ERCİYES AKADEMİ, 2025, 39(2), 452-467
https://doi.org/10.48070/erciyesakademi.1573483

EARLY POTENTIAL TRACES OF ALGEBRA IN JĀBIR IBN ḤAYYĀN’S WRITINGS*

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Öz

“Do not be angry, O my brother, if you find a discourse concerning religion in the middle of a discourse on alchemy without the latter having been completed; or if you find a discourse on alchemy after a discourse on religion before the principles of the latter have been fully established!” (Jābir Ibn Ḥayyān, qtd. in Haq, 1994, p. xxii).

This paper sheds light on the possibility that Jābir Ibn Ḥayyān (721-813), the famous alchemist, might have been the first intellectual in the Islamic world to practice algebra, long time before the foundations of this discipline were officially founded by al-Khwārizmī (780-850). It is reasonable to think that the early development of algebra might have sprung while early Muslims were studying the Arabic letters, the way Jābir did. Jābir believed in the perfection of the Arabic language, as a vehicle that would lead its practitioner to truth, after all, it is the unique language of the Qur’ān. And language, in the full sense of the word, was given by Allāh to Adam while he was in Paradise, as we are told in the Qur’ān. Consequently, the development of this field simultaneously with the flourish of religion is not a coincidence, for instance, Boyer and Merkelbach (2011) stated that both religion and mathematics developed at the same time, Pythagoras lived at the time of Buddha and Lao Tzu.
Keywords: Jabir Ibn Hayyan, Algebra, Alchemy, Al-Khwarizmi, Islamic Golden Age.

CÂBİR B. HAYYÂN’IN YAZILARINDA CEBİRİN ERKEN DÖNEM POTANSİYEL İZLERİ

Öz

”Ey kardeşim, eğer simya üzerine bir konuşmanın ortasında din hakkında bir söylem bulursan ve simya konusu tamamlanmadan geçiş yapılmışsa, ya da din hakkında bir söylem tam anlamıyla açıklanmadan simya üzerine bir söylem bulursan, sakın öfkelenme!” (Jābir Ibn Ḥayyān, aktaran Haq, 1994, syf. xxii).

Bu makale, ünlü simyacı Câbir bin Hayyân'ın (721-813), İslam dünyasında, resmi olarak el-Hârizmî (780-850) tarafından temellendirilmeden çok önce cebirle uğraşan ilk entelektüel olabileceği olasılığı hususuna değinmektedir. Cebirin ilk gelişiminin, erken dönem Müslümanların Arap harflerini incelemeleri sırasında ortaya çıkmış olabileceğini düşünmek pek de mantık dışı olarak değerlendirilemez, tıpkı Câbir’in yaptığı gibi. Câbir, Arap dilinin mükemmelliğine inanıyordu; sonuçta bu dil, Kur’ân’ın eşsiz

* This paper was presented as an oral presentation in the workshop named “Knowledge, Faith/Islam, Identity - Knowledge History as an Inclusive Culture of Memory?” held at the Faculty of Cultural and Social Sciences, Turkish-German University, Istanbul, Turkey, on 06.11.2022, but was not published.

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Makale Geliş Tarihi: 25.10.2024, Makale Kabul Tarihi: 26.03.2025

diliydi ve kullanıcılarını gerçeğe ulaştıracak bir araç olarak görülüyordu. Ve bu dil, kelimenin tam anlamıyla, Allah tarafından Âdem'e cennette iken verilmiştir, Kur'ân'da bize bildirildiği gibi. Dolayısıyla, bu alanın gelişiminin dinin yükselişiyle eş zamanlı olması tesadüf değildir; örneğin, Boyer and Merkelbach (2011), hem dinin hem de matematiğin aynı zamanda geliştiğini, Pisagor'un Buda ve Lao Tzu zamanında yaşadığını belirtmiştir.
Anahtar Kelimeler: Câbir bin Hayyân, Cebir, Simya, El-Hârizmî, İslam'ın Altın Çağı.

Introduction

Al-Khwārizmī (780-850) is one of the greatest intellectuals in the Islamic as well as in the universal history of science. One of his various original contributions was founding the very cornerstone of algebra as an independent science. Needless to say that some of the ideas used in his masterpiece *Al-kitāb al-mukhtaṣar fī hisāb al-jabr wal-muqābala* [*The Abbreviated Book on Calculation by Completion and Balancing*] have some ancient Mesopotamian, Egyptian, Indian, and Greek roots.[1] Nevertheless, the way al-Khwārizmī brought many different notions—after all, he never claimed that it was all his—in such a perfect synthesis and with such plain and articulated manner is spellbinding. Actually, in the very introduction of his book, al-Khwārizmī mentioned how he was encouraged by the caliph al-Maʾmūn (786-833) to write a simplified arithmetic book, where people would have an easy access to using it in solving their daily problems, specifically those which are related to inheritance, legacies, partition, law-suits, trade, measuring the lands, and the like (Al-Khwārizmī, 1937, p. 15-16).

However, many old scientists and historians from the ninth century up to the later centuries including Ibn Khaldūn (1332-1406) wrote that the first one to write about algebra was al-Khwārizmī. Likewise, al-Khwārizmī was also given the credit of being the first intellectual to introduce the Indian Numbers to the Islamic world, and from which it went later to the West. Nevertheless, there are some historical evidences showing earlier practices of algebra as well as the Indian Numbers and the zero in the Islamic world. This paper will focus on those earlier times before the emergence of al-Khwārizmī (780-850). This paper will mainly depend on Fuat Sezgin's (1924-2018) writings, since he is the first to bring serious proofs regarding the originality of Jābir's legacy. As a matter of fact, Sezgin asserted that some early scientists have written some mathematical works but their books were lost (Sezgin, 2002, p. 271). The best two examples are Jābir Ibn Ḥayyān (721-813) and his contemporary al-Farāhīdī (718-786). There are strong clues that al-Farāhīdī had used some algebraic equations in creating his dictionary *Kitāb al-ʻAyn* [*Book of the ʿAyn*] (Rashed, 2015, p. 150-156). Al-Farāhīdī had apparently talked about that in one of his now lost mathematical books. Thus, such lost books could have later indirectly encouraged al-Khwārizmī to create his unified formulas of algebra. Even though, it is not an easy task to prove this argument because of the lost chain of mathematics, still shedding light on such a possibility is worthwhile.

