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Enhancing Generalized Interpolative Contraction Through Simulation Functions

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Abstract
In the present manuscript, we elucidate a comprehensive framework for the generalized interpolative $\alpha-(\psi,\varphi)_Z-$contractive mapping, thereby extending the foundational theoretical constructs to augment its utility within the domain of advanced mathematical analysis. The investigation encompasses a meticulous examination of fixed point results within the context of non-Archimedean modular metric spaces, which are characterized by their distinctive structural properties that diverge from those of conventional metric spaces. Moreover, we apply the results attained to substantiate the existence and uniqueness of solutions pertaining to nonlinear Fredholm integral equations. This aspect of our inquiry underscores the practical implications of our theoretical advancements and provides a rigorous framework for the resolution of complex integral equations through the principles of established contractive mappings.

Keywords: *Admissible mappings, Fredholm integral equations, Interpolative contractions, Simulation functions*

AMS Subject Classification (2020): *47H10; 54H25; 37C25*

1. Introduction

This study designates the symbol $\mathbf{N}$ to represent the set of all positive natural numbers. Additionally, the sets of positive and non-negative real numbers are represented by $\mathbf{R}^+$ and $\mathbf{R}_0^+$, respectively.

The simulation function, introduced by Khojasteh et al. [1], has emerged as an invaluable innovative control function in metric spaces. Its application in defining a $\zeta-$contraction has not only facilitated the proof of pivotal fixed point theorems but also marks a noteworthy advancement in the discipline. Following this groundbreaking work, numerous researchers have expanded and refined this concept across various abstract spaces, as evidenced in [2–6] and [7].

Recently, Karapınar [8] made significant advancements in the field of fixed point theory by modifying the classical concept of Kannan contractions. He introduced an interpolative Kannan contraction, which was designed to enhance the convergence rate of operators toward a unique fixed point. This innovation aimed to refine the existing understanding of how operators behave in mathematical spaces. However, subsequent work by Karapınar

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and Agarwal [9] revealed a critical flaw in the assumptions laid out in Karapınar's initial paper. They presented a counter-example that highlighted the problematic assumption that the fixed point must be unique. Their findings showed that it is possible for fixed points to exist without uniqueness, thereby challenging the validity of this central premise in the original theory. Following this important correction, the researchers provided a revised framework that better accommodates situations where fixed points are not unique. This development opened the door to further exploration and prompted the investigation of various results related to different types of interpolative mappings. Consequently, a plethora of results for both single-valued and multivalued mappings have been established across diverse abstract spaces [10–13].

There is extensive interest in metric fixed point theory due to its compelling structural properties and broad applications across various fields, including mathematics, computer science, and economics. Within this theoretical framework, the Banach contraction mapping theorem, first introduced by Banach in 1922, occupies a pivotal position owing to its foundational significance and versatility. This seminal work provided a robust method for establishing the existence and uniqueness of fixed points in complete metric spaces, laying the groundwork for countless subsequent research efforts aimed at expanding and refining the understanding of this profound mapping.

The Banach contraction mapping theorem has not only deepened theoretical insights but also inspired practical applications, from solving differential equations to optimization problems. Over the years, the development of this field has witnessed a notable emergence of innovative structures concerning generalized metric spaces. These generalized spaces relax some of the traditional constraints, allowing for a broader class of mappings and facilitating the exploration of fixed point theorems within varied contexts [14–19].

Among the significant advancements in this domain is the introduction of the modular metric space. This new structure, which incorporates a modular function to define distance and convergence, offers a more flexible approach to analyzing fixed points and contracts. Its unique properties enable researchers to address more complex problems that may not fit within the confines of classical metric spaces. Consequently, modular metric spaces serve as a fertile ground for further theoretical exploration and practical application, potentially leading to new discoveries in fixed point theory and beyond.

