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Research Article

A New Kibria-Lukman-Type Estimator for Poisson Regression Models

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ABSTRACT

One of the most important models for the analysis of count data is the Poisson Regression Model (PRM). The parameter estimates of the PRM are obtained by the Maximum Likelihood Estimator (MLE). However, MLE is adversely affected in the presence of multicollinearity, which is known as the approximately linear relationship between the explanatory variables. Many shrinkage estimators have been proposed to reduce the effects of multicollinearity in PRMs. As an alternative to other biased estimators that are already in use in PRMs, we presented a novel estimator in this paper that is based on the Kibria-Lukman estimator. The superiority of the proposed new biased estimator over existing biased estimators is given by the asymptotic matrix mean square error. Furthermore, two separate Monte Carlo simulation studies are conducted to investigate the performance of the proposed biased estimators. Finally, real data is used to examine the superiority of the proposed estimator.

Keywords: Mean squared error, multicollinearity, poisson liu estimator, poisson regression, poisson ridge estimator

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1. INTRODUCTION

The Poisson Regression Model (PRM) is a basic model used to analyze count data (Hilbe, 2014). Let y_i be the response variable and follow a Poisson distribution with mean μ_i, the probability mass function is defined as

$$f(y_i) = \frac{e^{-\mu_i}\mu_i^{y_i}}{y_i!}, i = 1, 2, .., n \quad y_i = 0, 1, 2, ... \tag{1.1}$$

where the canonical log-link function and a linear combination of explanatory variables are used to describe μ_i as follows: $\mu_i = \exp(x_i^{'}\beta)$ where $x_i^{'}$ is the ith row of model matrix X, which is an $n \times (p+1)$ matrix with p explanatory variables and β is a $(p+1)\times$ n vector of parameters.

The most popular estimation approach for estimating parameters in PRMs is the Maximum Likelihood method. The following is the log-likelihood function for PRM:

$$l(\beta) = \sum_{i=1}^{n} y_i x_i^{'}\beta - \exp(x_i^{'}\beta) - \log(y_i!). \tag{1.2}$$

The log-likelihood function is maximized to obtain the Maximum Likelihood Estimator (MLE) of β , which yields the following equations:

$$S(\beta) = \frac{\partial l(\beta)}{\partial \beta} = \sum_{i=1}^{n} \left[y_i - \exp\left(x_i'\beta\right)\right] x_i = X'(y-\mu) = 0 \tag{1.3}$$

where μ is an $n \times 1$ dimensional vector with elements are $\mu_i = \exp\left(x_i'\beta\right), i = 1, 2, ..., n$. Since Equation (1.3) is nonlinear in β , the following iteratively reweighted least squares (IRLS) algorithm is used to find the solution of $S(\beta)$:

$$\hat{\beta}_{MLE} = (X^{'}\hat{W}X)^{-1}X^{'}\hat{W}Z \tag{1.4}$$

where Z is a vector with the ith element $z_i = \log(\hat{\mu}_i) + \frac{y_i - \hat{\mu}_i}{\hat{\mu}_i}$ and $\hat{W} = diag[\hat{\mu}_i]$, $i = 1, 2, ..., n$. The iterations end when the difference between successive estimates converges or is less than a specified small value, which is usulally 10^{-8} (Dunn & Smyth, 2018). The logic of the IRLS algorithm for PRM with a canonical link function is summarized in Table 1 (Hardin & Hilbe, 2018).

Table 1. IRLS estimation algorithm for PRM with canonical link function.

$Dev = 0$
$\mu_i = (y_i + \text{mean}(y))/2$, #where y is the response vector whose components are y_i, $i = 1, 2, \ldots, n$. Initialization of μ
$\eta = \ln(\mu)$ #initialization of η
While abs(ΔDev > tolerans) {
$W = \text{diag}(\mu_i)$ $i = 1, 2, \ldots, n$ #where W is the weighted matrix.
$z_i = \log(\mu_i) + (y_i - \mu_i)/\mu_i$ $i = 1, 2, \ldots, n$
$\beta = (X'WX)^{-1} X'WZ$ #where Z is a vector with the ith element $z_i, i = 1, 2, ..., n$.
$\eta = X'\beta$
$\mu = \exp(\eta)$
oldDev = Dev
$Dev = 2\Sigma\{y \ln(y/\mu) - (y - \mu)\}$
$\Delta Dev = Dev -$ oldDev } #Iterate until the change in deviance, log-likelihood, or estimated parameter values between two iterations is below a specified level of tolerance, or threshold.

An important drawback of the MLE is that estimated parameter values become unstable when multicollinearity occurs (Kibria et al. 2013; Türkan & Özel, 2016; Rashad & Algamal, 2019; Amin et al. 2022; Jadhav, 2022; Alkhateeb &

Algamal, 2020; Månsson & Kibria, 2020; Lukman et al., 2021; Akay & Ertan, 2022; Ertan & Akay, 2023). The estimates of model parameters in PRMs, as in linear regression models, are affected by the multicollinearity problem, which results from the approximately linear relationship between explanatory variables. The variance of MLE increases to such a degree that the estimates of model parameters become unstable due to multicollinearity between the explanatory variables Månsson & Shukur (2011), Månsson et al. (2012), Kibria et al. (2015), Asar & Genç (2018), Çetinkaya & Kaçıranlar (2019), Qasim et al. (2020b), Alheety et al. (2021).

Instead of using the MLE, alternative biased estimators are recommended to alleviate the negative impacts of multicollinearity. For instance, Månsson & Shukur (2011) defined the Poisson Ridge Estimator (PRE) as follows:

$$\hat{\beta}_{PRE} = \left(X'\hat{W}X + kI\right)^{-1} X'\hat{W}X\hat{\beta}_{MLE}, k > 0, \tag{1.5}$$

where k is a biasing parameter. The Ridge estimator (RE) proposed by Hoerl & Kennard (1970) for the linear regression model is generalized by the PRE.

The Poisson Liu Estimator (PLE) is proposed by Månsson et al. (2012), Amin et al. (2021), and Qasim et al. (2020a) as

$$\hat{\beta}_{PLE} = (X'\hat{W}X + I)^{-1}(X'\hat{W}X + dI)\hat{\beta}_{MLE}, 0 < d < 1, \tag{1.6}$$

where d is a biasing parameter. The Liu estimator (LE), which Liu (1993) proposed for the linear regression model, is extended by PLE.