At all events, the purpose of this investigation is not simply to contradict the typical mainstream convictions but to demonstrate the importance of alchemy in terms of its revolutionary impact on the different scientific disciplines. Especially when we know that the intellectual movement that paved the

[1] This view is widely accepted by historians of science like Boyer (Boyar, 2011, p. 208) and Rashed (Rashed, 2015, p. 109-113). Yet, some other scholars—like Gands—affirmed that al-Khwārizmī's algebra is much closer to the Babylonian algebra, and way too far from the Greek geometrical equations (Sayılı, 1985, p. 133).

way towards what is now called the 'Islamic Golden Age' (roughly 8th-14th centuries CE), had its very roots in alchemy.[2] In his *Al-Fihrist* [*Index*] Ibn al-Nadīm (ca. 932-990) mentioned that the translation movement started when Khālid Ibn Yazīd (ca. 668-704) ordered scholars to translate the ancient books of alchemy into Arabic (Ibn al-Nadīm, n.d., p. 243). This simply means that alchemy was somewhat a springhead of knowledge development at that time, which deserves more study in relation to its effect on other branches of science. Another significant reason of searching for the algebraic traces in the Jābir's lab, so to speak, is that the Islamic method of practicing science is directly connected with the religious and spiritual tendency of the Eastern people. And if Aristotle represents the cornerstone of the Western way of logical and secular thinking, we can likewise consider a polymath intellectual like Jābir Ibn Ḥayyān (whose corpus of writings, just like Aristotle's, covers many different areas) as the basic place from which to start tracing back to the roots of the relevant scientific theories which flourished in the Islamic world. So, it was no a coincidence that Sezgin considered Jābir as 'the Muslim Aristotle' when he talked about him. Jābir was a prolific author, who made contributions in several sciences. On the other hand, Jābir belonged to the early pathfinders of the Islamic scientific movement. And it is important to remember that Jābir's significant theories, which will be tackled later, are directly connected to language and numbers.[3]

In fact, the pattern of the interaction between language and mathematics can be seen in the Mesopotamian area from the ancient Babylonian times.[4] For instance, we know—based on studying the existing Mesopotamian equations—that the Mesopotamians didn't use the negative form in their algebraic calculations and there is no record that shows otherwise. But just because we don't have concrete evidence, doesn't mean that we should abandon the research, since the scarcity of evidence doesn't necessarily equate to an absence of evidence. In such cases we need to enlarge the scope of the research. A good example is the discovery of Koch-Westenholz, who had examined and studied the tablets and reports of the Assyrian and Babylonian astologers and magicians.[5] Her discovery forces us to think again about the possible existence of some algebraic formulas used in the Mesopotamians' speech, so to speak. Koch-Westenholz explained that:

"[a] simple rule that is common to all kinds of Babylonian divination is of almost mathematical rigor: within the same omen, a good sign combined with a good sign has a good prediction; good combined with bad means bad; bad combined with bad means good. Expressed algebraically, the rule is also familiar to us: + + = +; + — = —; — — = +. An often quoted example of this rule is found in the astrological texts: if a well-portending planet is bright: favorable (+ + = +); if it is faint: unfavorable (+ — = —), if an ill-portending planet is bright: unfavorable (— + = —); if it is faint: favorable (— — = +)" (Koch-Westenholz, 1995, p. 11).

[2] The interest in alchemy can obviously be regarded as the flame of the translation movement as well as the scientific revolution in the Islamic world. And by the same token, alchemy and the pursuit of the Philosopher's Stone created huge interest in Arabic science in the Western world during the so-called Renaissance. Frances Yates noticed such similar impact of alchemy on the West. She affirmed that the Italian Renaissance took place after the Arabic Hermetic literature arrived to Europe, which inspired people to engage in various branches of science (Yates, 1979, p. 191).

[3] Not to forget that it is a quite well-known fact that Jābir's interest in alchemy and spirituality had encouraged him to investigate and consequently to master many scientific fields.

[4] In fact, similar relation can be seen in different areas as well. Boyer mentioned that the development of language was essential to the rise of abstract mathematical thinking (Boyer, 2011, p. 5).

[5] Koch-Westenholz discussed that issue in her *Mesopotamian Astrology*. But those reports are also available in Thompson's *The Reports of the Magicians and Astrologers*.

In short, such a formula cannot be found in the old Mesopotamian algebra but it does in another branch, that is, astrology.

Even though the former hypothesis may not look so convincing, still the point to be reflected on here is the possibility that an idea could transfer from one field to another, especially when there is close relationship seen in these patterns. Actually, the reason of starting with such historical context is twofold. Firstly, to demonstrate the relation between language and mathematics which is what will be seen below in Jābir's system. Secondly, this example reminds us of the cultural heritage embedded in the area in which mathematics was developed later, that is to say, Mesopotamia.[6]

Jābir's Theory of Balance[7]

Jābir Ibn Ḥayyān is a well-known alchemist, though his books contain a mix of different branches of knowledge.[8] By and large, numbers were always important in Jābir's philosophical paradigm. According to him, everything is made of numbers but those numbers are hiding behind the names of the things that exist within the physical world. So, in the event that we follow back the initial root of a name and after we find its numerical value, we'll be able to know the nature of that particular thing. After all, the Qur'ān clearly stated that "He [Allāh] taught Adam the names" (*Qur'ān* 2:31-32). And Allāh's teaching is flawless therefore the connection between the name and the object must not have been made randomly. In spite of the fact that language was originally ideal, yet in the process of development, some of its words started to change (Jābir, 1935, p. 133-145). Subsequently, Jābir made a kind of mathematical and alphabetical system and by using this system correctly each name would arguably once again have a direct relation to the thing's essence.

