HACETTEPE ÜNİVERSİTESİ
İKTİSADİ VE İDARİ BİLİMLER FAKÜLTESİ DERGİSİ

CİLT 12/1994

MONEY DEMAND AND THE OPTIMAL INFLATION TAX IN TURKEY

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1. Introduction

In this study, the demand for money function is specified as a Cagan type semi-logarithmic function. Using Cagan's demand for money function we will attempt to determine the value of the revenue maximizing rate of inflation and the corresponding level of seigniorage. We will also investigate the characteristics of the demand for money in Turkey based on monthly data for the period 1982-1992. It will be demonstrated the hypothesis that the monetary experience of Turkey and the salient features of the demand for money during this period could be adequately characterized by the Cagan's demand for money model under hyperinflation. Although, the Turkish experience during this period does not exhibit a hyperinflation when one takes the strict definition of Cagan, Turkey have been facing sustained high rates of inflation for a long period.

In particular, we applied a test of Cagan's model which is not contingent on any particular assumption concerning expectations formation except that agents' error in forecasting inflation are stationary. Under this assumption, money demand estimation requires cointegration between real money balances and current inflation. Testing for cointegration between these

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variables, if cointegration is not rejected, a highly efficient estimate of the major parameter of interest in the model -the semi-elasticity of real money demand with respect to the inflation rate- is obtained as the cointegration parameter by applying Johansen maximum likelihood technique. This technique for estimating and testing the Cagan's model can, however, only be applied to a case where both real money balances and inflation rate are cointegrated according to I(1) process, although real money balances and inflation are non-stationary processes.

Closely related with stationarity is the degree of integration of a series. A series is said to be integrated of order one, I(1), if it has to be differenced once before becoming stationary. For this purpose, we will perform a stationarity test employing Dikey-Fuller unit root tests. The presence of a single unit root in the inflation process implies that the rate of acceleration of prices will tend to return to a constant average level. Hence, the presence of a unit root in the time series process for real money balances implies that the growth rate of real money stock is stationary.

Next to the unit root tests, cointegration tests particularly Johansen cointegration likelihood ratio test will be conducted. Testing for inflation tax as a revenue maximization will also be accomplished employing the likelihood ratio test. Next to inflation tax, rational expectations, naive expectations and adaptive expectations hypotheses will be investigated as well. Finally, autoregressive least square estimates of the Cagan money demand model under adaptive expectation formation will be analyzed.

2. Integration and Cointegration

Economic theory generally deals with equilibrium relationships. Most empirical econometric studies are an attempt to evaluate such relationships by summarizing economic time series using statistical analysis. To apply standard inference procedures in a dynamic time series model we need various variables to be stationary, since the majority of economic theory is built upon the assumption of stationarity. Integrated variables are a specific class of non-stationary variables with important economic and statistical properties(Dolado, Jenkinson and Rivero, 1990).

A series with no deterministic component which has a stationary, invertible, ARMA representation after d times, is said to be integrated of order d, denoted $x_t \sim I(d)$. Thus, the first difference of a time series integrated of order one is stationary. A white noise series and a stable first-order autoregressive process are examples of I(0) series, while a random walk process is an example of an I(1) series[Engle and Granger (1987)].

The components of the vector x_t are said to be cointegrated of order (d,b), denoted $x_t \sim CI(d,b)$, if (i) all components of x_t are I(d); (ii) there exists a vector $\alpha(\neq 0)$ so that
$z_t = \alpha x_t \sim I(d\text{-}b)$, $b>0$. The vector α is called the cointegration vector. On the other hand, if each elements of x_t first achieves stationarity after differencing, but α is already stationary, the time series x_t are said to be cointegrated with cointegrating vector α. Interpreting $\alpha = 0$ as a long run equilibrium, cointegration implies that deviations from equilibrium are stationary, with finite variance, even though the series themselves are nonstationary and have infinite variance.

Due to the fact that many economic time series contain seasonal components, there have been several developments in the concept of seasonal integration. Engle and Granger definition of integration to account for seasonality: a variable is said to be integrated of order (d,D) [I(d,D)], if it has a stationary, invertible, non-deterministic ARMA representation after one-period differencing d times and seasonally differencing D times.

