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Fixed Points Results for (α,β) - Contractive Mapping in Intuitionistic Fuzzy Metric-Like Spaces

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Abstract

Onbasioglu and Pazar Varol [17] successfully unified metric-like and intuitionistic fuzzy metric structures and establish a foundation for significant advances in fixed-point theory through their introduction of intuitionistic fuzzy metric-like spaces. Building upon this work, we propose a novel class of contraction mappings, called (α,β)-contraction mappings, within these spaces. Moreover, we contribute to the expanding knowledge base by presenting innovative results related to sequence convergence within this novel framework. We introduce a new concept, namely the $1-\mathscr{M}-$ Cauchy sequence and $1-\mathscr{M}-$ completeness within intuitionistic fuzzy metric-like spaces.

Keywords: *Intuitionistic fuzzy (α,β)-contractive mapping, $1-\mathscr{M}-$complete intuitionistic fuzzy metric-like spaces, fixed point.*
2010 Mathematics Subject Classification: *03B52, 47H10, 54H25.*

1. Introduction and Preliminaries

In classical set theory, the idea of member and non-member of an element in a set was exact, i.e., an element either belongs to a set or does not belong to the set. However, there was no knowledge about the transition of an element from member to non-member of the set and vice-versa. The fuzzy set theory, introduced by Zadeh [30], solved these ambiguities. Fuzzy set theory helps elements gradually become members of a set through evaluation using a membership function. This approach has provided a marked development in dealing with fields involving data that can be uncertain and imprecise, such as natural sciences and engineering. Indeed, fuzzy set theory is a mathematical framework dealing with uncertainty and data imprecision. It extends classical set theory by allowing elements to belong to a set to varying degrees.

Definition 1.1. *[30] A fuzzy set A in a nonempty set $\check{X}$ is defined by $A=\{\langle\check{x},\mu_A(\check{x})\rangle:\check{x}\in\check{X}\}$, where $\mu_A:\check{X}\to[0,1]$ denotes membership functions. $\mu_A(\check{x})$ is membership degree of each element $\check{x}\in\check{X}$ to the fuzzy set A.*

Although fuzzy set theory provides a powerful tool for modelling problems by assigning degrees of membership to the elements of a set, at one point it was inadequate for real-world problems because it only included degrees of membership. Atanassov [2] has solved this deficiency and gap in the literature through his new invention, the intuitionistic fuzzy set theory. This theory allows us to handle not only a degree of membership but also a degree of non-membership.

Definition 1.2. *[2] An intuitionistic fuzzy set A on $\check{X}$ is an object having the form $A=\{\langle\check{x},\mu_A(\check{x}),\nu_A(\check{x})\rangle:\check{x}\in\check{X}\}$ where $\mu_A:\check{X}\to[0,1]$ and $\nu_A:\check{X}\to[0,1]$ denote membership and nonmembership functions, respectively. $\mu_A(\check{x})$ and $\nu_A(\check{x})$ are membership and nonmembership degree of each element $\check{x}\in\check{X}$ to the intuitionistic fuzzy set A and $\mu_A(\check{x})+\nu_A(\check{x})\leq 1$ for each $\check{x}\in\check{X}$.*

In fuzzy set theory and logic, t-norms and t-conorms are mathematical operators used to define the intersection and union of fuzzy sets, respectively. These operators play an important role in operations involving fuzzy sets or intuitionistic fuzzy sets, allowing you to combine, separate or control membership degrees and non-membership degrees of an element to make informed decisions in the presence of uncertainty. They are fundamental tools in fuzzy logic for making decisions under uncertainty.

Definition 1.3. *[23] Let $\mathbb{I}=[0,1]$. The binary operation $*:\mathbb{I}\times\mathbb{I}\to\mathbb{I}$ is called a (continuous) t-norm if $*$ satisfies the following for all $u,v,w,t\in\mathbb{I}$:*
(1) $u*1=u$,
(2) $u*v=v*u$ and $u*(v*w)=(u*v)*w$,

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(3) If $u \leq w$ and $v \leq t$, then $u * v \leq w * t$,
(4) $*$ is continuous.

Definition 1.4. *[23] Let $\mathbb{I} = [0,1]$. The binary operation $\diamond : \mathbb{I} \times \mathbb{I} \to \mathbb{I}$ is called a (continuous) t-conorm if $\diamond$ satisfies the following for all $u,v,w,t \in \mathbb{I}$:*
(1) $u \diamond 0 = u$,
(2) $u \diamond v = v \diamond u$ and $u \diamond (v \diamond w) = (u \diamond v) \diamond w$,
(3) If $u \leq w$ and $v \leq t$, then $u \diamond v \leq w \diamond t$,
(4) $\diamond$ is continuous.

Below, we provide well-known examples by considering the relation between t-norms and t-conorms.

t-norm	t-conorm
$u *_m v = min\{u,v\}$	$u \diamond_m v = max\{u,v\}$
$u *_p v = uv$	$u \diamond_p v = u + v - uv$
$u *_l v = max\{u+v-1, 0\}$	$u \diamond_l v = min\{u+v, 1\}$

Here symbols $*_m$ minimum t-norm and $\diamond_m$ maximum t-conorm ; $*_p$, $\diamond_p$ product t-norm and t-conorm; $*_l$, $\diamond_l$ Lukasiewicz t-norm and t-conorm are describes respectively. Also, the following inequalities are satisfied:

$u *_l v \leq u *_p v \leq u *_m v$ and for each t-norm $u * v \leq u *_m v$
$u \diamond_l v \geq u \diamond_p v \geq u \diamond_m v$ and for each t-conorm $u \diamond_m v \leq u \diamond v$.

By the definitions, we have that if $r_1 > r_2$, then there exist $r_3, r_4 \in (0,1)$ such that $r_1 * r_3 \geq r_2$ and $r_2 \diamond r_4 \leq r_1$.
George and Veeramani(GV)[5] extended the original concept of fuzzy metric spaces, defined by Kramosil and Michalek [15], by incorporating continuous t-norms to refine and generalize the structure. This modification allowed many natural examples of fuzzy metrics, especially fuzzy metrics obtained from classical metrics. Fuzzy metric spaces in the sense of GV appeared to be more appropriate for the study of induced topological structures from classical metrics and also, it has a lot of applications ([3], [6], [9]-[11], [18], [24], [26]) in various fields where fuzzy distances are a more appropriate way to model the relationships between elements.

Definition 1.5. *[5] Let M be a fuzzy set on $\check{X} \times \check{X} \times (0,\infty)$ and $*$ be a continuous t-norm. If M fulfills the following features for all $\check{x}, \check{y}, \check{z} \in \check{X}$ and $t,s > 0$, we call that M is a fuzzy metric on $\check{X}$:*
(FM1) $M(\check{x},\check{y},t) > 0$,
(FM2) $M(\check{x},\check{y},t) = 1$ if and only if $\check{x} = \check{y}$,
(FM3) $M(\check{x},\check{y},t) = M(\check{y},\check{x},t)$,
(FM4) $M(\check{x},\check{y},t) * M(\check{y},\check{z},s) \leq M(\check{x},\check{z},t+s)$,
(FM5) $M(\check{x},\check{y},.) : (0,\infty) \to [0,1]$ is continuous.
The triplet $(\check{X}, M, *)$ is called fuzzy metric space (shortly, FMS).

The fuzzy metric of two points in a fuzzy metric space is measured by the degree of the nearness of two points concerning one parameter $t > 0$. In this definition, when $\check{x} = \check{y}$, the degree of nearness of the points $\check{x}$ and $\check{y}$ of the space is perfect, that is, the self fuzzy distance is unity.
In 2004, Park [19] introduced intuitionistic fuzzy metric spaces, which are an extension of fuzzy metric spaces as a generalization of classical metric spaces. Obtained by the idea of combining these three structures, intuitionistic fuzzy metric spaces are a more useful tool to deal with uncertainty, imprecision, and hesitation using the concept of metric spaces.

Definition 1.6. *[19] $(\check{X}, M, N, *, \diamond)$ is called intuitionistic fuzzy metric space (shortly, IFMS), if M and N are fuzzy sets on $\check{X}^2 \times (0,\infty)$, $*$ and $\diamond$ are continuous t-norm and t-conorm, satisfy the following conditions for all $\check{x}, \check{y}, \check{z} \in \check{X}$ and $t,s > 0$:*
(IFM1) $M(\check{x},\check{y},t) + N(\check{x},\check{y},t) \leq 1$,
(IFM2) $M(\check{x},\check{y},t) > 0$,
(IFM3) $M(\check{x},\check{y},t) = 1$ if and only if $\check{x} = \check{y}$,
(IFM4) $M(\check{x},\check{y},t) = M(\check{y},\check{x},t)$,
(IFM5) $M(\check{x},\check{y},t) * M(\check{y},\check{z},s) \leq M(\check{x},\check{z},t+s)$,
(IFM6) $M(\check{x},\check{y},.) : (0,\infty) \to (0,1]$ is continuous,
(IFM7) $N(\check{x},\check{y},t) < 1$,
(IFM8) $N(\check{x},\check{y},t) = 0$ if and only if $\check{x} = \check{y}$,
(IFM9) $N(\check{x},\check{y},t) = N(\check{y},\check{x},t)$,
(IFM10) $N(\check{x},\check{y},t) \diamond N(\check{y},\check{z},s) \geq N(\check{x},\check{z},t+s)$,
(IFM11) $N(\check{x},\check{y},.) : (0,\infty) \to (0,1]$ is continuous.
(M,N) is called intuitionistic fuzzy metric on $\check{X}$.