Jābir hoped that he was able to find the quality of the object whose name he was exploring. He was confident that 28 (the number of letters in the Arabic alphabet) was perfect, which was no simple coincidence. He classified the Arabic letters into 4 categories, which corresponded with the then predominant theory of the four components. This led to 4 categories (hot, cold, dry, wet), each containing 7 letters, two sacred numbers, which are exceptionally critical within the Qur'ān and which Jābir must have seen as another confirmation of the divine nature of the Arabic language. To realize this objective, Jābir embraced the Aristotelian hypothesis on the composition of matter, where everything is constructed from the components earth, water, air, and fire. So, this means that, when the four elementary qualities

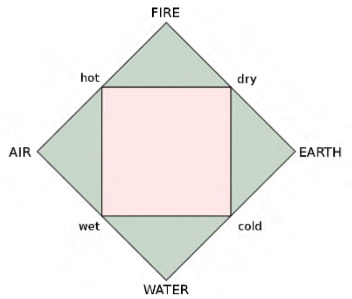
[6] Sezgin talked about the possible existence of some old Babylonian advanced algebra, which was eventually lost (Sezgin, 2002, p. 274). Neugebauer (1899-1990) is also of the opinion that "a continuous tradition must have existed, connecting Mesopotamian mathematics of the Hellenistic period with contemporary Semitic (Aramaic) and Greek writers and finally with the Hindu and Islamic mathematicians" (qtd. in Sezgin, 2002, p. 30-31).

[7] Most of the ideas discussed in this topic (including Figures 1, 2, and 4, which are also taken from there) were originally discussed, but in more detail, in my paper presented to The 4th Icon International Students' Conference at Johannes Gutenberg University, Mainz, Germany, "Looking through the Lens of Language: How Early Muslim Intellectuals Tried to Reach Hidden Truths by Examining the Arabic Language" (Al-Khaffaf, 2023, p. 29-42).

[8] According to Ibn al-Nadīm, Jābir wrote in various genres and that even al-Rāzī used to praise Jābir and call him "our teacher" (Ibn al-Nadīm, n.d., p. 355). In fact, the list of Jābir's books covered 3 pages of *al-Fihrist*. At the end of the paragraph we see how Jābir boasted of having written in almost every branch of science (Ibn al-Nadīm, n.d., p. 356-358). And even though alchemy is the first science to draw Muslims' attention during the Islamic Golden Age, we see that the section of alchemists was put at the very end of *al-Fihrist*, apparently for the author to freely elaborate on this topic. Moreover, in the very last paragraph of the book, Ibn al-Nadīm told us about his friend, an alchemist who said that he had succeeded in creating gold. Ibn al-Nadīm wrote that there are many books about alchemy that it is impossible to mention all of them, which implies that many of them were lost. All these hints assert the importance of alchemy.

dryness, coldness, humidity, and heat interact with a substance, they shape the qualities dry, wet, cold, and hot (see Fig. 1). For instance, fire is hot and dry. Air is hot and wet. Water is cold and wet. And earth is cold and dry (qtd. in Principe, 2013, p. 37).

Fig. 1. The four Aristotelian Primary Qualities and the Four Elements



Jābir utilized this theory in his research of letters. His علم الميزان [science of balance] is based on the thought that the properties of things, including names, can be weighed, as they are made according to particular numerical proportions. His ميزان الحروف [the balance of the letters] shows the mathematical dimension and arrangement of things. According to his adaptation, each of the Arabic letters is either hot, cold, dry, or wet. Moreover, each letter encompasses a particular weight based on its rank within the word. Jābir has given particular weights for each letter. Jābir made a chart with the four qualities organized in four groups and the seven grades of intensity arranged in seven lines, giving a table with the twenty-eight letters. In this manner, he assigned a quality and a degree to each letter (see Fig. 2). At this point, he created three more tables like that with all the weights of letters increasing separately along the ratio 1:3:5:8. When researching a word, he used the first chart (Fig. 2) for the first letter of the word, the second chart for the second letter, and so on (Mahmud, 2001, p. 120-135).

Fig. 2. The first of Jābir’s four detailed charts for finding the precise numerical qualities of letters. In this chart, the weight of each letter as well as its quality is given (Image taken from Jābir, 2006, p. 200; translated by the author)

المرتبة الأولى في الأربعة **First Rank of the Four**	**مرتبة** **Level**	الحرارة Heat	أ ā	1 dirham and daniq	البرودة Cold	ب b	1 dirham and daniq	اليبوسة Dry	ج j	1 dirham and daniq	الرطوبة Wet	د d	1 dirham and daniq
	درجة **Degree (as a measure)**		ه h	Half dirham		و w	Half dirham		ز z	Half dirham		ح ḥ	Half dirham
	دقيقة **Minute (as a measure)**		ط ṭ	2 and half daniqs		ي y	2 and half daniqs		ك k	2 and half daniqs		ل l	2 and half daniqs
	ثانية **Second (as a measure)**		م m	2 daniqs		ن n	2 daniqs		س s	2 daniqs		ع ʿ	2 daniqs
	ثالثة **Third (Grade)**		ف f	1 daniq and half		ص ṣ	1 daniq and half		ق q	1 daniq and half		ر r	1 daniq and half
	رابعة **Fourth (Grade)**		ش sh	1 daniq (1/6 of dirham)		ت t	1daniq (1/6 of dirham)		ث th	1 daniq (1/6 of dirham)		خ kh	1 daniq (1/6 of dirham)
	خامسة **Fifth (Grade)**		ذ dh	1 carat (half daniq)		ض ḍ	1 carat (half daniq)		ظ ẓ	1 carat (half daniq)		غ gh	1 carat (half daniq)

Jābir declared that everything is made of the four components which have diverse qualities, hence, by what he called ميزان (balance) we are able to know the precise sum of each qualities. For illustration, if we want to investigate the word أسرب (lead) which is composed with four letters (ا، س، ر، ب), we put each letter in one of the four charts. The first letter ا is hot within the most elevated review among the seven hot letters and its weight is 7 daniqs, that is, one dirham and one daniq. The second letter س which is in the second rank is dry and its weight is one dirham, that is, 6 daniqs. The third letter ر which is in the third rank is wet and its weight is one dirham and a quarter. The final letter ب which is in the fourth rank is cold and its weight is 9 dirhams and a third. In this manner, the entire weight of the four letters of the word أسرب is 12 dirhams and three-quarters.[9] In like manner, on the off chance that we take a mass of lead whose total weight is 12 dirhams and three-quarters, it will have the 4 mentioned qualities of dryness, coldness, humidity, and heat. And this ratio would be found in each mass of lead (Jābir, 1935, p. 185).