2.1.Testing for Unit Roots

In general, the procedure is to test whether the variable, Y_t, is stationary. If the hypothesis of stationarity is rejected, the first difference of the variable (ΔY) is formed and tested for stationarity. The relevant tests fall into three categories: informal examination of correlograms, Durbin-Watson statistic tests, and regression-based 't' tests. It is the last of these which, under the generic name "Dickey-Fuller" tests, has received the most attention in the literature. These tests are conducted within the context of three distinct types of data generating process (DGP) of a univariate series (Y):

$$Y_t = \alpha Y_{t-1} + u_t$$
$$Y_t = \alpha_0 + \alpha Y_{t-1} + u_t$$
$$Y_t = \alpha_0 + \alpha Y_{t+1} + \gamma T + u_t$$

The three processes differ according to whether the mean of the series zero or the mean is non-zero and a time variable, T, is included. Considering, if $\alpha=1$, then by repeated substitution, the variance of Y goes to infinity as t increases. What is needed is some transformation to remove the complication of non-stationarity:

$$\Delta Y_t = \beta Y_{t-1} + u_t$$
$$\Delta Y_t = \alpha_0 + \beta Y_{t-1} + u_t$$
$$\Delta Y_t = \alpha_0 + \beta Y_{t-1} + \gamma T + u_t \ .$$

The tests are directed towards determining whether the value of α is equal to 1 (in which case Y is not stationary) or less then one (in which case Y is stationary). Ordinary least squares (OLS) is valid for the transformed equations where β is α-1. So, the test of whether that $\alpha=1$ becomes the test of whether $\beta=0$ then, ΔY is stationary and so Y is I(1). If $\beta<0$, this implies $\alpha<1$ so that Y is I(0) and is stationary. However, a problem occurs with these three equations if the estimated residuals u_t are not free from autocorrelation since this invalidates the test. It is possible to overcome this difficulty by modifying the test procedures. This approach uses the modified Augmented Dickey-Fuller (ADF) test which involves lagged values of dependent variable:

$$\Delta Y_t = \alpha_0 + \beta Y_{t-1} + \gamma T + \delta_1 \Delta Y_{t-1} + \delta_2 \Delta Y_{t-2} + ... + \delta_k \Delta Y_{k-2} + u_t$$

Number of lags(i.e. the value of k) being determined by the minimum number to give the residuals free from autocorrelation. This can be tested by Lagrange Multiplier (LM) test. As before the appropriate test is again on the estimated coefficient of Y_t (i.e. β). Critical values for the 't' statistic are shown in Dickey and Fuller (1981).

A strategy for testing unit roots has been suggested by Dolado, Jenkinson and Sosvilla-Rivero (1990). It starts with general model (third model, or general ADF model). If the hypothesis of non-stationarity is rejected, the procedure terminates. Acceptance of non-stationarity is followed by test of significance of the time trend. If it is not significant, the model (without trend) is tested, and then, if necessary first model(without drift) can be tested. Table 1 and Table 2 contain the results of unit root tests applied to the alternative real money balances, actual inflation (π_t) and real LnGDP (y_t) series, with and without allowance for a trend in mean, while Table 3 summarizes our inferences from these tests.

We let π_t be the observed inflation series and assume that $\pi_t = \mu_t + \pi_t^+$, where $\mu_t = \mu_{t+1}$ is a sequence of monthly means and π_t^+ is the non-seasonal series which contains trend and purely stochastic components. The deterministic seasonality can be removed by weighted moving average procedures as follows: $\pi_t^+ = 1/12\ (0.5\ \pi_{t-6} + \pi_{t-5} + ... + \pi_{t+5} + 0.5\ \pi_{t+6}$). Likewise, we have obtained a smoothed series for LnGDP.

Dickey (1986) shows that the limiting distribution of the unit root statistics is not affected by removing seasonal means from autoregressive series that are stationary in first difference. Therefore, ADF statistic from the regression equation

$$\Delta Y_t = \beta Y_{t-1} + \gamma T + \delta_1 \Delta Y_{t-1} + \delta_2 \Delta Y_{t-2} + ... + \delta_k \Delta Y_{k-2} + u_t$$

can be used for testing the null hypothesis $H_0 = \pi_t^+ \sim I(1,0)$ versus the alternative $H_1 = \pi_t^+ \sim I(0,0)$. As shown in Table 2 that H_0 hypothesis can not be rejected at 5 percent level. On the basis of the test statistics, one may infer that smoothed inflation rate series is I(1), so that it can be used for testing of cointegration. The series of alternative real money balances are also I(1).

TABLE 1. Unit Root Tests of Alternative Real Money Balances

Alternative Money Balances	Δ^0 (no difference)		Δ^1 (first difference)	
	ADF (without trend)	ADF (with trend)	ADF** (without trend)	ADF** (with trend)
H	-2.10	-3.02	-7.31	-7.24
R	-1.14	-3.38	-7.90	-7.75
CBM	-1.70	-1.95	-7.52	-6.64

132 observations are used for the tests. Critical values for the ADF statistics are -2.88(w/o trend) and -3.44(with trend) at the 5% level. (*) denotes significance at %10 level and (**) denotes significance at the 5% level.