In 2010, Chistyakov [20, 21] made a significant advancement by establishing the concept of a modular metric space. This innovative framework not only extends the traditional metric space but also integrates the principles of modular linear space, paving the way for newfound research opportunities and applications in mathematical theory.

Let $\mathcal{X}$ be a nonempty set and $\Lambda : (0,\infty) \times \mathcal{X} \times \mathcal{X} \to [0,\infty]$ be a function. For the sake of brevity, we will denote the relationship as follows:

$$\Lambda_\chi(\iota,\jmath) = \Lambda(\chi,\iota,\jmath)$$

for all $\chi > 0$ and $\iota,\jmath \in \mathcal{X}$.

Definition 1.1. [20] Let $\mathcal{X}$ be nonempty set and $\Lambda : (0,\infty) \times \mathcal{X} \times \mathcal{X} \to [0,\infty]$ be a function satisfying the subsequent circumstances:

(Λ_1) $\iota = \jmath$ if and only if $\Lambda_\chi(\iota,\jmath) = 0$ for all $\chi > 0$ and and $\iota,\jmath \in \mathcal{X}$;

(Λ_2) $\Lambda_\chi(\iota,\jmath) = \Lambda_\chi(\jmath,\iota)$ for all $\chi > 0$ and $\iota,\jmath \in \mathcal{X}$;

(Λ_3) $\Lambda_{\chi+n}(\iota,\jmath) \leq \Lambda_\chi(\iota,z) + \Lambda_n(z,\jmath)$ for all $\chi, n > 0$ and $\iota,\jmath,z \in \mathcal{X}$.

Then, Λ is called modular metric in $\mathcal{X}$, and so Λ_χ is modular metric space. If the condition (Λ_1) is replaced by

(Λ_4) $\Lambda_\chi(\iota,\iota) = 0$ for all $\chi > 0$ and $\iota \in \mathcal{X}$,

then Λ is referred to as a pseudomodular metric on $\mathcal{X}$. A modular metric Λ defined on $\mathcal{X}$ is termed regular if it satisfies a weaker formulation of the condition denoted as (Λ_1).

(Λ_5) $\iota = \jmath$ if and only if $\Lambda_\chi(\iota,\jmath) = 0$ for some $\chi > 0$.

Moreover, Λ is called convex if for $\chi,\ n > 0$ and $\iota,\jmath,z \in \mathcal{X}$, the inequality holds:

(Λ_6) $\Lambda_{\chi+n}(\iota,\jmath) \leq \frac{\chi}{\chi+n}\Lambda_\chi(\iota,z) + \frac{n}{\chi+n}\Lambda_n(z,\jmath)$.

If we replace (Λ_3) by

(Λ_7) $\Lambda_{\max\{\chi,n\}}(\iota,\jmath) \leq \Lambda_\chi(\iota,z) + \Lambda_n(z,\jmath)$

for all $\chi,\ n>0$ and $\iota, \jmath, z \in \mathcal{X}_\Lambda$. Thus, we assert that $\mathcal{X}_\Lambda$ represents non-Archimedean modular metric space.

Definition 1.2. [20] Let $\mathcal{X}_\Lambda$ be a modular metric space, S be a subset of $\mathcal{X}_\Lambda$ and $(\iota_n)_{n\in\mathbf{N}}$ be a sequence in $\mathcal{X}_\Lambda$. Then,

(i) A sequence $(\iota_n)_{\kappa\in\mathbf{N}}$ is called $\Lambda-$convergent to $\iota \in \mathcal{X}_\Lambda$ if and only if $\Lambda_\chi(\iota_n,\iota)\to 0$ as $n\to\infty$ for all $\chi>0$, ι is said to be the $\Lambda-$limit of (ι_n).

(ii) A sequence $(\iota_n)_{\kappa\in\mathbf{N}}$ is called $\Lambda-$Cauchy if $\Lambda_\chi(\iota_n,\iota_p)\to 0$, as $p,n\to\infty$ for all $\chi>0$.