As an alternative to PRE and PLE, two biased estimators have been merged in recent years to produce innovative estimators with two biasing parameters. The Poisson–Liu-type estimator (PLTE) for PRM is defined in this context by Algamal (2018) as follows:

$$\hat{\beta}_{PLTE} = \left(X'\hat{W}X + kI\right)^{-1} \left(X'\hat{W}X - dI\right) \hat{\beta}_{MLE}, k > 0, d \in R \tag{1.7}$$

where k and d are the biasing parameters, respectively. The PLTE is a generalization of the Liu-type estimator introduced by Liu (2003). Moreover, Asar & Genç (2018) and Çetinkaya & Kaçıranlar (2019) proposed another estimator with two biasing parameters, which was defined by Özkale & Kaçıranlar (2007) for linear regression models. The Poisson Two-Parameter Estimator (PTPE) is defined as follows:

$$\hat{\beta}_{PTPE} = \left(X'\hat{W}X + kI\right)^{-1} \left(X'\hat{W}X + kdI\right) \hat{\beta}_{MLE},\ k > 0, 0 < d < 1, \tag{1.8}$$

where k and d are the biasing parameters, respectively.

However, as an alternative to estimators with two biasing parameters, Akay & Ertan (2022) proposed the improved Liu-type Estimator (ILTE). The following definitions of ILTE include MLE, PRE, PLE, PLTE, and PTPE:

$$\hat{\beta}_{ILTE} = (X'\hat{W}X + kI)^{-1}(X'\hat{W}X + f(k)I)\hat{\beta}^{*},\ k > 0, \tag{1.9}$$

where $\hat{\beta}^{*}$ is an estimator of β and $f(k)$ is a continuous function of k. The ILTE is a generalization of the Liu-type estimator defined by Kurnaz & Akay (2015) for linear regression models.

Let $f(k) = -k$ and $\hat{\beta}^{*} = \hat{\beta}_{MLE}$ as a special case of the estimator given by (1.9). In the literature, this estimator is known as Kibria-Lukman type estimator. Aladeitan et al. (2021) defined the Kibria-Lukman-type estimator for PRM as follows:

$$\hat{\beta}_{PKLE} = \left(X'\hat{W}X + kI\right)^{-1} \left(X'\hat{W}X - kI\right) \hat{\beta}_{MLE}, k > 0, \tag{1.10}$$

where k is a biasing parameter.

Numerous biased estimators for linear regression models have been adapted for use with PRMs in the literature. In recent investigations, researchers have concentrated on the Kibria-Lukman type estimator (Aladeitan et al., 2021; Dawoud et al. 2022; Lukman et al., 2023; Akay et al., 2023; Alrweili, 2024). Therefore, in addition to the estimators given above, in this paper, we focus on the application to PRMs of a new estimator based on the PKLE estimator given by (1.10). Additionally, as an alternative to the PRE and PLE, our goal in this study is to examine the performance of this new estimator with a single biasing parameter.

The remainder of this paper is organized as follows. A new biased estimator is defined, and some of its characteristics are described in Section 2. The conditions under which the proposed new estimator outperforms ILTE in terms of the matrix mean squared error are illustrated in Section 3. In Section 4, several estimators are proposed to determine the biasing parameter. In Section 5, two separate Monte Carlo simulation studies are conducted to evaluate the performance of the proposed estimator compared to other estimators. A real-world data application is presented in Section 6 to demonstrate how well the suggested biased estimators function. Finally, the conclusions of the study are given in Section 7.

2. A NEW KIBRIA-LUKMAN-TYPE ESTIMATOR

For PRMs, we can generalize the Kibria-Lukman estimator given in (1.10) as follows:

$$\hat{\beta} = (X^{'}WX + kI)^{-1}(X^{'}WX - kI)\hat{\beta}^{*} \tag{2.1}$$

where k is the biasing parameter and $\hat{\beta}^{*}$ is any estimator of β. As an approach to the case of nested estimators, we consider the estimator obtained when $\hat{\beta}^{*} = \hat{\beta}_{PRE}$ as follows:

$$\hat{\beta}_{PKLTEI} = (X^{'}WX + kI)^{-1}(X^{'}WX - kI)(X^{'}WX + kI)^{-1}X^{'}WX\hat{\beta}_{MLE} \tag{2.2}$$

where k is a biasing parameter. If $\hat{\beta}^{*} = \hat{\beta}_{PKLE}$, the estimator obtained is as follows:

$$\hat{\beta}_{PKLTEII} = (X^{'}WX + kI)^{-1}(X^{'}WX - kI)(X^{'}WX + kI)^{-1}(X^{'}WX - kI)\hat{\beta}_{MLE} \tag{2.3}$$

where k is a biasing parameter.

Using the estimator provided in (2.2) and (2.3), we can now determine the asymptotic scalar mean squared error (SMSE) and the asymptotic matrix mean squared error (MMSE). We indicate $\alpha = Q'\beta$, $\Lambda = diag\left(\lambda_1, ..., \lambda_{p+1}\right) = Q'\left(X'\hat{W}X\right)Q$, where $\lambda_1 \geq \lambda_2 \geq ...\lambda_{p+1} > 0$ are the ordered eigenvalues of $X'\hat{W}X$, the eigenvectors of $X'\hat{W}X$ are represented by the columns of Q and the ith element of $Q'\beta$ is denoted as α_j, $j = 1, 2, ..., p+1$.

The asymptotic SMSE and the asymptotic MMSE of $\hat{\beta} = A\hat{\beta}_{MLE}$ are defined as follows:

$$MSEM(\hat{\beta}) = E(\hat{\beta} - \beta)(\hat{\beta} - \beta)^{'} = A(\hat{\beta}_{MLE} - \beta)(\hat{\beta}_{MLE} - \beta)^{'}A^{'} + (A\beta - \beta)(A\beta - \beta)^{'} \tag{2.4}$$

$$SMSE(\hat{\beta}) = E(\hat{\beta} - \beta)^{'}(\hat{\beta} - \beta) = (\hat{\beta}_{MLE} - \beta)^{'}A^{'}A(\hat{\beta}_{MLE} - \beta) + (A\beta - \beta)^{'}(A\beta - \beta).$$

The relationship between the MMSE and SMSE is $SMSE(\hat{\beta}) = tr(MMSE(\hat{\beta}))$. Because of the relation of $\alpha = Q^{'}\beta$; $\hat{\beta}_{MLE}, \hat{\beta}_{PRE}, \hat{\beta}_{PLE}, \hat{\beta}_{PLTE}, \hat{\beta}_{ILTE}$ and $\hat{\beta}_{PKLTE}$ possess identical SMSE values to $\hat{\alpha}_{MLE}, \hat{\alpha}_{PRE}, \hat{\alpha}_{PLE}, \hat{\alpha}_{PLTE}, \hat{\alpha}_{ILTE}$ and $\hat{\alpha}_{PKLT}$, respectively.

