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Direct Synthesis Method-based PID Controller Design for Higher Order Systems

Mehmet Serhat Can [a], Tufan Dogruer [a,*]

[a] Tokat Gaziosmanpasa University, Department of Electrical and Electronics Engineering, Tokat, Türkiye – 60250

*Corresponding author

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ABSTRACT

This study presents a PID controller design using the Direct Synthesis (DS) method for higher order systems. The DS method offers an analytical approach to parameter tuning. PID controllers are widely favored due to their simple structure and the easy with which their parameters can be tuned. Classical controller performances may not be sufficient to control unstable, integrator-containing, or higher order systems. To overcome this problem in controlling higher order systems, higher order systems can be reduced to lower order systems using model reduction techniques. PID controller designs can be realized using reduced models. In the study, two higher order systems are taken as an example, and PID controller designs are made using second-order models of these systems. In both examples, time and frequency response analyses are presented with figures, and performance evaluations are interpreted. In the control of higher order systems, very successful results are obtained with the presented method.

Keywords: PID controller design, Direct Synthesis, Higher order systems

1. Introduction

The order of a system is defined as the highest power of the Laplace variable "s" in the denominator polynomial of its transfer function, also known as the characteristic equation. A system can thus be first-order, second-order, or of a higher order. The order of the system corresponds to the number of poles in the complex plane, and the locations of these poles in the complex plane determine the system's dynamic behavior. In a closed-loop control system, the controller can be configured in series or parallel compensation structures with the controlled system. The zeros and poles of the controller combine with those of the controlled system depending on the compensation structure, resulting in the overall closed-loop transfer function of the system. To achieve the desired system response, the controller parameters must be precisely adjusted, and the poles of the closed-loop system must be placed at appropriate locations in the complex plane, ensuring the desired system behavior.

Among the various types of controllers used in control systems, the Proportional-Integral-Derivative (PID) controllers are the most widely employed. The reason for this is its simplicity, as it only involves three gain parameters, making its design relatively easy, while also providing satisfactory performance for linear systems. Due to these advantages, PID controllers have extensive applications in industrial control systems [1, 2]. The tuning of the Proportional (K_p), Integral (K_i or T_i), and Derivative (K_d or T_d) parameters of a PID controller is crucial for achieving effective control performance. K_p increases system responds, but excessive values may lead to oscillations. Ki effectively eliminates steady-state error but can cause large amplitude oscillations if set too high. K_d helps suppress overshoot but may cause high-frequency oscillations if set too large, and it is also sensitive to noise. The process of determining the optimal PID controller parameters can be categorized into three main approaches: experimental methods (classical), optimization/computational methods, and analytical methods.

Well-known experimental methods for PID tuning include trial-and-error, Ziegler-Nichols (1940s) [3], Cohen-Coon (1953) [4], Tyreus-Luyben (1997) [5], and Åström-Hägglund (Relay) (1984) [6]. These methods represent some of the earliest approaches to PID controller design. The trial-and-error method is a simple approach. In this method, the proportional gain is gradually increased until the steady-state error is minimized, with the integral and derivative gains initially set to zero. The integral gain is then increased to eliminate the steady-state error, followed by tuning the derivative gain to suppress overshoot. This method requires minimal knowledge of the system but often takes longer to achieve satisfactory performance. The Ziegler-Nichols (ZN) method can be

* Corresponding author. e-mail address: tufan.dogruer@gop.edu.tr
ORCID : 0000-0002-0415-3042

applied either in open-loop or closed-loop configurations. In the open-loop configuration, certain parameters are measured from the system's step response, and the controller parameters are determined using the ZN table. In the closed-loop configuration, the proportional gain is increased until the system exhibits sustained oscillations, with the integral and derivative gains set to zero. Once oscillations occur, the controller gain and the oscillation period are recorded, and the ZN table is used to calculate the controller parameters. The Cohen-Coon method developed much later than the ZN method, is preferred for achieving faster system responses. The Tyreus-Luyben method is also based on the closed-loop ZN method. The Åström-Hägglund method offers better stability compared to the ZN method in terms of cycle instability risk.