The functions $M(\check{x},\check{y},t)$ and $N(\check{x},\check{y},t)$ denote the degree of nearness and the degree of non-nearness between $\check{x}$ and $\check{y}$ with respect to t, respectively.
In IFMS, distances between points with respect to one positive parameter are represented as intuitionistic fuzzy numbers called membership and non-membership, allowing for a more delicate description of vagueness. Many researchers use intuitionistic fuzzy sets to create proper mathematical models for their picked-up data. These results motivate many authors to study this concept in various areas such as fixed-point theorems and contraction mappings in this structure ([1], [13], [20], [22], [27], [28]).

Remark 1.7. *Let $(\check{X}, M, N, *, \diamond)$ be an IFMS, then $(\check{X}, M, *)$ is a FMS. Let $(\check{X}, M, *)$ is a FMS, then $(\check{X}, M, 1-M, *, \diamond)$ is an IFMS, where $u \diamond v = 1 - ((1-u) * (1-v))$, $\forall u, v \in [0,1]$.*

Metric functions and metric spaces are always important tools for mathematics. As a result of ongoing studies in this field, different metric structures have been discovered and the properties of the spaces obtained with these structures have been examined by many authors. Recently, Harandi [12] introduced the concept of metric-like spaces as a generalization of partial metric spaces and metric spaces to the literature and proved the results of the fixed point theorem in these spaces. The metric-like structure allows us to work in more relaxed conditions, as it suggests that in some cases, the self-distance of any point to itself is not necessarily zero. This is an interesting result compared to the traditional understanding of metric spaces.

Definition 1.8. *[12] A mapping $\sigma : \check{X} \times \check{X} \to IR^{+}$ is called metric-like on $\check{X}$ if the following hold:*
(ML1) $\sigma(\check{x},\check{y}) = 0 \Rightarrow \check{x} = \check{y}$*;*
(ML2) $\sigma(\check{x},\check{y}) = \sigma(\check{y},\check{x})$*;*
(ML3) $\sigma(\check{x},\check{z}) \leq \sigma(\check{x},\check{y}) + \sigma(\check{y},\check{z})$*.*
The pair $(\check{X},\sigma)$ is called a metric-like (or a dislocated metric or d-metric) space on $\check{X}$.

In 2014, Shukla [25] defined fuzzy metric-like spaces and presented the results of the fixed point theorem in this new setting by combining fuzzy metric structure and metric-like structure.

Definition 1.9. *[25] Let $*$ be a continuous t-norm,F be a fuzzy set on $\check{X} \times \check{X} \times (0,\infty)$, in this case $(\check{X},F,*)$ is called fuzzy metric-like space (shortly, FMLS) if the following properties hold for all $\check{x},\check{y},\check{z} \in \check{X}$ and $t,s > 0$;*
(FML1) $F(\check{x},\check{y},t) > 0$,
(FML2) $F(\check{x},\check{y},t) = 1 \Rightarrow \check{x} = \check{y}$,
(FML3) $F(\check{x},\check{y},t) = F(\check{y},\check{x},t)$,
(FML4) $F(\check{x},\check{y},t) * F(\check{y},\check{z},s) \leq F(\check{x},\check{z},t+s)$,
(FML5) $F(\check{x},\check{y},.) : (0,\infty) \to [0,1]$ *is continuous.*
Here, F with $*$ is called fuzzy metric-like on $\check{X}$.

In this settings, when $\check{x} = \check{y}$, the degree of the nearness of the points $\check{x}$ and $\check{y}$ of the space is not perfect, that is, the self-fuzzy distance may not necessarily be equal to 1.
Studies on fuzzy metric structures have been continued over time and extended to other sets such as neutrosophic sets as referred ([4], [14], [29]). In 2023, Onbasioglu and Pazar Varol [17] considered the intuitionistic fuzzy metric-like space which is a combination of intuitionistic fuzzy metric space and metric-like space and presented their new results and studied fixed point theorems in this new setting.

Definition 1.10. *[17] Let F,G be fuzzy sets on $\check{X}^2 \times (0,\infty)$, $*$ and $\diamond$ be continuous t-norm and t-conorm, respectively. We say that (F,G) is characterized as intuitionistic fuzzy metric-like on $\check{X}$ if F and G fulfills the following conditions for all $\check{x},\check{y},\check{z} \in \check{X}$ and $t,s > 0$;*
(IFML1) $F(\check{x},\check{y},t) + G(\check{x},\check{y},t) \leq 1$,
(IFML2) $F(\check{x},\check{y},t) > 0$,
(IFML3) $F(\check{x},\check{y},t) = 1 \Rightarrow \check{x} = \check{y}$,
(IFML4) $F(\check{x},\check{y},t) = F(\check{y},\check{x},t)$,
(IFML5) $F(\check{x},\check{y},t) * F(\check{y},\check{z},s) \leq F(\check{x},\check{z},t+s)$,
(IFML6) $F(\check{x},\check{y},.) : (0,\infty) \to (0,1]$ *is continuous,*
(IFML7) $G(\check{x},\check{y},t) < 1$,
(IFML8) $G(\check{x},\check{y},t) = 0 \Rightarrow \check{x} = \check{y}$,
(IFML9) $G(\check{x},\check{y},t) = G(\check{y},\check{x},t)$,
(IFML10) $G(\check{x},\check{y},t) \diamond G(\check{y},\check{z},s) \geq G(\check{x},\check{z},t+s)$,
(IFML11) $G(\check{x},\check{y},.) : (0,\infty) \to [0,1)$ *is continuous.*
The five-tuple $(\check{X},F,G,*,\diamond)$ is called intuitionistic fuzzy metric-like space (shortly, IFMLS).

If we compare the definitions of IFMS and IFMLS according to conditions (IFML3)-(IFM3) and (IFML8)-(IFM8), we observe that the value of $F(\check{x},\check{x},t)$ and $G(\check{x},\check{x},t)$ may be less than '1' and greater than '0', respectively, in an IFMLS. Besides, we can also conclude that every IFMS is IFMLS with unit self distance, which means $F(\check{x},\check{x},t) = 1$ and $G(\check{x},\check{x},t) = 0$ for all $t > 0$, $\check{x} \in \check{X}$.

Remark 1.11. *(1) Every FMLS $(\check{X},F,*)$ is an IFMLS of the form $(\check{X},F,1-F,*,\diamond)$, where $u \diamond v = 1 - [(1-u)*(1-v)]$ for all $u,v \in \mathbb{I}$.*
(2) If $(\check{X},F,G,*,\diamond)$ is an IFMLS, then $(\check{X},F,*)$ is a FMLS. [25].

The remark below shows that a metric-like space can construct an IFMLS.

Remark 1.12. *[17] Let $(\check{X},\sigma)$ be any metric-like space. Then, $(\check{X},F,G,*_m,\diamond_m)$ is an IFMLS, where (F,G) is given as $F(\check{x},\check{y},t) = \frac{t}{t+\sigma(\check{x},\check{y})}$ and $G(\check{x},\check{y},t) = \frac{\sigma(\check{x},\check{y})}{t+\sigma(\check{x},\check{y})}$ for all $\check{x},\check{y} \in \check{X}$, $t > 0$.*

This type of IFMLS $(\check{X},F,G,*_m,\diamond_m)$ is called the standard-IFMLS induced by metric-like.

Definition 1.13. *[17] Let $(\check{X},F,G,*,\diamond)$ be an IFMLS. We define the open ball with center $\check{x}$, radius r with respect to t like $B(\check{x},r,t) = \{\check{y} \in \check{X} : F(\check{x},\check{y},t) > 1-r,\ G(\check{x},\check{y},t) < r\}$, for $\check{x} \in \check{X}$, $r \in (0,1)$, $t > 0$.*

Therefore, $T_{(F,G)} = \{T \subset \check{X} : W \in T \Leftrightarrow \exists\, t > 0, r \in (0,1) : B(\check{x},\check{y},t) \subset T\}$ is a topology on $\check{X}$.