Using (1.4), (1.5), (1.6), (1.7), (1.9), (1.10), (2.2), and (2.3), we can calculate the MMSE of the considered estimators as follows:

$$MMSE\left(\hat{\beta}_{MLE}\right) = Q\Lambda^{-1}Q' \tag{2.5}$$

$$MMSE\left(\hat{\beta}_{PRE}\right) = Q\left((\Lambda + kI)^{-1}\Lambda(\Lambda + kI)^{-1} + k^2(\Lambda + kI)^{-1}\alpha\alpha'(\Lambda + kI)^{-1}\right)Q' \tag{2.6}$$

$$MMSE\left(\hat{\beta}_{PLE}\right) = Q\left((\Lambda + I)^{-1}(\Lambda + dI)\Lambda^{-1}(\Lambda + dI)(\Lambda + I)^{-1} + (d-1)^2(\Lambda + I)^{-1}\alpha\alpha'(\Lambda + I)^{-1}\right)Q' \tag{2.7}$$

$$MMSE\left(\hat{\beta}_{PLTE}\right) = Q\left((\Lambda + kI)^{-1}(\Lambda + dI)\Lambda^{-1}(\Lambda + dI)(\Lambda + kI)^{-1} + (d-k)^2(\Lambda + kI)^{-1}\alpha\alpha'(\Lambda + kI)^{-1}\right)Q' \tag{2.8}$$

$$MMSE\left(\hat{\beta}_{ILTE}\right) = Q\left((\Lambda + kI)^{-1}(\Lambda + f(k)I)\Lambda^{-1}(\Lambda + f(k)I)(\Lambda + kI)^{-1} + (f(k) - k)^2(\Lambda + kI)^{-1}\alpha\alpha'(\Lambda + kI)^{-1}\right)Q' \tag{2.9}$$

$$MMSE\left(\hat{\beta}_{PKLE}\right) = Q(\Lambda + kI)^{-1}(\Lambda - kI)\Lambda^{-1}(\Lambda - kI)(\Lambda + kI)^{-1}Q' + bias\left(\hat{\beta}_{PKLR}\right)bias\left(\hat{\beta}_{PKLR}\right)'$$
$$\text{where } bias\left(\hat{\beta}_{PKLE}\right) = \left((\Lambda + kI)^{-1}(\Lambda - kI) - I\right)Q\alpha \tag{2.10}$$

$$MMSE\left(\hat{\beta}_{PKLTEI}\right) = Q(\Lambda + kI)^{-1}(\Lambda - kI)(\Lambda + kI)^{-1}\Lambda(\Lambda + kI)^{-1}(\Lambda - kI)(\Lambda + kI)^{-1}Q'$$
$$+ bias\left(\hat{\beta}_{PKLTEI}\right)bias\left(\hat{\beta}_{PKLTEI}\right)' \text{where } bias\left(\hat{\beta}_{PKLTEI}\right) = \left((\Lambda + kI)^{-1}(\Lambda - kI)(\Lambda + kI)^{-1}\Lambda - I\right)Q\alpha \tag{2.11}$$

$$MMSE\left(\hat{\beta}_{PKLTEII}\right) = Q(\Lambda + kI)^{-1}(\Lambda - kI)(\Lambda + kI)^{-1}(\Lambda - kI)\Lambda^{-1}(\Lambda - kI)(\Lambda + kI)^{-1}(\Lambda - kI)(\Lambda + kI)^{-1}Q'$$
$$+ bias\left(\hat{\beta}_{PKLTEII}\right)bias\left(\hat{\beta}_{PKLTEII}\right)' \tag{2.12}$$

$$\text{where } bias\left(\hat{\beta}_{PKLTEII}\right) = \left((\Lambda + kI)^{-1}(\Lambda - kI)(\Lambda + kI)^{-1}(\Lambda - kI) - I\right)Q\alpha$$

Let $\hat{\beta}_1$ and $\hat{\beta}_2$ be any two estimators of β. Then, $\hat{\beta}_2$ is superior to $\hat{\beta}_1$ with respect to the MMSE criterion if and only if $MMSE\left(\hat{\beta}_1\right) - MMSE\left(\hat{\beta}_2\right)$ is a positive-definite (pd) matrix. If $MMSE\left(\hat{\beta}_1\right) - MMSE\left(\hat{\beta}_2\right)$ is a nonnegative definite matrix, then $SMSE\left(\hat{\beta}_1\right) - SMSE\left(\hat{\beta}_2\right) \geq 0$. However, the opposite is not always true (Theobald, 1974).

The following theorem can be used to compare the MMSEs of biased estimators.

Theorem 2.1. (Farebrother, 1976). Let c be a nonzero vector and A be a positive-definite matrix, namely $A > 0$. Then, $A - cc'$ is a positive-definite matrix iff $c'A^{-1}c \leq 1$.

3. THE SUPERIORITY OF THE PKLTE IN PRMS

In this part, we use the MMSE criterion to compare the PKLTE II and ILTE. As a result, using several $f(k)$ functions allows a more comprehensive assessment of the estimator's performance.

Theorem.3.1: Let be $k > 0$ and $(\lambda_j + k)^2(\lambda_j + f(k))^2 - (\lambda_j - k)^4 > 0$ where $j = 1, ..., p+1$. Then $MMSE\left(\hat{\beta}_{ILTE}\right) - MMSE\left(\hat{\beta}_{PKLTE\,II}\right) > 0$ iff

$$bias\left(\hat{\beta}_{PKLTE\,II}\right)'Q\left((\Lambda + kI)^{-1}(\Lambda + f(k)I)\Lambda^{-1}(\Lambda + kI)^{-1}(\Lambda + f(k)I)\right.$$
$$\left.- (\Lambda + kI)^{-1}(\Lambda - kI)(\Lambda + kI)^{-1}(\Lambda - kI)\Lambda^{-1}(\Lambda - kI)(\Lambda + kI)^{-1}(\Lambda - kI)(\Lambda + kI)^{-1}\right)^{-1}Q'bias\left(\hat{\beta}_{PKLTE\,II}\right) < 1 \tag{3.1}$$

where $bias\left(\hat{\beta}_{PKLTE\,II}\right) = \left((\Lambda + kI)^{-1}(\Lambda - kI)(\Lambda + kI)^{-1}\Lambda - I\right)Q\alpha.$

Proof. Using (2.9) and (2.12), we obtain

$$\begin{aligned}
&MMSE\left(\hat{\beta}_{ILTE}\right) - MMSE\left(\hat{\beta}_{PKLTE\,II}\right)\\
&= Q\left(AA' - BB'\right)Q' + bias\left(\hat{\beta}_{ILTE}\right)bias\left(\hat{\beta}_{ILTE}\right)' - bias\left(\hat{\beta}_{PKLTE\,II}\right)bias\left(\hat{\beta}_{PKLTE\,II}\right)'\\
&= Q\left((\Lambda + kI)^{-1}(\Lambda + f(k)I)\Lambda^{-1}(\Lambda + kI)^{-1}(\Lambda + f(k)I)\right.\\
&\left.- (\Lambda + kI)^{-1}(\Lambda - kI)(\Lambda + kI)^{-1}(\Lambda - kI)\Lambda^{-1}(\Lambda - kI)(\Lambda + kI)^{-1}(\Lambda - kI)(\Lambda + kI)^{-1}\right)^{-1}Q'\\
&+ bias\left(\hat{\beta}_{ILTE}\right)bias\left(\hat{\beta}_{ILTE}\right)' - bias\left(\hat{\beta}_{PKLTE\,II}\right)bias\left(\hat{\beta}_{PKLTE\,II}\right)'\\
&= Q\, diag\left\{\frac{(\lambda_j + f(k))^2}{(\lambda_j + k)^2\lambda_j} - \frac{(\lambda_j - k)^4}{\lambda_j(\lambda_j + k)^4}\right\}_{j=1}^{p+1} Q' + bias\left(\hat{\beta}_{ILTE}\right)bias\left(\hat{\beta}_{ILTE}\right)' - bias\left(\hat{\beta}_{PKLTE\,II}\right)bias\left(\hat{\beta}_{PKLTE\,II}\right)'.
\end{aligned}$$

We can observe that $AA' - BB' > 0$ if and only if $(\lambda_j + k)^2(\lambda_j + f(k))^2 - (\lambda_j - k)^4 > 0$ where $j = 1, 2, ..., p+1$. Therefore, $AA' - BB'$ is the pd matrix. The proof is completed by Theorem 2.1. completes the proof.