TABLE 2. Unit Root Test of the Original and Smoothed Series of Inflation and Gross Domestic Product

Original and Smooth Series	Δ^0 (no-difference)		Δ^1 (first difference)		Δ^2 (second difference)	
	ADF (w/o trend)	ADF (with trend)	ADF (w/o trend)	ADF (with trend)	ADF (w/o trend)	ADF (with trend)
(π_t)	-6.08**	-6.23**	-	-	-	-
(y_t)	-4.80**	-7.76**	-	-	-	-
Sm(π_t)	-1.69	-2.30	-5.10**	-6.05**	-	-
Sm(y_t)	-1.21	-3.32	-2.48	-3.22	-5.44**	-5.61**

*Critical values of the statistics are -2.88 (without trend) and -3.44(with trend) at the 5% level. ** denotes significance at the 5% level.*

TABLE 3. Summary of the Unit Roots

SERIES	Order of Integration
H	I(1)
R	I(1)
CBM	I(1)
π_t	SI(1,1)
Smooth(π_t)	I(1)
y_t	SI(2,1)
Smooth(y_t)	I(2)

2.2. Testing for Cointegration

In order to illustrate testing for cointegration, we will consider a bivariate case where say, y_t and x_t have been found to contain a single unit root (i.e. both are I(1)). Then the following part of the cointegration test is to estimate the cointegration regression,

$$y_t = \lambda_0 + \lambda x_t ,$$

and test whether the cointegrating residuals are I(0). Engle and Granger (1987) suggest seven alternative tests for determining if u_t is stationary. Here we consider only two of their tests, namely the DF and ADF statistics for the cointegrating residuals. The DF test for stationarity of the residuals follows the standard form by running the regression:

$$\Delta u_t = \beta u_{t-1}$$

The null hypothesis is that $\beta=0$, in which case acceptance of the null hypothesis implies that u_t is I(1) so that y and x are not cointegrated. Second test procedure involves two steps similar to DF test. ADF test procedure is applied as mentioned in the previous section. First step consists of estimating the long-run equilibrium relationship and second step is the estimation of the dynamic relationship.

Another method developed by Johansen (1988) and Johansen and Juselius (1990) is also used for estimating the cointegration vector. Johansen derived the maximum likelihood estimator of the space of cointegration vectors and the likelihood ratio test of the hypothesis that it has a given number of dimensions. Further, linear hypothesis can be tested for the cointegration vectors. This estimation procedure has several advantages on the two-step regression procedure suggested by Engle and Granger. It relaxes the assumption that the cointegrating vector is unique and takes into account the error structure of the underlying process.

The Johansen procedure for determining the number of cointegrating vector can be incorporated into a general modelling strategy as follows: a) Use economic theory to select the variable of interest b) Check the degree of integration of each variable using the DF or ADF test and critical values(Holden and Thompson, 1992).

The main purpose of the cointegration testing is to investigate long-run relationships between the real money balances and actual inflation.

The hypothesis that there are at most r cointegration vectors is tested for each money balances and the results are given in Table 4. As it is known that each vector includes real money balances and actual inflation coefficients and an intercept term. Results show that coefficients matrix has only one cointegration vector for all money balances. Thus, the hypothesis, r=0, is rejected for all cases at the 5% level.

TABLE 4. Johansen Cointegration Likelihood Ratio Test

Alternative Real Money Balances	Cointegration Likelihood Ratio Test Statistics	
	Test 1 H_0: r=0 H_a: r=1	Test 2 H_0: r<=1 H_a: r=2
H	49.44	5.06
R	29.75	1.20
CBM	21.51	2.74

The 5% critical values for Test 1 is 16.57 and for Test 2 is 9.24

In this cointegration analysis, particularly a test of Cagan's model of money demand which is not contingent on any specific assumption concerning expectation function except that agent's errors in forecasting inflation are stationary is applied. Regardless of the method used to form expectations, the expected rate of inflation must differ from the rate of actual inflation by a forecast error, the only assumption concerning this forecast error is that it is stationary. Under the circumstances of inflationary episodes, it would seem reasonable to suppose that real money balances and inflation are non-stationary processes with percentage change which do however approximate to stationary.