Optimization/computational methods are algorithms inspired by nature, including swarm intelligence, immune algorithms, physics-related algorithms, fuzzy logic algorithms, stochastic algorithms, evolutionary algorithms, neural algorithms, and probabilistic algorithms [7]. These methods are generally effective when the system's transfer function is unknown, difficult to estimate, or when the system parameters are variable. These methods are often iterative and algorithm-based. Optimization/computational methods have been extensively studied in the literature for PID tuning [8-11].

In cases where the transfer function of the controlled system is known or can be obtained through suitable estimation methods, the design of the controller, or in other words, the determination of appropriate controller parameters, can be achieved through analytical methods. The Direct Synthesis (DS) method is an analytical approach and requires knowledge of the transfer function of the system to be controlled. In this method, a desired transfer function is determined, and the poles and zeros of the closed-loop system are analytically matched to this model, allowing the controller parameters to be identified.

Numerous studies in the literature focus on DS. In [12], Jung et al. investigate the control of an unstable first-order system with time delay using a PI controller, and they design the controller based on the DS method. The proposed method retains the conventional PI control structure while allowing for a good overshoot ratio. In the proposed approach, a first-order set-point filter is used, and the controller parameters can be adjusted with simple rules without any tuning parameters. In [13], Rao et al. propose the DS method for the design of a series-connected lead/lag compensator and PID controller. In their study, they address overshoot by utilizing set-point weighting and provide guidelines for selecting the desired closed-loop tuning parameters and set-point weighting parameters.

Recent studies on DS-based PID design can be examine in [14-17]. In [14], Kula proposes the DS method for the design of PI/PID controllers. He tested the proposed method on two system models, FOPDT (First-Order plus Dead-Time) and SOPDT (Second-Order plus Dead-Time), and reported that the method successfully eliminates errors arising from Taylor series expansion or Padé approximations of the process model. In [17], Vilanova and colleagues discuss the importance of load disturbance rejection over set-point tracking in many industrial process control systems. For load disturbances, they propose robust tuning of PI/PID controllers designed using DS. In [16], the focus is on a two-degree-of-freedom (2-DOF) controller based on DS for integrating systems with time delays. This controller structure includes a PID controller to reject load disturbances and a set-point filter to improve servo response performance. In [15], Anwar et al. propose a DS-based design for a cascade control system, where the primary loop consists of an integrating and open-loop unstable process, while the secondary loop consists of a stable process.

When defining the desired closed-loop transfer function in the DS method, set-point changes are often taken into account. Although DS controllers perform well for set-point changes, they may not provide satisfactory results for disturbance inputs, which is a significant disadvantage of the DS approach. In many industrial processes, disturbances are prevalent, and eliminating them is sometimes more important than tracking set-point changes [18]. A fundamental challenge in controlling higher order systems with DS is the increased number of variables that need to be analytically determined. This makes the determination of controller parameters much easier for first-order and second-order systems, where all control parameters can be found by adjusting only one or two variables.

The reduction of higher order systems to lower order ones is known as model reduction. Model reduction methods can be categorized into time-domain, frequency-domain, and hybrid approaches. Several studies focusing on model reduction are listed in Table 1 [19, 20].

The main theme of this study is to present a literature review on the reduction of higher order systems and to realize PID controller design based on the DS method for these systems. The organization of the study is as follows: In the second section, the proposed DS method is explained in detail. The third section includes simulation studies of the proposed method on two different system models, one of third order and the other of fourth order. In the fourth section, the obtained findings are interpreted.

2. The Proposed Direct Synthesis Method

In the Direct Synthesis (DS) method, controller design begins by determining a desired closed-loop transfer function based on the desired dynamic behavior for the controlled system. Subsequently, the closed-loop transfer function of the system under the influence of the controller is analytically matched with the desired transfer function, and from the resulting equations, the controller parameters are derived.

Table 1. Some model reduction methods in literature.