4. SELECTION OF BIASING PARAMETER

In general, the most important parameter affecting the estimator performance is the biasing parameter. However, many techniques can be used to determine an appropriate statistic for estimating the biasing parameter. In general, values that minimize the SMSE function with respect to the biasing parameter are usually recommended as estimates of the biasing parameter. Initially, to find the optimal k for PKLTE I, the function $h_1(k) = SMSE(\hat{\beta}_{PKLTEI})$ is given as follows:

$$SMSE(\hat{\beta}_{PKLTEI}) = \sum_{j=1}^{p+1} \frac{\lambda_j(\lambda_j - k)^2}{(\lambda_j + k)^4} + \sum_{j=1}^{p+1} \frac{k^2(k + 3\lambda_j)^2\alpha_j^2}{(\lambda_j + k)^4} \tag{4.1}$$

The derivative of $h_1\ (k)$ in relation to parameter k is given as follows:

$$h_1^{'}(k) = \sum_{j=1}^{p+1} \frac{2\lambda_j(k - 3\lambda_j)(\lambda_j - k - k\alpha_j^2(k + 3\lambda_j))}{(k + \lambda_j)^5} \tag{4.2}$$

When it is $h_1'\ (k) = 0$, we have the following:

$$\begin{aligned} k_{PKLTEI(1)} &= 3\lambda_j \\ k_{PKLTEI(2)} &= -\frac{1 + 3\alpha_j^2\lambda_j + \sqrt{\left(1 + \alpha_j^2\lambda_j\right)\left(1 + 9\alpha_j^2\lambda_j\right)}}{2\alpha_j^2} \\ k_{PKLTEI(3)} &= \frac{-1 - 3\alpha_j^2\lambda_j + \sqrt{\left(1 + \alpha_j^2\lambda_j\right)\left(1 + 9\alpha_j^2\lambda_j\right)}}{2\alpha_j^2} \end{aligned} \tag{4.3}$$

where $j = 1, 2, ..., p + 1$.
Similarly, the $SMSE(\hat{\beta}_{PKLTEII})$ function of the PKLTE II estimator is as follows:

$$SMSE(\hat{\beta}_{PKLTEII}) = \sum_{j=1}^{p+1} \frac{(\lambda_j - k)^4}{\lambda_j(\lambda_j + k)^4} + \sum_{j=1}^{p+1} \frac{16k^2\lambda_j^2\alpha_j^2}{(\lambda_j + k)^4} \tag{4.4}$$

To determine the optimal k, the derivative of $h_2(k) = SMSE(\hat{\beta}_{PKLTEII})$ with respect to k is given as follows:

$$h'_2\ (k) = \sum_{j=1}^{p+1} \frac{8\left(k - \lambda_j\right)\left(k^2 - 2k\lambda_j + \left(1 - 4k\alpha_j^2\right)\lambda_j^2\right)}{\left(k + \lambda_j\right)^5} \tag{4.5}$$

When it is accepted $h_2^{'}(k) = 0$, we have:

$$\begin{aligned} k_{PKLTEII(1)} &= \lambda_j \\ k_{PKLTEII(2)} &= \lambda_j + 2\alpha_j^2\lambda_j^2 - 2\sqrt{\alpha_j^2\lambda_j^3\left(1 + \alpha_j^2\lambda_j\right)} \\ k_{PKLTEII(3)} &= \lambda_j + 2\alpha_j^2\lambda_j^2 + 2\sqrt{\alpha_j^2\lambda_j^3\left(1 + \alpha_j^2\lambda_j\right)} \end{aligned} \tag{4.6}$$

where the biasing parameter k depends on $\hat{\alpha}_j^2$, $j1, 2, ..., p + 1$. To find the estimators of k, we substitute their unbiased estimator $\hat{\alpha}_j^2$ for them for practical purposes. Note that $h_1(k)$ and $h_2(k)$ are nonlinear functions of k. Numerical methods are used to minimize the values of these functions relative to k. To determine the approximate minimum values of $h_1(k)$ or $h_2(k)$, we can make some approximations based on the obtained roots. The biasing parameter k can be estimated using the following estimators based on the simulation results:$\hat{k} = min(\lambda_j)$, $\hat{k} = median(\lambda_j)$, $\hat{k} = mean(\lambda_j)$ and $\hat{k} = quantile(\lambda_j, q)$ where q is the probability value used to generate sample quantiles and $j = 1, 2, ..., p + 1$.

5. THE MONTE CARLO SIMULATION STUDIES

In this section, we design two simulation designs to investigate the performance of PKLTE over other existing biased estimators in PRMs. In the first simulation design, we discuss the effects of sample size (n), the number of the explanatory variables (p) and the degree of the collinearity (ρ) on the behavior of the PRE, PLE, PLTE, PKLE, PKLTE I, and PKLTE II. In the second simulation scheme, we examine the behavior of the biasing parameter on the performances of PRE, PLE, PKLE, PKLTE I, and PKLTE II for each set of (n, ρ, p) . For both simulations, we generate the explanatory variables by following McDonald & Galarneau (1975), Asar & Genç (2018), & Akay & Ertan (2022):

$$x_{ij} = (1 - \rho^2)^{\frac{1}{2}} w_{ij} + \rho w_{ip+1}, i = 1, 2, .., n, j = 1, 2, ..., p \tag{5.1}$$

where ρ is defined such that the correlation between any two variables is given by ρ^2, and w_{ij} are independent standard normal pseudorandom numbers. Three correlation sets are examined, each of which corresponds to ρ=0.85,0.90, and 0.95. The number of explanatory variables selected is p=2,4, and 8. The sample sizes n were 100, 200, and 500. The parameter vector β is selected as the normalized eigenvector corresponding to the greatest eigenvalue of $X'X$ for every set of explanatory variables, so that $\beta'\beta = 1$. In addition, we set the intercept to zero.