Denoting the nominal money balances and prices by M and P respectively, the Cagan model can be written as follows:

$$Ln(M/P)_t = -\alpha \Delta LnP^e_{t+1} + \psi .$$

where superscript, e, indicates expectations formed at time t and ψ indicates elements of money demand not captured by the model. That is a random error term which may be serially correlated but is stationary. This disturbance term contains the effect of real variables such as income on demand for money. The parameter of interest in this equation is α, the semi-elasticity of real money demand with respect to expected inflation rate. By definition, elasticity of real money demand with respect to expected inflation is

$$| \alpha \Delta LnP^{e}_{t+1} |.$$

Actual inflation rate at time t+1 must be equal to the expected rate plus a forecasting error ϵ_{t+1}. Accordingly,

$$\Delta LnP_{t+1} = \Delta LnP^{e}_{t+1} + \epsilon_{t+1}.$$

The only restriction placed on the expectations formation is that the forecast error, ϵ_{t+1}, be stationary. Substituting this equation into above equation yields,

$$Ln(M/P)_t = -\alpha \Delta LnP_{t+1} + \epsilon_{t+1},$$

where $\epsilon_{t+1} = [\, \psi + \alpha(\Delta LnP_{t+1} - \Delta LnP^{e}_{t+1})\,]$.

Under the circumstances of inflationary episodes, growth rate of real money balances and the rate of change of inflation are stationary processes. This would imply that Ln(M/P) and ΔLnP_t are first difference stationary or, in other words, integrated of order one I(1). Adding $\alpha \Delta LnP_t$ to both sides of above equation we have

$$Ln(M/P)_t + \alpha \Delta LnPt = -\alpha \Delta^2 P_{t+1} + \epsilon_{t+1}$$

Since by assumption $\alpha \Delta^2 P_{t+1}$ and ϵ_{t+1} are stationary, this equation impMwes that the linear combination ($Ln(M/P)_t + \alpha \Delta LnP_t$) must also be stationary. Hence, real money balances and inflation are cointegrated with a cointegrating parameter (after the normalization on real money balances) which is just equal to the parameter of interest in the Cagan model, that is, the semi-elasticity of real money demand with respect to expected inflation rate(α). If the above assumptions are correct then this estimate of α, will be highly efficient (super consistent). Thus the test of applicability of the model lies in testing whether or not real money balances and inflation are cointegrated. If cointegration is found then a highly efficient and robust estimate of the main parameter of interest in the model can be obtained which is non specific with respect to expectations formation by applying ordinary least squares or a maximum likelihood technique to equation $Ln(M/P)_t = -\alpha \Delta LnP_{t+1} + \epsilon_{t+1}$. In this analysis we apply Johansen's maximum likelihood method for estimating α since this method has the added advantage of allowing tests of linear restrictions on the cointegrating parameters.

Tables 5, list point estimates of α. Those estimated values with the correct sign may be considered as estimates of the semi-elasticity coefficient of real money balances with respect to inflation rate. In the monetarist seigniorage literature, the optimal inflation rate that maximizes revenue from inflation is usually denoted by the reciprocal of the semi-elasticity coefficient. Therefore, we can compare the reciprocals of these estimated values ($-100\hat{\alpha}^{-1}$) with the average actual inflation rate of the period.

Those vectors that are found to be significant in LM test are given in Table 5. Here, the definition of inflation, $\pi_t=(LnP_t-LnP_{t-1})$, is employed. The estimated semi-elasticity coefficients, i.e. α coefficient in Cagan model, are as expected in terms of their sign.

TABLE 5. Johansen Cointegration Vector Estimates of Money Balances and Inflation with Intercept Terms

Alternative Real Money Balances	Estimated Cointegration Vector π_t		
	Real Money Balances	ΔLn(cpi) π_t	Intercept
H	-1	-26.28	4.87
R	-1	-27.67	4.86
CBM	-1	-24.91	4.93

Elements of the vector are normalized according to money balances so that the coefficients in the first column are equal to -1. Each coefficient in the second column corresponds to $\hat{\alpha}$, i.e. the semi-elasticity coefficient.

3. Testing for Optimal Inflation Rate

In this section, the percentage rate of increase in prices and base money which maximizes the revenue from the inflation tax that results from money creation is tested with the actual average monthly rate of inflation which prevails in an inflationary period by using the Likelihood ratio test statistics.

Actual average rate of inflation is calculated for Turkey adopting the procedure suggested by Cagan (1956); that is, value of π_0 is chosen such that,

$$(1 + \pi_a / 100)^N = P_T / P_1$$

where P_T is price level at end of the final month of inflation and P_1 is the price level at the beginning month of the inflation and N is the duration of the inflation. Price level used in this calculation is defined as the consumer price index, 1987=100. The value of actual monthly average rate of inflation is 3.546 that is calculated from the equation using the constants of N=132, P_1= 15.91, P_N=1588.3 for the period of 1982.01-1992.12. Then, the optimal inflation rates we obtained from the model is equal to $-100(1/\hat{\alpha})$ that is the inverse of the estimated semi-elasticity coefficient in cointegration vectors.