Example 1.14. *Let $(\check{X},F,G,*,\diamond)$ be a standard IFMLS and $\sigma(\check{x},\check{y}) = max\{\check{x},\check{y}\}$ be a metric-like function on $\check{X}$. From the definition of the open ball in IFMLS, when we choose the variables $\check{x},r,t$ specifically as $\check{x} = 0$, $r = 0.5$ and $t = 1$, then we obtain $B(0,0.5,1)$ open ball, which is a set of $\check{y} \in [-10,1]$ points which satisfy $F(0,\check{y},1) > 0.5$ and $G(0,\check{y},1) < 0.5$ inequalities. Below, we show the behaviours of F and G under these circumstances. The second graph also shows the interior points of the $B(0,0.5,1)$ open ball for $\check{y} \in [-10,10]$.*

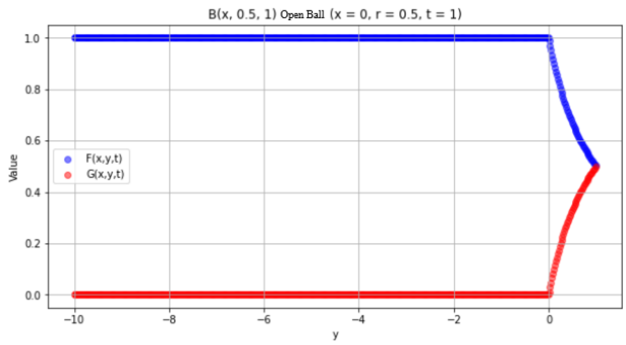

Figure 1.1: F and G for $\check{y} \in [-10,1]$.

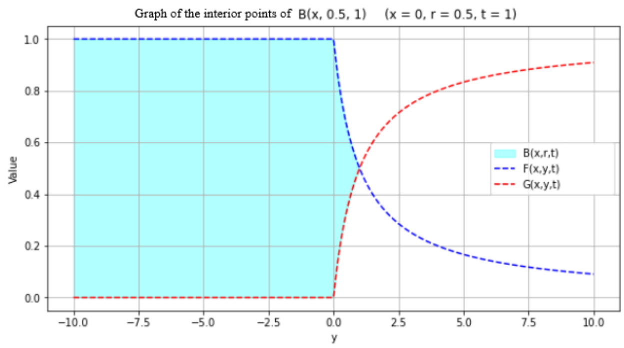

Figure 1.2: F and G for $\check{y} \in [-10,10]$.

Definition 1.15. *[17] Let $(\check{X},F,G,*,\diamond)$ be an IFMLS.*
(i) $(\check{x}_n)$ is called convergence to $\check{x}$ if $\lim_{n\to\infty} F(\check{x}_n,\check{x},t) = F(\check{x},\check{x},t)$ and $\lim_{n\to\infty} G(\check{x}_n,\check{x},t) = G(\check{x},\check{x},t)$ for all $t > 0$.
(ii) $(\check{x}_n)$ is called Cauchy sequence if $\lim_{n\to\infty} F(\check{x}_{n+p},\check{x}_n,t)$ and $\lim_{n\to\infty} G(\check{x}_{n+p},\check{x}_n,t)$ exist and finite for all $t > 0,\ p \geq 1$.
(iii) $(\check{X},F,G,*,\diamond)$ is called complete if every Cauchy sequence $(\check{x}_n)$ in $\check{X}$ converges to some $\check{x} \in \check{X}$ such that $\lim_{n\to\infty} F(\check{x}_n,\check{x},t) = F(\check{x},\check{x},t) = \lim_{n\to\infty} F(\check{x}_{n+p},\check{x}_n,t)$ and
$\lim_{n\to\infty} G(\check{x}_n,\check{x},t) = G(\check{x},\check{x},t) = \lim_{n\to\infty} G(\check{x}_{n+p},\check{x}_n,t)$ for all $t > 0,\ p \geq 1$.

Remark 1.16. *[17] In an IFMLS, a convergent sequence might not have a unique limit and might not be a Cauchy sequence.*

Definition 1.17. *[17] Let $(\check{X},F,G,*,\diamond)$ be an IFMLS. $\mathfrak{D} : \check{X} \to \check{X}$ is called an intuitionistic fuzzy contractive mapping if there exists $0 < \lambda < 1$ such that*

$$\frac{1}{F(\mathfrak{D}(\check{x}),\mathfrak{D}(\check{y}),t)} - 1 \leq \lambda.[\frac{1}{F(\check{x},\check{y},t)} - 1] \quad and \quad G(\mathfrak{D}(\check{x}),\mathfrak{D}(\check{y}),t) \leq \lambda.G(\check{x},\check{y},t)$$

for all $\check{x},\check{y} \in \check{X}$ and $t > 0$. Here, λ is called the intuitionistic fuzzy constant of $\mathfrak{D}$.

2. (α,β) - Contractive Mappings in Intuitionistic Fuzzy Metric-Like Spaces

The fixed-point theorems of Banach and Edelstein for contraction mapping are extended to FMS in the sense of Kramsoil and Michálek by Grabiec in 1988. Following that, Gregori and Sapena [8] and Mihet [16] extended the fixed point theorem of Banach for contraction mapping to fuzzy metric spaces in the sense of George and Veeramani(GV). In 2006, Rafi and Noorani [22] presented the fuzzy contraction mapping and fixed point theorem in IFMS. Onbaşıoğlu and Pazar Varol [17] introduced the intuitionistic fuzzy contraction mapping in IFMLS in 2023 and have proved that the distance between two same points is not necessary to be zero to find a fixed point in this new setting.
Priskillal and Thangavelu [21] defined the (ψ,η)-contraction mapping using special type functions in intuitionistic fuzzy metric spaces in 2017. Here, we present (α,β)- contraction mapping and extend our results considering important features of the metric-like structure.

Definition 2.1. *[21] Let B be the family of functions $\alpha : [0,1] \to [0,1]$ satisfying the following conditions:*
(1) α is non-decreasing;
(2) $\lim_{n\to\infty} \alpha^n(s) = 1$, for each $s \in (0,1)$.
If $\alpha \in B$, then $\alpha(s) > s$ for all $s \in (0,1)$ and $\alpha(1) = 1$.
Let B be the family of functions $\beta : [0,1] \to [0,1]$ satisfying the following conditions:
(1) β is non-decreasing;
(2) $\lim_{n\to\infty} \beta^n(r) = 0$, for each $r \in (0,1)$.
If $\beta \in B$, then $\beta(r) < r$ for all $r \in (0,1)$ and $\beta(0) = 0$.

Example 2.2. *The functions $\alpha : [0,1] \to [0,1]$ below belong to the class B.*
(i) $\alpha(s) = 1$
(ii) $\alpha(s) = \frac{2s}{1+s}$
The functions $\beta : [0,1] \to [0,1]$ below belong to the class B.
(i) $\beta(r) = 0$
(ii) $\beta(r) = \frac{r}{2-r}$

Below, we define B -contraction mapping in IFMLS using $\alpha, \beta \in B$ functions.

Definition 2.3. *Let $(\check{X}, F, G, *, \diamond)$ be an IFMLS and $\alpha, \beta \in B$. $\mathfrak{D} : \check{X} \to \check{X}$ is called an (α, β)-contractive mapping if the following implications hold:*
$F(\mathfrak{D}(\check{x}), \mathfrak{D}(\check{y}), t) \geq \alpha(F(\check{x}, \check{y}, t))$ *and* $G(\mathfrak{D}(\check{x}), \mathfrak{D}(\check{y}), t) \leq \beta(G(\check{x}, \check{y}, t))$ *for all* $\check{x}, \check{y} \in \check{X}$ *and all* $t > 0$.

In this case, we say that $\mathfrak{D}$ is a B -contraction with respect to (α, β).

Remark 2.4. *Every intuitionistic fuzzy contractive mapping with contractive constant λ is an (α, β) contraction where $\alpha(s) = \frac{s}{\lambda + (1-\lambda)s}$ and $\beta(r) = \frac{\lambda r}{1-(1-\lambda)r}$ for all $s, r \in [0,1]$.*

Theorem 2.5. *Let $(\check{X}, F, G, *, \diamond)$ be a complete IFMLS and $\mathfrak{D} : \check{X} \to \check{X}$ be an (α, β)-contractive mapping. Then $\mathfrak{D}$ has a unique fixed point $\check{x} \in \check{X}$.*

Proof. Let $\mathfrak{D}$ be an (α, β)-contractive mapping. Choose an arbitrary $\check{x}_0 \in \check{X}$ and define a sequence $(\check{x}_n)$ given by $\check{x}_{n+1} = \mathfrak{D}(\check{x}_n)$ for all $n \in IN$.
If there exists some $n_0 \in IN$ such that $\check{x}_{n_0+1} = \check{x}_{n_0}$, thus $\check{x}_{n_0}$ is a fixed point of $\mathfrak{D}$. Hence, the existence part is completed. On the contrary part, consider that $\check{x}_{n+1} \neq \check{x}_n$ for all $n \in IN$.
Then for a fixed positive parameter t ,

$$\begin{aligned} F(\check{x}_n, \check{x}_{n+1}, t) &= F(\mathfrak{D}(\check{x}_{n-1}), \mathfrak{D}(\check{x}_n), t) \\ &\geq \alpha(F(\check{x}_{n-1}, \check{x}_n, t)) \\ &= \alpha(F(\mathfrak{D}(\check{x}_{n-2}), \mathfrak{D}(\check{x}_{n-1}), t)) \\ &\geq \alpha^2(F(\check{x}_{n-2}, \check{x}_{n-1}, t)) \\ &\ldots\ldots\ldots \\ &\geq \alpha^n(F(\check{x}_0, \check{x}_1, t)). \end{aligned}$$