In the simulation and application sections, the works of Månsson & Shukur (2011), Månsson et al. (2012), Kibria et al. (2015), Asar & Genc (2018), Alanaz & Algamal (2018), Qasim et al. (2020a), and Akay & Ertan (2022) are used for optimal estimates of biasing parameters for PRE, PLE, and PLTE.

For the biasing parameter k in PRE, we used the optimal estimate of k as$\hat{k}_{PRE} = max\left(\frac{1}{m_j}\right)$ where

$$m_j = \sqrt{\frac{\hat{\sigma}^2}{\hat{\alpha}_j^2}}, j = 1, 2, ..., p \text{ and } \hat{\sigma}^2 = \frac{\sum_{i=1}^{n}(y_i - \hat{\mu}_i)^2}{(n - p - 1)} \text{which is given by Kibria et al. (2015).}$$

Based on the results of Qasim et al. (2020a), we use the optimal estimate of d in PLE as

$$\hat{d}_{PLE} = max\left(0, min\left(\frac{\hat{\alpha}_j^2 - 1}{max(\frac{1}{\lambda_j}) + \hat{\alpha}_{max}^2}\right)\right)$$

Three methods were considered to estimate the biasing parameters k and d of PLTE:

$$PLTEI : \hat{k}_{PLTE} = max\left(\frac{1}{m_j}\right) \text{ where } m_j = \sqrt{\frac{\hat{\sigma}^2}{\hat{\alpha}_j^2}}, j = 1, 2, ..., p \text{ and } \hat{d}_{PLTE} = \frac{\sum_{j=1}^{p} \frac{1-\hat{k}_{PLTE}\hat{\alpha}_j^2}{\left(\lambda_j+\hat{k}_{PLTE}\right)^2}}{\sum_{j=1}^{p} \frac{1+\lambda_j\hat{\alpha}_j^2}{\lambda_j\left(\lambda_j+\hat{k}_{PLTE}\right)^2}}$$

$$PLTEII : \hat{k}_{PLTE} = \frac{\lambda_1 - 100\lambda_p}{99} \text{ and } \hat{d}_{PLTE} = \frac{\sum_{j=1}^{p} \frac{1-\hat{k}_{PLTE}\hat{\alpha}_j^2}{\left(\lambda_j+\hat{k}_{PLTE}\right)^2}}{\sum_{j=1}^{p} \frac{1+\lambda_j\hat{\alpha}_j^2}{\lambda_j\left(\lambda_j+\hat{k}_{PLTE}\right)^2}}$$

$$PLTEIII : \hat{d}_{PLTE} = \frac{1}{2}min\left\{\frac{\lambda_j}{1 + \lambda_j\hat{\alpha}_j^2}\right\} \text{ and } \hat{k}_{PLTE} = \frac{1}{p}\sum_{j=1}^{p}\frac{\lambda_j - \hat{d}_{PLTE}(1 + \lambda_j\hat{\alpha}_j^2)}{(\lambda_j\hat{\alpha}_j^2)}$$

The k values for PKLE, PKLTE I, and PKLTE II are estimated using $\hat{k} = quantile\left(\lambda_j, q = \frac{8p-16}{100}\right)$ where p is the number of variables.

A comparison of the proposed estimators is based on the performance of the estimated MSEs (EMSEs), which are computed for an estimator $\hat{\beta}$ of β as

$$EMSE\left(\hat{\beta}\right) = \frac{1}{N}\sum_{r=1}^{N}\sum_{j=1}^{p+1}\left(\hat{\beta}_{rj} - \beta_j\right)^2 \tag{5.2}$$

where $\hat{\beta}_{rj}$ denotes the estimate of the j-th parameter in r-th replication, β_j are the true parameter values and N is the number of replications. The experiment is repeated 2000 times by creating response variables for each n, p and ρ. Using the R programing language, we conducted our Monte Carlo simulation studies, and Table 2 presents the results.

Table 2. The EMSE values of the estimators for the model with p = 2, 4, and 8

ID	p	n	ρ	MLE	PRE	PLE	PLTE I	PLTE II	PLTE III	PKLE	PKLTE I	PKLTE II
1	2	100	0.85	5.7538	0.4173	0.6121	2.7869	2.9650	0.9499	0.4039***	0.3767**	**0.3650***
2	2	100	0.9	6.3544	0.4055	0.5847	3.0471	3.2076	0.9489	0.3858***	0.3590**	**0.3455***
3	2	100	0.95	10.8516	0.4072***	0.4867	5.0765	5.2186	0.8905	0.4121	0.3873**	**0.3670***
4	2	200	0.85	3.7320	0.4376	0.6913	1.8306	2.0112	0.9749	0.3808**	**0.3790***	0.4124***
5	2	200	0.9	6.6718	0.4123	0.5603	3.2277	3.3813	0.9257	0.3893***	0.3640**	**0.3503***
6	2	200	0.95	10.4451	0.3984***	0.4955	5.0736	5.2371	0.9384	0.4030	0.3747**	**0.3523***
7	2	500	0.85	4.0880	0.4278	0.6749	2.0199	2.2165	0.9888	0.3707**	**0.3643***	0.3923***
8	2	500	0.9	6.4054	0.4043***	0.5793	3.0480	3.2379	0.9691	0.4096	0.3762**	**0.3570***
9	2	500	0.95	10.3092	0.4273***	0.5140	4.8168	4.9829	0.9360	0.4371	0.4077**	**0.3841***
10	4	100	0.85	8.7874	0.4113	0.8322	3.1125	4.5803	0.9791	0.3021***	0.2655**	**0.2613***
11	4	100	0.9	11.8191	0.3594***	0.7546	4.1368	5.5644	1.0489	0.4161	0.2991**	**0.2405***
12	4	100	0.95	33.6826	0.3430**	0.5055	12.0338	13.9604	1.2690	0.6725	0.3530***	**0.2318***
13	4	200	0.85	9.9565	0.3975	0.8404	3.6038	4.8777	1.0626	0.3100***	0.2621**	**0.2490***
14	4	200	0.9	17.4294	0.3327***	0.6551	6.2684	7.4587	1.1962	0.3395	0.2566**	**0.2183***
15	4	200	0.95	29.3244	0.3489***	0.5191	10.3539	11.4370	1.2341	0.4108	0.2857**	**0.2325***
16	4	500	0.85	10.7389	0.3837	0.8153	3.8824	4.3418	1.0971	0.2480***	**0.2356***	0.2387**
17	4	500	0.9	14.9318	0.3411	0.6729	5.1903	5.8912	1.0949	0.2803***	0.2435**	**0.2267***
18	4	500	0.95	34.2084	0.3582***	0.4690	11.7316	12.3691	1.1561	0.3883	0.2838**	**0.2371***
19	8	100	0.85	24.9982	0.353***	1.4524	7.3804	10.0366	1.1071	0.4055	0.2044**	**0.1371***
20	8	100	0.9	33.3556	0.2801**	1.2469	9.9121	13.6827	1.1946	1.0685	0.2989***	**0.1852***
21	8	100	0.95	75.5253	0.2181**	0.7280	22.0949	24.0267	1.6149	1.5832	0.4653***	**0.1764***
22	8	200	0.85	21.1630	0.4074***	1.5106	6.3536	9.4580	1.0062	0.4180	0.2081**	**0.1535***
23	8	200	0.9	37.3296	0.2736***	1.1506	11.233	13.9802	1.2988	0.7033	0.2419**	**0.1458***
24	8	200	0.95	70.3848	0.2282**	0.7576	21.1294	22.5549	1.6376	0.7787	0.3043***	**0.1282***
25	8	500	0.85	24.2082	0.3560	1.3791	7.2097	8.9182	1.0212	0.2346***	0.1616**	**0.1424***
26	8	500	0.9	37.9996	0.2722***	1.0581	11.2893	12.6393	1.1571	0.3217	0.1766**	**0.1337***
27	8	500	0.95	76.1531	0.2428***	0.6816	22.4592	23.1806	1.4282	0.4454	0.2118**	**0.1325***