(iii) A subset S is called $\Lambda-$closed if the $\Lambda-$limit of $\Lambda-$convergent sequence of S always belongs to S.

(iv) A subset S is called $\Lambda-$complete if any $\Lambda-$Cauchy sequence in S is $\Lambda-$convergent to a point of S.

(v) A subset S is called $\Lambda-$bounded if for all $\chi>0$, we have

$$\delta_\Lambda(S)=\sup\{\Lambda_\chi(\iota,\jmath);\iota,\jmath\in S\}<\infty.$$

Definition 1.3. [20] Let $\mathcal{X}_\Lambda$ be a modular metric space and $\coprod:\mathcal{X}_\Lambda\to\mathcal{X}_\Lambda$ be a mapping. It is said that $\coprod$ is a $\Lambda-$continuous when $\Lambda_\chi(\iota_n,\iota)\to 0 \Rightarrow \Lambda_\chi(\coprod\iota_n,\coprod\iota)\to 0$, as $n\to\infty$.

In recent years, the field of fixed point theory in modular metric spaces has witnessed significant developments and applications [22–25].

Khan et.al. [26] introduce the concept of altering distance function as follows.

Definition 1.4. [26] A continuous function $\varphi:[0,\infty)\to[0,\infty)$ is called an altering distance function if it is non-decreasing and $\varphi(r)=0$ if and only if $r=0$.

It is obvious that $\varphi(r)\geq 0$, for all $r\geq 0$. We denote Φ, the set of all altering distance functions.

Definition 1.5. [27] A function $\psi:[0,\infty)\to[0,\infty)$ is said to be a comparison function if it is monotonically increasing and $\psi^n(t)\to 0$ as $n\to\infty$ for all $t>0$.

If ψ is comparison function, then $\psi(t)<t$ for all $t>0$ and $\psi(0)=0$. The symbol Ψ denotes the set of all comparison functions.

Let $\mathcal{X}$ be a nonempty set and $\alpha:\mathcal{X}\times\mathcal{X}\to\mathbf{R}$. We collect the following concepts which are necessary for our subsequent discussion.

Definition 1.6. [28] A mapping $\coprod:\mathcal{X}\to\mathcal{X}$ is said to be a $\alpha-$admissible if

(α_1) $\alpha(\iota,\jmath)\geq 1$ implies $\alpha(\coprod\iota,\coprod\jmath)\geq 1$, for all $\iota,\jmath\in\mathcal{X}$.

Definition 1.7. [29] A mapping $\coprod:\mathcal{X}\to\mathcal{X}$ is called triangular $\alpha-$admissible if it satisfies (α_1) and

(α_2) $\alpha(\iota,z)\geq 1$ and $\alpha(z,\jmath)\geq 1$ imply $\alpha(\iota,\jmath)\geq 1$ for all $\iota,\jmath,z\in\mathcal{X}$.

In light of the aforementioned considerations, this study aims to integrate concepts such as interpolative contraction, simulation functions, admissible mappings, and modified distance functions to establish novel fixed point theorems within non-Archimedean modular metric spaces. Furthermore, we provide a comprehensive illustration demonstrating both the existence and uniqueness of a solution for a nonlinear Fredholm integral equation.

2. Main results

Definition 2.1. Let $\mathcal{X}_\Lambda$ be a non-Archimedean modular metric space and $\coprod:\mathcal{X}_\Lambda\to\mathcal{X}_\Lambda$ be a given mapping. It is said that $\coprod$ is a generalized interpolative $\alpha-(\psi,\varphi)_Z-$contractive mapping if there exist $\alpha:\mathcal{X}_\Lambda\times\mathcal{X}_\Lambda\to[0,\infty)$, $\psi\in\Psi$, $\varphi\in\Phi$ and $\zeta\in Z$ and $\mu_1,\mu_2\in(0,1)$ such that $\varphi(t)>\psi(t),\ t>0$ and $\mu_1+\mu_2<1$ providing the subsequent inequality

