OKU Fen Bilimleri Enstitüsü Dergisi
8(4): 1544-1571, 2025

OKU Journal of The Institute of Science and Technology, 8(4): 1544-1571, 2025

Osmaniye Korkut Ata Üniversitesi Fen Bilimleri Enstitüsü Dergisi

Osmaniye Korkut Ata University Journal of The Institute of Science and Technology

Genelleştirilmiş Küme Değerli Nötrosofik Beşli Sayılar Üzerine ANP Metodu

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Araştırma Makalesi

Makale Tarihçesi:
Geliş tarihi: 10.10.2024
Kabul tarihi:17.04.2025
Online Yayınlanma:16.09.2025

Anahtar Kelimeler:
ANP Metod
Yenilenebilir enerji kaynakları
Nötrosofik teori
Nötrosofik beşli sayılar
Karar verme uygulamaları

ÖZ

Bu çalışmada, nötrosofik beşli sayılar üzerinde çok kriterli karar verme uygulamalarında ilk defa Analytic Network Process (ANP) kullanılarak bir yöntem geliştirildi. Bu yöntem, nötrosofik küme teorisini entegre ettiği için klasik ANP'den farklıdır. Bu yöntemin uygulama adımları ve güncel hayatta karşılaşılan problemlere nasıl uygulanacağı hakkında bilgi verildi. Böylece nötrosofik küme teorisinin yeni yapılarından biri olan nötrosofik beşli kümeler ile çok kriterli karar verme uygulamalarından biri olan ANP yönteminin birlikte kullanıldığı yeni bir yöntem geliştirildi. Bu yeni yöntem, klasik ANP yönteminin tutarlılık hesaplaması ile 3. ve 6. adımlarını içermektedir. Ayrıca bu uygulamanın kullanıldığı güncel hayattan örnek bir uygulama yapıldı. Örnek uygulamada yenilenebilir enerji kaynaklarının seçiminde çevresel, sosyal ve ekonomik kriterlerin birbirleri üzerindeki etkisi kurgusal veriler kullanılarak analiz edildi. Bu uygulama sonucunda çevresel kriterin daha etkili olduğu, bunu sırasıyla sosyal kriterin ve ekonomik kriterin takip ettiği sonucuna varıldı. Ayrıca, kriterler arasındaki karşılıklı bağımlılıklar kriterlerin ağırlıklarını değiştirdi. Kriterlerin ağırlıklarının değişmesi, karar verme sürecindeki sıralamayı etkileyebileceği sonucuna varıldı. Ayrıca farklı operatörler ile elde edilen karşılaştırma matrislerinin tutarlılık oranlarının değişebileceği ve tutarsız bir matris ile tutarlı bir matris ile elde edilen önem sıralamalarının farklı olabileceği sonucuna da varıldı.

ANP Method on Generalized Set-Valued Neutrosophic Quintuple Numbers

Research Article

Article History:
Received: 10.10.2024
Accepted: 17.04.2025
Published online:16.09.2025

Keywords:
ANP method
Renewable energy sources
Neutrosophic theory
Neutrosophic quintuple numbers
Multi criteria decision-making applications

ABSTRACT

In this study, a method was proposed using the Analytic Network Process (ANP) for the first time in multi criteria decision-making applications on neutrosophic quintuple numbers. This method is different from classical ANP as it integrates neutrosophic set theory. Information about the application steps of this method and how to apply it to problems encountered in current life is given. Thus, a new method has been developed in which neutrosophic quintuple sets, one of the new structures of neutrosophic set theory, and the ANP method, one of the applications of multi-criteria decision-making, are used together. This new method includes consistency calculation, step 3 and step 6 of the classical ANP method. Also, an example application from current life is given. In the case study, the impact of environmental, social, and economic criteria on each other in selecting renewable energy sources was analyzed using fictitious data. As a result of this application, it was concluded that the environmental criterion was more effective, followed by the social criterion and the economic criterion, respectively. Moreover, the interdependencies between the criteria changed the weights of the criteria. It was concluded that changing the weights of the criteria may affect the ranking in the decision-making process. It is also concluded that the consistency ratios of the comparison matrices obtained with different operators may vary and the importance rankings obtained with an inconsistent matrix and a consistent matrix may be different.

To Cite: Kargın A. ANP Method on Generalized Set-Valued Neutrosophic Quintuple Numbers. Osmaniye Korkut Ata Üniversitesi Fen Bilimleri Enstitüsü Dergisi 2025; 8(4): 1544-1571.

1. Introduction

The neutrosophic set (NS) theory was obtained by Florentin Smarandache (Smarandache, 1998). In this theory, the elements are expressed with T, I, and F which are numerical value of truth, indeterminacy, and falsity. Then, Wang et al. introduced single-valued NS (Wang et al., 2010). In this set, the numerical values of T, I and F of each element are determined in [0, 1]. It provides a highly objective approach to mathematically express uncertain situations. Thanks to this advantage of NS, many researchers have conducted studies. Şahin studied neutron-algebras based on NS (Şahin, 2022). Also, Şahin and Kargın defined neutrosophic crisp sets on bipolar NS (Şahin and Kargın, 2022a). Garg introduced single-valued NS and applications (Garg, 2024). Vaz-Patto el at. Introduced the DEMATEL method for NS (Vaz-Patto et al., 2024). Voskoglou and Smarandache defined q-rung NS (Voskoglou et al., 2024).

Florentin Smarandache defined neutrosophic quadruple sets (NQS) (Smarandache; 2015). NQS, like NS, have truth, indeterminacy, and falsity values but NQS, unlike NS, have known and unknown parts. Therefore, NQS is a general case of NS. The inclusion of known and unknown parts in these sets, which provide the properties of NS, allows NS to be more useful. Thus, researchers studied NS theory (Karaaslan, 2015; Karaaslan, 2017; Jana et al., 2020). Also, in 2020, Şahin et al. defined generalized set-valued neutrosophic set (GSVNQS) (Şahin et al., 2020). Kargın et al. studied similarity measure based on NQS (Kargın et al., 2021). Şahin et al. obtained Euclidean measures based on NQS (Şahin et al., 2021). Tahan et al. defined hyperstructures-groups based on NQS (Al-Tahan et al., 2023).

Chatterjee et al. defined quadri-partitioned neutrosophic sets (QPNS) (Chatterjee et al., 2016). In QPNS, contradiction and uncertainty values are used instead of indeterminacy values in NS. Thus, by dividing the indeterminacy value into contradiction and uncertainty values, it has been an approach to facilitate the mathematical expression of uncertain situations. Thanks to this advantage of QPNS, they have been used in many studies. Panimalar et al. defined mathematical morphological operations for QPNS (Panimalar et al., 2023). Mary defined soft QPNS (Mary, 2021). Hussain et al., defined quadri-partitioned neutrosophic soft graphs (Hussain et al., 2022). Debnath et al. defined fuzzy QPNS soft matrix and applications (Debnath et al., 2022).

Şahin et al. studied the neutrosophic quintuple set (NQS) theory (Şahin et al., 2022). Neutrosophic quintuple sets are a specific form of NQS and QPNS. Şahin and Kargın defined interval neutrosophic quintuple sets and interval SVNQS, bipolar SVNQS (Şahin and Kargın, 2022b; 2022c). Şahin et al. defined generalized set-valued neutrosophic quintuple sets (GSVNQS) (Şahin et al., 2023). These sets are a generalized form of SVNQS.

The Analytic Network Process (ANP) method was developed by Saaty in 1986 for MCDM (Saaty, 1986). In this method, there is a network structure instead of a top-down unidirectional hierarchy. In the network structure, there may be interactions between criteria and alternatives within a set as well as interactions between sets. With its special network structure, the ANP method shows the interdependencies and feedback between the criteria and thus produces highly objective results. Lyu et al. studied a sensitivity analysis using ANP-GIS (Lyu et al., 2024). Mousavi et al. presented a framework using a combined DEMATEL-ANP model (Mousavi et al., 2024). Borujeni et al. studied the factors affecting using ANP with AHP (Borujeni et al., 2024). Li et al. introduced underground infrastructure on D-ANP (Li et al, 2024).

Fuzzy set theory is a widely used construct, especially in decision-making applications. Thus, researchers studied fuzzy set theory (Özer, 2022; Mahmood, 2025). Also, Mihaylov et al. developed the fuzzy ANP method (Mikhailov and Singh et al., 2003). Thus, new solutions were obtained with the ANP method for situations expressed with fuzzy sets. Büyüközkan et al. studied DEMATEL, ANP, and VIKOR methods and applications based on fuzzy set (Büyüközkan et al., 2024). Alkabaa et al used fuzzy ANP about CNC lathe components (Alkabaa et al., 2024). Nalbant et al. evaluated climate factors using ANP based on fuzzy logic (Nalbant et al., 2024). Pouyakian et al. used fuzzy ANP for construction projects (Pouyakian et al., 2024). Liao et al. obtained the intuitionistic fuzzy ANP method using intuitionistic fuzzy sets and the ANP method together (Liao et al., 2018). Thanks to this method, a new solution to MCDM problems involving uncertainty has been introduced. Şahin and Soylu conducted a study on intuitionistic fuzzy ANP models for maritime supply chains (Şahin and Soylu, 2020). Yang et al. obtained the ANP method based on fuzzy logic (Yang et al., 2020). Bakhshizadeh et al. used the ANP method to extract and prioritize the attractiveness parameters of shopping centers based on intuitionistic fuzzy numbers (Bakhshizadeh et al., 2024). Zhang et al. introduced the ANP method based on triangular intuitionistic fuzzy sets (Zhang et al., 2024).

Zaied et al. developed the neutrosophic ANP method (Zaied et al., 2019). By using the ANP method on NS theory, more objective results can be obtained in MCDM applications. Abdel-Basset et al. presented an ANP method for bipolar NS and its applications (Abdel-Basset et al., 2020). Slamaa et al. studied neutrosophic ANP and applications (Slamaa et al., 2021). Kar and Rai used the neutrosophic ANP method in Industry 4.0 (Kar and Rai, 2024). Kungumaraj used the neutrosophic ANP method for real life project time and cost estimation (Kungumaraj, 2024).