The estimators with the lowest EMSE values are indicated in the table by bolded numerals. The second and third smallest EMSE values are denoted by the signs (**) and (***), respectively.

The results obtained in Table 2 are listed below:

1) When p and ρ are kept constant, PRE, PLE, PKLE, and PKLTE II exhibit stable behavior as the number of observations increases. In contrast, PKLTE I shows a decreasing effect for large variables and high correlation values.

2) When the number of observations (n) and ρ are kept constant, the EMSE values of PRE, PKLTE I, and II decrease as the number of variables increases, whereas PLE and PKLE increase.

3) When n and p in the model are kept constant, PRE, PKLTE I, and PKLTE II are more robust than the other estimators as the correlation increases. In contrast, the EMSE of PKLE increased as the correlation increased, whereas PKLE decreased as the number of observations increased for large observation values.

As a result, when all cases are analyzed, the estimator with the smallest EMSE values is PKLTE II.

In the second simulation scheme, we investigated the performances of PRE, PLE, PKL, PKLTE I, and PKLTE II for each n, ρ, and p. The purpose of this simulation is to examine the performances of PRE, PLE, PKLE, PKLTE I, and PKLTE II at various values of k according to the EMSE values given in (5.2). The second simulation approach did not estimate the biasing parameter k. Only the EMSE values derived by increasing k values in the [0,1] range by 0.1 are compared. Depending on these n, ρ, and p values, the explanatory variables are generated according to equation (5.1). The simulation is conducted 2000 times for each k value. Figures 1, 2, and 3 graphically show the results.

The following results can be obtained based on Figures 1-3:

1) The EMSE values of PRE tend to decrease with increasing values of k. On the contrary, PLE tends to increase as the biasing parameter increases.

2) The EMSE values of PKLE, PKLTE I, and PKLTE II generally decreased faster than in PRE.

3) The EMSE values of the PKLTE I and PRE behave in almost the same way, whereas PKLE and PKLTE II show an increase in the EMSE values after a certain value of k.

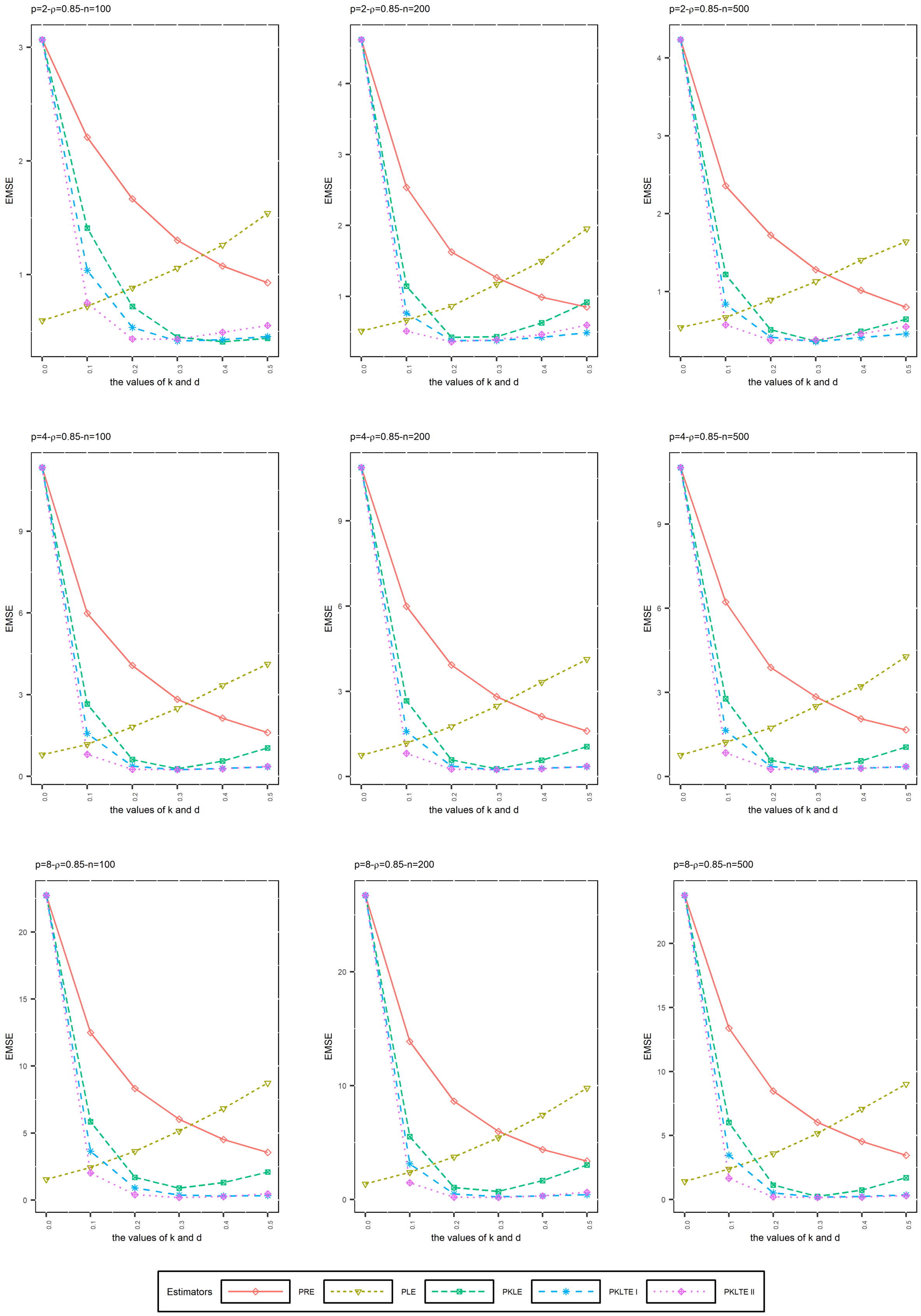

Figure 1. The EMSE values of PRE, PLE, PKLE, PKLTE I, and PKLTE II as a function k and d where ρ=0.85