The elasticity coefficients and the corresponding optimal inflation rates (π^*) for various monetary base and inflation definitions are presented in Table 6. Also, the actual monthly inflation rate, 3.55, calculated according to the above formula is given in
Table 6.

As it can be seen from Table 6, there is a great similarity between the optimal inflation rate which is derived immediately by taking the inverse of the Johansen cointegration coefficient and the average inflation rates of the period. The optimal inflation rates that are obtained for various monetary base definitions and alternative inflation rates are slightly higher than the average actual inflation rate of the period.

The main implication of the seigniorage model is that generating inflation through a persistent increase in the money supply can be viewed as a means of raising revenue for the monetary authorities or an inflation tax. In the context of this model optimal inflation rate which maximizes the revenue from the inflation tax that results from money creation is equal to $(-100/\hat{\alpha})\%$.

The statistical significance of these differences are analyzed with the help of the Likelihood Ratio Test and the results are presented in Table 7. Table 7 lists the values of the several estimates of α, together with the actual average monthly rate of inflation and likelihood ratio test statistics for the null hypothesis that $-100/\hat{\alpha}$ is in fact equal to the average inflation rate over the period.

TABLE 6. The Estimated Coefficients of the Optimal Inflation Rate vs Actual Average Inflation Rate

Alternative Real Money Balances	Johansen Cointegration Coefficient (semi-elasticity coefficient) $\hat{\alpha}$	Optimal Monthly Inflation Rate $\pi^* = -100(1/\hat{\alpha})$	Actual Average Monthly Inflation Rate π_a
H	-26.28	3.81	3.55
R	-27.67	3.61	3.55
CBM	-24.91	4.01	3.55

TABLE 7. The Likelihood Ratio Test of Significance Between the Semi-Elasticity Coefficients

Alternative Real Money Balances	Johansen Cointegration Coefficient $\hat{\alpha}$	Inverse of Average Monthly Inflation Rate $\alpha_a = 100\pi_a^{-1}$	Likelihood Ratio Test $H_0 : \hat{\alpha} = \pi_a$ $H_a : \hat{\alpha} \neq \pi_a$
H	-26.28	-28.20	1.42
R	-27.67	-28.20	0.14
CBM	-24.91	-28.20	3.21

95% critical value of LR statistic, $\chi^2(2)$, is 7.38

Using alternative money balances, the unrestricted estimate of $-100/\hat{\alpha}$ is numerically close to the average actual monthly inflation rate and more formally the hypothesis that the authorities expand base money on average in such a way as to maximize the inflation tax revenue can not be rejected at any of the standard levels of statistical significance.

Our results show that authorities expanded monetary base to maximize inflation tax revenue and since government receives only the revenue accruing from the inflation tax on notes and coin which is equal to the real value of seigniorage on new money issue plus the decline in real value of notes and coin outstanding, monetary policy over the sample period was in effect tantamount to maximization of the inflation tax revenue from monetary base creation.

4. Testing The Rational Expectations Hypothesis

The Cagan model implies that the current level of real money balances can be viewed as proportional to agents' expectation of next period's inflation rate:

$$Ln(M_t/P_t) = \alpha \Delta Ln(P^e_{t+1})$$

Here, M denotes nominal money balances, α is the semi-elasticity coefficient of expected rate of inflation, ΔLnP^e_{t+1}. Actual rate of inflation at time t+1 must be equal to the expected rate plus a forecasting error, ${}^{n}{}_{t+1}$ say :

$$\Delta Ln(P_{t+1}) = \Delta Ln(P^e_{t+1}) + {}^{n}{}_{t+1}$$

If expectations are formed according to the rational expectations model, we assume that one-step-ahead rational expectations forecasting errors should be orthogonal to information available at time t, I_t (i.e. forecast errors are independent from I_t). It can be written statistically as follows:

$$E({}^{n}{}_{t+1} / I_t) = 0$$

Assuming that the Cagan model holds, a measure of agents' forecasting errors, ${}^{n}{}_{t+1}$, can be extracted from equation,