The limited reserves and environmental impacts of fossil fuels have made the need for clean and sustainable energy solutions more important than ever. At this point, renewable energy sources (RES) have great potential for environmental sustainability and energy security. They play a critical role in combating climate change by reducing carbon dioxide emissions while contributing to local economies by increasing energy independence and lowering energy costs in the long run. Researchers have conducted studies on the impacts and utilization of RES. Rather et al. studied RES for Asia-Pacific economies (Rather et al., 2024). Gayen et al. presented a study for RES based sustainable development (Gayen et al.; 2024). Wilberforce et al. studied status of RES in urban areas (Wilberforce et al.; 2024).

Sayed et al. studied the RES and energy storage systems (Sayed et al., 2023). Qing et al. presented a study for microgrids with the RES based on uncertainty set (Qing et al., 2024).

Today, the energy sector is undergoing a rapid transformation driven by advanced technologies such as artificial intelligence. These technologies have great potential to increase the efficiency of RES and optimize energy production processes. Artificial intelligence algorithms and data analytics methods are widely used to ensure the efficient use of RES such as solar, wind, hydroelectricity, and biomass. Artificial intelligence adapts energy production processes to increase efficiency, reduce costs, and minimize environmental impacts. Moreover, the integration of energy storage systems with artificial intelligence enables the uninterrupted and reliable transfer of RES to the grid. These developments make significant contributions to achieving sustainability and efficiency goals in the energy sector while encouraging wider adoption of RES. The applications of artificial intelligence in the renewable energy sector provide significant advantages in terms of both economic efficiency and environmental sustainability. Researchers have conducted studies on the use of artificial intelligence in RES. Talaat et al. obtained artificial intelligence applications for hybrid RES (Talaat et al., 2023). Shoaei et al. studied an overview of artificial intelligence applications in RES (Shoaei et al., 2024). Hanafi et al. studied development of sustainable energy management (Hanafi et al., 2024). Onwusinkwue et al. introduced artificial intelligence in RES: a review of predictive maintenance and energy optimization (Onwusinkwue et al., 2024). Karaaslan and Aydın conducted a study on the RES for Türkiye (Karaaslan and Aydın; 2020).

In the Preliminary Information Section, the basic definitions and properties used in this study are given. In the Research and Results Section, score and accuracy functions for GSVNQN are defined. In addition, the ANP method on GSVNQN using these functions is defined and the steps of this method are given. This new method also includes the consistency calculation and global weight calculation in the classical ANP method. In the application section, an example application from current life is given. In this example, the impact of environmental, social, and economic criteria on each other in the selecting RES were analyzed. As a result of this application, it was concluded that the social criterion was more effective than the others, while the environmental and economic criteria followed the social criterion respectively. It was also observed that the weights of the interdependencies between the criteria significantly affect the results, which may lead to a certain ranking change in the MCDM process. In the comparison section, it was shown that the consistency ratios of the comparison matrices obtained by using different operators are different, and that the results obtained as a result of a consistent matrix, an inconsistent matrix may be different and the importance of using a consistent matrix was emphasized. In the Conclusion Section, recommendations for future studies are given to researchers, and study's results are introduced.

2. Preliminaries

Definition 2.1. (Wang et al., 2010) Let $\acute{S} \neq \emptyset$ be a set. For $\forall \acute{s} \in \acute{S}$, with the functions

$$T_{\acute{S}_{\breve{D}}}: \acute{S} \to [0,1]\ ,\ I_{\acute{S}_{\breve{D}}}: \acute{S} \to [0,1] \text{ and } F_{\acute{S}_{\breve{D}}}: \acute{S} \to [0,1]$$

a single valued NS on $\acute{S}$ is given by

$$\breve{D} = \left\{\langle \acute{s}, T_{\acute{S}_{\breve{D}}}(\acute{s}), I_{\acute{S}_{\breve{D}}}(\acute{s}), F_{\acute{S}_{\breve{D}}}(\acute{s})\rangle : \acute{s} \in \acute{S}\right\}.$$

Here, $T_{\acute{S}_{\breve{D}}}(\acute{s}), I_{\acute{S}_{\breve{D}}}(\acute{s})$, and $F_{\acute{S}_{\breve{D}}}(\acute{s})$; for $\acute{s} \in \acute{S}$, truth, indeterminacy, and falsity functions, respectively are such that

$$0 \leq T_{\acute{S}_{\breve{D}}}(\acute{s}) + I_{\acute{S}_{\breve{D}}}(\acute{s}) + F_{\acute{S}_{\breve{D}}}(\acute{s}) \leq 3.$$

Definition 2.2. (Chatterjee et al., 2016) Let $\acute{S} \neq \emptyset$ be a set. For $\forall \acute{s} \in \acute{S}$, with the functions

$$T_{\acute{S}_{\breve{D}}}: \acute{S} \to [0,1]\ ,\ U_{\acute{S}_{\breve{D}}}: \acute{S} \to [0,1]\ , C_{\acute{S}_{\breve{D}}}: \acute{S} \to [0,1] \text{ and } F_{\acute{S}_{\breve{D}}}: \acute{S} \to [0,1]$$

a QPNS on $\acute{S}$ is given by

$$\boldsymbol{\breve{D}} = \left\{\langle \boldsymbol{\acute{s}}, \boldsymbol{T}_{\acute{S}_{\breve{D}}}(\boldsymbol{\acute{s}}), \boldsymbol{U}_{\acute{S}_{\breve{D}}}(\boldsymbol{\acute{s}}), \boldsymbol{C}_{\acute{S}_{\breve{D}}}(\boldsymbol{\acute{s}}), \boldsymbol{F}_{\acute{S}_{\breve{D}}}(\boldsymbol{\acute{s}})\rangle : \boldsymbol{\acute{s}} \in \boldsymbol{\acute{S}}\right\}.$$

Here, for $\boldsymbol{\acute{s}} \in \boldsymbol{\acute{S}}$

$$\boldsymbol{T}_{\acute{S}_{\breve{D}}}(\boldsymbol{\acute{s}}), \boldsymbol{U}_{\acute{S}_{\breve{D}}}(\boldsymbol{\acute{s}}), \boldsymbol{C}_{\acute{S}_{\breve{D}}}(\boldsymbol{\acute{s}}) \textbf{ and } \boldsymbol{F}_{\acute{S}_{\breve{D}}}(\boldsymbol{\acute{s}})$$

truth, uncertainty, contradiction, and falsity functions, respectively are such that

$$0 \leq T_{\acute{S}_{\breve{D}}}(\acute{s}) + U_{\acute{S}_{\breve{D}}}(\acute{s}) + C_{\acute{S}_{\breve{D}}}(\acute{s}), + F_{\acute{S}_{\breve{D}}}(\acute{s}) \leq 4.$$

Definition 2.3. (Şahin et al., 2022) Let $\acute{\mathcal{Y}} \neq \emptyset$ be a set and $\mathrm{P}(\acute{\mathcal{Y}})$ be a power set. A SVNQN is given by

$$\breve{D}^N = \langle \acute{\mathcal{Y}}_{1_{\breve{D}}}, \acute{\mathcal{Y}}_{2_{\breve{D}}} T_{\acute{\mathcal{Y}}_{2_{\breve{D}}}}, \acute{\mathcal{Y}}_{3_{\breve{D}}} U_{\acute{\mathcal{Y}}_{3_{\breve{D}}}}, \acute{\mathcal{Y}}_{4_{\breve{D}}} C_{\acute{\mathcal{Y}}_{4_{\breve{D}}}}, \acute{\mathcal{Y}}_{5_{\breve{D}}} F_{\acute{\mathcal{Y}}_{5_{\breve{D}}}}\rangle.$$

Here,

$$T_{\acute{\mathcal{Y}}_{2_{\breve{D}}}},\ U_{\acute{\mathcal{Y}}_{3_{\breve{D}}}},\ C_{\acute{\mathcal{Y}}_{4_{\breve{D}}}} \text{ and } F_{\acute{\mathcal{Y}}_{5_{\breve{D}}}}$$

are components in QPNS and

$$\acute{\mathcal{Y}}_{1_{\breve{D}}},\ \acute{\mathcal{Y}}_{2_{\breve{D}}},\ \acute{\mathcal{Y}}_{3_{\breve{D}}},\ \acute{\mathcal{Y}}_{4_{\breve{D}}}, \acute{\mathcal{Y}}_{5_{\breve{D}}} \in \mathrm{P}(\acute{\mathcal{Y}}).$$

Also, SVNQS is defined by

$$\breve{D} = \{\langle \acute{\mathcal{Y}}_{1_{\breve{D}}}, \acute{\mathcal{Y}}_{2_{\breve{D}}} T_{\acute{\mathcal{Y}}_{2_{\breve{D}}}}, \acute{\mathcal{Y}}_{3_{\breve{D}}} U_{\acute{\mathcal{Y}}_{3_{\breve{D}}}}, \acute{\mathcal{Y}}_{4_{\breve{D}}} C_{\acute{\mathcal{Y}}_{4_{\breve{D}}}}, \acute{\mathcal{Y}}_{5_{\breve{D}}} F_{\acute{\mathcal{Y}}_{5_{\breve{D}}}}\rangle ; \acute{\mathcal{Y}}_{1_{\breve{D}}},\ \acute{\mathcal{Y}}_{2_{\breve{D}}},\ \acute{\mathcal{Y}}_{3_{\breve{D}}},\ \acute{\mathcal{Y}}_{4_{\breve{D}}}, \acute{\mathcal{Y}}_{5_{\breve{D}}} \in \mathrm{P}(\acute{\mathcal{Y}})\}.$$

Here,

$$\langle \acute{\mathcal{Y}}_{1_{\breve{D}}}, \acute{\mathcal{Y}}_{2_{\breve{D}}} T_{\acute{\mathcal{Y}}_{2_{\breve{D}}}}, \acute{\mathcal{Y}}_{3_{\breve{D}}} U_{\acute{\mathcal{Y}}_{3_{\breve{D}}}}, \acute{\mathcal{Y}}_{4_{\breve{D}}} C_{\acute{\mathcal{Y}}_{4_{\breve{D}}}}, \acute{\mathcal{Y}}_{5_{\breve{D}}} F_{\acute{\mathcal{Y}}_{5_{\breve{D}}}}\rangle$$

represents a number, idea, object, community, etc. Furthermore,

$$\text{“ } \acute{\mathcal{Y}}_{1_{\breve{D}}} \text{ ”}$$

is called known part and

$$\text{‘‘} \acute{\mathcal{Y}}_{2_{\breve{D}}} T_{\acute{\mathcal{Y}}_{2_{\breve{D}}}}, \acute{\mathcal{Y}}_{3_{\breve{D}}} U_{\acute{\mathcal{Y}}_{3_{\breve{D}}}}, \acute{\mathcal{Y}}_{4_{\breve{D}}} C_{\acute{\mathcal{Y}}_{4_{\breve{D}}}}, \acute{\mathcal{Y}}_{5_{\breve{D}}} F_{\acute{\mathcal{Y}}_{5_{\breve{D}}}} \text{’’}$$

is called unknown part.