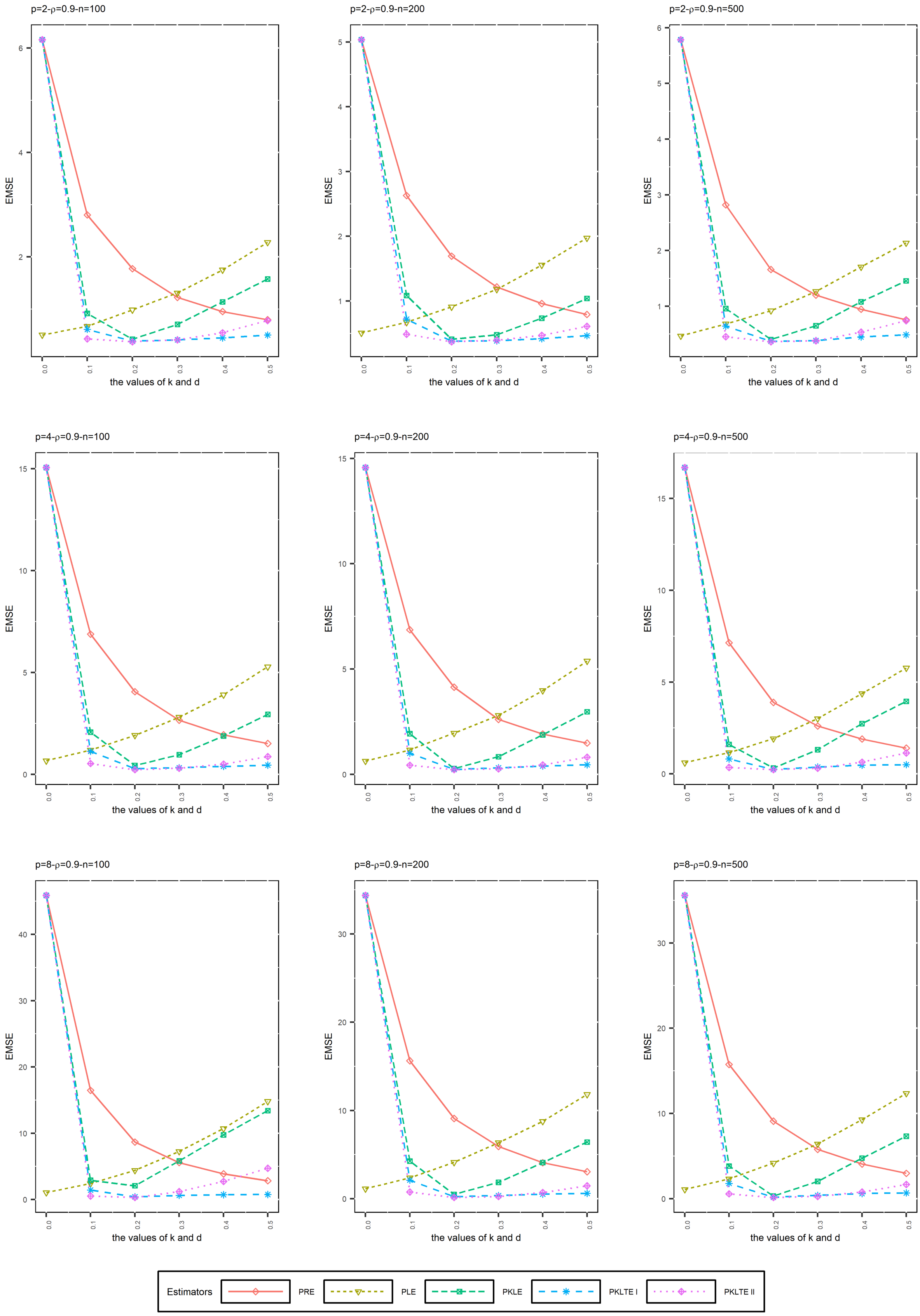

Figure 2. The EMSE values of PRE, PLE, PKLE, PKLTE I, and PKLTE II as functions of k and d where ρ=0.90