$$Ln(M_t/P_t) = \alpha \Delta Ln(P_{t+1}) + \epsilon_{t+1}$$

$${}^{n}{}_{t+1} = \Delta Ln(P_{t+1}) - \alpha^{-1} Ln(M_t/P_t)$$

where $\epsilon_{t+1} = -\alpha {}^{n}{}_{t}$

This suggest... a very simple test of the Cagan model under rational expectations can be obtained by extracting the forecast errors according to above equation(using the cointegration estimate of α) and examining their serial correlation properties. Two sets of prediction errors, ${}^{n}{}_{t+1}$, were constructed for each real money balances. One set is constructed by using the Johansen cointegration estimate of α as reported in Table 4 and second set is constructed by assuming inflation tax revenue maximization(i.e. $\alpha = -100\ \pi^{-1}$, where π_a is average monthly inflation rate over the period). For each series we calculated the autocorrelation function for up to twelve lags and also tested for serial correlation for up to order twelve using Lagrange multiplier tests(Phylaktis and Taylor(1993), Taylor(1991)).

The results in Table 8 show that all of the lagrange multiplier statistics are significant. In other words, the forecast errors are not independent of I_t. Thus, on the basis of this evidence, the Cagan model under the rational expectations is strongly rejected for each type of money balances.

TABLE 8. Tests of the Cagan Model Under Rational Expectations

Alternative Real Money Balances	χ^2 - Lagrange Multiplier Statistics for Two Sets of Prediction Errors	
	For Cointegration Estimates	For Average Monthly Inflation Rate
	$\pi_c = -100\alpha^{-1}$	π_a
H	113.376	120.822
R	120.821	120.822
CBM	121.600	121.605

Test statistics is distributed $\chi^2(12)$ under the null hypothesis of rational expectations. The 5% critical value is 23.3.

5. Testing The Naive Model of Expectations

Expectation of next period's inflation rate, π^e_{t+1} [ΔLn(P^e_{t+1})], may assume to be generated by a naive model presented as, $\pi^e_{t+1} = \pi_t$. That is, agents believe that the inflation of the next period will be the same as the inflation of the current period. A simple extrapolative model would be to say that inflation of the next period will increase by the same amount as the latest increase in the actual inflation rate(Maddala, 1988:338-40). This gives

$$\pi^e_{t+1} - \pi_t = \pi_t - \pi_{t-1}$$

or

$$\pi^e_{t+1} = 2\pi_t - \pi_{t-1}.$$

Expectations from naive model are used for estimating the α (semi-elasticity coefficient) of the Cagan model. Regression equations have been constructed for each type money balances series. Alternative each money balances series has been regressed on expected inflation series from naive model.

Expected inflation series, namely $\pi^{e}_{t+1} = 2\pi_t - \pi_{t-1}$, are generated and each of them tried in Cagan money demand regression model. The results that are given in Table 9 show the coefficients of OLS regressions and "t" values which are insignificant at the 95% confidence level. Thus, we strongly rejected the Cagan model under the naive expectations mechanism.

TABLE 9. Tests of the Cagan Model Under Naive Expectations

Alternative Money Balances	Coefficients of Regressions
	π^{e}_{t+1}
H	-0.28 (-1.07)
R	-0.51 (-1.48)
CBM	-0.61 (-1.70)

t values are given in parentheses below coefficients, critical value is 1.96 at the %5 significance level.

6. Testing Adaptive Expectations Hypothesis

One of the most frequently encountered models of inflationary expectations is the adaptive model. In each period an individual calculates expected inflation by adding ß of his most recent error to his previous period's expectation(Maddala, 1988:340-47).

$\pi^{e}_{t+1} = \pi^{e}_{t} + \beta(\pi_t - \pi^{e}_{t})$

The adaptive coefficient, ß, is between 0 and 1, Expectations can be expressed as a simple distributed lag on past actual inflation, in which the weights decline exponentially.

The optimal ß coefficient can be chosen by using the alternative expected inflation series in the Cagan model. Each alternative series is generated by altering the ß coefficient systematically. At the end of treating processes we have decided that the coefficient of ß is equal to 0.03 making the residuals sum of squared minimum and the determination coefficient maximum. In all the treatments, "monetary base" and "reserve money" have been used as money balances. The results were almost the same for the two money balances series. Let us illustrate this process step by step:

$$\pi^e_1 = \pi_1 * b$$
$$\pi^e_2 = \pi^e_1 + \beta(\pi_1 - \pi^e_1)$$
$$\pi^e_3 = \pi^e_2 + \beta(\pi_2 - \pi^e_2)$$
$$\cdot \quad \cdot \quad \cdot$$
$$\pi^e_N = \pi^e_{N-1} + \beta(\pi_{N-1} - \pi^e_{N-1})$$

where b is a constant between 0 and 1.