Definition 2.4. (Şahin et al., 2023) Let $\acute{\mathcal{Y}} \neq \emptyset$ be a set and $\mathrm{P}(\acute{\mathcal{Y}})$ be a power set. A GSVNQS is defined by

$$\breve{D} = \{< \acute{\mathcal{Y}}_{1_{\breve{D}_1}}, \acute{\mathcal{Y}}_{2_{\breve{D}_1}} T_{\acute{\mathcal{Y}}_{2_{\breve{D}_1}}}, \acute{\mathcal{Y}}_{3_{\breve{D}_1}} U_{\acute{\mathcal{Y}}_{3_{\breve{D}_1}}}, \acute{\mathcal{Y}}_{4_{\breve{D}_1}} C_{\acute{\mathcal{Y}}_{4_{\breve{D}_1}}}, \acute{\mathcal{Y}}_{5_{\breve{D}_1}} F_{\acute{\mathcal{Y}}_{5_{\breve{D}_1}}}; \acute{\mathcal{Y}}_{1_{\breve{D}_2}}, \acute{\mathcal{Y}}_{2_{\breve{D}_2}} T_{\acute{\mathcal{Y}}_{2_{\breve{D}_2}}}, \acute{\mathcal{Y}}_{3_{\breve{D}_2}} U_{\acute{\mathcal{Y}}_{3_{\breve{D}_2}}}, \acute{\mathcal{Y}}_{4_{\breve{D}_2}} C_{\acute{\mathcal{Y}}_{4_{\breve{D}_2}}}, \acute{\mathcal{Y}}_{5_{\breve{D}_2}} F_{\acute{\mathcal{Y}}_{5_{\breve{D}_2}}};$$
$$\ldots \acute{\mathcal{Y}}_{1_{\breve{D}_l}}, \acute{\mathcal{Y}}_{2_{\breve{D}_l}} T_{\acute{\mathcal{Y}}_{2_{\breve{D}_l}}}, \acute{\mathcal{Y}}_{3_{\breve{D}_l}} U_{\acute{\mathcal{Y}}_{3_{\breve{D}_l}}}, \acute{\mathcal{Y}}_{4_{\breve{D}_l}} C_{\acute{\mathcal{Y}}_{4_{\breve{D}_l}}}, \acute{\mathcal{Y}}_{5_{\breve{D}_l}} F_{\acute{\mathcal{Y}}_{5_{\breve{D}_l}}} >; \acute{\mathcal{Y}}_{1_{\breve{D}_n}}, \acute{\mathcal{Y}}_{2_{\breve{D}_n}}, \acute{\mathcal{Y}}_{3_{\breve{D}_n}}, \acute{\mathcal{Y}}_{4_{\breve{D}_n}}, \acute{\mathcal{Y}}_{5_{\breve{D}_n}} \in \mathrm{P}(\acute{\mathcal{Y}}), n \in \{1,2,3, \ldots, l\}\}.$$

Here,

$$T_{\acute{y}_{2_{\breve{D}_n}}}, U_{\acute{y}_{3_{\breve{D}_n}}}, C_{\acute{y}_{4_{\breve{D}_n}}} \text{ and } F_{\acute{y}_{5_{\breve{D}_n}}}$$

are components in QPNS. Also, a GSVNQN is defined by

$$\left(\breve{D}_1^N\right)=\{<\acute{y}_{1_{\breve{D}_1}}, \acute{y}_{2_{\breve{D}_1}} T_{\acute{y}_{2_{\breve{D}_1}}}, \acute{y}_{3_{\breve{D}_1}} U_{\acute{y}_{3_{\breve{D}_1}}}, \acute{y}_{4_{\breve{D}_1}} C_{\acute{y}_{4_{\breve{D}_1}}}, \acute{y}_{5_{\breve{D}_1}} F_{\acute{y}_{5_{\breve{D}_1}}}>\}.$$

Here,

$$``\acute{y}_{1_{\breve{D}_1}}"$$

is called known part and

$$``\acute{y}_{2_{\breve{D}_1}} T_{\acute{y}_{2_{\breve{D}_1}}}, \acute{y}_{3_{\breve{D}_1}} U_{\acute{y}_{3_{\breve{D}_1}}}, \acute{y}_{4_{\breve{D}_1}} C_{\acute{y}_{4_{\breve{D}_1}}}, \acute{y}_{5_{\breve{D}_1}} F_{\acute{y}_{5_{\breve{D}_1}}}"$$

is called unknown part. Furthermore, a GSVNQS is represented by GSVNQN such that

$$\breve{D} = \left\{\left(\breve{D}_n^N\right): n \in \{1,2,3,\dots,l\}\right\}.$$

Definition 2.5. (Şahin et al., 2023) Let

$$\breve{D} = \{<\acute{y}_{1_{\breve{D}_1}}, \acute{y}_{2_{\breve{D}_1}} T_{\acute{y}_{2_{\breve{D}_1}}}, \acute{y}_{3_{\breve{D}_1}} U_{\acute{y}_{3_{\breve{D}_1}}}, \acute{y}_{4_{\breve{D}_1}} C_{\acute{y}_{4_{\breve{D}_1}}}, \acute{y}_{5_{\breve{D}_1}} F_{\acute{y}_{5_{\breve{D}_1}}}; \acute{y}_{1_{\breve{D}_2}}, \acute{y}_{2_{\breve{D}_2}} T_{\acute{y}_{2_{\breve{D}_2}}}, \acute{y}_{3_{\breve{D}_2}} U_{\acute{y}_{3_{\breve{D}_2}}}, \acute{y}_{4_{\breve{D}_2}} C_{\acute{y}_{4_{\breve{D}_2}}}, \acute{y}_{5_{\breve{D}_2}} F_{\acute{y}_{5_{\breve{D}_2}}};$$
$$\dots \acute{y}_{1_{\breve{D}_l}}, \acute{y}_{2_{\breve{D}_l}} T_{\acute{y}_{2_{\breve{D}_l}}}, \acute{y}_{3_{\breve{D}_l}} U_{\acute{y}_{3_{\breve{D}_l}}}, \acute{y}_{4_{\breve{D}_l}} C_{\acute{y}_{4_{\breve{D}_l}}}, \acute{y}_{5_{\breve{D}_l}} F_{\acute{y}_{5_{\breve{D}_l}}}>; \acute{y}_{1_{\breve{D}_n}}, \acute{y}_{2_{\breve{D}_n}}, \acute{y}_{3_{\breve{D}_n}}, \acute{y}_{4_{\breve{D}_n}}, \acute{y}_{5_{\breve{D}_n}} \in \mathrm{P}(\acute{Y}), n \in \{1,2,3,\dots,l\}\}$$

and

$$\breve{E} = \{<\acute{y}_{1_{\breve{E}_1}}, \acute{y}_{2_{\breve{E}_1}} T_{\acute{y}_{2_{\breve{E}_1}}}, \acute{y}_{3_{\breve{E}_1}} U_{\acute{y}_{3_{\breve{E}_1}}}, \acute{y}_{4_{\breve{E}_1}} C_{\acute{y}_{4_{\breve{E}_1}}}, \acute{y}_{5_{\breve{E}_1}} F_{\acute{y}_{5_{\breve{E}_1}}}; \acute{y}_{1_{\breve{E}_2}}, \acute{y}_{2_{\breve{E}_2}} T_{\acute{y}_{2_{\breve{E}_2}}}, \acute{y}_{3_{\breve{E}_2}} U_{\acute{y}_{3_{\breve{E}_2}}}, \acute{y}_{4_{\breve{E}_2}} C_{\acute{y}_{4_{\breve{E}_2}}}, \acute{y}_{5_{\breve{E}_2}} F_{\acute{y}_{5_{\breve{E}_2}}};$$
$$\dots \acute{y}_{1_{\breve{E}_l}}, \acute{y}_{2_{\breve{E}_l}} T_{\acute{y}_{2_{\breve{E}_l}}}, \acute{y}_{3_{\breve{E}_l}} U_{\acute{y}_{3_{\breve{E}_l}}}, \acute{y}_{4_{\breve{E}_l}} C_{\acute{y}_{4_{\breve{E}_l}}}, \acute{y}_{5_{\breve{E}_l}} F_{\acute{y}_{5_{\breve{E}_l}}}>; \acute{y}_{1_{\breve{E}_n}}, \acute{y}_{2_{\breve{E}_n}}, \acute{y}_{3_{\breve{E}_n}}, \acute{y}_{4_{\breve{E}_n}}, \acute{y}_{5_{\breve{E}_n}} \in \mathrm{P}(\acute{Y}); n \in \{1,2,3,\dots,l\}\}$$

be two GSVNQN. For $\breve{D}$ and $\breve{E}$, "average ∩'' operator is defined by $\tilde{\cap}_O$ and