Jābir's Fascination with the Magical Square

Indeed, Jābir was obsessed with numbers and balance. His obsession of putting each thing in its correct place can be seen in the inspiration he conceived in the magical square (Jābir, 2006, p. 191). Here, the 9 base numbers are organized in a way that each three of the numbers included, horizontally,

[9] A shortcoming of this theory is the existence of Arabic words that consist of more than four letters (Kâhya, 1995, p. 160).

vertically or diagonally, come up to the same value of 15 (see Fig. 3). Jābir believed that numbers established within this square represented the foundation of all matter (Anawati, 2005, p. 1107). The numbers 4, 9, 2, 7, and 6 (the Gnomon of the magic square, which can be seen highlighted in Fig. 4) may be added up to 28—the number of letters in the Arabic alphabet—leaving the numbers 3, 5, 8, and 1 (found inside the foot, cleared out corner cells), which are the ratio of the weight of the letters based on their rank within the word, as clarified prior. These numbers were considered the seed of creation by Jābir and his adherents. According to Jābir's method, the numbers were related with the four components: 1=fire, 3=earth, 5=water, and 8=air. Thus, 5 numbers of this magical square represent the number of the Arabic alphabet (28). On the other hand, the remaining four numbers represent the added increase of the weight of the letters according to their position in the word (1, 3, 5, 8). Additionally, the amount of these four numbers (17) also represents the number of the shapes in the early Arabic script before the addition of dots.[10] That means that the strict number of the Arabic alphabet and the weights of letters were not made arbitrary. Everything was made in a perfect form and by putting everything in its right place, it will regain its divine order once again. In short, according to Jābir "what is written refers to what is spoken, and what is spoken refers to what is in the mind, and likewise what is in the mind refers to what is in the real things" (Mahmud, 2001, p. 125, my translation).[11] Thus, such a deep examination of language and numbers in the Jābirian ideas discussed above, tells us a lot about his revolutionary advanced approach to mathematics as well as his extreme interest in balance which is the backbone of algebra itself.

Fig. 3. The Magical Square. As seen below, the total amount of the numbers when added from all directions makes the same quantity (15)

[10] In other words, these numbers (1, 3, 5, 8) which represent the essence of Jābir's theory equal 17, an important number to Jābir because it is by turn the number of the original unique shapes of the Arabic script before the addition of dots (see Fig. 5). In this context we have to remember that based on some unique Arabic manuscript attributed to the famous alchemist Zosimos (third-fourth century), there is a strange relevant idea. According to a specific prophecy quoted by the ancient alchemist Mary: the truth—about the natural world—will become clear in the time of Arabs "sons of Ismail" (Zosimos, 2007, p. 10). It would not be so illogical to think that the ancient alchemists and magicians have reached this conclusion based on the fact that the two pivotal parts of the magical square perfectly represent the number of the basic shapes of the Arabic alphabet (1, 3, 5, 8=17) the way they are written without dots as well as the number of the Arabic letters (4, 9, 2, 7, 6=28) the way they are uttered (see Fig. 6).

[11] "الكتابة دالة على ما في اللفظ المنطوق، واللفظ دال على ما في الفكر، وما في الفكر دال على ماهية الأشياء" (محمود، 2001، ص 125).

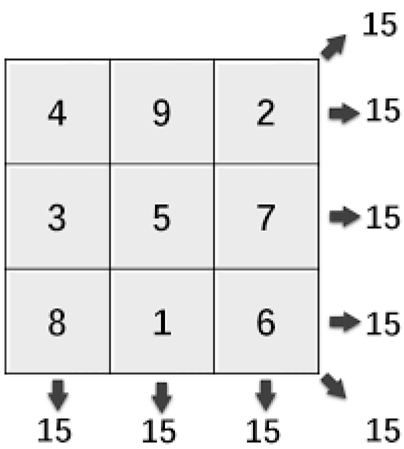


Fig. 4. The Magical Square explained in connection with Arabic. The numbers checked in yellow make 28—the number of letters in the Arabic alphabet—leaving the numbers 3, 5, 8, and 1—the ratio of increment for the weight of the letters, concurring to Jābir's hypothesis (Picture taken from Mysterious Writings, 2012, n.p.)

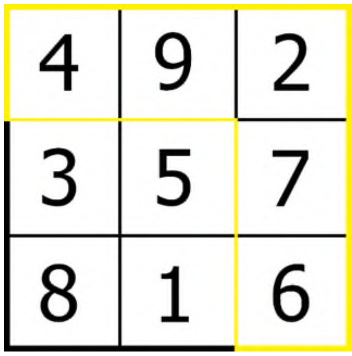


Fig. 5. The 17 unique shapes in the early Arabic script before the addition of dots

1. ا – Represented ا (’alif).

2. ٮ – Represented ب (bā’), ت (tā’), ث (thā’).

3. ح – Represented ج (jīm), ح (ḥā’), خ (khā’).

4. د – Represented د (dāl), ذ (dhāl).

5. ر – Represented ر (rā’), ز (zāy).

6. س – Represented س (sīn), ش (shīn).

7. ص – Represented ص (ṣād), ض (ḍād).

8. ط – Represented ط (ṭā’), ظ (ẓā’).

9. ع – Represented ع (‘ayn), غ (ghayn).

10. ڡ – Represented ف (fā’), ق (qāf).

11. ک – Represented ك (kāf).

12. ل – Represented ل (lām).

13. م – Represented م (mīm).

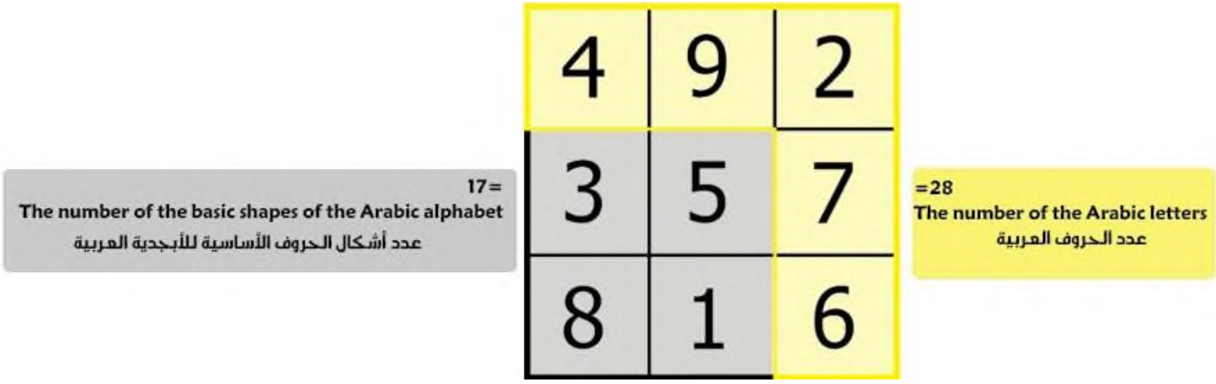
14. ں – Represented ن (nūn).