Researchers	Used Method
Davison, 1966 [21]	Dominant Eigenvalue and eigenvectors Approach
Bosley and Lees, 1972 [22]	Continued Fraction Expansion, Fitting the Moments of the Impulse Response
Shamash, 1974 [23]	Padé Approximation
Hutton and Friedland, 1975 [24]	Routh Approximation
Krishnamurthy and Seshadri, 1978 [25]	Routh Stability
Chen et al., 1979 [26]	Stability Equation
Mishra and D.A.Wilson, 1980 [27]	Error Minimization
Moore, 1981 [28]	Balanced Realization
Gutman et al., 1982 [29]	Differentiation Method
Sinha and Kuszta, 1983 [30]	Moment Matchin Techniques
Glover, 1984 [31]	Hankel Norm Approximation
Lucas, 1986 [32]	Factor Division Method
Sinha and Pal, 1990 [33]	Pole Clustering
Antoulas, 2004 [34]	Krylov Subspace
Panda et al., 2009 [35]	Continued Fraction
Kumar and Nagar, 2014 [36]	Hankel Norm Approximation
Suman, 2020 [37]	Singular Perturbation
Prajapati and Rajendra Prasad, 2018 [38]	Dominate Pole
Suman and Kumar, 2021 [39]	Hybrid
Kumari and Vishwakarma, 2021 [40]	Hybrid
Yüce [41]	Experimental Technique Using Stability Boundary Locus Method

As mentioned in the introduction, the DS method relies on analytically determining the unknown controller parameters. However, when there are numerous unknown equation variables, the analytical solution becomes more challenging. For higher order systems, the use of model reduction methods can simplify the problem by approximating the system with lower order models. In this way, the controlled system can be reduced to a first- or second-order approximation. Once this reduced model is obtained, the DS method can be easily applied. In this study, third-order and fourth-order systems are considered as examples. For these two systems, reduced-order approximations were directly taken from [42] using model reduction methods. Figure 1 provides a block diagram of a closed-loop control system.

In Figure 1, *R(s)* represents the reference signal, *E(s)* denotes the control error, *U(s)* is the control signal, *D(s)* represents the input disturbance signal, and *Y(s)* is the system output. In this study, the desired transfer function given in Equation 1 has been selected.

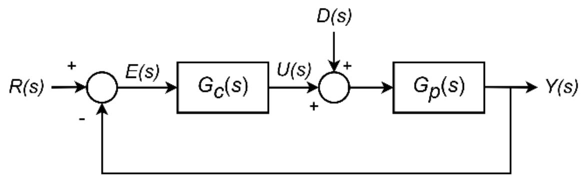

Figure 1. Closed-Loop control system block diagram

$$G_d(s) = \frac{e^{-\theta s}}{\tau_c s + 1} \tag{1}$$

The transfer function of the PID controller is given in Equation 2, the reduced-order approximate model of the system to be controlled is presented in Equation 3, and the equality of the closed-loop transfer function of the unit feedback system under the influence of the PID controller and the reference transfer function is provided in Equation 4.

$$G_c(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right) \tag{2}$$

$$\tilde{G}(s) = \frac{K}{(\tau_1 s + 1)(\tau_2 s + 1)} \tag{3}$$

$$G_d(s) = \frac{G_c(s)\tilde{G}(s)}{1 + G_c(s)\tilde{G}(s)} \tag{4}$$

By rearranging Equation 4 to solve for $G_c(s)$, the controller parameters are obtained as follows.

$$K_c = \frac{1}{K}\frac{\tau_1 + \tau_2}{\tau_c + \theta} \tag{5}$$

$$T_i = \tau_1 + \tau_2 \tag{6}$$

$$T_d = \frac{\tau_1 \tau_2}{\tau_1 + \tau_2} \tag{7}$$

When examining Equations 5-7, it is clearly evident that the controller gain K_c is dependent on the design parameter τ_c, while the integral time constant T_i and the derivative time constant T_d are not dependent on the design parameter τ_c. Furthermore, it can be clearly observed from Equation 5 that K_c is dependent on the system gain K. As a result, the proposed design method for this DS allows access to all PID controller parameters solely through the tuning parameter τ_c, based on the identification of a high-accuracy approximate system model. This indicates that the proposed design method is a straightforward and practical approach. In the following sections, the proposed method has been tested on two different systems, one of third order and the other of fourth order, and the obtained results are presented.

3. Simulation Studies

In this section, two simulation examples are presented to evaluate the performance of the PID controller designed for higher order systems. In each simulation, the controller parameters are tested with different time parameters to examine the changes in the behavior of the system and target performance improvements. The aim is to ensure that the system reaches the desired output in the fastest and most stable way.