$$\begin{aligned}&\zeta\left(\alpha(i,j)\,\varphi\left(\Lambda_\chi(\coprod i,\coprod j)\right),\psi\left(\Xi(i,j)\right)\right)\geq 0,\\&\Xi(i,j)=\Lambda_\chi(i,j)^{\mu_1}.\Lambda_\chi(i,\coprod i)^{\mu_2}.\Lambda_\chi(j,\coprod j)^{1-\mu_1-\mu_2}\end{aligned}\tag{2.1}$$

for all $i,j\in\mathcal{X}_\Lambda$.

Theorem 2.1. *Let $\mathcal{X}_\Lambda$ be a complete non-Archimedean modular metric space. Let $\coprod$ be a generalized interpolative $\alpha-(\psi,\varphi)_Z-$contractive mapping satisfying the following conditions:*

- (i) $\coprod$ *is a triangular $\alpha-$admissible mapping,*
- (ii) *there exists $i_0 \in \mathcal{X}_\Lambda$ such that $\alpha(i_0, \coprod i_0) \geq 1$,*
- (iii) $\coprod$ *is a continuous mapping.*

Then, $\coprod$ admits a fixed point in $\mathcal{X}_\Lambda$.

Proof. Let $i_0 \in \mathcal{X}_\Lambda$ such that $\alpha(i_0, \coprod i_0) \geq 1$. Construct the sequence $\{i_\kappa\}$ in $\mathcal{X}_\Lambda$ by $i_{\kappa+1} = \coprod i_\kappa$, for all $\kappa \in \mathbf{N}$. If $i_{\kappa+1} = i_\kappa$, for some $\kappa \in \mathbf{N}$, then $i^* = i_\kappa$ is a fixed point for $\coprod$ and the proof completed. Hence, we presume that $i_{\kappa+1} \neq i_\kappa$, for all $\kappa \in \mathbf{N}$. Due to the fact that $\coprod$ is triangular $\alpha-$ admissible, we have:

$$\alpha(i_0, i_1) = \alpha\left(i_0, \coprod i_0\right) \geq 1 \Rightarrow \alpha\left(\coprod i_0, \coprod i_1\right) = \alpha\left(i_1, \coprod i_2\right) \geq 1.$$

By induction, we get

$$\alpha(i_\kappa, i_{\kappa+1}) \geq 1, \tag{2.2}$$

for all $\kappa \in \mathbf{N}$. Regarding (2.1), we derive that

$$\begin{aligned} 0 &\leq \zeta\left(\alpha(i_{\kappa-1}, i_\kappa)\varphi\left(\Lambda_\chi\left(\coprod i_{\kappa-1}, \coprod i_\kappa\right)\right), \psi\left(\Xi(i_{\kappa-1}, i_\kappa)\right)\right) \\ &= \zeta\left(\alpha(i_{\kappa-1}, i_\kappa)\varphi\left(\Lambda_\chi(i_\kappa, i_{\kappa+1})\right), \psi\left(\Xi(i_{\kappa-1}, i_\kappa)\right)\right) \\ &< \psi\left(\Xi(i_{\kappa-1}, i_\kappa)\right) - \alpha(i_{\kappa-1}, i_\kappa)\varphi\left(\Lambda_\chi(i_\kappa, i_{\kappa+1})\right), \end{aligned} \tag{2.3}$$

where

$$\begin{aligned} \Xi(i_{\kappa-1}, i_\kappa) &= \Lambda_\chi(i_{\kappa-1}, i_\kappa)^{\mu_1}.\Lambda_\chi(i_{\kappa-1}, \coprod i_{\kappa-1})^{\mu_2}.\Lambda_\chi(i_\kappa, \coprod i_\kappa)^{1-\mu_1-\mu_2} \\ &= \Lambda_\chi(i_{\kappa-1}, i_\kappa)^{\mu_1}.\Lambda_\chi(i_{\kappa-1}, i_\kappa)^{\mu_2}.\Lambda_\chi(i_\kappa, i_{\kappa+1})^{1-\mu_1-\mu_2}. \end{aligned}$$