By taking limit as $n \to \infty$, we reach $\lim_{n\to\infty} F(\check{x}_n, \check{x}_{n+1}, t) = 1$.

$$\begin{aligned} F(\check{x}_{n+1}, \check{x}_{n+2}, t) &= F(\mathfrak{D}(\check{x}_n), \mathfrak{D}(\check{x}_{n+1}), t) \\ &\geq \alpha(F(\check{x}_n, \check{x}_{n+1}, t)) \\ &= \alpha(F(\mathfrak{D}(\check{x}_{n-1}), \mathfrak{D}(\check{x}_n), t)) \\ &\geq \alpha^2(F(\check{x}_{n-1}, \check{x}_n, t)) \\ &\ldots\ldots\ldots \\ &\geq \alpha^n(F(\check{x}_1, \check{x}_2, t)). \end{aligned}$$

By taking limit as $n \to \infty$, we reach $\lim_{n\to\infty} F(\check{x}_{n+1}, \check{x}_{n+2}, t) = 1$.

$$\begin{aligned} F(\check{x}_n, \check{x}_{n+p}, t) &\geq F(\check{x}_n, \check{x}_{n+1}, \tfrac{t}{p}) * \ldots * F(\check{x}_{n+p-1}, \check{x}_{n+p}, \tfrac{t}{p}) \\ &\geq \alpha^n(F(\check{x}_0, \check{x}_1, \tfrac{t}{p})) * \ldots * \alpha^n(F(\check{x}_{n+p-1}, \check{x}_{n+p}, \tfrac{t}{p})) \end{aligned}$$

By taking limit $n \to \infty$, we get

$$\begin{aligned} \lim_{n\to\infty} F(\check{x}_n, \check{x}_{n+p}, t) &\geq \lim_{n\to\infty} \alpha^n(F(\check{x}_n, \check{x}_{n+1}, \tfrac{t}{p})) * \ldots * \lim_{n\to\infty} \alpha^n(F(\check{x}_{n+p-1}, \check{x}_{n+p}, \tfrac{t}{p})) \\ &\geq 1 * \ldots * 1 = 1 \end{aligned}$$

Hence, $\lim_{n\to\infty} F(\check{x}_n, \check{x}_{n+p}, t) = 1$.
Now, again $t > 0$,

$$\begin{aligned} G(\check{x}_n, \check{x}_{n+1}, t) &= G(\mathfrak{D}(\check{x}_{n-1}), \mathfrak{D}(\check{x}_n), t) \\ &\leq \beta(G(\check{x}_{n-1}, \check{x}_n, t)) \\ &= \beta(G(\mathfrak{D}(\check{x}_{n-2}), \mathfrak{D}(\check{x}_{n-1}), t)) \\ &\leq \beta^2(G(\check{x}_{n-2}, \check{x}_{n-1}, t)) \\ &\ldots\ldots\ldots \\ &\leq \beta^n(G(\check{x}_0, \check{x}_1, t)). \end{aligned}$$

By taking limit as $n \to \infty$, we get $\lim_{n\to\infty} G(\check{x}_n, \check{x}_{n+1}, t) = 0$.
Similarly, we obtain $\lim_{n\to\infty} G(\check{x}_{n+1}, \check{x}_{n+2}, t) = 0$.

$$\begin{aligned} G(\check{x}_n, \check{x}_{n+p}, t) &\leq G(\check{x}_n, \check{x}_{n+1}, \tfrac{t}{p}) \diamond \ldots \diamond G(\check{x}_{n+p-1}, \check{x}_{n+p}, \tfrac{t}{p}) \\ &\leq \beta^n(G(\check{x}_0, \check{x}_1, \tfrac{t}{p})) \diamond \ldots \diamond \beta^n(G(\check{x}_{n+p-1}, \check{x}_{n+p}, \tfrac{t}{p})) \end{aligned}$$

By taking limit $n \to \infty$, we get

$$\begin{aligned} \lim_{n\to\infty} G(\check{x}_n, \check{x}_{n+p}, t) &\leq \lim_{n\to\infty} \beta^n(G(\check{x}_n, \check{x}_{n+1}, \tfrac{t}{p})) \diamond \ldots \diamond \lim_{n\to\infty} \beta^n(G(\check{x}_{n+p-1}, \check{x}_{n+p}, \tfrac{t}{p})) \\ &\leq 0 \diamond \ldots \diamond = 0 \end{aligned}$$

Hence, $\lim_{n\to\infty} G(\check{x}_n, \check{x}_{n+p}, t) = 0$.
That is, $(\check{x}_n)$ is a Cauchy sequence in $\check{X}$. Since $(\check{X}, F, G, *, \diamond)$ is a complete IFMLS, there exists $\check{x} \in \check{X}$ such that $\lim_{n\to\infty} F(\check{x}_n, \check{x}, t) = \lim_{n\to\infty} F(\check{x}_{n+p}, \check{x}_n, t) = F(\check{x}, \check{x}, t) = 1$ and $\lim_{n\to\infty} G(\check{x}_n, \check{x}, t) = \lim_{n\to\infty} G(\check{x}_{n+p}, \check{x}_n, t) = G(\check{x}, \check{x}, t) = 0$, for all positive parameter t and $p \geq 1$.
Now, we prove that $\check{x}$ is a fixed point for $\mathfrak{D}$. Using Definition 2.3, we obtain
$F(\check{x}, \mathfrak{D}(\check{x}), t) \geq F(\check{x}, \check{x}_{n+1}, \frac{t}{2}) * F(\check{x}_{n+1}, \mathfrak{D}(\check{x}), \frac{t}{2}) = F(\check{x}, \check{x}_{n+1}, \frac{t}{2}) * F(\mathfrak{D}(\check{x}_n), \mathfrak{D}(\check{x}), \frac{t}{2}) \geq F(\check{x}, \check{x}_{n+1}, \frac{t}{2}) * \alpha.F(\check{x}_n, \check{x}, \frac{t}{2})$
and $G(\check{x}, \mathfrak{D}(\check{x}), t) \leq G(\check{x}, \check{x}_{n+1}, \frac{t}{2}) \diamond G(\check{x}_{n+1}, \mathfrak{D}(\check{x}), \frac{t}{2}) = G(\check{x}, \check{x}_{n+1}, \frac{t}{2}) \diamond G(\mathfrak{D}(\check{x}_n), \mathfrak{D}(\check{x}), \frac{t}{2}) \leq G(\check{x}, \check{x}_{n+1}, \frac{t}{2}) \diamond \beta.G(\check{x}_n, \check{x}, \frac{t}{2})$.
Taking limit as $n \to \infty$ and using above equalities, we get $F(\check{x}, \mathfrak{D}(\check{x}), t) = 1$ and $G(\check{x}, \mathfrak{D}(\check{x}), t) = 0$. Hence, $\mathfrak{D}(\check{x}) = \check{x}$.

We now proceed to prove that the $\breve{x}$ is a unique fixed point of $\mathfrak{D}$. Let $\breve{z}$ be another fixed point of $\mathfrak{D}$. For any positive t, we get $F(\breve{x},\breve{z},t) = F(\mathfrak{D}(\breve{x}),\mathfrak{D}(\breve{z}),t) \geq \alpha(F(\breve{x},\breve{z},t))... \geq \alpha^n(F(\breve{x},\breve{z},t))$.
By taking limit $n \to \infty$, we have $F(\breve{x},\breve{z},t) \geq \lim_{n\to\infty}\alpha^n(F(\breve{x},\breve{z},t)) = 1$ and hence $F(\breve{x},\breve{z},t) = 1$.
Now, again for $t > 0$, $G(\breve{x},\breve{z},t) = G(\mathfrak{D}(\breve{x}),\mathfrak{D}(\breve{z}),t) \leq \beta(G(\breve{x},\breve{z},t))... \leq \beta^n(G(\breve{x},\breve{z},t))$.
By taking limit $n \to \infty$, we have $G(\breve{x},\breve{z},t) \leq \lim_{n\to\infty}\beta^n(G(\breve{x},\breve{z},t)) = 0$ and hence $G(\breve{x},\breve{z},t) = 0$. Hence, $\breve{x} = \breve{z}$. □