$$\breve{D} \tilde{\cap}_O \breve{E} = \{<\acute{y}_{1_{\breve{D}_1\breve{E}_1}}, \acute{y}_{2_{\breve{D}_1\breve{E}_1}} T_{\acute{y}_{2_{\breve{D}_1\breve{E}_1}}}, \acute{y}_{3_{\breve{D}_1\breve{E}_1}} U_{\acute{y}_{3_{\breve{D}_1\breve{E}_1}}}, \acute{y}_{4_{\breve{D}_1\breve{E}_1}} C_{\acute{y}_{4_{\breve{D}_1\breve{E}_1}}}, \acute{y}_{5_{\breve{D}_1\breve{E}_1}} F_{\acute{y}_{5_{\breve{D}_1\breve{E}_1}}};$$
$$\acute{y}_{1_{\breve{D}_2\breve{E}_2}}, \acute{y}_{2_{\breve{D}_2\breve{E}_2}} T_{\acute{y}_{2_{\breve{D}_2\breve{E}_2}}}, \acute{y}_{3_{\breve{D}_2\breve{E}_2}} U_{\acute{y}_{3_{\breve{D}_2\breve{E}_2}}}, \acute{y}_{4_{\breve{D}_2\breve{E}_2}} C_{\acute{y}_{4_{\breve{D}_2\breve{E}_2}}}, \acute{y}_{5_{\breve{D}_2\breve{E}_2}} F_{\acute{y}_{5_{\breve{D}_2\breve{E}_2}}}$$
$$\dots \acute{y}_{1_{\breve{D}_l\breve{E}_l}}, \acute{y}_{2_{\breve{D}_l\breve{E}_l}} T_{\acute{y}_{2_{\breve{D}_l\breve{E}_l}}}, \acute{y}_{3_{\breve{D}_l\breve{E}_l}} U_{\acute{y}_{3_{\breve{D}_l\breve{E}_l}}}, \acute{y}_{4_{\breve{D}_l\breve{E}_l}} C_{\acute{y}_{4_{\breve{D}_l\breve{E}_l}}}, \acute{y}_{5_{\breve{D}_l\breve{E}_l}} F_{\acute{y}_{5_{\breve{D}_l\breve{E}_l}}}>;$$
$$\acute{y}_{1_{\breve{D}_n\breve{E}_n}}, \acute{y}_{2_{\breve{D}_n\breve{E}_n}}, \acute{y}_{3_{\breve{D}_n\breve{E}_n}}, \acute{y}_{4_{\breve{D}_n\breve{E}_n}}, \acute{y}_{5_{\breve{D}_n\breve{E}_n}} \in \mathrm{P}(\acute{Y}); n \in \{1,2,3,\dots,l\}\}.$$

Here,

$$\acute{y}_{1_{\breve{D}_n\breve{E}_n}} = \left(\acute{y}_{1_{\breve{D}_n}} \cap \acute{y}_{1_{\breve{E}_n}}\right), \acute{y}_{2_{\breve{D}_n\breve{E}_n}} = \left(\acute{y}_{2_{\breve{D}_n}} \cap \acute{y}_{2_{\breve{E}_n}}\right), \acute{y}_{3_{\breve{D}_n\breve{E}_n}} = \left(\acute{y}_{3_{\breve{D}_n}} \cap \acute{y}_{3_{\breve{E}_n}}\right)$$
$$\acute{y}_{4_{\breve{D}_n\breve{E}_n}} = \left(\acute{y}_{4_{\breve{D}_n}} \cap \acute{y}_{4_{\breve{E}_n}}\right), \acute{y}_{5_{\breve{D}_n\breve{E}_n}} = \left(\acute{y}_{5_{\breve{D}_n}} \cap \acute{y}_{5_{\breve{E}_n}}\right)$$

and

$$T_{\acute{y}_{2_{\breve{D}_n\breve{E}_n}}} = \frac{T_{\acute{y}_{2_{\breve{D}_n}}} + T_{\acute{y}_{2_{\breve{E}_n}}}}{2}, U_{\acute{y}_{2_{\breve{D}_n\breve{E}_n}}} = \frac{U_{\acute{y}_{2_{\breve{D}_n}}} + U_{\acute{y}_{2_{\breve{E}_n}}}}{2}, C_{\acute{y}_{2_{\breve{D}_n\breve{E}_n}}} = \frac{C_{\acute{y}_{2_{\breve{D}_n}}} + C_{\acute{y}_{2_{\breve{E}_n}}}}{2}, F_{\acute{y}_{2_{\breve{D}_n\breve{E}_n}}} = \frac{F_{\acute{y}_{2_{\breve{D}_n}}} + F_{\acute{y}_{2_{\breve{E}_n}}}}{2}.$$

Definition 2.6. (Saaty, 1986) The ANP method consists of 7 steps.

Step 1: The MCDM problem is described.

The criteria related to our case study are determined. Let

$$K = \{K_1, K_2, \dots, K_m\}$$

be a set of criteria.

Step 2. Pairwise comparisons are made between the criteria. To calculate the criteria's weights and alternatives, comparison matrices should be created using the Saaty 1-9 scale in Table 1.

Table 1. Saaty 1-9 Scale

Level of Importance	Definition	Description
1	Equally Important	Both criteria are equally important.
3	Somewhat Important	Experiences and judgments make one criterion slightly more important than the other.
5	Very Important	Experience and judgment strongly favor one criterion over the other.
7	Very Highly Important	The criterion is very strongly superior to the other.
9	Extremely Important	Information and experience indicate that one criterion is very strongly superior to the other.
2, 4, 6, 8	Intermediate Importance	Intermediate figures can be used when necessary.

In addition, each criterion's local weight is obtained using the function

$$YA(K_a) = \frac{1}{1 + \sum_{i=1}^{m} \left(\check{d}^N{}_{ij}\right)}; a, i, j \in \{1,2, \dots, m\}.$$

The net value matrix and each criterion's local weight are in Table 2.

Table 2. The Criteria's Net Value Matrix and Local Weight Values

	K_1	K_2	...	K_m
K_1	1	$(\check{d}_{12})$	...	$(\check{d}_{1m})$
K_2	$\frac{1}{(\check{d}_{12})}$	1	...	$(\check{d}_{2m})$
...	...	...	...	...
K_m	$\frac{1}{(\check{d}_{1m})}$	$\frac{1}{(\check{d}_{2m})}$	...	1
Local Weight Values	$YA(K_1)$	$YA(K_2)$	...	$YA(K_m)$

Using the criteria's local weight

$$W_{YA} = \begin{bmatrix} YA(K_1) \\ YA(K_2) \\ \vdots \\ YA(K_m) \end{bmatrix}$$

local weight matrix is obtained.

Step 3: The consistency ratio (CR) of the comparison matrices is obtained according to the consistency index (CI) and the random consistency index (RI) such that

$$CR = \frac{CI}{RI}.$$

Here, $CI = \frac{\lambda_{max} - n}{n-1}$, RI is in Table 3, it must CR ≤ 0.1 for each comparison matrix, λ_{max} is the largest eigenvalue of the comparison matrix and n is the criteria's number.

Table 3. RI

n	1	2	3	4	5	6	7	8	9
RI	0	0	0.58	0.90	1.12	1.24	1.32	1.41	1.45

Step 4: Eigenvectors are obtained to create a super matrix. The eigenvectors are the comparison matrix of the internal dependencies of each criterion and the weights of the criteria are obtained. Eigen vectors are obtained using these weights. For criteria K_1 and K_m, the values are in Table 4 and Table 5, respectively.

Table 4. Comparison Net Value Matrix of Internal Dependencies According to Criterion K_1 and Criteria Weights

	K_2	...	K_m
K_2	1	...	$(\check{d}^N{}_{2m})$
...	...	...	...
K_m	$\frac{1}{(\check{d}^N{}_{2m})}$	...	1
Weights	$(A(K_2))_{K_1}$	...	$A(K_m)_{K_1}$

Table 5. Comparison Net Value Matrix of Internal Dependencies According to Criterion K_m and Criteria Weights

	K_1	...	$K_{(m-1)}$
K_1	1	...	$(\check{d}^N{}_{1(m-1)})$
...	...	...	...
$K_{(m-1)}$	$\frac{1}{(\check{d}^N{}_{1(m-1)})}$	...	1
Weights	$(A(K_1))_{K_m}$	...	$A(K_{(m-1)})_{K_m}$

Where the criteria's weights are obtained using the function

$$(A(K_\alpha))_{K_\beta} = \frac{1}{1+\sum_{j=1}^{m}\left(\check{d}_{ij}^N\right)}.$$

Eigenvectors are created using the weights obtained in Stage 4.2. The eigenvectors are created using the weights in the comparison net value matrices of the criteria's internal dependencies. For the criterion K_1, the eigenvector

$$V_{K_1} = \begin{bmatrix} 1 \\ A(K_2)_{K_1} \\ \vdots \\ A(K_m)_{K_1} \end{bmatrix}$$

is found in the form. Similarly, other eigenvectors are also found.

Step 5: Super matrix is created.

The eigenvectors obtained in Step 4.3 are placed in the column where the relevant criterion is located and a super matrix is obtained and this matrix is in Table 6.

Table 6. Super Matrix

	K_1	K_2	...	K_m
K_1	1	$(A(K_1))_{K_2}$	...	$(A(K_1))_{K_m}$
K_2	$A(K_2)_{K_1}$	1	...	$(A(K_2))_{K_m}$
...	...	...	...	...
K_m	$A(K_m)_{K_1}$	$(A(K_m))_{K_2}$	...	1

Super matrix is normalized using the function

$$N\left((A(K_i))_{K_j}\right) = \frac{(A(K_i))_{K_j}}{\sum_{i=1}^{m}(A(K_i))_{K_j}};\ i,j = 1,2,\dots,m.$$

The normalized super matrix is in Table 7.

Table 7. The Normalized Super Matrix

	K_1	K_2	...	K_m
K_1	$N\big((A(K_1))_{K_1}\big)$	$N\big((A(K_1))_{K_2}\big)$	...	$N\big((A(K_1))_{K_m}\big)$
K_2	$N\big((A(K_2))_{K_1}\big)$	$N\big((A(K_2))_{K_2}\big)$	...	$N\big((A(K_2))_{K_m}\big)$
...	...	...	...	...
K_m	$N\big((A(K_m))_{K_1}\big)$	$N\big((A(K_m))_{K_2}\big)$	...	$N\big((A(K_m))_{K_m}\big)$

Step 6: The weight values are obtained by multiplying the super matrix and the local weight matrix.

$$\begin{bmatrix} N\big((A(K_1))_{K_1}\big) & N\big((A(K_1))_{K_2}\big) & \dots & N\big((A(K_1))_{K_m}\big) \\ N\big((A(K_2))_{K_1}\big) & N\big((A(K_2))_{K_2}\big) & \dots & N\big((A(K_2))_{K_m}\big) \\ \vdots & \vdots & \ddots & \vdots \\ N\big((A(K_m))_{K_1}\big) & N\big((A(K_m))_{K_2}\big) & \dots & N\big((A(K_m))_{K_m}\big) \end{bmatrix} \times \begin{bmatrix} YA(K_1) \\ YA(K_2) \\ \vdots \\ YA(K_m) \end{bmatrix} = \begin{bmatrix} GA(K_1) \\ GA(K_2) \\ \vdots \\ GA(K_m) \end{bmatrix}$$

Step 7: Finally, the best criterion is selected based on the weight values from Step 6.