After generating the expected inflation rates in accordance with the selected ß, the series is used in Cagan model as an explanatory variable π_t^e in addition to monthly real income. Cagan money demand in semi-logarithmic form is written as

$$Ln(M/P) = \alpha_0 + \alpha^e_t + \gamma Ln(y_t) + u_t$$

where M=stock of money balances, P_t=level of prices, y_t=income and u_t is random error term. The OLS method is applied to estimate regression coefficients and the results are given in Table 10. Here π^e_t and y_t are taken to be $\Delta Ln(P_t)$ and Real GDP, and P_t denotes CPI. The regression coefficient of the expected inflation rate is the parameter of interest which corresponds to the semi-elasticity of the real money demand. This parameter is obtained for various monetary aggregates such as reserve money, base money and Central Bank Money. Semi-elasticity value for base money (H) is slightly different from the other two.

The diagnostic statistics included in OLS regression results are for testing the following hypotheses: Residual serial correlation (RSC), functional form mis-specification, normality of residuals and heteroscedasticity. Lagrange Multiplier(LM) statistics are computed for each of these hypotheses. The LM statistic is asymptotically distributed as a χ^2 variate. The LM statistics are reported in the diagnostic tests columns in Table 11.

TABLE 10.A Ordinary Least Square Estimates of the Cagan Money Demand Model Under Adaptive Expectations

Alternative Real Money Balances	Coefficients of Explanatory Variables		
	$\hat{\alpha}_0$	$\hat{\alpha}$	γ
H	3.61 (9.97)	-16.81 (-13.4)	0.0556 (2.29)
R	4.18 (12.35)	-24.38 (-20.8)	0.032 (1.43)
CBM	5.74 (11.58)	-20.75 (-12.1)	-0.064 (-1.95)

TABLE 10.B Diagnostic Tests of and Indicators of the Cagan Money Demand Model Under Adaptive Expectations

Diagnostic Tests				Indicators	
RCS test $\chi^2(12)$	RESET $\chi^2(1)$	Bera-Jarque $\chi^2(2)$	Hetero-scadas LM $\chi^2(1)$	R^2	RSS
97.62	11.59	0.08	28.68	0.61	0.600
97.50	33.81	4.68	30.11	0.80	0.524
112.71	6.80	6.76	0.17	0.62	1.123

132 observations are used for estimation from sample 1982.01 to 1992.12 . "t" ratios are given in brackets. The 5% critical values for test statistics are 1.96 for the "t" statistics, 23.3 for the serial correlation LM (Lagrange Multiplier) test ($\chi^2(12)$), 5.02 for the heteroscedasticity and Ramsey's RESET tests, 7.38 for Bera and Jarque's normality test ($\chi^2(2)$).

It is possible to test the hypothesis that the residuals, u_t, are serially uncorrelated against the alternative hypothesis that they are autocorrelated of order p. RESET test is also reported as a diagnostic test for testing functional form mis-specification. The test statistic of Bera and Jarque is given for testing normality assumption. Test for heteroscedasticity is also reported.

LM test for RSC shows that residuals of each model have a significant serial correlation. At the same time, functional forms of each model have a mis-specification according to RESET test. Normality of residuals are tested by using Bera-Jarque test. The null hypothesis of normality can not be rejected for the residuals of the models. Finally, money balances series except CBM indicates a heteroscadasticity.

Because of the distortions in the assumptions underlying the OLS method, it is not possible to interpret and use these results. But we can say that there are significant relationships between inflation and money balances. Correlation coefficients show that this relationships are sufficiently strong.

After the analysis of the autocorrelations of residuals for alternative models, it can be shown that the autocorrelation coefficients at first and second lags are significant. In order to relax the assumption of the serial correlation, we consider the estimation procedures with autocorrelated residuals. The estimation of the regression model with autocorrelated errors have been done by Gauss-Newton iterative method.

Here, we also take $\pi_t=\Delta Ln(P_t)$ and y_t=Ln(Real GDP) as we have done in the OLS method. As Table 11 reveals, the estimated coefficients of the expected inflation series which is used money demand function for H (base money) and R (reserve money) are remarkably close to each other. In the models that adopted H and CBM monetary base definitions, the explanatory power of income variable has been found to insignificant and hence it has not been included to the model. For the model related to R, although the 't value' of income variable is below the critical value at 95% confidence level, income is included to the model by considering it significant at 97%. The money demand equation for each money balances are given as:

$$Ln(H_t) = 4.97 - 28.89\Delta Ln(P^e_t) + u_t$$
$$u_t = 0.775u_{t-1} + 0.215u_{t-2}$$

$$Ln(R_t) = 4.38 - 28.84\Delta Ln(P^e_t) + 0.029Ln(GDP_t) + v_t$$
$$v_t = 0.665v_{t-1} + 0.219v_{t-2}$$

$$Ln(CBM_t) = 4.88 - 25.20\Delta Ln(P^e_t) + w_t$$
$$w_t = 0.782w_{t-1} + 0.167w_{t-2}$$

Here, the variables u_t, v_t, w_t are second order stationary autoregressive processes.