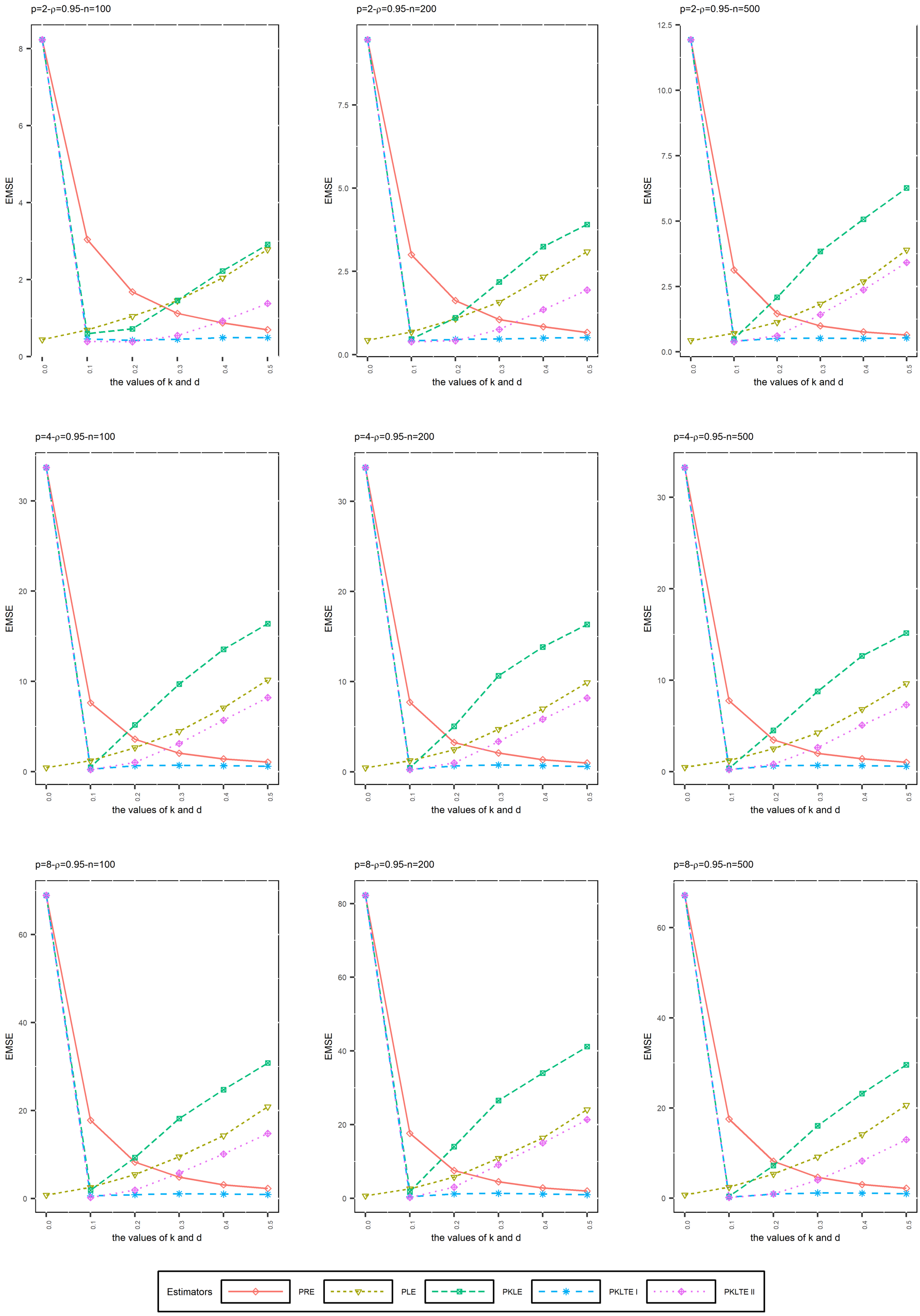

Figure 3. The EMSE values of PRE, PLE, PKLE, PKLTE I, and PKLTE II as functions of k and d where ρ=0.95.

6. AN EXAMPLE: THE AIRCRAFT DAMAGE DATA

This section examines the performance of PKLTE by considering aircraft damage data. Asar & Genc (2018), Myers et al. (2012), Lukman et al. (2021), Lukman et al. (2022), Amin et al. (2022), Akay & Ertan (2022), and Ertan & Akay (2023) also used these data. There are three explanatory variables and thirty observations in this data set. The kind of aircraft is indicated by the dichotomous explanatory variable (x_1) . The bomb load in tons and the total number of months of aircrew experience the explanatory variables (x_2) and (x_3), respectively. The number of locations where the aircraft was damaged is represented by the count variable y.

Asar & Genc (2018), Amin et al. (2022), and Akay & Ertan (2022) described the effects and solutions due to multicollinerity in the following model: $\mu = \exp(\beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3)$. The eigenvalues of $X'X$ are 2085.2251, 374.8961 and 4.3333. As a result, condition number 219.3654 suggests that there is a problem with multicollinearity among the explanatory variables. Additionally,$\lambda_1 = 283543.5$, $\lambda_2 = 789.85$, $\lambda_3 = 4.2887$ and $\lambda_4 = 1.2585$ are the eigenvalues of $X'\hat{W}X$. The condition number is 474.653, which is significantly greater than 30, suggesting that multicollinearity continues to have an impact on MLE.

Table 3 summarizes the numerical results for comparing the PKLTEs with the other existing estimators. The average of the MLE values determined by the bootstrap sampling technique is considered as a true parameter to compute the SMSE values of the biased estimators. Table 3 shows that compared with the other estimators under consideration, PKLTEs produce good results in terms of variance and SMSE values.

Table 3. The parameter and SMSE values of the estimators

	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\mathrm{var}\left[\hat{\beta}\right]$	$SMSE\left[\hat{\beta}\right]$
$\hat{\beta}_{MLE}$	0.1262	1.5576	2.6710	-1.4157	3.8847	
$\hat{\beta}_{PRE}\ \left(\hat{k}_{PRE} = 2.7044\right)$	0.4200	0.7970	1.2620	-0.5504	0.2329	0.2391
$\hat{\beta}_{PLE}\ \left(\hat{d}_{PLE} = 0\right)$	0.2994	1.1554	1.8704	-0.8852	0.6714	0.6736
$\hat{\beta}_{PLTE\,\mathrm{I}}\ \left(\hat{k} = 2.7044, \hat{d} = -1.2946\right)$	0.2793	1.1611	1.9365	-0.9646	1.3472	1.3489
$\hat{\beta}_{PLTE\,\mathrm{II}}\ \left(\hat{k} = 0.1088, \hat{d} = 0.3268\right)$	0.2329	1.3552	2.1716	-1.0640	0.6947	0.6961
$\hat{\beta}_{PLTE\,\mathrm{III}}\ \left(\hat{k} = 36.3139, \hat{d} = 0.0484\right)$	0.3820	0.1383	0.2081	-0.0787	0.0093	0.0494
$\hat{\beta}_{PKLE}\ \left(\hat{k} = 0.8681\right)$	0.4409	0.8377	1.2194	-0.4451	0.5924	0.5998
$\hat{\beta}_{PKLTE\,I}\ \left(\hat{k} = 0.8681\right)$	0.5029	0.6212	0.9187	-0.3035	0.1200	0.1295
$\hat{\beta}_{PKLTE\,II}\ \left(\hat{k} = 0.8681\right)$	0.5649	0.4047	0.6179	-0.1619	0.1109	0.1231

We now wish to examine, in terms of MMSE, the performance of the PKLTE II and ILTE that were derived from the selection of different $f(k)$ functions. We replace with the estimates obtained from the bootstrap sampling approach. Let us take $f(k) = 0.05k + 0.05$ for ILTE. In this instance, $cov(\hat{\beta}_{ILTE}) - cov(\hat{\beta}_{PKLTEII})$ is the pd matrix for $0 < k \leq 1.499$. In addition, the values of k satisfying the inequality in (3.1) are $0 < k \leq 1.245$. Consequently, $MMSE\left(\hat{\beta}_{ILTE}\right) - MMSE\left(\hat{\beta}_{PKLTEII}\right)$ is the pd matrix where $0 < k \leq 1.245$. Let us take $f(k) = 0.05k - 0.01$ for another comparison. In this case, $cov(\hat{\beta}_{ILTE}) - cov(\hat{\beta}_{PKLTEII})$ is pd matrix for $0 < k \leq 1.1679$. Additionally, $0 < k \leq 1.1676$ are the values of k that satisfy the inequality in (3.1). Thus, $MMSE(\hat{\beta}_{ILTE}) - MMSE(\hat{\beta}_{PKLTEII})$ is the pd matrix where $0 < k \leq 1.1676$.

7. CONCLUSION

In this paper, we propose a new biased estimator for PRMs called PKLTE as an alternative to MLE and other existing biased estimators in the presence of multicollinearity. The PKLTE is a general estimator that includes PKLE and its variations. We investigated the properties of PKLTE and proposed several estimators to estimate the biasing parameter. The performance of the proposed PKLTEs was evaluated using Monte Carlo simulations. The findings demonstrate that in the scenario of low-moderate-high multicollinearity, the proposed PKLTE II performs better than the existing estimators. Additionally, a generic simulation study is provided to compare PRE, PLE, PKLE, PKLTE I, and PKLTE II. It can be observed that PKLTE I and PKLTE II exhibit a faster decrease in EMSE values than PRE when the biasing parameter k is varied in the range [0,1]. We can also say that PKLE and PKLTE II reach minimum EMSE values in this range. Furthermore, the considered estimators were applied to real data, and the results were found to be consistent with the simulation study. Therefore, based on the simulation and application results, PKLTE II is recommended when there is multicollinearity in PRMs.

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