Example 1: In this example, the following higher order process, which has been studied in the literature by [42], is analyzed.

$$G(s) = \frac{9}{(s+1)(s^2+2s+9)} \tag{8}$$

For the third-order process model given in Equation 8, a reduction to a second-order system is performed by the model reduction technique in [42]. The reduced model is shown in Equation 9.

$$G_r(s) = \frac{5.3871}{(s^2+4.7886s+5.3871)} e^{-0.2650s} \tag{9}$$

In the reduced model, Equation 10 is obtained by writing the denominator as multiplications. Equation 10 is then written as Equation 11, a more convenient form for applying the direct synthesis method.

$$G(s) = \frac{5.3871}{(s+2.9822)(s+1.8064)} e^{-0.2650s} \tag{10}$$

$$G(s) = \frac{1}{(0.335s+1)(0.553s+1)} e^{-0.2650s} \tag{11}$$

In Figure 2, the Bode diagrams of the higher order transfer function and the reduced approximate model are compared. It is seen that the amplitude and phase responses are quite close to each other in the low frequency region. This indicates that a good reduced approximate model has been obtained.

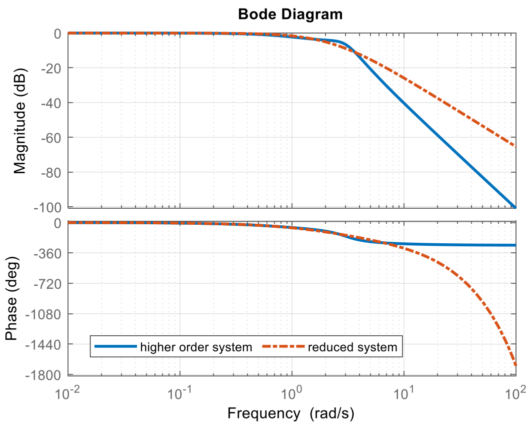

Figure 2. Comparison of the Bode diagrams of the higher order system and the model order-reduced system model

The unit step responses of the higher order transfer function and the reduced order approximate model are compared in Figure 3. It is seen that the unit step responses are quite close, with only a small difference in the rise time.

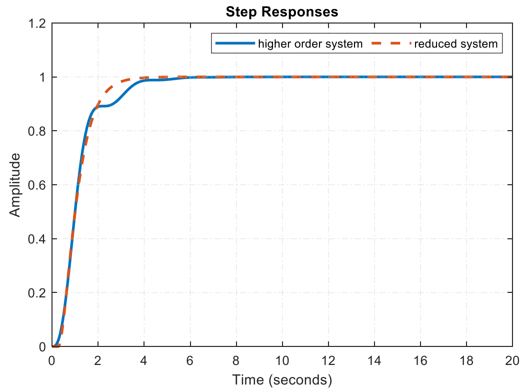

Figure 3. Comparison of the unit step responses of the higher order system and the model order-reduced system model

It is understood from both the Bode diagram and the overlap in the unit step responses that the PID controller designed for the reduced model can also give appropriate responses for the higher order system.

According to the method described in the previous section, PID controller parameters are determined for various values of the time parameter. The determined PID controller parameters are given in Table 2.

Table 2. PID controller parameters for τ_c for Example 1.

τ_c	K_p	K_i	K_d	OS (%)	Settling time (s)
3	0.2720	0.3063	0.0567	0	11.57
2.5	0.3212	0.3617	0.0670	0	9.57
2	0.3921	0.4415	0.0818	0	7.57
1.5	0.5031	0.5666	0.1050	0.0694	5.59
1	0.7020	0.7905	0.1464	0.8325	3.56

The unit step responses of the closed-loop system obtained by applying the PID controller parameters obtained according to different time constant parameters to the higher order system are given in Figure 4. When the figure is analyzed, it is seen that the response accelerates as the τ_c parameter decreases, but after a certain τ_c value, the response starts to exceed. It is seen from both the table and the figure that there is almost no overshoot at τ_c=1.5 and the settling time is 5.59 seconds.