Example 2.6. *Let $\breve{X} = [0,\infty)$ with the metric-like σ defined by $\sigma(\breve{x},\breve{y}) = |\breve{x}-\breve{y}|$, define $F(\breve{x},\breve{y},t) = \frac{t}{t+\sigma(\breve{x},\breve{y})}$ and $G(\breve{x},\breve{y},t) = \frac{\sigma(\breve{x},\breve{y})}{t+\sigma(\breve{x},\breve{y})}$, for all $\breve{x},\breve{y} \in \breve{X}$ and $t > 0$.*
Then, $(\breve{X},F,G,*,\diamond)$ is a complete IFMLS where $u*v = uv$ and $u \diamond v = min\{1,u+v\}$.
Let $\mathfrak{D} : \breve{X} \to \breve{X}$, $\mathfrak{D}(\breve{x}) = \frac{9-\breve{x}}{4}$.
Define the maps $\alpha,\beta : [0,1] \to [0,1]$ by $\alpha(s) = \frac{3s}{2s+1}$ and $\beta(r) = \frac{r}{3-2r}$ for each $s,r \in [0,1]$ and $\alpha,\beta \in B$.

$$F(\mathfrak{D}(\breve{x}),\mathfrak{D}(\breve{y}),t) \geq \alpha(F(\breve{x},\breve{y},t)) \Rightarrow F(\tfrac{9-\breve{x}}{4},\tfrac{9-\breve{y}}{4},t) \geq \frac{3.F(\breve{x},\breve{y},t)}{2.F(\breve{x},\breve{y},t)+1} \Rightarrow \frac{t}{t+\sigma(\frac{9-\breve{x}}{4},\frac{9-\breve{y}}{4})} \geq \frac{\frac{3t}{t+\sigma(\breve{x},\breve{y})}}{\frac{2t}{t+\sigma(\breve{x},\breve{y})}+1}$$

$$\Rightarrow \frac{t}{t+|\frac{9-\breve{x}}{4}-\frac{9-\breve{y}}{4}|} \geq \frac{\frac{3t}{t+|\breve{x}-\breve{y}|}}{\frac{2t}{t+|\breve{x}-\breve{y}|}+1} \Rightarrow \frac{t}{t+\frac{|\breve{x}-\breve{y}|}{4}} \geq \frac{3t}{3t+|\breve{x}-\breve{y}|} = \frac{t}{t+\frac{|\breve{x}-\breve{y}|}{3}}$$

$$\Rightarrow t+\frac{|\breve{x}-\breve{y}|}{3} \geq t+\frac{|\breve{x}-\breve{y}|}{4} \Rightarrow 4 \geq 3.$$

$$G(\mathfrak{D}(\breve{x}),\mathfrak{D}(\breve{y}),t) \leq \beta(G(\breve{x},\breve{y},t)) \Rightarrow G(\tfrac{9-\breve{x}}{4},\tfrac{9-\breve{y}}{4},t) \leq \frac{G(\breve{x},\breve{y},t)}{3-2G(\breve{x},\breve{y},t)} \Rightarrow \frac{\sigma(\frac{9-\breve{x}}{4},\frac{9-\breve{y}}{4})}{t+\sigma(\frac{9-\breve{x}}{4},\frac{9-\breve{y}}{4})} \leq \frac{\frac{\sigma(\breve{x},\breve{y})}{t+\sigma(\breve{x},\breve{y})}}{3-2\frac{\sigma(\breve{x},\breve{y})}{t+\sigma(\breve{x},\breve{y})}}$$

$$\Rightarrow \frac{|\frac{9-\breve{x}}{4}-\frac{9-\breve{y}}{4}|}{t+|\frac{9-\breve{x}}{4}-\frac{9-\breve{y}}{4}|} \leq \frac{\frac{|\breve{x}-\breve{y}|}{t+|\breve{x}-\breve{y}|}}{3-2(\frac{|\breve{x}-\breve{y}|}{t+|\breve{x}-\breve{y}|})} \Rightarrow \frac{\frac{|\breve{x}-\breve{y}|}{4}}{t+\frac{|\breve{x}-\breve{y}|}{4}} \leq \frac{|\breve{x}-\breve{y}|}{3t+|\breve{x}-\breve{y}|} \Rightarrow \frac{|\breve{x}-\breve{y}|}{4t+|\breve{x}-\breve{y}|} \leq \frac{|\breve{x}-\breve{y}|}{3t+|\breve{x}-\breve{y}|}$$

$$\Rightarrow 3t+|\breve{x}-\breve{y}| \leq 4t+|\breve{x}-\breve{y}| \Rightarrow 3 \leq 4.$$

Therefore $\mathfrak{D}$ is the intuitionistic fuzzy (α,β)-contractive mapping. Then 3 is the unique fixed point.

3. $1-\mathscr{M}-$Complete Intuitionistic Fuzzy Metric-like Space and Fixed-Point Results

Fuzzy metric spaces present two distinct concepts regarding Cauchy sequences and completeness: G-Cauchy sequence and G-completeness [7] (Grabiec, 1988), M -Cauchy sequence and M-completeness [5] (George and Veeramani, 1994). George and Veeramani's definition of completeness is more encompassing compared to Grabiec's. Within this section, we study a modification and generalization of the ideas surrounding Cauchy sequences and completeness in intuitionistic fuzzy metric-like spaces given by Onbasıoglu et al. [17]. This involves introducing the notions of 1-Cauchy sequence and 1-complete fuzzy metric-like spaces, considering both Grabiec and George and Veeramani. Within this novel framework, we establish several fixed-point theorems that expand and consolidate previous findings.

Definition 3.1. *Let $(\breve{X},F,G,*,\diamond)$ be an IFMLS and $(\breve{x}_n)$ be a sequence in $\breve{X}$.*
(i) $(\breve{x}_n)$ is called a $1-\mathscr{G}-$ Cauchy sequence if $\lim_{n\to\infty}F(\breve{x}_{n+p},\breve{x}_n,t) = 1$ and $\lim_{n\to\infty}G(\breve{x}_{n+p},\breve{x}_n,t) = 0$ for all $t > 0$, $p \geq 1$.
(ii) $(\breve{X},F,G,*,\diamond)$ is called $1-\mathscr{G}-$ complete if every $1-\mathscr{G}-$ Cauchy sequence $(\breve{x}_n)$ in $\breve{X}$ converges to some $\breve{x} \in \breve{X}$ such that $F(\breve{x},\breve{x},t) = 1$ and $G(\breve{x},\breve{x},t) = 0$ for all $t > 0$.

Every $1-\mathscr{G}-$ Cauchy sequence is a Cauchy sequence (in the sense of Onbasıoglu et.al. [17] (2023)) in $(\breve{X},F,G,*,\diamond)$, and every complete IFMLS (in the sense of Onbasıoglu et.al.) is $1-\mathscr{G}-$complete.
With the following examples, we see that a Cauchy sequence (in the sense of Onbasıoglu et.al) need not be a $1-\mathscr{G}-$Cauchy sequence and that there exists an IFMLS which is $1-\mathscr{G}$- complete, but it is not complete (in the sense of Onbasıoglu et.al).

Example 3.2. *Let $\breve{X} = IN$, $u*v = uv$ and $u \diamond v = min\{1,u+v\}$. F and G in $\breve{X}^2 \times (0,\infty)$ are defined by*

$$F(\breve{x},\breve{y},t) = \frac{min(\breve{x},\breve{y})}{max(\breve{x},\breve{y})^2} = \begin{cases} \frac{\breve{x}}{\breve{y}^2}, & \breve{x} \leq \breve{y}; \\ \frac{\breve{y}}{\breve{x}^2}, & \breve{y} \leq \breve{x}. \end{cases} \qquad G(\breve{x},\breve{y},t) = \frac{max(\breve{x},\breve{y})^2 - min(\breve{x},\breve{y})}{max(\breve{x},\breve{y})^2} = \begin{cases} \frac{\breve{y}^2-\breve{x}}{\breve{y}^2}, & \breve{x} \leq \breve{y}; \\ \frac{\breve{x}^2-\breve{y}}{\breve{x}^2}, & \breve{y} \leq \breve{x}. \end{cases}$$

Then $(\breve{X},F,G,*,\diamond)$ is IFMLS. Consider the sequence $(n) \subset \breve{X}$.From the definition, $\lim_{n\to\infty}F(\breve{x}_n,\breve{x}_{n+p},t) = \lim_{n\to\infty}\frac{n}{(n+p)^2} = 0$ and $\lim_{n\to\infty}G(\breve{x}_n,\breve{x}_{n+p},t) = \lim_{n\to\infty}(1-\frac{n}{(n+p)^2}) = 1$ for each $p \geq 1$. Since this limit exists, (n) is a Cauchy sequence (in the sense of Onbasıoglu et. al) in $(\breve{X},F,G,*,\diamond)$. However, it is not a $1-\mathscr{G}$ Cauchy sequence.