3. Main Results

3.1. Score and Accuracy Functions

In this part, we define score and accuracy functions for GSVNQN. Furthermore, ANP method on GSVNQN using these functions is defined and the steps of this new method are detailed.

Definition 3.1. Let

$$\breve{D}^N = \langle \acute{\mathcal{Y}}_{1_{\breve{D}}}, \acute{\mathcal{Y}}_{2_{\breve{D}}} T_{\acute{\mathcal{Y}}_{2_{\breve{D}}}}, \acute{\mathcal{Y}}_{3_{\breve{D}}} U_{\acute{\mathcal{Y}}_{3_{\breve{D}}}}, \acute{\mathcal{Y}}_{4_{\breve{D}}} C_{\acute{\mathcal{Y}}_{4_{\breve{D}}}}, \acute{\mathcal{Y}}_{5_{\breve{D}}} F_{\acute{\mathcal{Y}}_{5_{\breve{D}}}} \rangle$$

be a GSVNQN. The score and the accuracy functions for $\breve{D}^N$ are defined by respectively

$$S(\breve{D}^N) = \frac{|\, s(\acute{\mathcal{Y}}_{1_{\breve{D}}} \cap \acute{\mathcal{Y}}_{2_{\breve{D}}}) - s(\acute{\mathcal{Y}}_{1_{\breve{D}}} \cap \acute{\mathcal{Y}}_{3_{\breve{D}}}) - s(\acute{\mathcal{Y}}_{1_{\breve{D}}} \cap \acute{\mathcal{Y}}_{4_{\breve{D}}}) - s(\acute{\mathcal{Y}}_{1_{\breve{D}}} \cap \acute{\mathcal{Y}}_{5_{\breve{D}}})|}{s(\acute{\mathcal{Y}}_{1_{\breve{D}}})} + \frac{\left| T_{\acute{\mathcal{Y}}_{2_{\breve{D}}}} - U_{\acute{\mathcal{Y}}_{3_{\breve{D}}}} - C_{\acute{\mathcal{Y}}_{4_{\breve{D}}}} - F_{\acute{\mathcal{Y}}_{5_{\breve{D}}}} \right|}{3}$$

and

$$K(\breve{D}^N) = \frac{|s(\acute{\mathcal{Y}}_{2_{\breve{D}}} / \acute{\mathcal{Y}}_{3_{\breve{D}}}) + s(\acute{\mathcal{Y}}_{2_{\breve{D}}} / \acute{\mathcal{Y}}_{4_{\breve{D}}}) + s(\acute{\mathcal{Y}}_{2_{\breve{D}}} / \acute{\mathcal{Y}}_{5_{\breve{D}}})|}{2.s(\acute{\mathcal{Y}}_{2_{\breve{D}}})} + T_{\acute{\mathcal{Y}}_{2_{\breve{D}}}}.$$

Here, the function

$$s: \mathrm{P}(\acute{\mathcal{Y}}) \to \mathbb{N}$$

gives number of elements of set in $\mathrm{P}(\acute{\mathcal{Y}})$.

3.2. Algorithm

In this part, the steps of the ANP method on GSVNQN are given.

Step 1. The MCDM problem is identified. A goal for the MCDM problem and the criteria and alternatives to be used to achieve this goal is determined.

Stage 1.1. Experts experienced in the field are selected. Let

$$U = \{U_1, U_2, \dots, U_l\}$$

be the set of experts.

Stage 1.2. The criteria related to our case study are determined. Let

$$K = \{K_1, K_2, \dots, K_m\}$$

be the set of criteria.

Stage 1.3. The alternatives to be used in our case study are identified. Let

$$A = \{A_1, A_2, A_3, \dots, A_t\}$$

be the set of alternatives.

This step is identical to step 1 of the classical ANP method (Saaty, 1986) given in Definition 2.6.

Step 2. Pairwise comparison matrices are created between the relevant criteria. In these matrices, the criteria in the columns are the influencing criteria and the criteria in the rows are the influenced criteria.

Stage 2.1. Each expert makes pairwise comparisons between the criteria and expresses each comparison with GSVNQN. These numbers are defined by values for accuracy, uncertainty, contradiction, and inaccuracy, allowing for a more comprehensive evaluation of expert opinions in MCDM processes. For example, when asked about the importance of criterion A relative to criterion B, the expert can express this comparison with values for accuracy, uncertainty, contradiction, and falsity, and alternatives for these values. Thus, instead of expressing each expert opinion with a single value using the 1-9 scale in Table 1 in Step 2 of the Classical ANP method (Saaty, 1986) in Definition 2.6, it can be expressed in more detail with these five components, making decisions more objective and reliable.

Elements of the comparison matrix created for each expert is defined by

$$\left(\breve{D}^N_{n_{ij}}\right) = \{<\acute{y}_{1_{\breve{D}_{n_{ij}}}}, \acute{y}_{2_{\breve{D}_{n_{ij}}}} T_{\acute{y}_{2_{\breve{D}_{n_{ij}}}}}, \acute{y}_{3_{\breve{D}_{n_{ij}}}} U_{\acute{y}_{3_{\breve{D}_{n_{ij}}}}}, \acute{y}_{4_{\breve{D}_{n_{ij}}}} C_{\acute{y}_{4_{\breve{D}_{n_{ij}}}}}, \acute{y}_{5_{\breve{D}_{n_{ij}}}} F_{\acute{y}_{5_{\breve{D}_{n_{ij}}}}}\}.$$

Here, $n \in \{1,2,3, \dots, l\}$; i and $j \in \{1,2,3, \dots, m\}$, l is the number of experts, i and j are the row and column of the element in the comparison matrix. For example,

$$\left(\breve{D}^N_{1_{12}}\right)$$

shows the effect of criterion K_1 on criterion K_2 according to expert U_1 and is located in row 1 and column 2. Furthermore, the pairwise comparison matrices for expert $U_1$and expert U_n are given in Table 8 and Table 9.

Table 8. Comparison Matrix for U_1

	K_1	K_2	...	K_m
K_1	1	$\left(\breve{D}^N_{1_{12}}\right)$	...	$\left(\breve{D}^N_{1_{1m}}\right)$
K_2	$\left(\breve{D}^N_{1_{21}}\right)$	1	...	$\left(\breve{D}^N_{1_{2m}}\right)$
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
K_m	$\left(\breve{D}^N_{1_{m1}}\right)$	$\left(\breve{D}^N_{1_{m2}}\right)$	...	1

Table 9. Comparison Matrix for U_n

	K_1	K_2	...	K_m
K_1	1	$\left(\breve{D}^N_{1_{n_{12}}}\right)$	...	$\left(\breve{D}^N_{n_{1m}}\right)$
K_2	$\left(\breve{D}^N_{n_{21}}\right)$	1	...	$\left(\breve{D}^N_{n_{2m}}\right)$
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
K_m	$\left(\breve{D}^N_{n_{m1}}\right)$	$\left(\breve{D}^N_{n_{m2}}\right)$	...	1

Stage 2.2. The pairwise comparison matrices are obtained by different experts converted into a single pairwise comparison matrix between the criteria using one of the operators in Şahin et al.'s study (Şahin et al., 2023) and the pairwise comparison matrix of these criteria are in Table 10. This process aims to combine the opinions of the experts. Here,

$$\breve{D}^N{}_{ii} = 1.$$

Table 10. Pairwise Comparison Matrix of Criteria

	K_1	K_2	...	K_m
K_1	1	$(\breve{D}^N{}_{12})$	...	$(\breve{D}^N{}_{1m})$
K_2	$(\breve{D}^N{}_{21})$	1	...	$(\breve{D}^N{}_{2m})$
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
K_m	$(\breve{D}^N{}_{m1})$	$(\breve{D}^N{}_{m2})$	...	1

By using the average intersection operator in Definition 2.5, the pairwise comparison matrix of the criteria element $(i, j \in \{1,2,3, \dots, m\})$

$$(\breve{D}^N{}_{ij})$$

is obtained by

$$(\breve{D}^N{}_{ij}) = \left((\breve{D}^N_{1_{ij}} \,\tilde{\cap}_O\, \breve{D}^N_{2_{ij}}) \,\tilde{\cap}_O \dots \breve{D}^N_{l-1_{ij}}) \,\tilde{\cap}_O\, \breve{D}^N_{l_{ij}}\right).$$

Stage 2.3. The net value matrix of the criteria are created.

The net value matrix is obtained by using the score and the accuracy functions given in Definition 3.1 for each GSVNQN in the pairwise comparison matrix of the criteria.

In addition, the local weight of each criterion is obtained using the function $(a, i, j \in \{1,2,3, \dots, m\}$ and i ≠j)

$$YA(K_a) = \frac{1}{1+\sum_{i=1}^{m} S\left((\breve{D}^N{}_{ij})\right)}.$$

The local weight of the criterion is obtained by dividing the value in the diagonal by the sum of the columns. Here,

$$S\left((\breve{D}^N{}_{ij})\right)$$

denotes the score value of the GSVNQN $(\breve{D}^N{}_{ij})$ using score function in Definition 3.1.

The net value matrix and local weight values of the criteria are given in Table 11.

Table 11. The Net Value Matrix and Local Weight Values of the Criteria

	K_1	K_2	...	K_m
K_1	1	$S\left((\breve{D}^N{}_{12})\right)$	...	$S\left((\breve{D}^N{}_{1m})\right)$
K_2	$S\left((\breve{D}^N{}_{21})\right)$	1	...	$S\left((\breve{D}^N{}_{2m})\right)$
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
K_m	$S\left((\breve{D}^N{}_{m1})\right)$	$S\left((\breve{D}^N{}_{m2})\right)$	...	1
Local Weight Values	$YA(K_1)$	$YA(K_2)$	...	$YA(K_m)$

Stage 2.4. Using the local weights of the criteria in Table 11,

$$W_{YA} = \begin{bmatrix} YA(K_1) \\ YA(K_2) \\ \vdots \\ YA(K_m) \end{bmatrix}$$

local weight matrix is obtained.

Step 3. This step is the same as step 3 of the classical ANP method (Saaty, 1986) given in Definition 2.6.

Step 4. Eigenvectors are obtained to create a super matrix.

This step is basically the same as in the classical ANP method (Saaty, 1986). However, unlike the classical ANP method, GSVNQN is used in this step of the algorithm.