15. ه – Represented ه (hā’).

16. و – Represented و (wāw).

17. ى – Represented ي (yā’) and, sometimes in certain regions, ى (’alif maqṣūrah).

Fig. 6. A more detailed explanation of the Magic Square in terms of its interesting connection to Arabic, where its two pivotal parts perfectly represent both the number of basic shapes of the Arabic alphabet and the total number of Arabic letters



Algebraic traces in Jābir's Writings

In the previous two topics we had some examples regarding the dominant connection between letters and numbers in Jābir's paradigm but now we will have a look at some indications that imply his algebraic practices. Indeed, Jābir wrote several books about ميزان (balance) because he was fixated with re-establishing the original state of balance, either within the physical, scientific, or indeed within the chemical substances.[12] Actually, this can be compared to algebra's goal. We know today the meaning of al-Khwārizmī's terms (*Al-Jabr wal-Muqābala*) in the title of his book, where "al-Jabr" apparently means completion, reunion, or restoration of broken parts whereas "al-Muqābala" means balancing (Boyer & Merkelbach, 2011, p. 207). Needless to say that al-Khwārizmī's aim was to solve linear and quadratic equations by removing the negatives, using a process of balancing both sides of an equation. So, Jābir's surviving ideas are pointing to his tendency to create such a balance. In *Kashf al-ẓunūn* [*The Removal of Doubts*] where Çelebi (1609-1657) talked about alchemy he mentioned an old couplet which was also cited in much older books, criticizing Jābir: "O you, who have deceived many generations, you are but a breaker, he lied who named you bone-setter" (Çelebi, 1971, p. 133, my translation).[13] This interesting critique shows that Jābir was attacked during and after his life, consequently, it makes sense that many of his ideas were not given credit and it also confirms the credibility of the historical reports that say that Jābir (the one who fixes) was named so because he was the one who fixed the sciences. In this sense, even the Arabic name of جبر (Jabr=algebra) might have derived from the name of جابر (Jābir).

Moreover, the title of al-Khwārizmī's book contains the word "al-Mukhtaṣar" (abbreviation) and we should also keep in mind that according to what he mentioned in his introduction regarding how the caliph encouraged him to write the book, it implies the existence of some older comprehensive book of algebra. So, just because al-Khwārizmī's is the oldest available book about algebra doesn't necessary mean that it was the first work on the subject. Furthermore, al-Khwārizmī didn't even discuss the

[12] Jābir wrote many books with the term balance in the title, such as: كتاب الميزان (*Book of Balance*), كتاب الميزان الصغير (*Little Book of Balance*), كتاب ترتيب الأوزان (*Book of the Arrangement of the Scales*), كتاب الموازين الصغيرة (*Book of Small Scales*), موازين الحق (*Correct Balances*), كتاب المقابلة (*Book of Balance*), etc.

[13] "هذا الذي بمقاله غرّ الأوائل والأواخر / ما أنت الا كاسر كذب الذي سماك جابر" (جلبي، 1971، ص 133).

meaning of the two new terms "al-Jabr wal-Muqābala" that lie in the title of his book, which is not typical to a Muslim scientist writing about a new field of science at the time. Hence, if al-Khwārizmī's book about algebra was the first written book in Arabic, that would have been somehow indicated in the book. Al-Khwārizmī seems to have written his book without any claims of originality of any kind (Sayılı, 1985, p. 94). Knowing that the oldest Arabic use of the terms: al-Jabr (algebra) and al-Muqābala (balance) are found in Jābir's writings, even though, in Jābir's relevant surviving books they had different meaning from al-Khwārizmī's (Sezgin, 2002, p. 31).

Additionally, there are surviving fragments of some algebraic book attributed to ʿAbd al-Hamīd Ibn Turk, who based on few reports, had published his book at around the same time or even potentially before al-Khwārizmī published his book.[14] This view is not so strong, because this argument was already challenged by al-Khwārizmī's contemporaries few years after his death. So, a few years after al-Khwārizmī, a man appeared claiming that Ibn Turk was the first to write about algebra. But this claim was harshly criticized, for instance, according to Çelebi's *Kashf al-ẓunūn*, in a book written by Abū Kāmil Shujāʿ Ibn Aslam (850-930) he proved the priority and antecedence of al-Khwārizmī and refuted the argument of Abū Barza (Ibn Turk's descendant). Yet, his book in which he discussed that issue "is probably lost" (Sayılı, 1985, p. 89-90). In short, Abū Kāmil's lost book became the proof that al-Khwārizmī was the first who wrote about algebra. And apparently being himself the best authority on algebra—after al-Khwārizmī—he (Abū Kāmil) must have similarly shut all other claims. So, it can be said that Abū Barza wasn't the one who started that claim, but it seems that there was an older discussion about the first who wrote about algebra. But as mentioned before, with Abū Kāmil's strong critique to such a claim, the discussion was silenced forever.

Indeed, Sayılı also shared a similar view. He remarked that from that time there was a rivalry in the matter of antecedence in algebra publication (Sayılı, 1985, p. 90).[15] Regarding this issue, it seems that Sezgin held the opinion that both of Ibn Turk and al-Khwārizmī must have used some ancient algebraic books (Sezgin, 2002, p. 30). At any rate, all what survived from Ibn Turk's works are just two incomplete manuscripts of his algebraic work,[16] which have some similarities with al-Khwārizmī's. In addition, we have a couple of references made by old historians about him. Except that, not much is known about his life (Sayılı, 1985, p. 87). But in light of that story, it would be more logical to trace back to the roots of algebra among Jābir's giant corpus, which is mostly not seriously examined by far. In short, the existent similarity between Ibn Turk's and al-Khwārizmī's work implies that algebra might have been much more developed in the Islamic world before that time.

Another interesting example is Jābir's كتاب الخواص [*The Book of Specific Properties*], in which he discussed the secrets of numbers according to Homer and in which he dwelt on how the number three

[14] Sayılı's publication in Türkiye raised some questions about the originality and importance of ʿAbd al-Hamīd Ibn Turk's work (*Al Jabr wa'l Muqābala*). Additionally, there is another work with the same title attributed to another contemporary of al-Khwārizmī, named Sanad ibn ʻAlī (see: Sezgin, 2002, p. 30).

