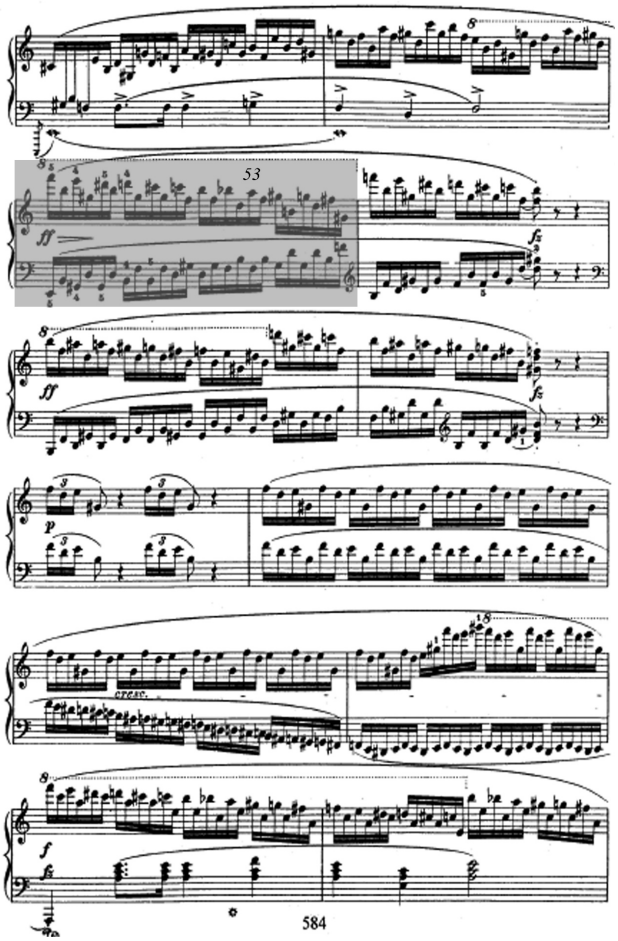

Figure 14. Op.25 no:11 Sheet Music

Franz Liszt Sonata in B Minor

The Sonata in B minor, completed in February 1853, constitutes one of the most distinctive formal contributions to 19th-century music. In addition to the incorporation of multiple sonata movements into a single composition, the work exhibits an unconventional and innovative approach to form. Specifically, the Sonata Form (exposition-development-sequel) is distributed across the four movements, with the form based on two simultaneous functions. This approach to form is reminiscent of that seen in Schoenberg's first chamber symphony, which emerged approximately fifty years later.

Prior to analysing the Golden Ratio, it is essential to undertake a detailed examination of the thematic elements within the work. This will facilitate a comprehensive understanding of the intricate structural nuances. The majority of the structure is derived from a series of minor thematic cavities that are discernible in the introduction but do not indicate the presence of a tonal center. The initial material, the descending melody in the Gypsy scale, is employed at structurally pivotal points in the piece, in a manner evocative of the pitch at the conclusion of the scene. The initial thematic elements can be analysed in figure 15-17.

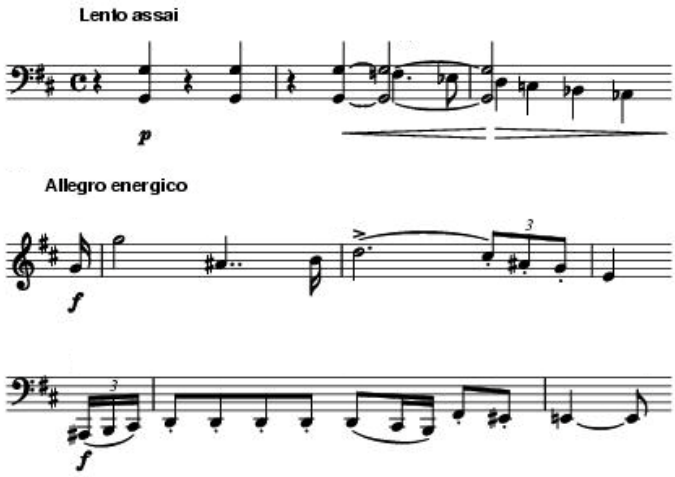

Figure 15. Initial Thematic Elements

Initial thematic elements the initial theme, which exemplifies Liszt's thematic metamorphosis technique and accentuates the axis tone, is also derived from the second and third compartments.

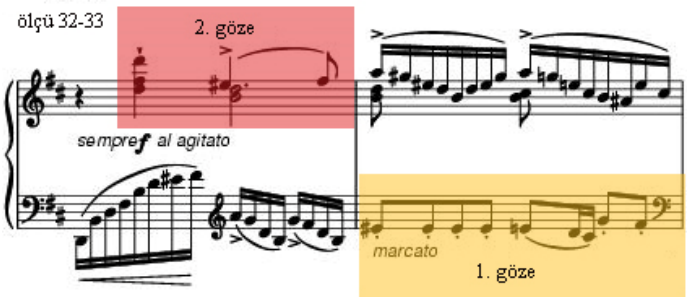

Figure 16. Theme 1

The second theme group's themes also emerge from the relative major introductory theme.

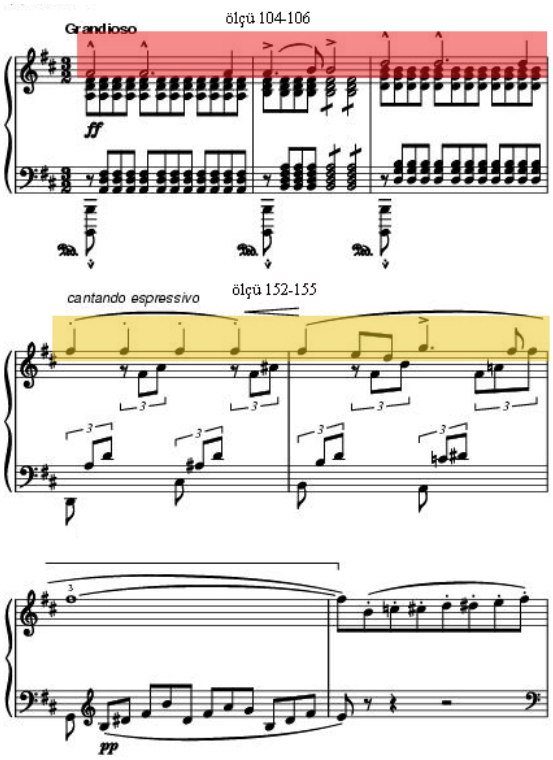

Figure 17. Other Themes

In consideration of these themes and structural material, the principal moments of the work and the relationships to the Golden Ratio can be demonstrated. Following the introduction and exposition (which commences in measure 32 and concludes in measure 331), the development in F-sharp major begins. Additionally, the development fulfils the function of the slow second movement within the broader context of the sonata conception. Composed in the same key as the slow movement Sonata (which may have had a religious significance for Liszt), 'Paradiso', from Liszt's Dante, this movement is linked to a fugato featuring a theme from the second and third sections, evoking Beethoven. The formal flexibility of the work makes it challenging to ascertain whether the fugato represents the start of a third movement or a reintroduction with a distinct formal structure.