Stage 4.1. Comparison matrices of the internal dependencies of each criterion are created. These matrices are obtained by deleting the row and column containing the relevant criterion. The comparison matrix of internal dependencies for criterion K_i is obtained by deleting the ith row and ith column in which criterion K_i is located. The comparison matrices of internal dependencies according to criteria K_1 and K_m are given in Table 12 and Table 13.

Table 12. The comparison matrices of internal dependencies according to criteria K_1

	K_2	K_3	...	K_m
K_2	1	$(\breve{D}^N{}_{23})$	...	$(\breve{D}^N{}_{2m})$
K_3	$(\breve{D}^N{}_{32})$	1	...	$(\breve{D}^N{}_{3m})$
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
K_m	$(\breve{D}^N{}_{m2})$	$(\breve{D}^N{}_{m3})$	...	1

Table 13. The comparison matrices of internal dependencies according to criteria K_m

	K_1	K_2	...	$K_{(m-1)}$
K_1	1	$(\breve{D}^N{}_{12})$	...	$(\breve{D}^N{}_{1(m-1)})$
K_2	$(\breve{D}^N{}_{21})$	1	...	$(\breve{D}^N{}_{2(m-1)})$
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
$K_{(m-1)}$	$(\breve{D}^N{}_{(m-1)1})$	$(\breve{D}^N{}_{(m-1)2})$	...	1

Stage 4.2. Using the obtained internal dependencies comparison matrix, net value matrices are created according to the process in Stage 2.3 and the weights of the criteria are obtained. The internal dependencies comparison net value matrix is created for m criteria. In Table 14 and Table 15, the net

value matrix of comparison of internal dependencies and criteria weights for criteria K_1 and K_m are given. Here, the weights of the criteria are obtained using the function

$$(A(K_\alpha))_{K_\beta} = \frac{1}{1+\sum_{j=1}^{m} S\left(\left(\breve{D}_{ij}^{N}\right)\right)}.$$

Here,

$$(A(K_\alpha))_{K_\beta}$$

is the weight of the K_α criterion with respect to the internal dependence of the K_β criterion. In Table 14 and Table 15, the K_1 and K_m criteria weights are given.

Table 14. Comparison Net Value Matrix of Internal Dependencies According to Criterion K_1 and Criteria Weights

	K_2	K_3	...	K_m
K_2	1	$S\left(\left(\breve{D}^{N}{}_{23}\right)\right)$	...	$S\left(\left(\breve{D}^{N}{}_{2m}\right)\right)$
K_3	$S\left(\left(\breve{D}^{N}{}_{32}\right)\right)$	1	...	$S\left(\left(\breve{D}^{N}{}_{3m}\right)\right)$
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
K_m	$S\left(\left(\breve{D}^{N}{}_{m2}\right)\right)$	$S\left(\left(\breve{D}^{N}{}_{m3}\right)\right)$	...	1
Weights	$(A(K_2))_{K_1}$	$(A(K_3))_{K_1}$	...	$A(K_m)_{K_1}$

Table 15. Comparison Net Value Matrix of Internal Dependencies According to Criterion K_m and Criteria Weights

	K_1	K_2	...	K_{m-1}
K_1	1	$S\left(\left(\breve{D}^{N}{}_{22}\right)\right)$	...	$S\left(\left(\breve{D}^{N}{}_{1(m-1)}\right)\right)$
K_2	$S\left(\left(\breve{D}^{N}{}_{21}\right)\right)$	1	...	$S\left(\left(\breve{D}^{N}{}_{2(m-1)}\right)\right)$
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
K_{m-1}	$S\left(\left(\breve{D}^{N}{}_{(m-1)1}\right)\right)$	$S\left(\left(\breve{D}^{N}{}_{(m-1)2}\right)\right)$	...	1
Weights	$(A(K_1))_{K_m}$	$(A(K_2))_{K_m}$	...	$A(K_{m-1})_{K_m}$

Stage 4.3. Eigenvectors are created using the weights obtained in Stage 4.2. The eigenvectors are created using the weights in the comparison net value matrices of the internal dependencies of the criteria. For example, the eigenvector of criterion K_1

$$V_{K_1} = \begin{bmatrix} 1 \\ A(K_2)_{K_1} \\ \vdots \\ A(K_m)_{K_1} \end{bmatrix}$$

is found in the form. Similarly, other eigenvectors are also found.

Step 5. The super matrix of the criteria are created using the eigenvectors obtained in Step 4.3.

Step 5.1. The eigenvectors obtained in Step 4.3 are placed in the column. Here, the relevant criterion is located and a super matrix is obtained and this matrix is in Table 16.

Table 16. Super Matrix

	K_1	K_2	...	K_m
K_1	1	$(A(K_1))_{K_2}$	...	$(A(K_1))_{K_m}$
K_2	$A(K_2)_{K_1}$	1	...	$(A(K_2))_{K_m}$
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
K_m	$A(K_m)_{K_1}$	$(A(K_m))_{K_2}$	...	1

Step 5.2. The super matrix is normalized as in step 5 of the classical ANP method [22] given in Definition 2.6.

Super matrix is normalized using the function

$$N\left((A(K_i))_{K_j}\right) = \frac{(A(K_i))_{K_j}}{\sum_{i=1}^{m}(A(K_i))_{K_j}}.$$

Here,

$$N\left((A(K_i))_{K_j}\right)$$

represents the normalized value of each element in the matrix and $i, j \in \{1,2,3,\ldots,m\}$. The normalized super matrix is in Table 17.

Table 17. The Normalized Super Matrix

	$\boldsymbol{K_1}$	$\boldsymbol{K_2}$	...	$\boldsymbol{K_m}$
$\boldsymbol{K_1}$	$N((A(K_1))_{K_1})$	$N((A(K_1))_{K_2})$	...	$N((A(K_1))_{K_m})$
$\boldsymbol{K_2}$	$N((A(K_2))_{K_1})$	$N((A(K_2))_{K_2})$	...	$N((A(K_2))_{K_m})$
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
$\boldsymbol{K_m}$	$N((A(K_m))_{K_1})$	$N((A(K_m))_{K_2})$	...	$N((A(K_m))_{K_m})$

Step 6. To calculate the global weight of the criteria as in Step 6 of the classical ANP method (Saaty, 1986) given in Definition 2.6, the normalized super matrix obtained in Step 5.2 is multiplied by the local weight matrix obtained in Step 2.4. 3.3.

$$\begin{bmatrix} N\big((A(K_1))_{K_1}\big) & N\big((A(K_1))_{K_2}\big) & \dots N\big((A(K_1))_{K_m}\big) \\ N\big((A(K_2))_{K_1}\big) & N\big((A(K_2))_{K_2}\big) & \dots N\big((A(K_2))_{K_m}\big) \\ \vdots & \vdots & \ddots \quad \vdots \\ N\big((A(K_m))_{K_1}\big) & N\big((A(K_m))_{K_2}\big) & \dots N\big((A(K_m))_{K_m}\big) \end{bmatrix} \times \begin{bmatrix} YA(K_1) \\ YA(K_2) \\ \vdots \\ YA(K_m) \end{bmatrix} = \begin{bmatrix} GA(K_1) \\ GA(K_2) \\ \vdots \\ GA(K_m) \end{bmatrix}$$

Step 7. As in Step 7 of the classical ANP method (Saaty, 1986) given in Definition 2.6, the global weights obtained in Step 6 are used to rank the impact of the criteria. The global weights obtained by multiplying the normalized super matrix with the local weight matrix are ranked. The criterion with the largest global weight is determined as the criterion with the most influence on the other criteria, and the criterion with the smallest global weight is determined as the criterion with the least influence on the other criteria.

3.3. Application

In this part, the ANP method defined on the GSVNQN in Definition 3.2 was used to evaluate the impact of environmental, social, and economic criteria on each other in the selecting RES. In this application, fictitious data was used to demonstrate the usability of the algorithm. Researchers can use this application to generate solutions to their problems with real data. The criteria in the application were taken from Karaaslan and Aydın's study (Karaaslan and Aydın; 2020). In addition, 3 experts, 7 the selecting RES alternatives and 3 criteria were used in this application.

Step 1. The aim is to assess the impact of environmental, social and economic criteria on each other in the selecting RES.

Stage 1.1. 3 experts experienced in the field are selected.

Let

$$U = \{U_1, U_2, U_3\}$$

be the set of experts.

Stage 1.2. Criteria related to the problem are identified.

Let

$$K = \{K_1, K_2, K_3\}$$

be set consisting of criteria such that

K_1 = Environmental,

K_2 = Social,

K_3 = Economic

Stage 1.3. The alternatives to be used in the problem are identified. Let

A={*S*, *Wi*, *H*, *J*, *B*, *CG*,*W*}

be set of alternatives such that

S = Solar Energy

Wi = Wind

H = Hydropower

J = Geothermal

B = Biomass

CG = Come and go

W = Wave

Step 2. A pairwise comparison matrix is created between the criteria with the experts.