Consequently, we arrive at

$$\begin{aligned} \varphi\left(\Lambda_\chi(i_\kappa, i_{\kappa+1})\right) &\leq \alpha(i_{\kappa-1}, i_\kappa)\varphi\left(\Lambda_\chi(i_\kappa, i_{\kappa+1})\right) \\ &< \psi\left(\Xi(i_{\kappa-1}, i_\kappa)\right) \\ &= \psi\left(\Lambda_\chi(i_{\kappa-1}, i_\kappa)^{\mu_1}.\Lambda_\chi(i_{\kappa-1}, i_\kappa)^{\mu_2}.\Lambda_\chi(i_\kappa, i_{\kappa+1})^{1-\mu_1-\mu_2}\right). \end{aligned} \tag{2.4}$$

Suppose that $\Lambda_\chi(i_{\kappa-1}, i_\kappa) < \Lambda_\chi(i_\kappa, i_{\kappa+1})$ for all $\kappa \in \mathbf{N}$, then from (2.4), we obtain

$$\varphi\left(\Lambda_\chi(i_\kappa, i_{\kappa+1})\right) \leq \psi\left(\Lambda_\chi(i_\kappa, i_{\kappa+1})\right) < \varphi\left(\Lambda_\chi(i_\kappa, i_{\kappa+1})\right),$$

which causes a contradiction. Accordingly, we obtain

$$\Lambda_\chi(i_\kappa, i_{\kappa+1}) \leq \Lambda_\chi(i_{\kappa-1}, i_\kappa), \tag{2.5}$$

for all $\kappa \in \mathbf{N}$. Hence, $\{\Lambda_\chi(i_\kappa, i_{\kappa+1})\}$ is a monotone decreasing sequence of positive real numbers and bounded below by zero. So, there exists $r \geq 0$ such that $\lim\limits_{n\to\infty} \Lambda_\chi(i_\kappa, i_{\kappa+1}) = r$. We claim that $r > 0$, otherwise from (2.3), (2.4) together with (2.5) we procure

$$\begin{aligned} 0 &\leq \zeta\left(\alpha(i_{\kappa-1}, i_\kappa)\varphi\left(\Lambda_\chi\left(\coprod i_{\kappa-1}, \coprod i_\kappa\right)\right), \psi\left(\Xi(i_{\kappa-1}, i_\kappa)\right)\right) \\ &= \zeta\left(\alpha(i_{\kappa-1}, i_\kappa)\varphi\left(\Lambda_\chi(i_\kappa, i_{\kappa+1})\right), \psi\left(\Xi(i_{\kappa-1}, i_\kappa)\right)\right) \\ &< \psi\left(\Xi(i_{\kappa-1}, i_\kappa)\right) - \alpha(i_{\kappa-1}, i_\kappa)\varphi\left(\Lambda_\chi(i_\kappa, i_{\kappa+1})\right). \end{aligned} \tag{2.6}$$

Consequently, we achieve

$$\begin{aligned}\varphi\left(\Lambda_{\chi}\left(i_{\kappa}, i_{\kappa+1}\right)\right) &\leq \alpha\left(i_{\kappa-1}, i_{\kappa}\right) \varphi\left(\Lambda_{\chi}\left(i_{\kappa}, i_{\kappa+1}\right)\right) \\ &\leq \psi\left(\Xi\left(i_{\kappa-1}, i_{\kappa}\right)\right) \\ &\leq \varphi\left(\Xi\left(i_{\kappa-1}, i_{\kappa}\right)\right) \\ &\leq \varphi\left(\Lambda_{\chi}\left(i_{\kappa-1}, i_{\kappa}\right)\right).\end{aligned} \tag{2.7}$$