Example 3.3. *Let $\breve{X} = [0,1] \cap Q$ and define $\sigma : \breve{X} \times \breve{X} \to IR^+$by $\sigma(\breve{x},\breve{y}) = max\{\breve{x},\breve{y}\}$. Hence, $(\breve{X},\sigma)$ is a metric-like space. The standard IFMLS $(\breve{X},F,G,*_p,\diamond)$ is a $1-\mathscr{G}$ complete IFMLS.*
If $(\breve{x}_n)$ is a $1-\mathscr{G}$- Cauchy sequence in $\breve{X}$, then we have $\breve{x}_n \to 0$ as $n \to \infty$, and so $F(\breve{x}_n,0,t) \to 1$ and $G(\breve{x}_n,0,t) \to 0$ as $n \to \infty$ for all $t > 0$. Hence, every $1-\mathscr{G}$- Cauchy sequence in $\breve{X}$ converges to $0 \in \breve{X}$ and $(\breve{X},F,G,*,\diamond)$ is not a complete IFMLS (in the sense of Onbasıoglu et.al.).

Definition 3.4. *Let $(\breve{X},F,G,*,\diamond)$ be an IFMLS and $(\breve{x}_n) \subset \breve{X}$.*
(i) $(\breve{x}_n)$ is called a $1-\mathscr{M}$- Cauchy sequence if $\lim_{n\to\infty}F(\breve{x}_n,\breve{x}_m,t) = 1$ and $\lim_{n\to\infty}G(\breve{x}_n,\breve{x}_m,t) = 0$ for all $t > 0$.
(ii) If every $1-\mathscr{M}$ Cauchy sequence $(\breve{x}_n)$ in $\breve{X}$ converges to some $\breve{x}$ in $\breve{X}$ such that $F(\breve{x},\breve{x},t) = 1$ and $G(\breve{x},\breve{x},t) = 0$ for all $t > 0$, the space $(\breve{X},F,G,*,\diamond)$ is called $1-\mathscr{M}$- complete.

Remark 3.5. *Every complete IFMS in the sense of Grabiec is $1-\mathscr{G}$- complete as an IFMLS and every complete IFMS in the sense of George and Veeramani is $1-\mathscr{M}$- complete as an IFMLS.*

The $1-\mathscr{G}$ completeness implies $1-\mathscr{M}$ completeness. Every IFMS which is complete in the sense of Grabiec [7] is also complete in the sense of George and Veeramani [5]. Hence, by the previous Remarks, we can say that the notion of $1-\mathscr{M}$ completeness is more general than that of $1-\mathscr{G}$ completeness.
The figures below illustrate the relationships among these concepts, with arrows indicating their implications.

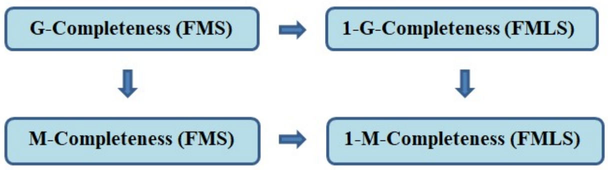

Figure 3.1: Relation of completeness between FMS and FMLS

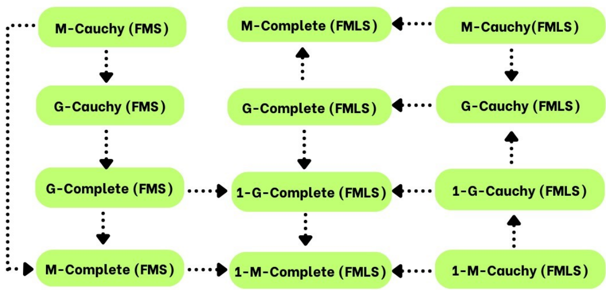

Figure 3.2: Relation of concepts between FMS and FMLS

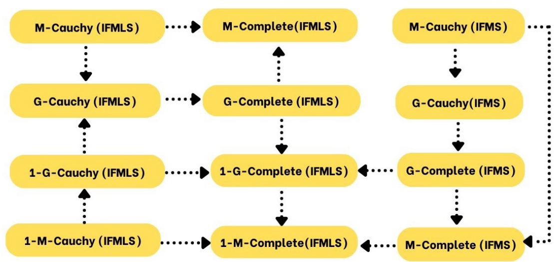

Figure 3.3: Relation of concepts between IFMS and IFMLS

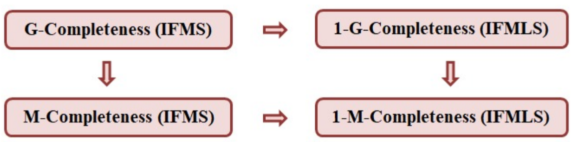

Figure 3.4: Relation of completeness between IFMS and IFMLS

Theorem 3.6. *Let $(\check{X},F,G,*,\diamond)$ be a $1-\mathscr{M}$ complete IFMLS and $\mathfrak{D}:\check{X}\to\check{X}$ be a B-contraction. $\mathfrak{D}$ has a unique fixed point $a\in\check{X}$ and $F(a,a,t)=1$ and $G(a,a,t)=0$ for all $t>0$.*

Proof. Let $\mathfrak{D}$ be a B-contraction with respect to $\alpha,\beta\in B$. For any arbitrary $\check{x}_0\in\check{X}$, set up a sequence $(\check{x}_n)\subset\check{X}$ by $\check{x}_1=\mathfrak{D}(\check{x}_0)$, $\check{x}_2=\mathfrak{D}(\check{x}_1)$,..., $\check{x}_n=\mathfrak{D}(\check{x}_{n-1})$ for all $n\in IN$. If $\check{x}_n=\check{x}_{n-1}$ for some $n\in IN$, then $\check{x}_{n-1}$ is a fixed point of $\mathfrak{D}$. On the contrary part, consider that $\check{x}_n\neq\check{x}_{n-1}$ for all $n\in IN$.
For a fixed $t>0$, let $I_n=\inf_{m\geq n}F(\check{x}_n,\check{x}_m,t)$ and $J_n=\sup_{m\geq n}G(\check{x}_n,\check{x}_m,t)$, $n\in IN$.