Although the arrival of the main theme in measure 533 in the key of B minor requires the fugato to be treated as a third movement (in a similar manner to the scherzos of sonatas), aural perception of the fugato is such that it is heard as the antecedent of the recapitulation, with the latter being regarded as part of the former. Furthermore, in a considerable number of recordings, the work is divided into three movements.

The remarkable second theme of the second theme group arrives in measure 616, following the arrival of the main theme. The reintroduction is connected to the section where the coda, the most enthusiastic and joyful moment of the work, begins in measure 682. After the second eye in measure 729 and the opening theme in measure 750, the work ends in measure 760.

Figure 18 clearly shows that all these measures and key moments are in the Golden Section of the piece. Measure 331 is the approximate inverse Golden Section (760 ÷ 331 = 2.3), while measure 460 is the exact Golden Section (760 ÷ 460 =) and all are shown on score at figures 19-21.

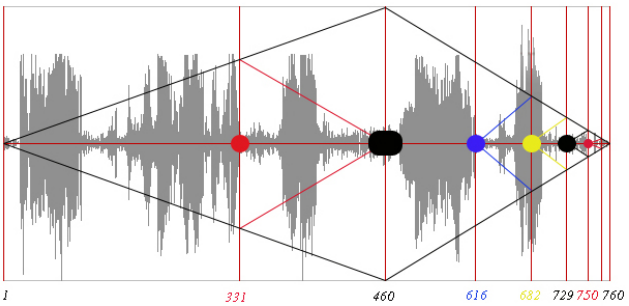

Figure 18. Sonata in B Minor Analysis

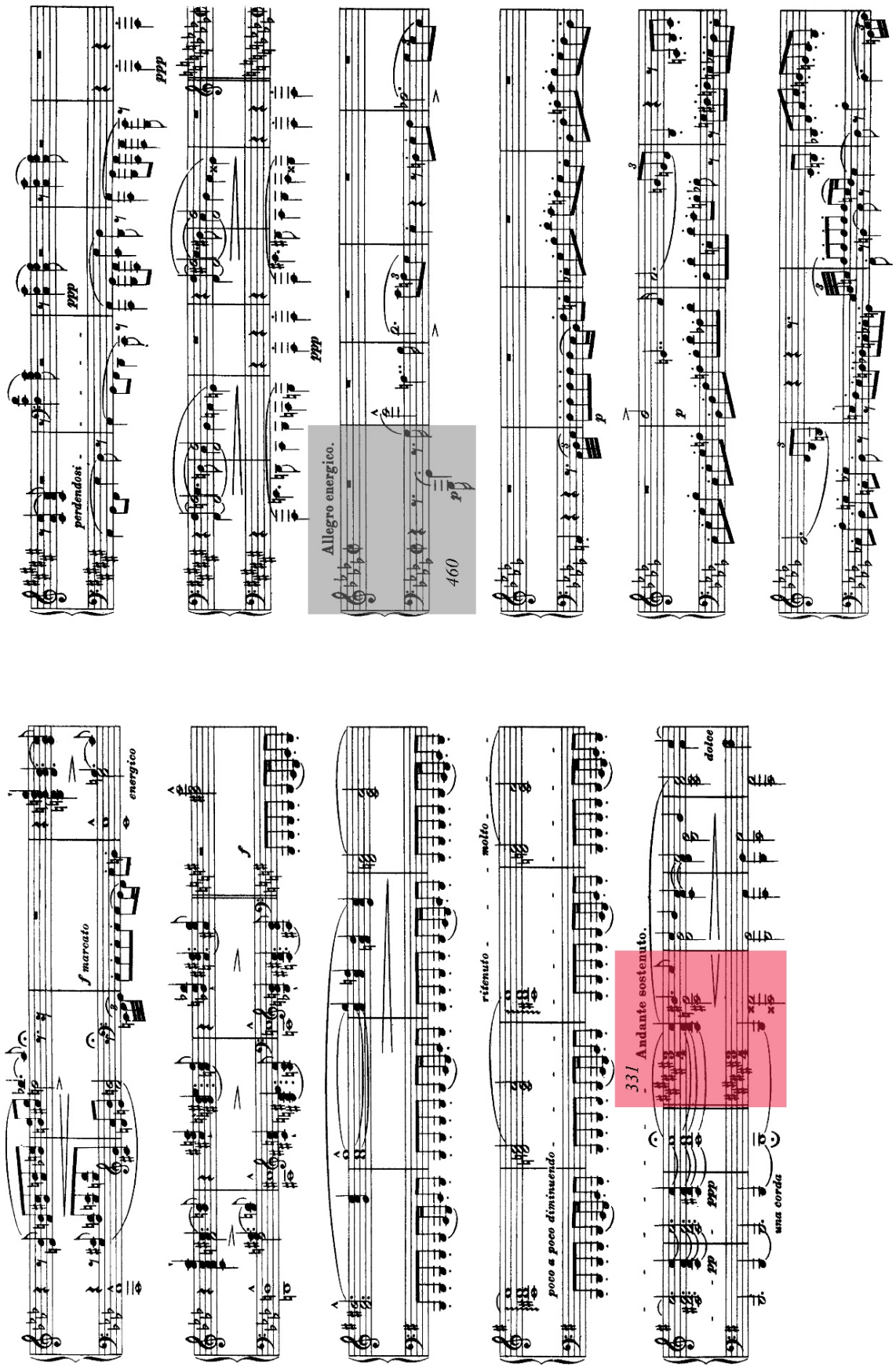

Figure 19. Sonata in B Minor Sheet Music 1

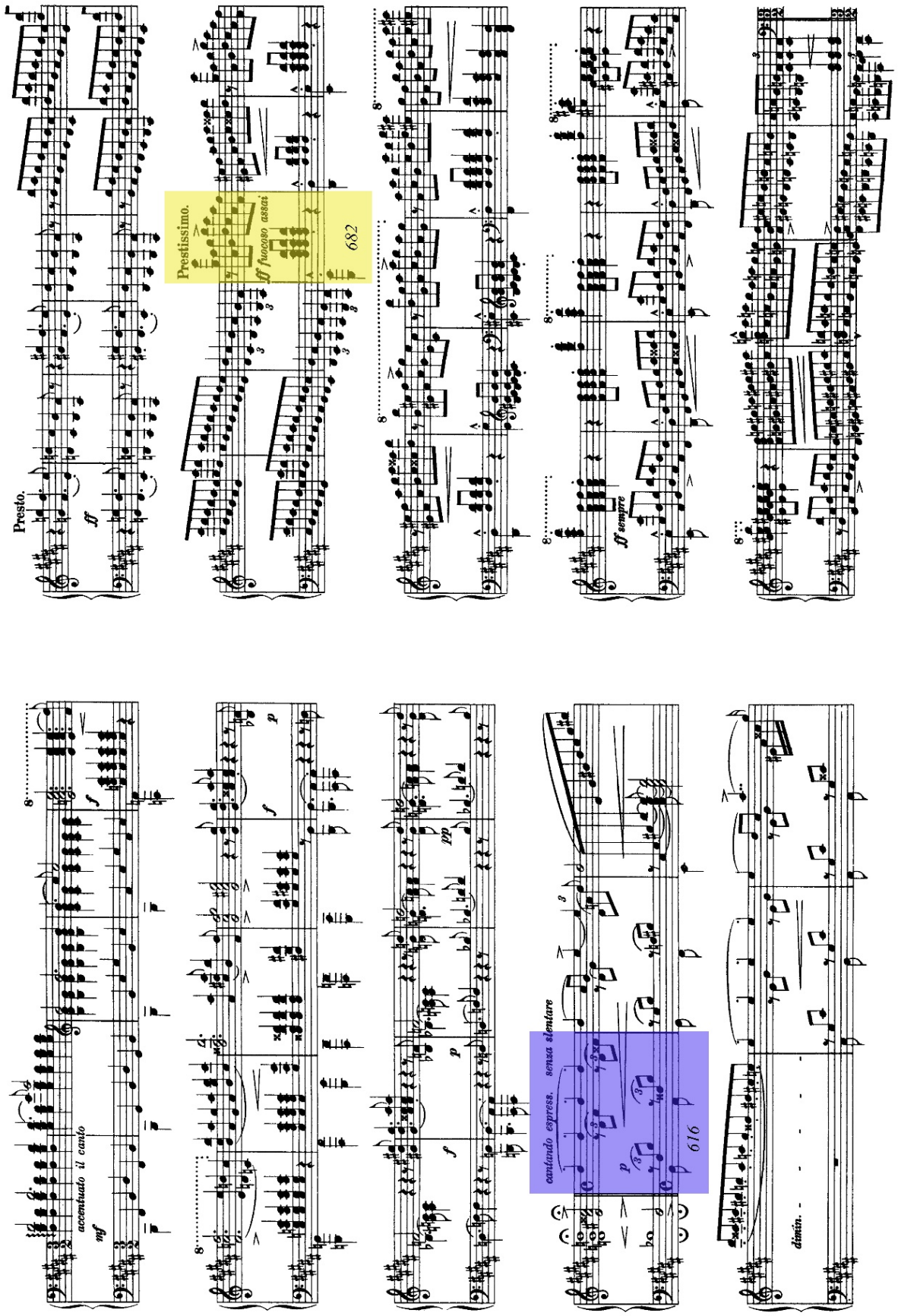

Figure 20. Sonata in B Minor Sheet Music 2

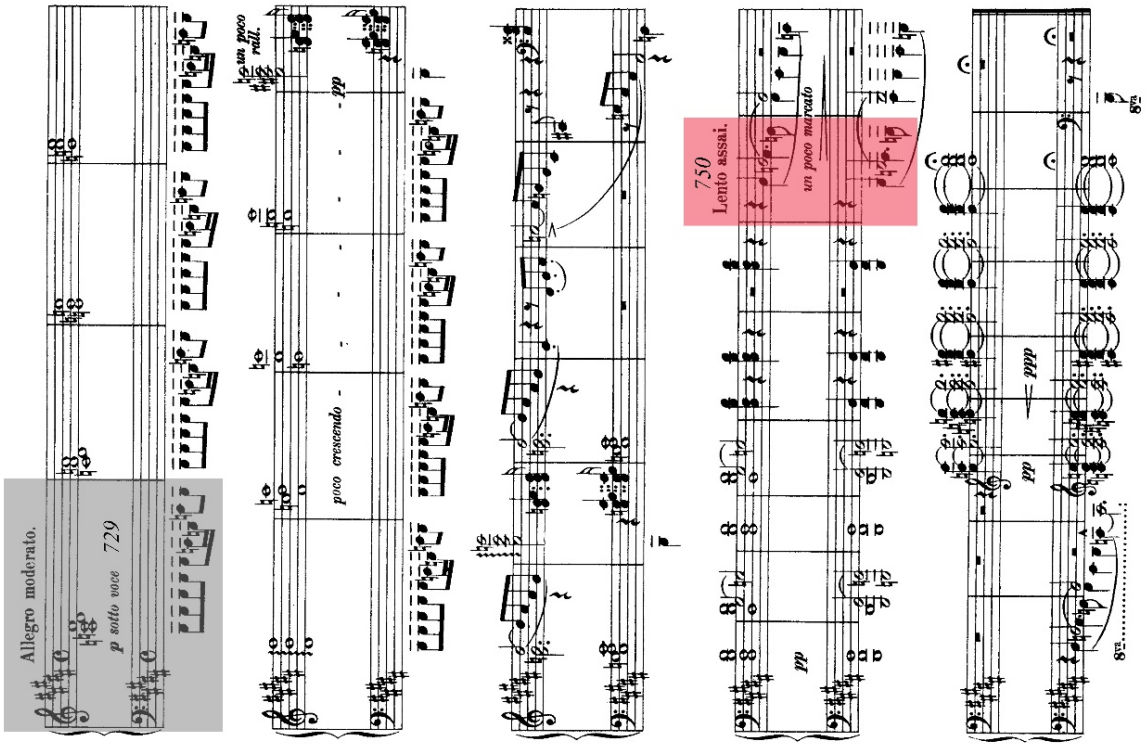

Figure 21. Sonata in B Minor Sheet Music 3

Maurice Ravel String Quartet in F Major 1st Episode