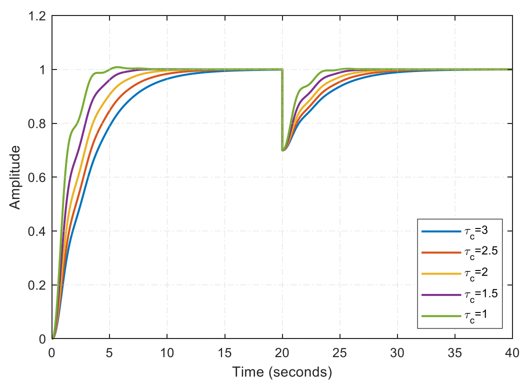

Figure 4. The unit-step responses of the closed-loop system for different time constant

Figure 4 also illustrates the disturbance rejection performance of the systems. A step disturbance input of 0.3 units was applied to the closed-loop system at the 20th second. It can be observed from the figure that the fastest disturbance rejection performance occurs at the smallest value of the time constant.

Figure 5 shows the Bode diagrams of the closed-loop systems for different time constants. The Bode diagram shows a wide flat region in the phase curve, especially around the gain cross-over frequencies, leading to an iso-damped time response. This means that the system exhibits similar damping behavior across different frequencies, resulting in consistent and predictable time-domain responses. In other words, the flatness in the phase curve ensures that the system has controlled and uniform damping, regardless of the frequency.

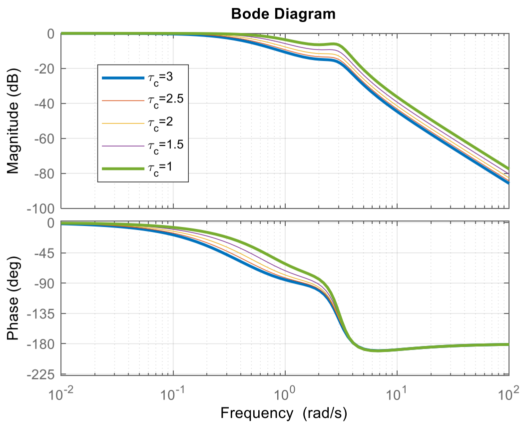

Figure 5. Bode diagrams of the closed-loop system for different time constant

Example 2: Consider a fourth-order plant as follows [42]

$$G(s) = \frac{1}{(s+1)(0.2s+1)(0.04s+1)(0.008s+1)} \tag{12}$$

In accordance with the fourth-order process model presented in Equation 12, a model reduction technique described in [42] is applied to simplify it into a second-order system. The reduced model is shown below.

$$G_r(s) = \frac{4.6812}{(s^2+5.6676s+4.6812)} e^{-0.0421s} \tag{13}$$

$$G(s) = \frac{1}{(0.9963s+1)(0.2144s+1)} e^{-0.0421s} \tag{14}$$

Figures 6 and 7 present a comparison of the higher order transfer function and the reduced-order approximate model. Figure 6 displays the Bode diagrams, where the amplitude and phase responses are similar in the low frequency region, indicating that a good reduced approximate model has been achieved. Figure 7 shows the unit step responses of both models, revealing that they overlap. Both the Bode diagram and the overlap in the unit step responses indicate that the PID controller designed for the reduced model can also provide suitable responses for the higher order system.

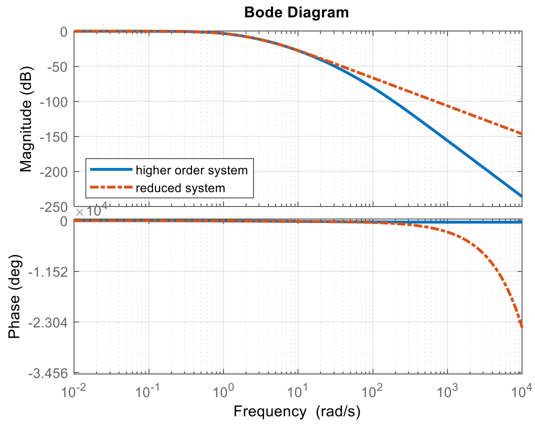

Figure 6. Comparison of the Bode diagrams of the higher order system and the model order-reduced system model