Since $\mathfrak{D}$ is a B-contraction with respect to $\alpha,\beta \in B$, then by properties of α and β for all $m,n \in IN$, $m \geq n$, we have
$F(\check{x}_n,\check{x}_{m-1},t) \leq \alpha(F(\check{x}_n,\check{x}_{m-1},t)) \leq F(\mathfrak{D}(\check{x}_n),\mathfrak{D}(\check{x}_{m-1}),t) = F(\check{x}_{n+1},\check{x}_m,t)$
and
$G(\check{x}_n,\check{x}_{m-1},t) \geq \beta(G(\check{x}_n,\check{x}_{m-1},t)) \geq G(\mathfrak{D}(\check{x}_n),\mathfrak{D}(\check{x}_{m-1}),t) = G(\check{x}_{n+1},\check{x}_m,t)$.
Now, take infimum and supremum over $m \geq n+1$, we have $I_n \leq I_{n+1}$ and $J_n \geq J_{n+1}$ for all $n \in IN$. Then, there exists I and J such that $lim_{n\to\infty} I_n = I$ and $lim_{n\to\infty} J_n = J$.
Since $\mathfrak{D}$ is a B-contraction with respect to $\alpha,\beta \in B$, then for all $m,n \in IN$, $m \geq n$, we get
$\alpha(F(\check{x}_{n-1},\check{x}_{m-1},t)) \leq F(\mathfrak{D}(\check{x}_{n-1}),\mathfrak{D}(\check{x}_{m-1}),t) = F(\check{x}_n,\check{x}_m,t)$ and
$\beta(G(\check{x}_{n-1},\check{x}_{m-1},t)) \geq G(\mathfrak{D}(\check{x}_{n-1}),\mathfrak{D}(\check{x}_{m-1}),t) = G(\check{x}_n,\check{x}_m,t)$.
Therefore, if $n \geq 2$, $m > n$, then $\alpha(F(\check{x}_{n-2},\check{x}_{m-2},t)) \leq F(\check{x}_{n-1},\check{x}_{m-1},t)$ and $\beta(G(\check{x}_{n-2},\check{x}_{m-2},t)) \geq G(\check{x}_{n-1},\check{x}_{m-1},t)$.
Since α,β are non-decreasing, it follows that
$\alpha^2(F(\check{x}_{n-2},\check{x}_{m-2},t)) \leq \alpha(F(\check{x}_{n-1},\check{x}_{m-1},t)) \leq F(\check{x}_n,\check{x}_m,t)$ and
$\beta^2(G(\check{x}_{n-2},\check{x}_{m-2},t)) \geq \beta(G(\check{x}_{n-1},\check{x}_{m-1},t)) \geq G(\check{x}_n,\check{x}_m,t)$.
Hence, by induction, we get $\alpha^n(F(\check{x}_0,\check{x}_{m-n},t)) \leq F(\check{x}_n,\check{x}_m,t)$ and $\beta^n(G(\check{x}_0,\check{x}_{m-n},t)) \geq G(\check{x}_n,\check{x}_m,t)$ for all $n \in IN$.
Now, take infimum and supremum over $m (> n)$, we have
$inf\{\alpha^n(F(\check{x}_0,\check{x}_1,t)),\alpha^n(F(\check{x}_0,\check{x}_2,t)),\ldots\} \leq I_n$ and $sup\{\beta^n(G(\check{x}_0,\check{x}_1,t)),\beta^n(G(\check{x}_0,\check{x}_2,t)),\ldots\} \geq J_n$ for all $n \in IN$.
Taking into account (2) in Definition 2.1, we obtain from the above inequalities that
$lim_{n\to\infty} I_n = I = 1$ and $lim_{n\to\infty} J_n = J = 0$.
By the definition, we get $I_n \leq F(\check{x}_m,\check{x}_n,t)$ and $J_n \geq G(\check{x}_n,\check{x}_m,t)$ for all $n,m \in IN, m > n$ and from the above equalities $lim_{n,m\to\infty} F(\check{x}_n,\check{x}_m,t) = 1$ and $lim_{n,m\to\infty} G(\check{x}_n,\check{x}_m,t) = 0$.
Hence, $(\check{x}_n)$ is $1-\mathscr{M}$- Cauchy sequence in $\check{X}$. Since $(\check{X},F,G,*,\diamond)$ is a $1-\mathscr{M}$-complete IFMLS, there exists $\check{x} \in \check{X}$ such that
$lim_{n\to\infty} F(\check{x}_n,\check{x},t) = lim_{n\to\infty} F(\check{x}_n,\check{x}_m,t) = F(\check{x},\check{x},t) = 1$ and
$lim_{n\to\infty} G(\check{x}_n,\check{x},t) = lim_{n\to\infty} G(\check{x}_n,\check{x}_m,t) = G(\check{x},\check{x},t) = 0$ for all $t > 0$.
Now, we prove that $\check{x}$ is the unique fixed point for $\mathfrak{D}$. By Definition 2.3, we have that $\alpha(F(\check{x}_n,\check{x},t)) \leq F(\mathfrak{D}(\check{x}_n),\mathfrak{D}(\check{x}),t) = F(\check{x}_{n+1},\mathfrak{D}(\check{x}),t)$ and $\beta(G(\check{x}_n,\check{x},t)) \geq G(\mathfrak{D}(\check{x}_n),\mathfrak{D}(\check{x}),t) = G(\check{x}_{n+1},\mathfrak{D}(\check{x}),t))$ for all $n \in IN$ and $t > 0$.
From above equalities and properties of α,β, we obtain that $lim_{n\to\infty} F(\check{x}_{n+1},\mathfrak{D}(\check{x}),t) = 1$ and
$lim_{n\to\infty} G(\check{x}_{n+1},\mathfrak{D}(\check{x}),t) = 0$ for all $t > 0$.
By (IFML5) and (IFML10), we get
$F(\check{x},\mathfrak{D}(\check{x}),t) \geq F(\check{x},\check{x}_{n+1},\frac{t}{2}) * F(\check{x}_{n+1},\mathfrak{D}(\check{x}),\frac{t}{2})$ and $G(\check{x},\mathfrak{D}(\check{x}),t) \leq G(\check{x},\check{x}_{n+1},\frac{t}{2}) \diamond G(\check{x}_{n+1},\mathfrak{D}(\check{x}),\frac{t}{2})$.
Using the above last two equalities and inequality, we obtain $F(\check{x},\mathfrak{D}(\check{x}),t) = 1$ and $G(\check{x},\mathfrak{D}(\check{x}),t) = 0$ for all $t > 0$, which means that $\mathfrak{D}(\check{x}) = \check{x}$.
We proceed by investigating whether the fixed point $\check{x}$ of $\mathfrak{D}$ is unique. Let $\check{z} \in \check{X}$ be another fixed point of $\mathfrak{D}$, that is $\mathfrak{D}(\check{z}) = \check{z}$ and there is $k > 0$ such that $F(\check{x},\check{z},k) < 1$ and $G(\check{x},\check{z},k) > 0$. Then, by the Definition 2.3 and the fact that $s > \alpha(s)$ and $\beta(r) < r$ for all $s,r \in (0,1)$, we get
$F(\check{x},\check{z},k) < \alpha(F(\check{x},\check{z},k)) \leq F(\mathfrak{D}(\check{x}),\mathfrak{D}(\check{z}),k) = F(\check{x},\check{z},k)$ and
$G(\check{x},\check{z},k) > \beta(G(\check{x},\check{z},k)) \geq G(\mathfrak{D}(\check{x}),\mathfrak{D}(\check{z}),k) = G(\check{x},\check{z},k)$, which is a contradiction. Then, we must have $F(\check{x},\check{z},k) = 1$ and $G(\check{x},\check{z},k) = 0$ for all $k > 0$ and therefore $\check{x} = \check{z}$.

□

Corollary 3.7. *Let $(\check{X},F,G,*,\diamond)$ be a $1-\mathscr{M}$-complete IFMLS. $\mathfrak{D} : \check{X} \to \check{X}$ be a (α,β)-contraction, meaning that $F(\mathfrak{D}(\check{x}),\mathfrak{D}(\check{y}),t) \geq \alpha(F(\check{x},\check{y},t))$ and $G(\mathfrak{D}(\check{x}),\mathfrak{D}(\check{y}),t) \leq \beta(G(\check{x},\check{y},t))$ for all $\check{x},\check{y} \in \check{X}$ and $t > 0$, where $\alpha,\beta \in B$. Thus, $\mathfrak{D}$ has a unique fixed point $a \in \check{X}$ and $F(a,a,t) = 1$ and $G(a,a,t) = 0$ for all $t > 0$.*

Corollary 3.8. *Let $(\check{X},F,G,*,\diamond)$ be a $1-\mathscr{M}$-complete IFMLS and $\mathfrak{D} : \check{X} \to \check{X}$ be a self mapping. Suppose that there exist a positive integer n and $\alpha,\beta \in B$ such that*
$\alpha(F(\check{x},\check{y},t)) \leq F(\mathfrak{D}^n(\check{x}),\mathfrak{D}^n(\check{y}),t)$ and $\beta(G(\check{x},\check{y},t)) \geq G(\mathfrak{D}^n(\check{x}),\mathfrak{D}^n(\check{y}),t)$ for all $\check{x},\check{y} \in \check{X}$ and $t > 0$,
Then $\mathfrak{D}$ has a unique fixed point $a \in \check{X}$ and $F(a,a,t) = 1$ and $G(a,a,t) = 0$ for all $t > 0$.

Proof. The proof is similar to the proof of Corollary 1 in [17].

□

Theorem 3.9. *$(\check{X},F,G,*,\diamond)$ be a $1-\mathscr{M}$ complete IFMLS and $\mathfrak{D} : \check{X} \to \check{X}$ be a mapping satisfying*
$F(\mathfrak{D}(\check{x}),\mathfrak{D}(\check{y}),t) \geq F(\check{x},\check{y},g(t))$ and $G(\mathfrak{D}(\check{x}),\mathfrak{D}(\check{y}),t) \leq G(\check{x},\check{y},h(t))$ for all $\check{x},\check{y} \in \check{X}$ and $t > 0$, where $g,h : (0,\infty) \to (0,\infty)$ is such
$F(\check{x},\check{y},g(t)) \geq F(\check{x},\check{y},t)$ and $\lim_{n\to\infty} F(\check{x},\check{y},g^n(t)) = 1$,
$G(\check{x},\check{y},h(t)) \leq G(\check{x},\check{y},t)$ and $\lim_{n\to\infty} G(\check{x},\check{y},h^n(t)) = 0$.
Then $\mathfrak{D}$ has a unique fixed point $a \in \check{X}$ and $F(a,a,t) = 1$ and $G(a,a,t) = 0$ for all $t > 0$.

Proof. Let $\check{x}_0 \in \check{X}$ arbitrary and define $(\check{x}_n)$ by $\check{x}_n = \mathfrak{D}(\check{x}_{n-1})$ for all $n \in IN$. We can take $\check{x}_n \neq \check{x}_{n-1}$ for all $n \in IN$. We need to show that $(\check{x}_n)$ is a $1-\mathscr{M}$- Cauchy sequence.
Let $I_n = \inf_{m\geq n} F(\check{x}_n,\check{x}_m,t)$ and $J_n = \sup_{m\geq n} G(\check{x}_n,\check{x}_m,t)$, for a fixed $t > 0$ and $n \in IN$.
For all $n,m \in IN, m \geq n$, we have from the hypothesis that
$F(\check{x}_{n+1},\check{x}_m,t) = F(\mathfrak{D}(\check{x}_n),\mathfrak{D}(\check{x}_{m-1}),t) \geq F(\check{x}_n,\check{x}_{m-1},g(t)) \geq F(\check{x}_n,\check{x}_{m-1},t)$ and
$G(\check{x}_{n+1},\check{x}_m,t) = G(\mathfrak{D}(\check{x}_n),\mathfrak{D}(\check{x}_{m-1}),t) \leq G(\check{x}_n,\check{x}_{m-1},h(t)) \leq G(\check{x}_n,\check{x}_{m-1},t)$.
Taking infimum and supremum over $m (\geq n+1)$, respectively, we get $I_n \leq I_{n+1}$ and $J_{n+1} \geq J_n$ for all $n \in IN$.
Thus, there exists I and J such that $lim_{n\to\infty} I_n = I$ and $lim_{n\to\infty} J_n = J$. For $k \in IN$, from the hypothesis again, we obtain
$F(\check{x}_{n+k},\check{x}_{m+k},t) = F(\mathfrak{D}(\check{x}_{n+k-1}),\mathfrak{D}(\check{x}_{m+k-1}),t) \geq F(\check{x}_{n+k-1},\check{x}_{m+k-1},g(t)) \geq F(\check{x}_{n+k-1},\check{x}_{m+k-1},t)$.
$G(\check{x}_{n+k},\check{x}_{m+k},t) = G(\mathfrak{D}(\check{x}_{n+k-1}),\mathfrak{D}(\check{x}_{m+k-1}),t) \leq G(\check{x}_{n+k-1},\check{x}_{m+k-1},h(t)) \leq G(\check{x}_{n+k-1},\check{x}_{m+k-1},t)$.
From the above inequalities, it follows that
$F(\check{x}_{n+k-1},\check{x}_{m+k-1},g(t)) \geq F(\check{x}_{n+k-2},\check{x}_{m+k-2},g^2(t)) \geq F(\check{x}_{n+k-2},\check{x}_{m+k-2},t)$ and