Following the introduction and bridge in F major, the second theme is introduced in the relative minor in measure 55. After the presentation of the second theme and the closing section, the development section commences in measure 84.

Following the adroit deployment of thematic material, the work reaches its zenith with the mounting tension and accelerated tempo. After a brief fade-out, the exposition section resumes in the primary tonality in measure 129. The coda is marked by the simultaneous presentation of both themes, and the work concludes in measure 213.

From the examples presented thus far, it can be surmised that the recapitulation occurs within the context of the Golden Section (213/129 =). In contrast, the initial phase of the development is situated within the inverse Golden Section (213/84 = 2). The ratio of the divisions in which the initial and secondary themes are presented is approximately the Golden Ratio (84/55 = 1.53). Measure 180 represents the Golden Section between the reopening and the conclusion of the work (213-129)/1.618+129 = 180) as illustrated at figure 22. It appears that this is the optimal method for balancing the contrasting sections of a Sonata Allegro through the application of mathematical principles.

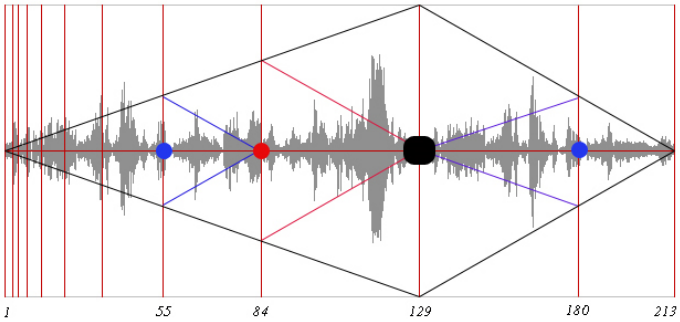

Figure 22. Quartet in F Major 1st Episode

In his book, Orenstein quotes Ravel as saying of this quartet: ‘My string quartet symbolises the concept of musical construction, incompletely executed, insecure, but much more meticulous and flawlessly written than my earlier works’ (Orenstein, 1990, p. 225) The golden points of work can also be examined on score at figures 23 and 24.