Taking the limit as $n \to \infty$ in (2.7), we attain

$$\lim_{n\to\infty} \alpha\left(i_{\kappa-1}, i_{\kappa}\right) \varphi\left(\Lambda_{\chi}\left(i_{\kappa}, i_{\kappa+1}\right)\right) = \lim_{n\to\infty} \psi\left(\Xi\left(i_{\kappa-1}, i_{\kappa}\right)\right) = \varphi\left(r\right). \tag{2.8}$$

Setting $s_n = \alpha\left(i_{\kappa-1}, i_{\kappa}\right) \varphi\left(\Lambda_{\chi}\left(i_{\kappa}, i_{\kappa+1}\right)\right), \quad t_n = \psi\left(\Xi\left(i_{\kappa-1}, i_{\kappa}\right)\right)$ in (2.3), then by the property of simulation function and (2.8), it is yielded that

$$0 \leq \limsup_{n\to\infty} \zeta\left(\alpha\left(i_{\kappa-1}, i_{\kappa}\right) \varphi\left(\Lambda_{\chi}\left(i_{\kappa}, i_{\kappa+1}\right)\right), \psi\left(\Xi\left(i_{\kappa-1}, i_{\kappa}\right)\right)\right) < 0.$$

This is a contradiction and thus we have $\lim_{n\to\infty} \Lambda_{\chi}\left(i_{\kappa}, i_{\kappa+1}\right) = 0.$

Now, we show that $\{i_{\kappa}\}$ is a $\Lambda-$Cauchy sequence. Suppose that, there exist $\varepsilon > 0$, for which one can find two sequences $\{m_{\rho}\}$ and $\{\kappa_{\rho}\}$, for all $\rho \geq 1$ with $i_{m_{\rho}} > i_{\kappa_{\rho}} \geq \rho$ such that $\Lambda_{\chi}\left(i_{\kappa_{\rho}}, i_{m_{\rho}}\right) \geq \varepsilon$. Further, we assume that m_{ρ} is the smallest number greater than κ_{ρ}, then $\Lambda_{\chi}\left(i_{\kappa_{\rho}}, i_{m_{\rho}-1}\right) < \varepsilon$. By triangular inequality of non-Archimedean quasi modular metric space, we gain

$$\begin{aligned}\varepsilon \leq \Lambda_{\chi}\left(i_{\kappa_{\rho}}, i_{m_{\rho}}\right) &= \Lambda_{\max\{\chi,\chi\}}\left(i_{\kappa_{\rho}}, i_{m_{\rho}}\right) \\ &\leq \Lambda_{\chi}\left(i_{\kappa_{\rho}}, i_{m_{\rho}-1}\right) + \Lambda_{\chi}\left(i_{m_{\rho}-1}, i_{m_{\rho}}\right) \\ &< \varepsilon + \Lambda_{\chi}\left(i_{m_{\rho}-1}, i_{m_{\rho}}\right).\end{aligned}$$

Taking the limit as $\rho \to \infty$, we get

$$\lim_{\rho\to\infty} \Lambda_{\chi}\left(i_{\kappa_{\rho}}, i_{m_{\rho}}\right) = \varepsilon. \tag{2.9}$$

Again by triangular inequality of non-Archimedean quasi modular metric space, we have