[15] Al-Khwārizmī didn't speak in detail about the existence of special cases of the equation x2+c=bx. He apparently considered them to be sufficiently well-known, perhaps "because they were available in an earlier text" (Sayılı, 1985, p. 93). Unlike this humble study which somehow focuses on the philological indications, Sayılı's study was more in-depth employing algebraic technical comparison between Ibn Turk's and al-Khwārizmī's works. But due to the fact that Jābir's mathematical books were lost consequently such mathematical comparison is impossible.

[16] Only one chapter called "Logical Necessities in Mixed Equations" (which is on the solution of quadratic equations) has survived.

and four are the source of all numbers (Sezgin, 2002, p. 81-82). In this regard, it is no surprise that in 1900 Heinrich Suter gave Jābir the third place among the most important Muslim/Arabic contributors in the Islamic Mathematics and Astrology. In his *History of Arabic-Islamic Heritage* Sezgin repeatedly asserted that Jābir had mastery in mathematics (Sezgin, 2008, p. 162). For instance, in Jābir's كتاب التجميع [*Book of Gathering*], he talked about the function of zero among numbers, and he stated that he had mentioned it earlier in his astronomical books where he wrote about the Indian Numbers (Sezgin, 2008, p. 164). Besides using many algebraic terms, particularly in his كتلب التجميع [*Book of Gathering*] Jābir used a small circle as a symbol to the zero, whereas the Indians used a dot as a symbol to that number (Sezgin, 2002, p. 271). When Julius Ruska (1867-1949) studied Jābir's astrological notions, it was clear to him that Jābir's mathematical knowledge was so advanced. Likewise, Paul Kraus (1904-1944) asserted that Jābir's whole natural philosophy was built based on numbers. Nevertheless, as will be tackled soon, both Kraus and Ruska had different explanations to this interestingly early algebraic terminology found among Jābir's works.

As Sezgin remarked, Jābir often encouraged his readers to arm themselves with logic, arithmetic and geometry (Sezgin, 2002, p. 270). And based on Ibn al-Nadīm, Jābir had made an important zīj (astronomical book used for astronomical calculations) and he wrote commentaries about Ptolemy's and Euclid's works. And the implications found in Jabir's كتاب السموم [*Book of Poisons*] and كتاب السبعين [*The Book of Seventy*] tell us that Jābir knew very sophisticated mathematical theories (Sezgin, 2002, p. 10). Jābir also wrote a very long book regarding how to make an astrolabe, which contained a thousand complex matters but alas that book was also lost (Sezgin, 2002, p. 268).

Jābir's idea about the zero is significant. As was minutely discussed before, Jābir affirmed that everything in this world is made out of the four elements (fire, air, water and soil) and the source of those four elements is heat, cold, wet and dry. And that the latter four qualities can be understood with reason but they can never be seen with the eyes.[17] Thus, those four qualities have no higher roots/source, just like the zero, which can never be seen yet it has its essential function within numbers (Sezgin, 2002, p. 269). Jābir created a shift in mathematics, he changed the mystical secretive science of numbers into a rather strict mathematics (Sezgin, 2002, p. 272). Yet, finding references about the Indian Numbers and the zero in Jābir's writings, pushed Ruska and Kraus to conclude that given that these notions were not introduced to the Islamic world until the time of al-Khwārizmī, that is to say, the ninth century, that simply means that those books were written in a later period and were then attributed to Jābir (qtd. in Sezgin, 2002, p. 15). But this view was strongly challenged by Sezgin. According to him, Muslims knew about the Indian Numbers and about the zero from relatively early times, long before they reached al-Khwārizmī through India. Sezgin was in the opinion that Muslims could have received those ideas from the Persians who have taken them earlier from the Indians, not to forget that caliph al-Manṣūr's (ruled 754-775) astrologers were Persians (Sezgin, 2002, p. 15). Thus, many Indian books had arrived to the East through Syriac translations. Accordingly, the Indian Numbers were known in the Middle East as well as the Near East from the middle of the seventh century AD (Sezgin, 2002, p. 21-22). So, we are not required to take al-Khwārizmī as the first one to introduce the Indian Numbers, rather we should think about the

[17] In fact, Jābir's theory about the creation of the world is deeply meta-scientific. Jābir believed that everything was originally made from abstract ideas. And that it was then the qualities and souls came to existence, followed by the material elements, such as water and so on. Jābir related his theory to the Qur'ānic verses (such as 22:5, 18:109, 13:17) about water as the origin of the creation (Jābir, 2006, p. 108, 472, 540). In other words, he believed that his theory of creation is proven by the Qur'ān.

possibility of achieving that through the old Persians astrologers who had a relationship with the Indians (Sezgin, 2002, p. 270).

In order to pave the way towards such a possibility, Sezgin provided some ancient reports indicating that early Muslims had a good deal of mathematical knowledge. For instance, he highlighted that the famous religious figure, Sufyān al-Thawrī (715/716-778), was considered as an authority on mathematics (Sezgin, 2002, p. 17). Sezgin provided another report about an alchemical book attributed to Sufyān al-Thawrī, and another report claiming that he worked on natural science (Sezgin, 2002, p. 263). Actually, Sezgin tried to dig into a considerable depth in order to trace back the early Muslims' users of mathematics. He provided another report about the prophet's companion, 'Abd Allāh Ibn 'Abbās (619-687/688), who apparently had a good grasp on arithmetic (Sezgin, 2002, p. 26). According to Sezgin's hypothesis, Muslims' philosophical debates starting from the second half of the 8th century between the Atomists and the Muʿtazilites had helped developing many mathematical concepts, because religious intellectuals at the time worked on issues that were not connected to their main field such as thinking about the minimum number of atoms which could produce an entity (Sezgin, 2002, p. 31-33). Sezgin asserted that the algebraic term "square" was used by some early Muslim scholar when that scholar talked about a person who drew the foundations of the Ka'ba (Sezgin, 2002, p. 27). Also, Sezgin emphasized that the oldest name of a mathematical book that we ever know in the Islamic civilization is Jābir's تعاليم الهندسة [*Teachings of Geometry*] (Sezgin, 2002, p. 17).[18]