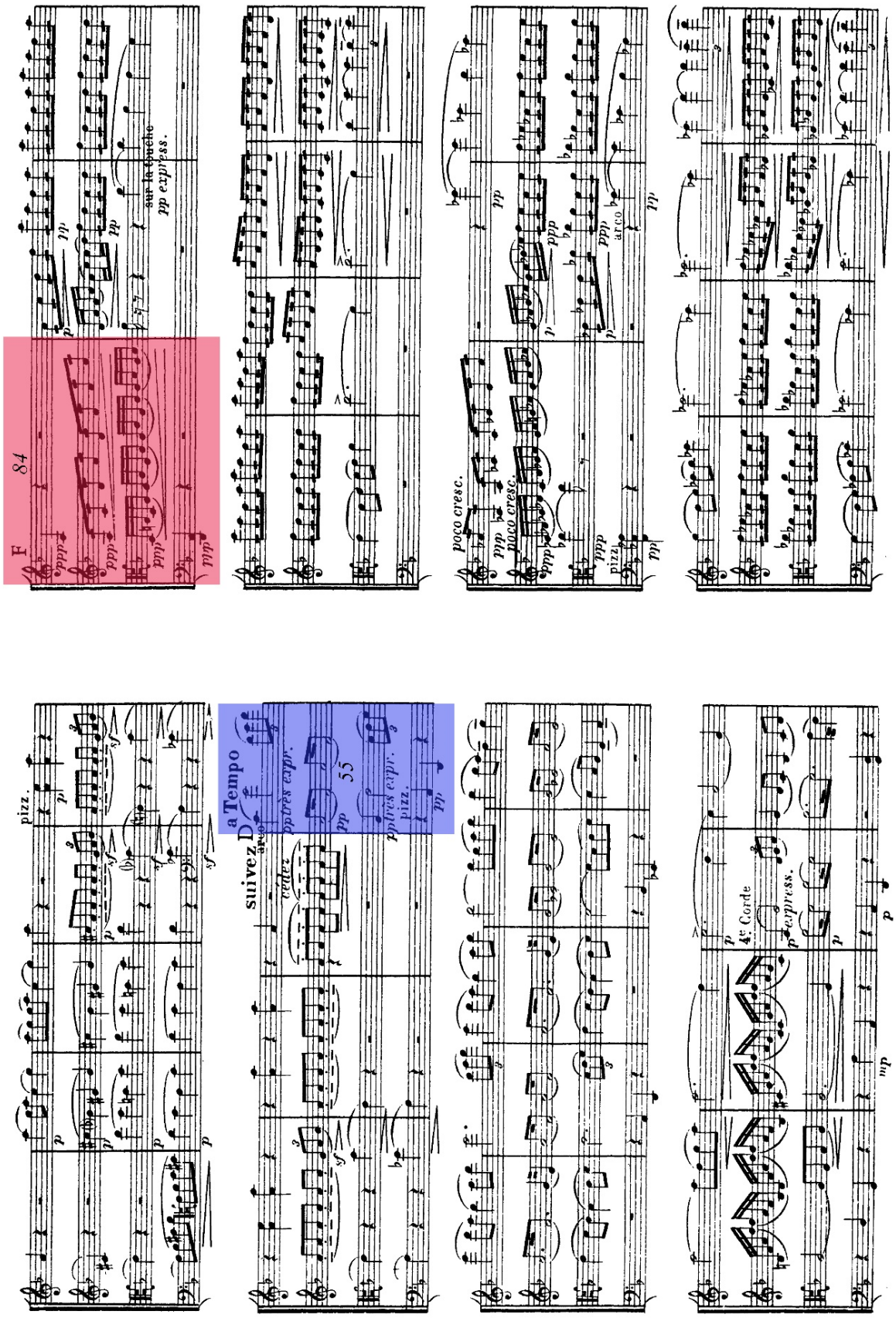

Figure 23. Quartet in F Major 1st Episode Sheet Music 1

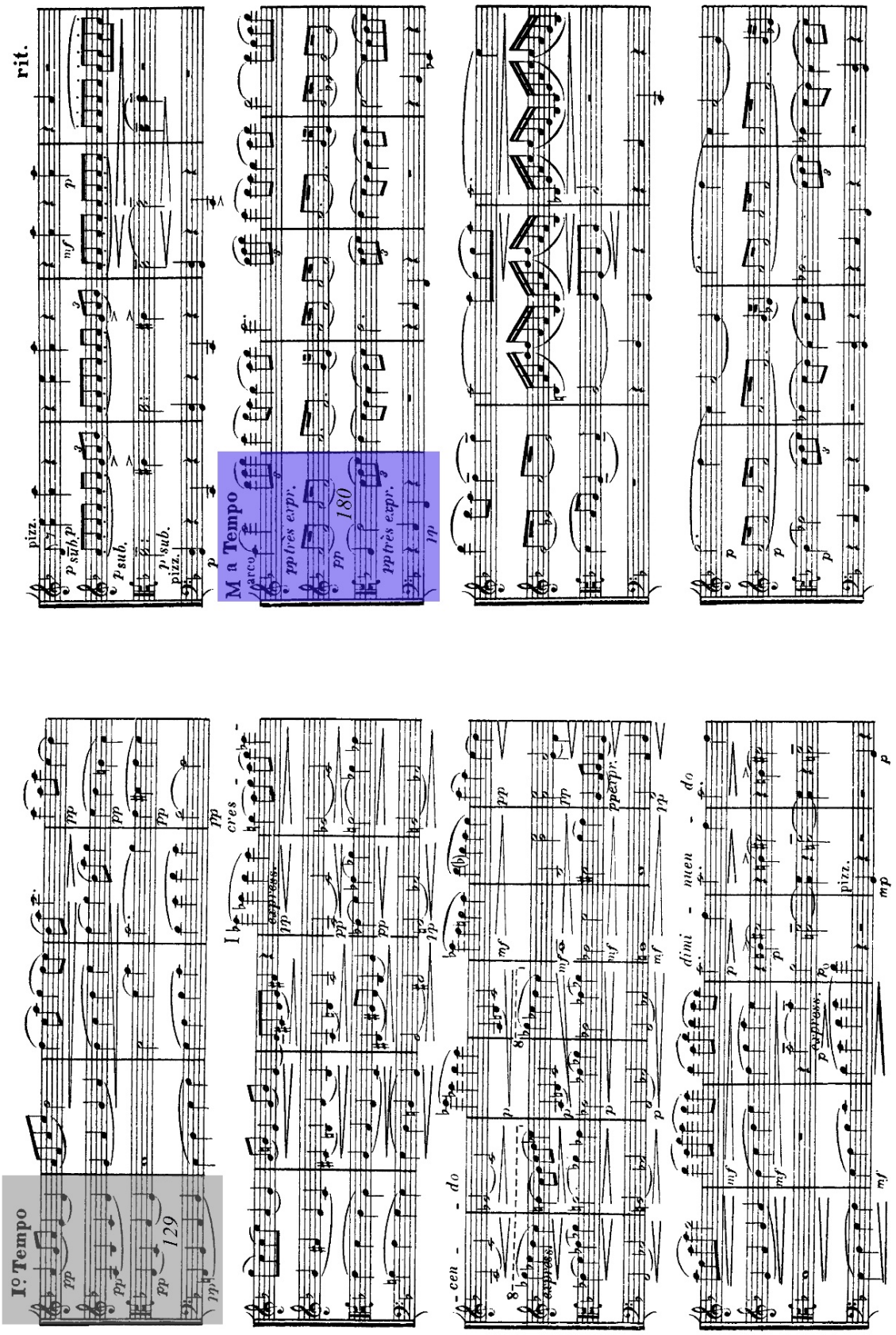

Figure 24. Quartet in F Major 1st Episode Sheet Music 2

Igor Stravinsky Symphony in C 1st Episode

The initial movement of the symphony is structured in accordance with the classical form of a sonata allegro. However, it also exhibits a distinctive feature in that it is divided into three symmetrical sections, with the lengths of the sections surrounding the development being symmetrically equalised. This symmetrical circular structure can be succinctly summarised as follows, with the corresponding measure lengths provided for clarity:

Introduction. 25
Theme. 34
Bridge. 34
Second Theme. 58
Development. 67,5
Recapitulation 1st Theme. 56,5
Recapitulation, 2nd Theme..34
1st Coda. 34
2nd Coda. 25

Following an ascending crescendo that commences at the exposition section, the reintroduction of the second theme is initiated in measure 227. This is followed by the reintroduction of the first theme in measure 278. Subsequently, the initial coda commences in measure 310 and is linked to the subsequent coda in measure 344, concluding the movement in measure 368. These measures, which can be characterised as pivotal points in the musical structure, represent the Golden Sections of the entire composition, as illustrated in Figure 25. These elements can also be observed in the graphical analysis.

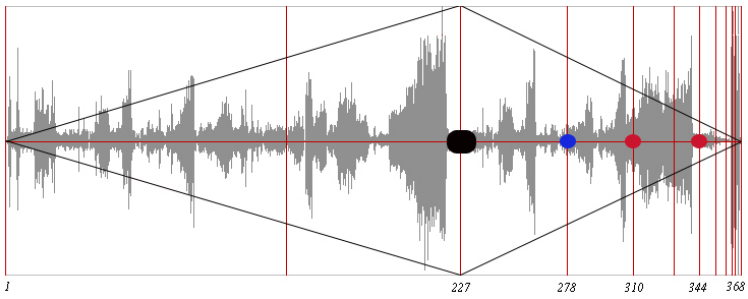

Figure 25. Symphony in C Analysis

As is frequently the case, the recapitulation is situated within the Golden Section of the entire composition, which is represented by the ratio of 367 to 227, equivalent to the Golden Ratio (ϕ). The point at which the recapitulation ends and the initial coda commences is also situated within the Golden Section between the recapitulation and the conclusion of the piece (368-227/1,618+227 = 314). Similarly, the second coda also forms the Golden Section between the first coda and the end of the piece (368-310/1,618+310 = 345). Furthermore, the ratio of the measures covered by the first and second themes to each other is also the Golden Ratio (310-227/1,618+227 = 278). Figures 26-29 illustrates these measures on score.