$G(\check{x}_{n+k-1},\check{x}_{m+k-1},h(t)) \leq G(\check{x}_{n+k-2},\check{x}_{m+k-2},h^2(t)) \leq G(\check{x}_{n+k-2},\check{x}_{m+k-2},t)$.
Using the obtained inequalities and repeating this process, we obtain
$F(\check{x}_{n+k},\check{x}_{m+k},t) \geq F(\check{x}_n,\check{x}_m,g^k(t)) \geq F(\check{x}_n,\check{x}_m,t)$ and
$G(\check{x}_{n+k},\check{x}_{m+k},t) \leq G(\check{x}_n,\check{x}_m,h^k(t)) \geq G(\check{x}_n,\check{x}_m,t)$.
Now, let take infimum and supremum over $m(\geq n)$, respectively, we get
$I_n = \inf\{F(\check{x}_n,\check{x}_{n+1},g^k(t)),F(\check{x}_n,\check{x}_{n+2},g^k(t)),...\} \leq I_{n+p}$ and
$J_n = \sup\{G(\check{x}_n,\check{x}_{n+1},h^k(t)),G(\check{x}_n,\check{x}_{n+2},h^k(t)),...\} \geq J_{n+p}$.
Since $lim_{k\to\infty}F(\check{x},\check{y},g^k(t)) = 1$ and $lim_{k\to\infty}G(\check{x},\check{y},h^k(t)) = 0$ for all $\check{x},\check{y} \in \check{X}, t > 0$, we get from the above inequality, that $lim_{n\to\infty}I_n = I = 1$ and $lim_{n\to\infty}J_n = J = 0$. By definition, we get $I_n \leq F(\check{x}_n,\check{x}_m,t)$ and $J_n \geq G(\check{x}_n,\check{x}_m,t)$ for all $n,m \in IN, m > n$ and then $lim_{n,m\to\infty}F(\check{x}_n,\check{x}_m,t) = 1$, $lim_{n,m\to\infty}G(\check{x}_n,\check{x}_m,t) = 0$. Hence, $(\check{x}_n)$ is a $1-\mathscr{M}$- Cauchy sequence in $\check{X}$. Since $(\check{X},F,G,*,\diamond)$ is a $1-\mathscr{M}$-complete IFMLS, there exists $a \in \check{X}$ such that
$lim_{n\to\infty}F(\check{x}_n,a,t) = lim_{n,m\to\infty}F(\check{x}_n,\check{x}_m,t) = F(a,a,t) = 1$ and
$lim_{n\to\infty}G(\check{x}_n,a,t) = lim_{n,m\to\infty}G(\check{x}_n,\check{x}_m,t) = G(a,a,t) = 0$, for all positive parameter t.
We need to show that a is the unique fixed point of $\mathfrak{D}$.

$$\begin{aligned} F(a,\mathfrak{D}(a),t) &\geq F(a,\check{x}_{n+1},\tfrac{t}{2}) * F(\check{x}_{n+1},\mathfrak{D}(a),\tfrac{t}{2}) \\ &= F(a,\check{x}_{n+1},\tfrac{t}{2}) * F(\mathfrak{D}(\check{x}_n),\mathfrak{D}(a),\tfrac{t}{2}) \\ &\geq F(a,\check{x}_{n+1},\tfrac{t}{2}) * F(\check{x}_n,a,g(\tfrac{t}{2})) \end{aligned}$$

and

$$\begin{aligned} G(a,\mathfrak{D}(a),t) &\leq G(a,\check{x}_{n+1},\tfrac{t}{2}) \diamond G(\check{x}_{n+1},\mathfrak{D}(a),\tfrac{t}{2}) \\ &= G(a,\check{x}_{n+1},\tfrac{t}{2}) \diamond G(\mathfrak{D}(\check{x}_n),\mathfrak{D}(a),\tfrac{t}{2}) \\ &\leq G(a,\check{x}_{n+1},\tfrac{t}{2}) \diamond G(\check{x}_n,a,h(\tfrac{t}{2})) \end{aligned}$$

Using the inequalities mentioned above, we find $F(a,\mathfrak{D}(a),t) = 1$ and $G(a,\mathfrak{D}(a),t) = 0$ for all $t > 0$, hence $\mathfrak{D}(a) = a$.
We shall show that the fixed point is unique, for this let $a \neq b \in \check{X}$ be another fixed point of $\mathfrak{D}$, this means $\mathfrak{D}(b) = b \neq a = \mathfrak{D}(a)$. Given the hypothesis, we get
$F(a,b,s) = F(\mathfrak{D}(a),\mathfrak{D}(b),s) \geq F(a,b,g(s))$ and $G(a,b,s) = G(\mathfrak{D}(a),\mathfrak{D}(b),s) \leq G(a,b,h(s))$.
By repeating this process we have $F(a,b,s) \geq F(a,b,g^n(s))$ and $G(a,b,s) \leq G(a,b,h^n(s))$.
Taking $n \to \infty$ and using the properties of the functions g and h, we get $F(a,b,s) = 1$ and $G(a,b,s) = 0$, which implies $a = b$. Therefore, a is the unique fixed point of $\mathfrak{D}$, which verifies $F(a,a,t) = 1$ and $G(a,a,t) = 0$ for all positive parameter t.

□

Example 3.10. *Let a,b,c be rationals such that $0 < a < b < c$ and $\check{X} = [0,c] \cap Q$. Let define intuitionistic fuzzy sets F and G in $\check{X}^2 \times (0,\infty)$ by $F(\check{x},\check{y},t) = 1 - \frac{max\{\check{x},\check{y}\}}{c+t}$, $G(\check{x},\check{y},t) = \frac{max\{\check{x},\check{y}\}}{c+t}$ for all $\check{x},\check{y} \in \check{X}$ and $t > 0$.*
Then $(\check{X},F,G,*_l,\diamond_m)$ is a $1-\mathscr{M}$-complete IFMLS, but it is not a IFMS, as $F(\check{x},\check{x},t) = 1-\check{x}/(c+t) \neq 1$ and $G(\check{x},\check{x},t) = \check{x}/(c+t) \neq 0$ for all $\check{x} \in \check{X} \setminus \{0\}$.
Let the mapping $\mathfrak{D} : \check{X} \to \check{X}$ defined by

$$\mathfrak{D}(\check{x}) = \begin{cases} 0, & \check{x} \in [0,b] \cap Q; \\ a, & \check{x} \in (b,c) \cap Q. \end{cases}$$

Define the mappings $g,h : (0,\infty) \to (0,\infty)$ by $g(t) = (\frac{b-a}{a})c + (\frac{b}{a})t$ and $h(t) = t(\frac{a}{b}) - c(\frac{b-a}{b})$ for all $t > 0$. Hence, we obtain $F(\check{x},\check{y},g(t)) \geq F(\check{x},\check{y},t)$, $G(\check{x},\check{y},h(t)) \leq G(\check{x},\check{y},t)$ and $lim_{n\to\infty}F(\check{x},\check{y},g^n(t)) = 1$, $lim_{n\to\infty}G(\check{x},\check{y},h^n(t)) = 0$ for all $\check{x},\check{y} \in \check{X}$ and $t > 0$. Therefore, $\mathfrak{D},g,h$ satisfy all the conditions of Theorem 3.9 and $\mathfrak{D}$ has a unique fixed point $0 \in \check{X}$ with $F(0,0,t) = 1$ and $G(0,0,t) = 0$ for all $t > 0$.

4. Conclusion

Onbasıoglu et. al. [17] defined intuitionistic fuzzy metric-like space to combine the concepts of intuitionistic fuzzy metric space and metric-like space. As a continuation [17], this article improves the definitions of Cauchy sequences and completeness in IFMLS and proves several fixed-point results. This study can be extended to other fuzzy metric structures.

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