Another relevant case is that of al-Khalīl Ibn Aḥmad al-Farāhīdī (718-791), the famous language and literature scholar who systematized syntax and prosody. He is credited for creating the first surviving dictionary in the world كتاب العين [*Kitāb al-'Ayn*]. He was also a mathematician and he wrote a book on computation but unfortunately that book hasn't survived.[19] Rashed, the famous modern historian on mathematics also believes that Islamic algebra must have been practiced before al-Khwārizmī, that is to say, by al-Farāhīdī (Rashed, 2015, p. 150-156). Al-Farāhīdī asserted that the roots of all Arabic words can be put under 4 categories (Al-Farāhīdī, 2003, p. 35). Furthermore, in his other work, which was lost but some of its ideas were referred to by other scientists, he provided the number of all Arabic possible words: two-lettered words (756), three-lettered words (19.656), four-lettered words (491.400), and five-lettered words (11.793.600). Accordingly, the total number of the Arabic potential words is 12.305.412. Even though most of those words were neglected or have not even been used, nevertheless employing this discovery made it possible for him to write down the still used words and put them in his dictionary according to their logical order. In fact, the numbers given by al-Farāhīdī are very correct and exact, and they cannot be obtained by normal calculation. There is no doubt that al-Farāhīdī must have consulted some algebraic formula. Without going into mathematical details, al-Farāhīdī must have used factorial permutation by applying it to the 28 letters, and on each of the 4 previously mentioned groups of words at the time. In fact, Rashed discussed many important theories that were proposed after al-Farāhīdī's time, regarding a logical explanation to the question of how could al-Farāhīdī arrive to that specific number of Arabic words at a time when apparently there was no such apparatus available in the Islamic world. Such theories proposed by various intellectuals such as Al-Suyūṭī, Ibn al-Bannāʾ, Hamza al-Aṣfahānī, al-Layth, Ibn Durayd. They almost all agreed that al-Farāhīdī

[18] In Jābir's surviving writings, we also come across other proofs of mathematical knowledge, such as his calculation of pi and aspects of geometry.

[19] It is not coincidence that the linguist al-Farāhīdī who has made the greatest contribution in Arabic language, was also a mathematician.

must have used some algebraic formulas, namely a combinatorial analysis in creating his dictionary *Kitāb al-'Ayn* (*The Book of al-'Ayn*).

However, what concerns us regarding the above-mentioned case is its indirect possible relation with Jābir's ideas. In other words, al-Farāhīdī's four categories of Arabic original roots have some resemblance to Jābir's division of Arabic words (Jābir, 2006, p. 294). Was al-Farāhīdī familiar with his contemporary's works? If Jābir had preceded al-Farāhīdī in discovering the original state of the Arabic words, then that would make us assume that al-Farāhīdī has taken the four categories as well as his above-mentioned algebraic ideas from Jābir. What supports this claim is the fact that al-Farāhīdī has started his dictionary [*Kitāb al-'Ayn*] with the letter ع ['Ayn] and not by the typical first letter ا [ʾAlif] because al-Farāhīdī argued that the ع ['Ayn] when spelled, produces a balanced amount of sound compared to all other Arabic letters. So, according to al-Farāhīdī, the ع ['Ayn] is "the most stable letter".[20] One could find several references glorifying this specific letter in Jābir's books, since it is the first letter of علي [ʿAlī Ibn Abī Ṭālib], the cousin and the son-in-law of the Prophet. Therefore, no wonder that Jābir too had a book named كتاب العين [*Kitāb al-'Ayn*].[21] The Shi'ite Jābir claimed that his knowledge was given to him by Ja'far [The Truthful One] (702-765) who had received it from his ancestor who likewise had received it through ʿAlī, who in the same example had obtained it from the Prophet, who had taken it directly from Allāh.

Sezgin mentioned that there is no way to contradict the fact that al-Farāhīdī's arrangement of the Arabic alphabet was attained based on the Indian method (Sezgin, 1988, p. 22). But it is also possible that such a notion had come to al-Farāhīdī through Jābir's now lost books. Many historians including Sezgin joined the opinion that it was al-Farāhīdī's friend who later completed his dictionary (Sezgin, 1988, p. 159). So, since there are doubts about who could have completed al-Farāhīdī's dictionary, there should be a discussion about the sources which were possibly used in discovering the exact number of Arabic words, and who knew about the mystcial nature of Arabic letters more than Jābir? Especially when we remember how Jābir boasted about his mastery of the Indian sciences that he claimingly was able to write a whole book about the Indian philosophical ideas (Sezgin, 1986, p. 242).[22]

Conclusion

In conclusion, the availability of some ancient algebraic elements as well as Muslims' daily-life demands had facilitated and encouraged al-Khwārizmī to produce his book of algebra. Yet, there could have been other contributing factors that had indirectly helped the robust emergence of this branch of knowledge, such as Jābir's alchemical interest that pushed him to investigate many different fields. The

[20] In this context, it must be said that the letter "ayn" (ع) in Arabic is not just a letter; it rather carries rich importance, be it linguistically or phonetically. It is a unique and emphatic consonant that plays a pivotal role in Arabic phonology. Being a pharyngeal voiced fricative, it is produced deep in the throat, making it one of the most distinctive sounds in the Arabic language. It is linked to the articulation point of the throat (Makhārij al-ḥurūf), which is considered the first point of articulation in the Arabic language. The letter "ayn" (ع) is produced from the middle of the throat. That is to say, it is a voiced sound intermediate between the emphatic and the soft sounds. Furthermore, the use of this particular letter as a title might also carry symbolic weight, meaning the core principles of language.

[21] Needless to say, Jābir's *Kitāb al-'Ayn* (*The Book of al-'Ayn*) had different contents and topics from al-Farāhīdī's eponymous book—which was written after Jābir's.

[22] Jābir's theory of balance is directly connected to language, that is to say, to the relation between numbers, music and language. And ʿArūḍ (study of prosody)—which was founded by al-Farāhīdī—can be considered as the first stage of music science (Sezgin, 1986, p. 213).

abundance of the relevant terms in Jābir's writings, as explored so far, is interesting. Unfortunately, we lack almost all of his mathematical books, and that makes this hypothesis far from presenting conclusive evidence on the issue. But what this article has hopefully achieved is highlighting an area which was not yet seriously studied in terms of its real effect on Islamic algebra and this might fill up some of the relevant existing lacunas.