Stage 2.1. The experts made pairwise comparisons between the criteria and each comparison was expressed as a GSVNQN. Here,

for expert U_1,

$$(D_1^N)_{12} = \{< \{S, Wi, H, J, B, CG, W\}, \{S, Wi, H, J, B, CG, W\}(0{,}6), \{B, H\}(0{,}7), \{J, CG\}(0{,}6), \{CG, W, J\}(0{,}6) >\}$$
$$(D_1^N)_{13} = \{< \{S, Wi, H, J, B, CG, W\}, \{S, Wi, H, J, B, CG, W\}(1{,}0), \{CG, H, J\}(0{,}9), \{B\}(0{,}7), \{CG, W\}(1{,}0) >\}$$
$$(D_1^N)_{21} = \{< \{S, Wi, H, J, B, CG, W\}, \{H, Wi, J, B, S\}(0{,}8), \{S, Wi, H\}(0{,}1), \{CG, W, J, Wi, S\}(1{,}0), \{CG, W, B\}(0{,}5) >\}$$
$$(D_1^N)_{22} = \{< \{S, Wi, H, J, B, CG, W\}, \{S, Wi, H, J, B, CG, W\}(1{,}0), \emptyset(0{,}25), \emptyset(0{,}25), \emptyset(0{,}5) >\}$$
$$(D_1^N)_{23} = \{< \{S, Wi, H, J, B, CG, W\}, \{H, Wi, J, B, S\}(0{,}2), \{S, B\}(0{,}7), \{W, J, H\}(0{,}6), \{CG, S\}(0{,}8) >\}$$
$$(D_1^N)_{31} = \{< \{S, Wi, H, J, B, CG, W\}, \{H, Wi, J, B, S\}(0{,}9), \{B, G, W, CG\}(0{,}7), \{H, Wi, J, CG, W, B\}(0{,}3), \{W\}(0{,}3) >\}$$
$$(D_1^N)_{32} = \{< \{S, Wi, H, J, B, CG, W\}, \{Wi, H, J\}(0{,}5), \{S, B\}(0{,}8), \{J, B\}(0{,}6), \{CG, W, B, J\}(0{,}75) >\}$$
$$(D_1^N)_{33} = \{< \{S, Wi, H, J, B, CG, W\}, \{S, Wi, H, J, B, CG, W\}(1{,}0), \emptyset(0{,}25), \emptyset(0{,}25), \emptyset(0{,}5) >\}$$

for expert U_2,

$$(D_2^N)_{12} = \{< \{S, Wi, H, J, B, CG, W\}, \{S, Wi, H, J, B, CG, W\}(0{,}4), \{B, S\}(0{,}9), \{CG, Wi\}(0{,}8), \{W\}(0{,}4) >\}$$
$$(D_2^N)_{13} = \{< \{S, Wi, H, J, B, CG, W\}, \{S, Wi, H, J, B, CG, W\}(1{,}0), \{CG\}(1{,}0), \{J\}(0{,}8), \{W, J, H\}(0{,}6) >\}$$
$$(D_2^N)_{21} = \{< \{S, Wi, H, J, B, CG, W\}, \{H, Wi, J, B, S\}(0{,}6), \{S, Wi, H, J\}(0{,}1), \{CG, W, J, Wi, S, B\}(0{,}8), \{CG, W, S\}(0{,}1) >\}$$
$$(D_2^N)_{22} = \{< \{S, Wi, H, J, B, CG, W\}, \{S, Wi, H, J, B, CG, W\}(1), \emptyset(0{,}25), \emptyset(0{,}25), \emptyset(0{,}5) >\}$$
$$(D_2^N)_{23} = \{< \{S, Wi, H, J, B, CG, W\}, \{H, Wi, J\}(0{,}1), \{S, B, W\}(0{,}6), \{W, J, CG\}(0{,}7), \{CG, W\}(0{,}7) >\}$$
$$(D_2^N)_{31} = \{< \{S, Wi, H, J, B, CG, W\}, \{H, Wi, J, B\}(0{,}8), \{B, S, CG, W, H\}(0{,}8), \{H, Wi, J, CG, W, S\}(0{,}7), \{W, S\}(0{,}1) >\}$$
$$(D_2^N)_{32} = \{< \{S, Wi, H, J, B, CG, W\}, \{H, Wi, J, S\}(0{,}7), \{S, B, W\}(0{,}9), \{J, B, W\}(0.5), \{CG, W\}(1{,}0) >\}$$
$$(D_2^N)_{33} = \{< \{S, Wi, H, J, B, CG, W\}, \{S, Wi, H, J, B, CG, W\}(1{,}0), \emptyset(0{,}25), \emptyset(0{,}25), \emptyset(0{,}5) >\}$$

and for expert U_3,

$$(D_3^N)_{12} = \{< \{S, Wi, H, J, B, CG, W\}, \{S, Wi, H, J, B, CG, W\}(0{,}5), \{B, J\}(0{,}8), \{CG\}(0{,}7), \{W, H\}(0{,}5) >\}$$
$$(D_3^N)_{13} = \{< \{S, Wi, H, J, B, CG, W\}, \{S, Wi, H, J, B, CG, W\}(1{,}0), \{CG, W, B\}(0{,}9), \{B\}(0{,}9), \{W\}(0{,}8) >\}$$
$$(D_3^N)_{21} = \{< \{S, Wi, H, J, B, CG, W\}, \{H, Wi, J, B, S, W\}(0{,}8), \{S, Wi, H, B\}(0{,}1), \{CG, W, J, Wi, S\}(1{,}0), \{CG, W\}(0{,}4) >\}$$
$$(D_3^N)_{22} = \{< \{S, Wi, H, J, B, CG, W\}, \{S, Wi, H, J, B, CG, W\}(1{,}0), \emptyset(0{,}25), \emptyset(0{,}25), \emptyset(0{,}5) >\}$$
$$(D_3^N)_{23} = \{< \{S, Wi, H, J, B, CG, W\}, \{H, Wi, J, W\}(0{,}3), \{S, B, J\}(0{,}8), \{W, J, H\}(0{,}55), \{CG\}(0{,}85) >\}$$
$$(D_3^N)_{31} = \{< \{S, Wi, H, J, B, CG, W\}, \{H, Wi, J, B, S\}(0{,}95), \{B, S, CG, W, J\}(0{,}85), \{H, Wi, J, CG, W\}(0{,}5), \{W\}(0{,}2) >\}$$
$$(D_3^N)_{32} = \{< \{S, Wi, H, J, B, CG, W\}, \{H, Wi, J, B\}(0{,}9), \{S, B, CG\}(0{,}95), \{J, B, S\}(0{,}85), \{CG, W\}(0{,}85) >\}$$
$$(D_3^N)_{33} = \{< \{S, Wi, H, J, B, CG, W\}, \{S, Wi, H, J, B, CG, W\}(1{,}0), \emptyset(0{,}25), \emptyset(0{,}25), \emptyset(0{,}5) >\}$$

the comparison matrices elements of experts U_1, U_2 and U_3 are given in Table 18, Table 19 and Table 20; respectively.

Table 18. The Comparison Matrices of experts U_1

	K_1	K_2	K_3
K_1	1	$(D_1^N)_{12}$	$(D_1^N)_{13}$
K_2	$(D_1^N)_{21}$	1	$(D_1^N)_{23}$
K_3	$(D_1^N)_{31}$	$(D_1^N)_{32}$	1

Table 19. The Comparison Matrices of experts U_2

	K_1	K_2	K_3
K_1	1	$(D_2^N)_{12}$	$(D_2^N)_{13}$
K_2	$(D_2^N)_{21}$	1	$(D_2^N)_{23}$
K_3	$(D_2^N)_{31}$	$(D_2^N)_{32}$	1

Table 20. The Comparison Matrices of experts U_3

	K_1	K_2	K_3
K_1	1	$(D_3^N)_{12}$	$(D_3^N)_{13}$
K_2	$(D_3^N)_{21}$	1	$(D_3^N)_{23}$
K_3	$(D_3^N)_{31}$	$(D_3^N)_{32}$	1

Stage 2.2. A single pairwise comparison matrix is obtained by using the comparison matrices of the experts. While obtaining the comparison matrix of the criteria, the average intersection operator given in Definition 2.5 was used and the order of the experts was taken as U_1 , U_2 and U_3. The comparison matrix of the criteria obtained using the average intersection operator $\widetilde{\cap}_O$ in Definition 2.5 is in Table 21. With the help of the GSVNQN of experts U_1 , U_2 and U_3 in Table 18, Table 19 and Table 20, which show the effect of criterion K_1 on criterion K_2, the GSVNQN

$$(D^N)_{12} =((D_1^N)_{12} \widetilde{\cap}_O (D_2^N)_{12}) \widetilde{\cap}_O (D_3^N)_{12}$$

given in Table 21, is obtained such that

$$\{S, Wi, H, J, B, CG, W\} = \left((\{S, Wi, H, J, B, CG, W\} \cap \{S, Wi, H, J, B, CG, W\}) \cap \{S, Wi, H, J, B, CG, W\}\right)$$

$$\{S, Wi, H, J, B, CG, W\} = \left((\{S, Wi, H, J, B, CG, W\} \cap \{S, Wi, H, J, B, CG, W\}) \cap \{S, Wi, H, J, B, CG, W\}\right)$$

$$\{B\} = \left((\{B, J\} \cap \{B, S\}) \cap \{B, H\}\right)$$

$$\{CG\} = \left((\{CG\} \cap \{CG, R\}) \cap \{J, CG\}\right)$$

$$\{W\} = \left((\{W, H\} \cap \{W\}) \cap \{CG, W, J\}\right)$$

and

$$0{,}5 = \frac{\frac{0{,}6+0{,}4}{2}+0{,}5}{2};\ 0{,}8 = \frac{\frac{0{,}7+0{,}9}{2}+0{,}8}{2};\ 0{,}7 = \frac{\frac{0{,}6+0{,}4}{2}+0{,}7}{2} \text{ and } 0{,}5 = \frac{\frac{0{,}6+0{,}4}{2}+0{,}5}{2}.$$

Thus, we obtained that

$$(D^N)_{12} = \{< \{S, Wi, H, J, B, CG, W\}, \{S, Wi, H, J, B, CG, W\}(0{,}5), \{B\}(0{,}8), \{CG\}(0{,}7), \{W\}(0{,}5) >\}$$

Similarly, the other elements of Table 21 are obtained such that

$$(D^N)_{13} = \{< \{S, Wi, H, J, B, CG, W\}, \{S, Wi, H, J, B, CG, W\}(1{,}0), \{CG\}(0{,}92), \emptyset(0{,}82), \{W\}(0{,}8) >\}$$

$$(D^N)_{21} = \{< \{S, Wi, H, J, B, CG, W\}, \{H, Wi, J, B, S\}(0{,}75), \{S, Wi, H\}(0{,}1), \{CG, W, J, R, S\}(0{,}95), \{CG, W\}(0{,}35) >\}$$

$$(D^N)_{23} = \{< \{S, Wi, H, J, B, CG, W\}, \{H, Wi, J\}(0{,}22), \{S, B\}(0{,}72), \{W, J\}(0{,}6), \{CG\}(0{,}8) >\}$$

$$(D^N)_{31} =< \{S, Wi, H, J, B, CG, W\}, \{H, Wi, J, B\}(0{,}9), \{B, S, CG, W\}(0{,}8), \{H, Wi, J, CG, W\}(0{,}5), \{W\}(0{,}2) >\}$$

$$(D^N)_{32} = \{< \{S, Wi, H, J, B, CG, W\}, \{H, R, J\}(0{,}75), \{S, B\}(0{,}9), \{J, B\}(0{,}7), \{CG, W\}(0{,}86) >\}$$

Table 21. Single Pairwise Comparison Matrix According to $\tilde{\cap}_O$ Operator

	K_1	K_2	K_3
K_1	1	$(D^N)_{12}$	$(D^N)_{13}$
K_2	$(D^N)_{21}$	1	$(D^N)_{23}$
K_3	$(D^N)_{31}$	$(D^N)_{32}$	1

Stage 2.3. The net value matrix of the criteria was created. The net value matrix was obtained using the score function in Definition 3.1 and given in Table 22. For example, the net value for GSVNQN $(D_N)_{12}$ in Table 21 is obtained such that

$$S((D_N)_{12}) = \frac{|7 - 1 - 1 - 1|}{7} + \frac{|0{,}5 - 0{,}8 - 0{,}7 - 0{,}5|}{3}$$
$$= 1{,}071.$$

Similarly, the net values for other GSVNQN are obtained and Table 22 is obtained. Also, the local weight of the K_1 criterion is obtained such that

$$YA(K_1) = \frac{1}{1 + 0{,}930 + 1{,}057}$$
$$= 0{,}344.$$

Similarly, local weights for K_2 and K_3 criteria are obtained and given in Table 22.