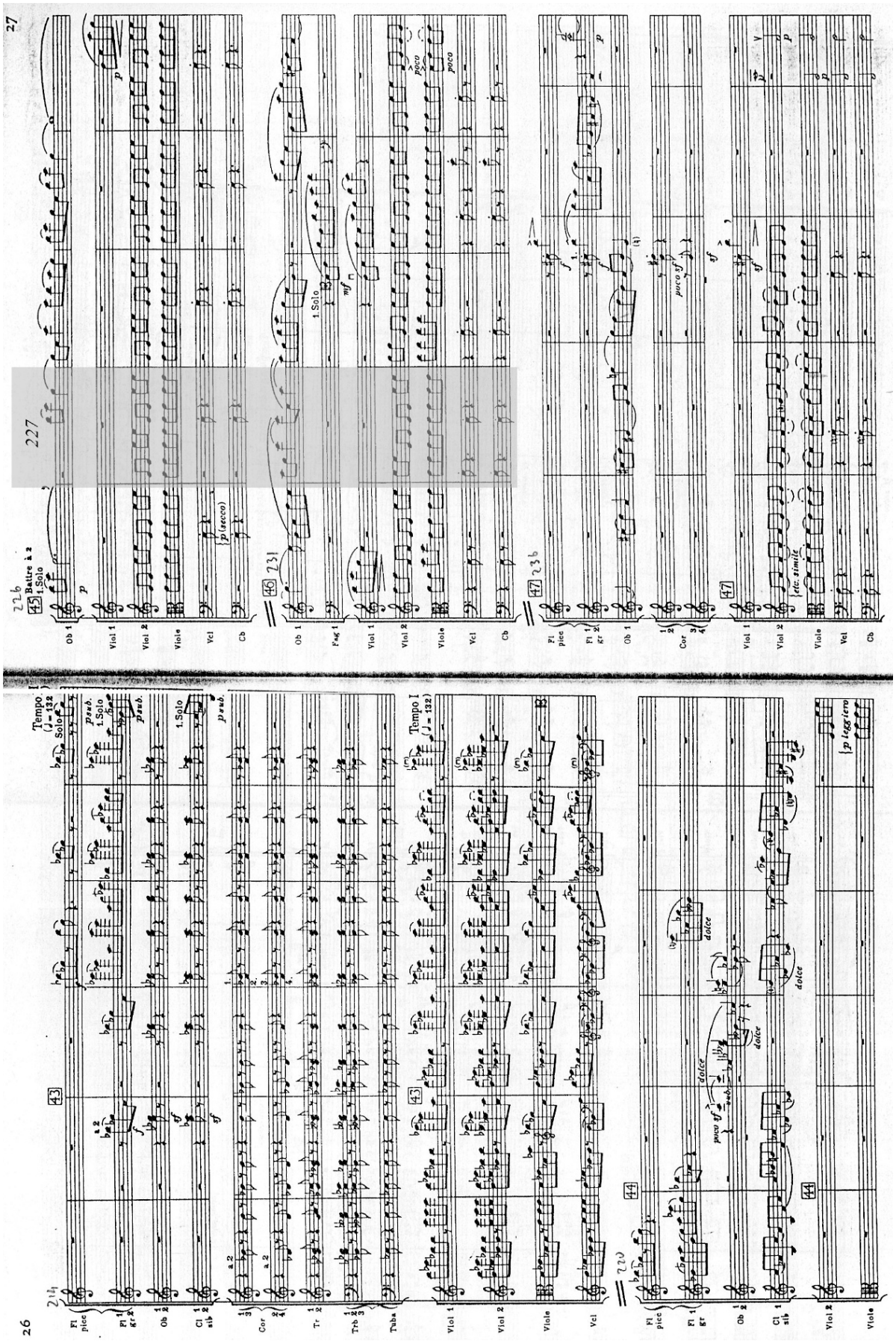

Figure 26. Symphony in C Sheet Music 1

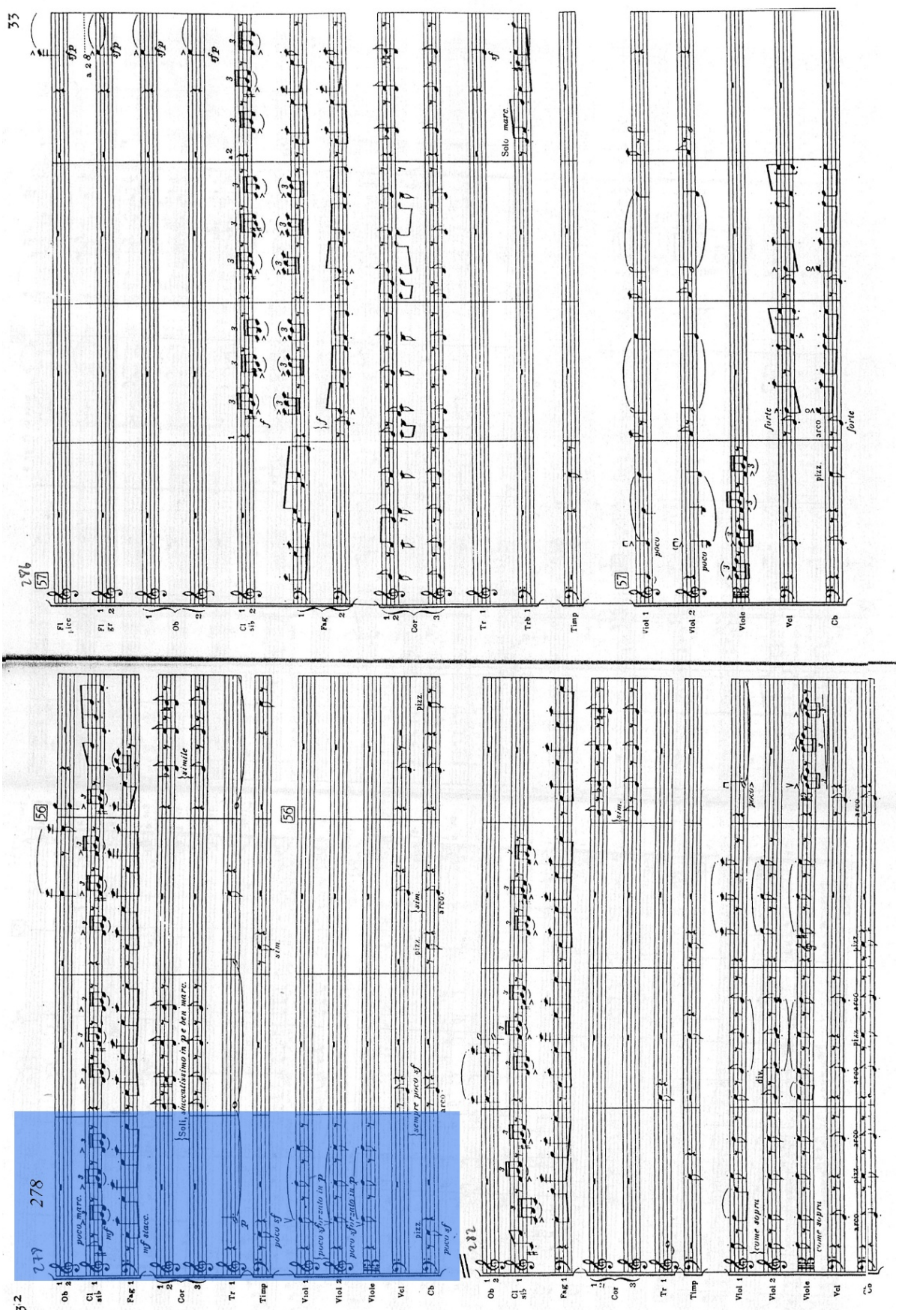

Figure 27. Symphony in C Sheet Music 2

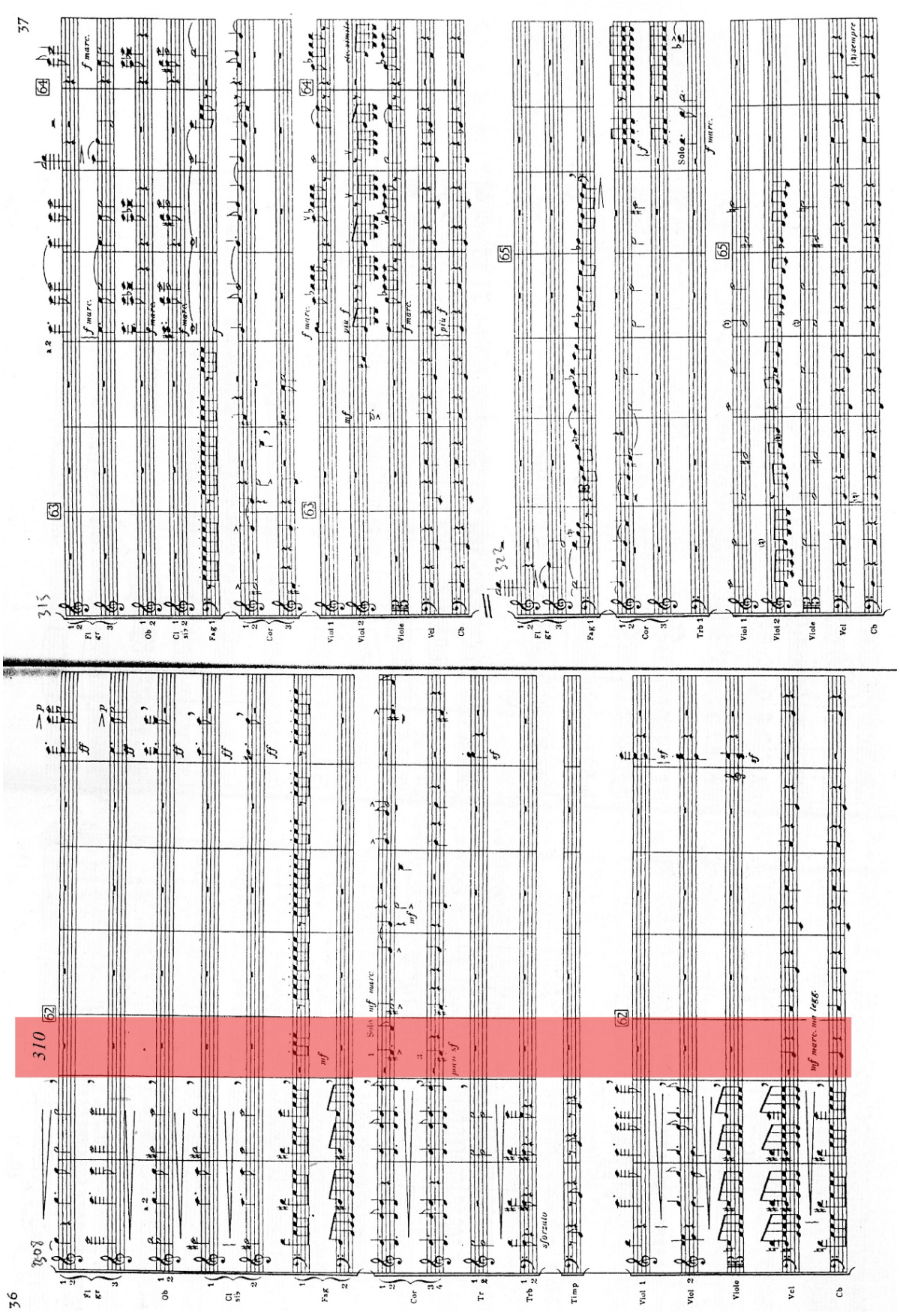

Figure 28. Symphony in C Sheet Music 3

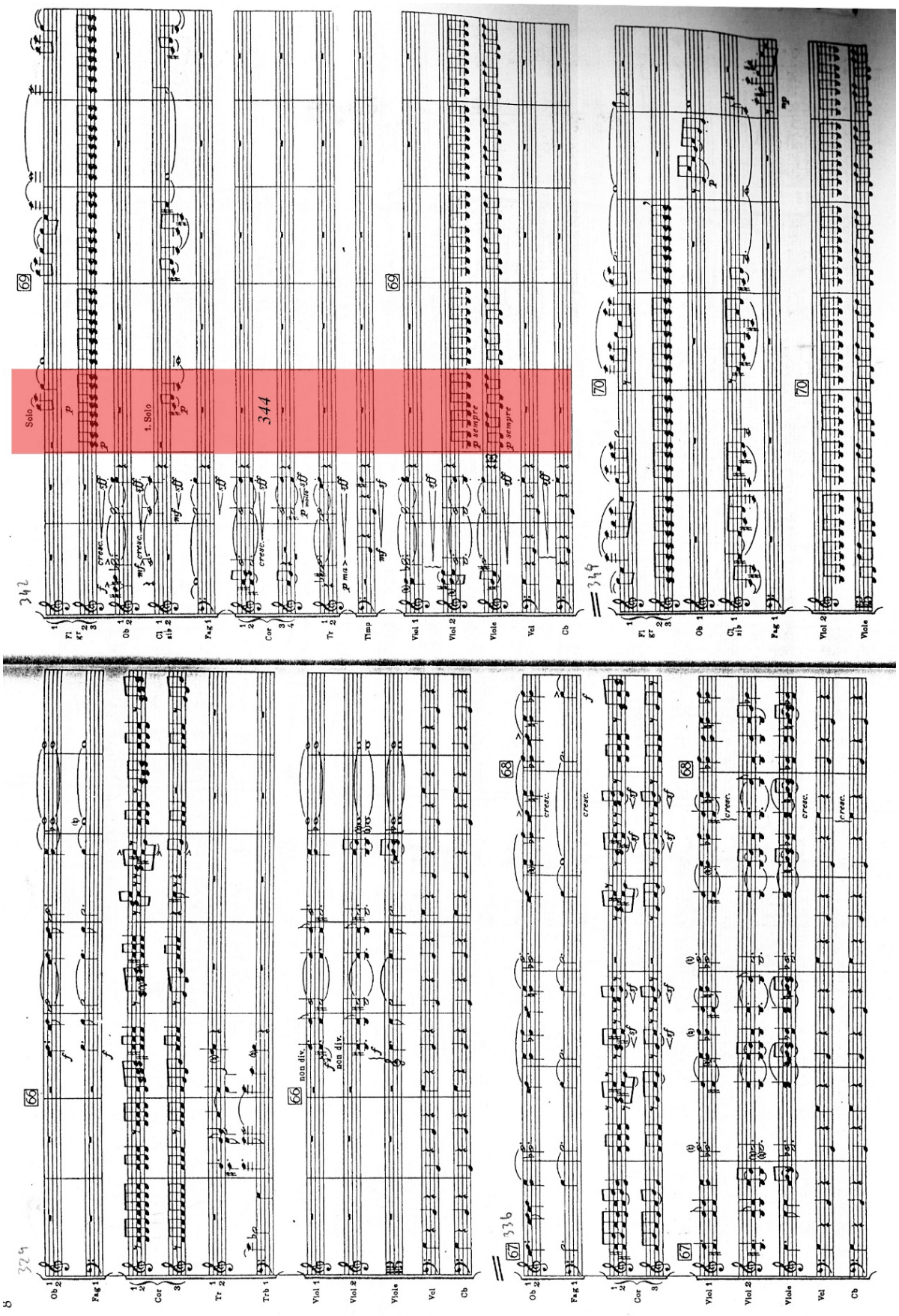

Figure 29. Symphony in C Sheet Music 4

Anton Webern Variations for Piano op. 27

The Variations for Piano, Op. 27, composed in 1936, exhibits a structural similarity to Bartók's Music for Strings, Percussion and Celesta, composed in the same year. Both compositions focus on the same tonal center, A.

It is widely acknowledged that both Bartók and Webern were fascinated by the natural world and its underlying scientific principles. They both held a profound appreciation for plants and their intricate forms, as evidenced by Sadie (2001, pp. 179–189).