Finally, while reflecting on this hypothesis we should remind ourselves that unlike al-Khwārizmī who worked in the prestigious well-known institute "House of Wisdom" (Bayt al-Ḥikmah), the poor Jābir used to work underground far away from that time's academia, so to speak. That reason accompanied with Jābir's negative reputation of being a so-called trickster and a practitioner of pseudo-science as well as being a shi'ite at that politically heated atmosphere, must have helped denying the credit of his many revolutionary ideas. And in this sense, some of his relevant significant books were not read, let alone praised, by his contemporaries. Therefore, this paper invites the readers—especially if they are interested in searching for algebra's lost chain—towards that great alchemical heritage, which was deservedly the springboard of the various fields of science at the time.

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This article is not part of a working group that requires ethical committee approval.

Kaynakça

Al-Farāhīdī (2003). *Kitāb al-'Ayn* [*The Book of 'Ayn*], first ed., Beirut: Dār al-Kutub al-ʿIlmīya.

Al-Khaffaf, E. (2023). Looking through the lens of language: How early Muslim intellectuals tried to reach hidden truths by examining the Arabic language, p. 29-42. *Identities and representations, reaching one another: language as interface and performance, joint proceedings of the 3rd and 4th International Students' Conference ICON 2020 and 2021*, edited by Jan Jokisch in collaboration with Dr. Daniel Schmicking. Mainz: Johannes Gutenberg-Universität Mainz. Retrieved from (2 February 2024): http://doi.org/10.25358/openscience-8988

Al-Khwārizmī (1937). *Al-kitāb al-mukhtaṣar fī hisāb al-Jabr wal-muqābala* [*The Abbreviated Book on Calculation by Completion and Balancing*], edited by Ali Mustafa Mashrafa and Muhammed Mursi Ahmed. Egypt: Matba't Paul Parbi.

Anawati, Georges (2005). Arabic alchemy. *Mawsū'at tārīkh al-'ulūm al-'arabīyah* [*Encyclopedia of History of Arabic Science*], supervised by Roshdi Rashed, vol. 3, Beirut: Markaz Dirāsāt al-Waḥdah al-'Arabīyah, p. 1089-1126.

Boyer, C. & Merkelbach, U. (2011). *A history of mathematics*. foreword by Asimov, I. New Jersey.

Çelebi, K. (1971). *Kashf al-ẓunūn ʿan asāmi al-kutub wa al-funūn* [*The Removal of Doubts from the Names of Books and the Arts*], 3rd volume, edited by Mohammed Abdul-Kadir Ata. Beirut: Dar Al-Kutub Al-Ilmiya.

Haq, S. N. (1994). *Names, natures and things: The alchemist jābir ibn Hayyān and his Kitāb al-Aḥjār (Book of Stones)*, edited by Robert S. Cohen. Boston: Boston University.

Ibn al-Nadīm (n.y.). *Al-fihrist* [*Index*]. Retrieved from (1 January 2022): https://www.alarabimag.com/books/13601-%D8%A7%D9%84%D9%81%D9%87%D8%B1%D8%B3%D8%AA.html

Jābir Ibn Ḥayyān (1935) *Mukhtarat Rasa'il Jābir Ibn Ḥayyān* [*Selected Epistles of Jābir Ibn Ḥayyān*], edited by Paul Kraus, Cairo: El-Khandgi Library.

Jābir Ibn Ḥayyān (2006). *Rasa'il Jābir Ibn Ḥayyān [Epistles of Jābir Ibn Ḥayyān]*, edited by al- Ahmad Farid Miziyadi, Beirut: Dār al-Kutub al-ʿIlmīya.

Kâhya, E. (1995) *Modern Kimyanın Kurucusu Câbir Bin Hayyan* [*The Founder of Modern Chemistry: Jābir Ibn Ḥayyān*]. Ankara: TDV. Yayınları.

Koch-Westenholz, U. (1995). *Mesopatomian astrology*. No City: The Carsten Niebuhr Institute of Near Eastern Studies Museum Tusculanum. University of Copenhagen.

Mahmud, Z. N. (2001). *Jābir Ibn Ḥayyān*, Alexandria: Maktabat Masir.

Principe, L. M. (2013). *The secrets of Alchemy*, Chicago UP.

Mysterious Writings (2012). "Geber, the Number 17, and the 3×3 Magic Square". Retrieved from (2 July 2022): https://mysteriouswritings.com/geber-the-number-17-and-the-3x3-magic-square/#:~:text=Considered%20to%20be%20one%20of,the%20magic%20constant%20of%2015.

Qur'ān (2007). *The Interpretation of the Meaning of The Noble Qur'ān*, translated by Muhammad Taqi-ud-Din Al-Hilālī & Muhammad Muhsin Khān, Riyadh: Darussalam.

Rashed, R. (2015). *Classical mathematics from al-Khwārizmī to Descartes*, translated by Shank, Michael H., London & New York: Routledge.

Sayılı, A. (1985). *Logical necessities in mixed equations by ʿAbd al-Hamīd Ibn Turk and the Algebra of his time*. Retrieved from (6 February 2022): https://en.calameo.com/read/003359034efeae7442045

Sezgin, F. (1986). *Tarikh al-Turath al-'Arabi* [*History of Arabic-Islamic Heritage*], volume 4, Alchemy et al, translated by Abdullah Hijazi, Riyadh: King Saud University.

Sezgin, F. (1988). *Tarikh al-Turath al-'Arabi* [*History of Arabic-Islamic Heritage*], volume 8, Langauge, translated by Arafa Mustafa, Riyadh: Imam Mohammed Ibn Saud's Islamic University.

Sezgin, F. (2002). *Tarikh al-Turath al-'Arabi* [*History of Arabic-Islamic Heritage*], volume 5, Mathematics, translated by Abdullah Hijazi et al, Riyadh: King Saud University.

Sezgin, F. (2008). *Tarikh al-Turath al-'Arabi* [*History of Arabic-Islamic Heritage*], volume 6, Astronomy, translated by Abdullah Hijazi, Riyadh: King Saud University.

Thompson, R. C. (1900). *The Reports of the magicians and astrologers*. London: British Museum, Luzac and Co.

Yates, F. (1979). *The occult philosophy in the Elizabethan Age*. London & New York: Routledge & Kegan Paul.

Zosimos of Panopolis (2007). "Mushaf as-Suwar" [The Book of Pictures], edited by Theodor Abt and Wilferd Madelung. *Corpus Alchemicum Arabicum*, volume 2, no. 1, Zurich: Living Human Heritage Publications.