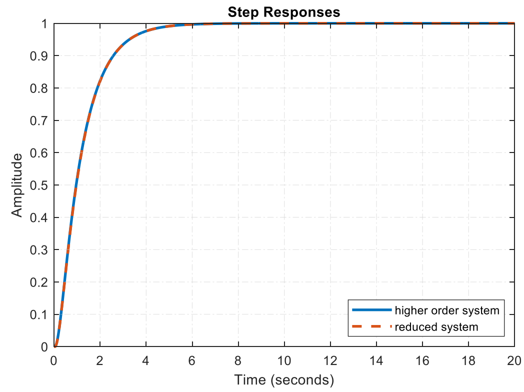

Figure 7. Comparison of the unit step responses of the higher order system and the model order-reduced system model

PID controller designs are made for five different values of the time constant parameter, and the parameter values are provided in Table 3. The table also presents important time parameters, including the settling time and overshoot values.

Table 3. PID controller parameters for $\boldsymbol{\tau_c}$ for Example 2.

τ_c	K_p	K_i	K_d	OS (%)	Settling time (s)
1	1.1618	0.9596	0.2050	0	3.9691
0.8	1.4377	1.1875	0.2537	0	3.1936
0.6	1.8855	1.5574	0.3327	8.8e-04	2.4197
0.4	2.7385	2.2619	0.4832	0.0065	1.6504
0.2	5.0008	4.1305	0.8823	0.0145	0.8752

When the PID parameters from Table 3 are substituted into the closed-loop system, the unit-step responses are obtained as shown in Figure 8. It is clearly observed from the figure and the table that the fastest response occurs at τ_c=0.2, with a settling time of less than 1 second and an overshoot of approximately 0.01%. Additionally, a disturbance input is applied at the 5th second, and it is observed that the controller demonstrated successful disturbance rejection performance.

The Bode diagrams for the closed-loop system with varying time constants are presented in Figure 9. The Bode diagram reveals a broad flat section in the phase curve near the gain crossover frequencies. This characteristic creates an iso-damped time response, meaning that the system maintains similar damping behavior across various frequencies. As a result, the system achieves uniform and predictable responses in the time domain.

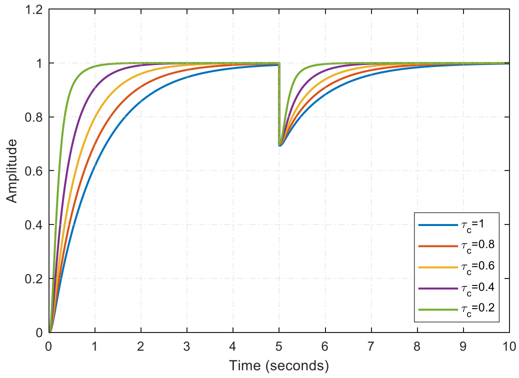

Figure 8. The unit-step responses of the closed-loop system for different time constant

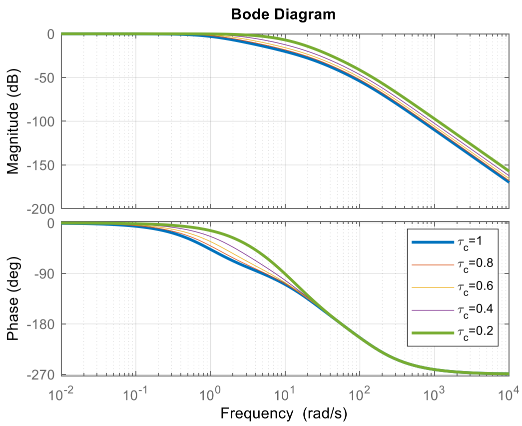

Figure 9. Bode diagrams of the closed-loop system for different time constant

4. Conclusions

In this study, a PID controller design methodology based on the DS method was implemented to control higher order systems. In the Direct Synthesis technique, the controller is designed by utilizing a process model and selecting a desired closed-loop transfer function. In study, a first order process model is selected as a desired model and PID controller designs are made according to the τ_c time constant of this model.

In order to evaluate the performance of the method, PID controller designs for two higher order system models are carried out. For both simulation examples, PID parameters are obtained based on five different time constant. It is seen that the fastest settling time is obtained at the smallest τ_c value among the obtained time constant values. Furthermore, an increase in τ_c increases the settling time and decreases the overshoot value.

As a result, no-overshoot and fast step responses can be obtained with the presented method to control higher order systems.

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