In addition, Webern was interested in the structural similarities between architecture and music. In a letter to Josef Humplik dated 5 March 1933, he articulated this concept as follows:

Dear Pepo, I recently had the opportunity to view the Parthenon Frieze, which I found to be an astonishing feat of design. It is a remarkable reflection of our compositional method, manifesting in a multitude of forms. Its sheer scale is awe-inspiring, and it bears resemblance to Bach's "Fugue in D Minor".

Webern did not explicitly identify the design element that remained constant across the numerous forms. However, an analysis of the examples and the accompanying analyses suggests that he was likely alluding to the Golden Ratio.

Given that the work is written in serial method, an analysis employing Pitch Class Set Theory, a method developed in the 20th century, would be appropriate. However, to avoid digressing from the subject and to focus on the Golden Ratio, we will not delve into this detail.

The variations are treated in a single section and their structural form can be summarised as follows:

A B A B | B A B || A B A | A B B A
A1 B1 B2 A2

These sections can be created according to the following criteria:

A1 and A2 are identified in the form according to their correspondence with each other, their sectional content, their tempo and low dynamics.

The distinctive feature of B1 and B2 is that they are single 8-note divisions. The second part is defined by its ternary structure, which varies from the first part with extreme dynamics, often alternating between ***f*** and ***p***.

The sole section that features 16ths and 32nds, as well as uninterrupted alternations between '*ritardando*' and '*a tempo*', is B1.

The center of the splits is defined by the pause between B1 and B2, which divides the sequence in the middle.