Table 22. Net Values of Criteria

	K_1	K_2	K_3
K_1	1,000	1,071	1,222
K_2	0,930	1,000	0,919
K_3	1,057	0,998	1,000
Local Weights	0,334	0,325	0,318

Stage 2.4. Using the local weights in Table 22, the local weight matrix is

$$W_{YA} = \begin{bmatrix} 0{,}334 \\ 0{,}325 \\ 0{,}318 \end{bmatrix}$$

created.

Step 3. Using the CR formulas in Step 4 of the Classical ANP (Saaty, 1986) in Definition 2.6, the CR of the net value matrix of the criteria in Table 22 is obtained. Here,

λ_{max} = 3,058;

n = 3 criteria;

RI= 0,58;

CI = ((3,067-3))/2=0,025;

CR = 0,029/0,58=0,051;

and

0 ≤ 0,051 ≤ 0,1;

Thus, the matrix is consistent.

Step 4. Eigenvectors are obtained to create a super matrix.

Step 4.1. Comparison matrices of the internal dependencies of the criteria were created according to each criterion. Table 23, Table 24, and Table 25 show the internal dependency matrices according to the criteria.

Table 23. Comparison Matrix of Internal Dependencies According to Criterion K_1

	K_2	K_3
K_2	1	$(D^N)_{23}$
K_3	$(D^N)_{32}$	1

Table 24. Comparison Matrix of Internal Dependencies According to Criterion K_2

	K_1	K_3
K_1	1	$(D^N)_{13}$
K_3	$(D^N)_{31}$	1

Table 25. Comparison Matrix of Internal Dependencies According to Criterion K_3

	K_2	K_3
K_2	1	$(D^N)_{23}$
K_3	$(D^N)_{32}$	1

Stage 4.2. Net value matrices of the matrices obtained in Stage 4.1 are created. Similar to Stage 2.3, net values and weights are obtained. Internal dependency comparison net value matrices according to the criteria are given in Table 26, Table 27, and Table 28.

Table 26. Comparison Net Matrix of Internal Dependencies According to Criterion K_1

	K_2	K_3
K_2	1,000	0,919
K_3	0,998	1,000
Weights	0,500	0,521

Table 27. Comparison Net Matrix of Internal Dependencies According to Criterion K_2

	K_1	K_3
K_1	1,000	1,222
K_3	1,057	1,000
Weights	0,486	0,450

Table 28. Comparison Net Matrix of Internal Dependencies According to Criterion K_3

	K_1	K_2
K_1	1,000	1,071
K_2	0,930	1,000
Weights	0,518	0,482

Stage 4.3. Eigenvectors were created using the weights obtained in Stage 4.2.

Eigenvectors of criteria K_1, K_2 and K_3 is obtained such that

$$V_{K_1} = \begin{bmatrix} 1,000 \\ 0,500 \\ 0,521 \end{bmatrix}, V_{K_2} = \begin{bmatrix} 0,486 \\ 1,000 \\ 0,450 \end{bmatrix} \text{ and } V_{K_3} = \begin{bmatrix} 0,518 \\ 0,482 \\ 1,000 \end{bmatrix}$$

Step 5. Super matrix created.

Step 5.1. Using the eigenvectors obtained in Step 4.3, the super matrix in Table 29 is obtained.

Table 29. Super Matrix

	K_1	K_2	K_3
K_1	1,000	0,486	0,518
K_2	0,500	1,000	0,482
K_3	0,521	0,450	1,000

Step 5.2. The super matrix is normalized.

For example, using the values (in the matrix in Table 29)

$$N\big((A(K_1))_{K_1}\big) = \frac{1,000}{1,000 + 0,500 + 0,521} = 0,494$$

is obtained. Similarly, the other elements are obtained to obtain the normalized super matrix given in Table 30.

Table 30. Normalized Super Matrix

	K_1	K_2	K_3
K_1	0,494	0,251	0,259
K_2	0,274	0,516	0,241
K_3	0,257	0,232	0,500

Step 6. To calculate the global weight of the criteria, the local weight matrix is obtained in Step 2.4 is multiplied by the normalized super matrix obtained in Step 5.2.

$$\begin{bmatrix} 0,494 & 0,251 & 0,259 \\ 0,274 & 0,516 & 0,241 \\ 0,257 & 0,232 & 0,500 \end{bmatrix} \times \begin{bmatrix} 0,334 \\ 0,325 \\ 0,318 \end{bmatrix} = \begin{bmatrix} 0,329 \\ 0,327 \\ 0,320 \end{bmatrix}$$

Step 7. With the global weights obtained in Step 6, an impact ranking is made among the criteria. The obtained weights of the criteria are given in Table 31.

Table 31. Weights of the Criteria

Criteria	Weights
K_1	0,329
K_2	0,327
K_3	0,320

As can be seen in Table 26, it is clear that environmental criteria have the greatest impact on other criteria when choosing RES with a weight of 0,329; followed by social criteria with a weight of 0,327 and economic criteria with a weight of 0,320. Also, attention should be paid to the impact of the internal dependencies of the criteria on the weights. Because, the internal dependencies changed the weights of the main criteria from (0,334; 0,325; 0.318) to (0,329; 0,327; 0,320). This change in the weights of the main criteria reveals a certain change in the order of importance in the MCDM process. Therefore, taking into account the interdependencies of the criteria in future applications will provide more objective results in MCDM.

4. Comparison Method

The comparison matrices created by experts U_1, U_2, and U_3 were obtained by using the average intersection, optimistic intersection, pessimistic intersection, average union, optimistic union, and pessimistic union operators (Şahin et al., 2023) defined for GSVNQN. The CR of the obtained comparison matrices are obtained and these CR are given in Table 32.

Table 32. CR of Comparison Matrices Obtained from Operators (Şahin et al., 2023)

Operator	CR
Average Intersection	0,051
Optimistic Intersection	0,116
Pessimistic Intersection	-0,086
Average Union	0,419
Optimistic Union	0,491
Pessimistic Union	0,288

As can be seen in Table 32, only the matrix obtained using the average intersection operator is found to be consistent. However, the comparison matrices obtained with other operators were not consistent. Operations cannot be performed with inconsistent matrices. If it is done, the results obtained may contradict the real results.

Although the comparison matrix obtained with the optimistic intersection operator is not consistent, calculations are performed. The ranking obtained with the inconsistent optimistic intercept operator and the ranking obtained with the consistent average intercept operator are given in Table 33.

Table 33. Rankings Obtained from Operators (Şahin et al., 2023)

Operator	**Ranking**
Average Intersection	$K_1 > K_2 > K_3$
Optimistic Intersection	$K_3 > K_2 > K_1$

As can be seen in Table 33, K_1 is found to be the most important criterion in the ranking of the importance of the criteria with the consistent comparison matrix obtained using the average intersection operator, while K_3 is found to be the most important criterion in the ranking of the importance of the criteria with the inconsistent comparison matrix obtained using the optimistic intersection operator. It was also seen that the rankings obtained were opposite to each other. Therefore, the choice of the operator to be used in MCDM applications and the consistency of the matrix obtained as a result of the selected operator are of great importance. Because the different rankings obtained will greatly affect the outcome of MCDM applications.

5. Conclusion

In this article, score and accuracy functions for GSVNQN are determined. The steps of the ANP method using these numbers and how it can be applied in practice are explained. The ANP method defined on GSVNQN also includes the consistency ratio calculation and global weight calculation steps in the classical ANP method. An application was made using fictitious data on a sample problem from current life. In the example problem, an MCDM application was carried out by considering the environmental, social, and economic criteria in the selecting RES and deciding which criterion is more effective than other criteria. As a result of this application, it was concluded that the environmental criterion was more effective than the other criteria, followed by the social and economic criteria.

Although the example was conducted with fictitious data, researchers can obtain more objective results by using this method with real data. It was also observed that the interdependencies between the criteria significantly affect the weight results and this effect can lead to a certain change of priority in the MCDM process. In addition, the comparison matrix obtained must be consistent, otherwise the results obtained will be different from the results obtained with a consistent matrix.

Researchers can use interval GSVNQS and bipolar GSVNQS in the ANP Method thanks to the obtained algorithm, score and accuracy functions. Furthermore, researchers can define similarity or distance measure for using the in the obtained algorithm instead of score and accuracy functions. Researchers also can use GSVNQS in other specialized MCDM methods (The DEMATEL Method, AHP Method, TOPSIS Method, ...) thanks to the obtained algorithm, score, and accuracy functions. In addition, the algorithm will be converted into a computer program by the researchers so that they can try the algorithm directly on their own.

Abbreviations

NS: Neutrosophic Sets

NQS: Neutrosophic Quadruple Sets

NQS: Neutrosophic Quintuple Sets

SVNQS: Set-Valued Neutrosophic Quintuple Sets

SVNQN: Set-Valued Neutrosophic Quintuple Numbers

QPNS: Quadri-Partitioned Neutrosophic Sets

QSVNQS: Generalized Set-Valued Neutrosophic Quintuple Sets

QSVNQN: Generalized Set-Valued Neutrosophic Quintuple Numbers

MCDM: Multi Criteria Decision-Making

RES: Renewable Energy Sources

Conflict of Interest Statement

The author of the article declares that there is no conflict of interest.

Contribution Rate Statement Summary of Researchers

The author declares that he contributed 100% to the article.

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