TABLE 11. Autoregressive Least Square Estimates of the Cagan Money Demand Model Under Adaptive Expectations

Alternative Money Balances	Coefficients of Explanatory Variables			Autoregressive Error Specification	
	Constant	$\hat{\alpha}^e_t$	y_t	ϕ_1	ϕ_2
H	4.97 (18.73)	-28.89 (-5.83)	-	0.755 (8.66)	0.215 (2.37)
R	4.38 (13.13)	-28.84 (-8.71)	0.029 (1.57)	0.665 (7.83)	0.219 (2.58)
CBM	4.88 (31.50)	-25.20 (-6.04)	-	0.782 (8.88)	0.167 (1.89)

t values are given in parentheses below coefficients, critical value is 1.96 at the 5% significance level.

The results of the diagnostic tests on the residuals of the models are demonstrated in Table 12. As it can be seen from the table the Box-Pierce statistic which shows the validity of the models has been calculated with 16 lags and the validity of the models can not be rejected. The LM test which is carried out to test significance of the autocorrelations in residuals demonstrated that the autocorrelations are not significant. The ARCH test shows that the Autoregressive Conditional Heteroscedasticity effect is not significant in all models. According to the results of the normality test the residuals are normally distributed for the models with H and CBM. For the model with R the hypothesis of the normality of residuals can not be rejected at 99% level. Therefore, it is founded out that there is no heteroscedasticity in the residuals of all the models except the model with H. Furthermore, a mis-specification in functional form of the model with H is involved whereas for the other two models no mis-specification is detected.

TABLE 12. The Results of the Diagnostic Tests on the Residuals of the Models

Real Money Balances	Diagnostic Test					
	Box-Pierce $\chi^2(16)$	Serial Corr. LM Test $\chi^2(12)$	ARCH LM Test $\chi^2(7)$	Bera-Jarque's Normal. Test $\chi^2(2)$	Hetero-scedastic Test F(2,127)	Functional Form White's Test F(2,127)
H	13.05	10.15	9.40	3.90	5.01	5.01
R	17.90	15.05	9.43	7.92	2.11	2.15
CBM	20.04	14.30	2.75	5.18	2.51	2.51

132 observations are used for estimation from the sample 1982.01 to 1992.12 . "t" ratios are given in brackets. The 5% critical values at the 5% level: 28.8 for the Box-Pierce, 23.3 for the serial correlation, 16 for the ARCH test, 7.38 for Bera and Jarque's normality test, 3.00 for the heteroscedasticity and functional form tests.

TABLE 13. The Likelihood Ratio Test of Significance Between Johansen Cointegration and Autoregressive Least Squares (ALS) Estimates of the Semi-Elasticity Coefficients

Alternative Real Money Balances	Johansen Cointegration Coefficient $\hat{\alpha}_j$	ALS Estimate of Semi-Elasticity Coefficient From Adaptive Expected Inflation $\hat{\alpha}_a$	Likelihood Ratio Test H0 : $\hat{\alpha}_j = \pi_a$ H_a : $\hat{\alpha}_j \neq \pi_a$
H	-26.28	-28.89	0.39
R	-27.67	-32.67	1.32
CBM	-24.91	-25.20	1.44

At 95% significance level the critical value of LR statistic, $\chi^2(2)$, *is 7.38.*

7. Conclusion

In this study we have accomplished two main jobs. Firstly, we have estimated the cointegration vector with a view to determine and specify the linear form of the long-run relation between demand for money and inflation. We have compared the inverse of the cointegration parameter with the average inflation rate and showed that there has been no significant difference between them. Secondly, we have used the OLS and ALS methods to build the semi-logarithmic linear model of money demand. This time, the parameters related to the inflation variables in the models have been compared using the LR test. The results of LR test which are given in Table 13 show that there is no significant difference among the coefficients. Hence, it follows that there is no difference among the parameters that are estimated by both ways and the results are almost the same. It seems satisfactory for us to have estimated parameters and other results be conferred by the different techniques. So, we regard the results that are reached throughout the analysis are substantially reliable and use them with a high confidence in seigniorage analyses.

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