Figure 30 exhibits the section reaches its zenith in the middle of Section 9 or Measure 33. In this instance, the dynamic ***p*** is situated between the two ***f***'s. The presence of this p does not result in a reduction of the dynamic level at the peak; rather, it creates a contrast between the ***f***'s, thereby drawing attention to the peak even more. Given that the weighing is constant, a calculation can be made for the section with the measure numbers, with the sole exception of the issue of delays. There are 54 measures in the movement, each comprising three 16ths. Their product thus yields 162 semiquavers. Upon multiplying this figure by the golden ratio number 0.618, the result is 100 quavers (162/100 = ϕ).

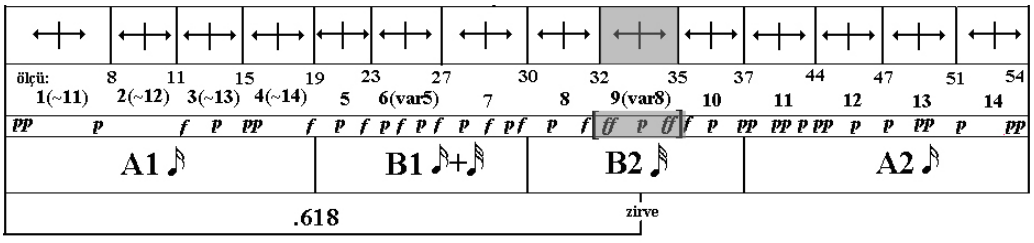

Figure 30. Op.27 Analysis (Solomon, 2002, p. 11)

The 33rd measure, which corresponds to the Golden Section, can also be calculated by the number of bars (54/33 = ϕ). Furthermore, the delays are also approximately evenly balanced, with 18 occurring before the climax and 17 occurring after, and do not disturb the overall proportion. In order to more accurately reflect the sequential structure of the work and the information provided, a different graphic representation has been selected than that used previously. The Golden Ratio point within the work are also illustrated in Figure 31.

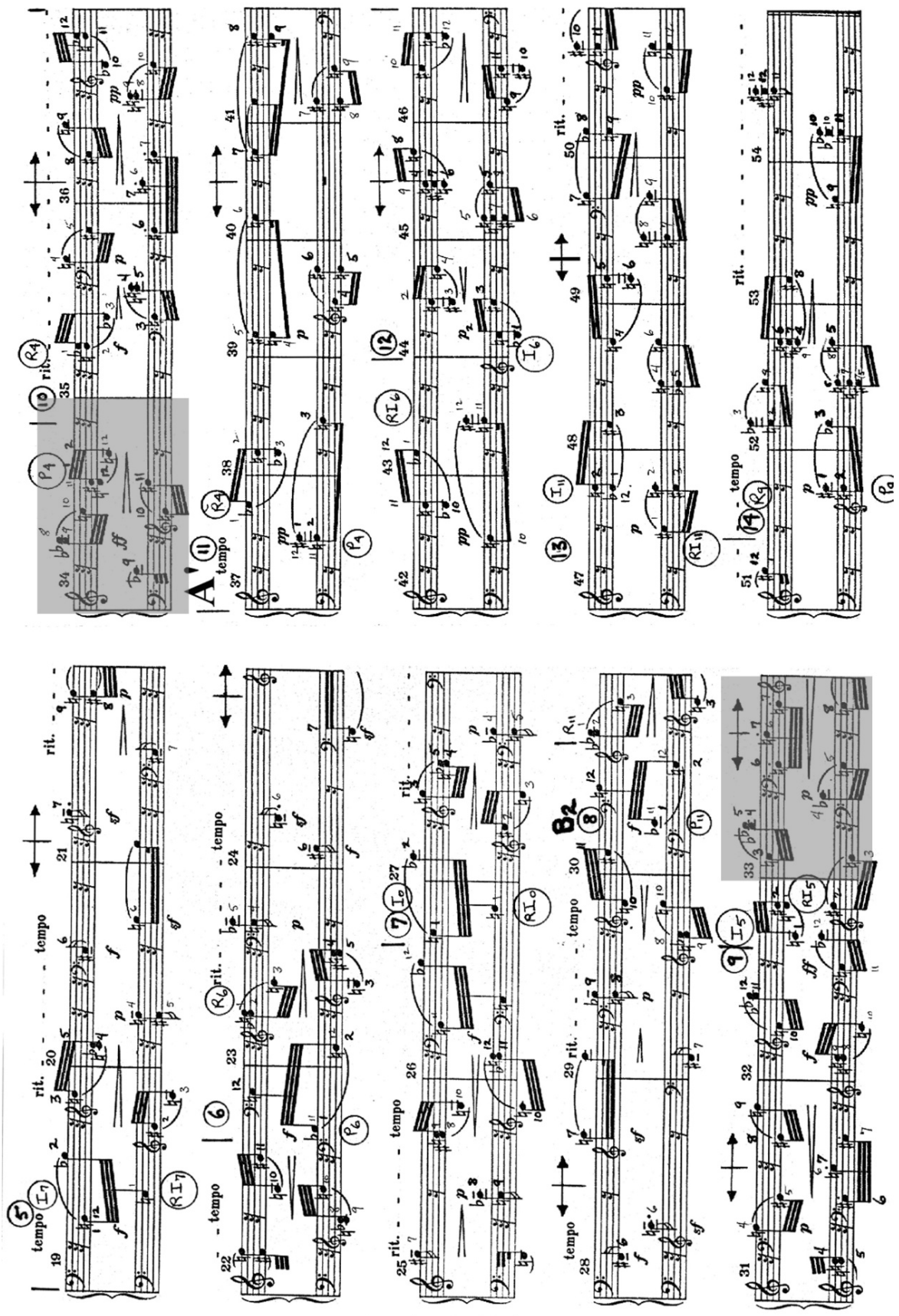

Figure 31. Op. 27 Sheet Music

Conclusion

In consideration of the aforementioned analyses, it can be observed that the Golden Ratio is employed in a functional and structural capacity within the domain of musical art. As an effective shaping and balancing element in musical composition, it has been demonstrated that the Golden Ratio is employed in a variety of contexts. A review of the literature on the Golden Ratio reveals that, based on historical evidence, its use was largely intuitive and instinctive, rather than based on mathematical calculations.

However, as evidenced in the work of Webern, it is evident that the Golden Ratio was a deliberate and intentional choice that evolved from being a mere means to an end, particularly in the 20th century. This may have been influenced by Schoenberg's musical system, which largely based its composition on mathematical and logical principles.

Concurrently with the aforementioned analyses, the composer and candidates of composition, are provided with guidance on matters of form and balance in their studies of composition. It is evident that there is no definitive, universally accepted methodology in art. However, from a formal perspective, the Golden Ratio represents a reliable and established approach.

A comparable advantage is observed in the case of performers. The application of the Golden Ratio enables the reliable identification of turning points in musical works, the substantiation of interpretations, and the formulation of opinions regarding disputed bars, unspecified dynamics, and expressions. Since the Golden Ratio can also be shown on a temporal graph, it can be a guide for the performer, especially for the common tempo changes (rubato) in the performance of romantic period music, and it can prevent overdoing in tempo interpretation by giving the opportunity to comply with the golden sections formed according to the tempo and number of measures specified by the composer.

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Anton Webern, Variations for piano - op. 27, for piano, Universal Edition.

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