$$\begin{aligned}\Lambda_{\chi}\left(i_{\kappa_{\rho}}, i_{m_{\rho}}\right) &= \Lambda_{\max\{\chi,\chi\}}\left(i_{\kappa_{\rho}}, i_{m_{\rho}}\right) \\ &\leq \Lambda_{\chi}\left(i_{\kappa_{\rho}}, i_{\kappa_{\rho}+1}\right) + \Lambda_{\chi}\left(i_{\kappa_{\rho}+1}, i_{m_{\rho}}\right) \\ &= \Lambda_{\chi}\left(i_{\kappa_{\rho}}, i_{\kappa_{\rho}+1}\right) + \Lambda_{\max\{\chi,\chi\}}\left(i_{\kappa_{\rho}+1}, i_{m_{\rho}}\right) \\ &\leq \Lambda_{\chi}\left(i_{\kappa_{\rho}}, i_{\kappa_{\rho}+1}\right) + \Lambda_{\chi}\left(i_{\kappa_{\rho}+1}, i_{m_{\rho}+1}\right) + \Lambda_{\chi}\left(i_{m_{\rho}+1}, i_{m_{\rho}}\right).\end{aligned} \tag{2.10}$$

Also, we get

$$\begin{aligned}\Lambda_{\chi}\left(i_{\kappa_{\rho}+1}, i_{m_{\rho}+1}\right) &= \Lambda_{\max\{\chi,\chi\}}\left(i_{\kappa_{\rho}+1}, i_{m_{\rho}+1}\right) \\ &\leq \Lambda_{\chi}\left(i_{\kappa_{\rho}+1}, i_{\kappa_{\rho}}\right) + \Lambda_{\chi}\left(i_{\kappa_{\rho}}, i_{m_{\rho}+1}\right) \\ &= \Lambda_{\chi}\left(i_{\kappa_{\rho}+1}, i_{\kappa_{\rho}}\right) + \Lambda_{\max\{\chi,\chi\}}\left(i_{\kappa_{\rho}}, i_{m_{\rho}+1}\right) \\ &\leq \Lambda_{\chi}\left(i_{\kappa_{\rho}+1}, i_{\kappa_{\rho}}\right) + \Lambda_{\chi}\left(i_{\kappa_{\rho}}, i_{m_{\rho}}\right) + \Lambda_{\chi}\left(i_{m_{\rho}}, i_{m_{\rho}+1}\right).\end{aligned} \tag{2.11}$$

Combining the expressions (2.10) and (2.11) and taking the limit as $\rho \to \infty$ together with (2.9), we attain

$$\lim_{\rho\to\infty} \Lambda_{\chi}\left(i_{\kappa_{\rho}+1}, i_{m_{\rho}+1}\right) = \varepsilon. \tag{2.12}$$

As $\coprod$ is a triangular $\alpha-$admissible mapping, we obtain $\alpha\left(i_{\kappa_{\rho}}, i_{m_{\rho}}\right) \geq 1$, for all numbers $m_{\rho}, \ \kappa_{\rho}$ such that $m_{\rho} > \kappa_{\rho}$, where $\rho \geq 1$. From (2.1), we get

$$
\begin{aligned}
0 &\leq \zeta\left(\alpha\left(i_{\kappa_\rho}, i_{m_\rho}\right)\varphi\left(\Lambda_\chi\left(\coprod i_{\kappa_\rho}, \coprod i_{m_\rho}\right)\right), \psi\left(\Xi\left(i_{\kappa_\rho}, i_{m_\rho}\right)\right)\right) \\
&= \zeta\left(\alpha\left(i_{\kappa_\rho}, i_{m_\rho}\right)\varphi\left(\Lambda_\chi\left(i_{\kappa_\rho+1}, i_{m_\rho+1}\right)\right), \psi\left(\Xi\left(i_{\kappa_\rho}, i_{m_\rho}\right)\right)\right) \\
&< \psi\left(\Xi\left(i_{\kappa_\rho}, i_{m_\rho}\right)\right) - \alpha\left(i_{\kappa_\rho}, i_{m_\rho}\right)\varphi\left(\Lambda_\chi\left(i_{\kappa_\rho+1}, i_{m_\rho+1}\right)\right).
\end{aligned}
$$

Consequently, it can be inferred that

$$
\begin{aligned}
\varphi\left(\Lambda_\chi\left(i_{\kappa_\rho+1}, i_{m_\rho+1}\right)\right) &\leq \alpha\left(i_{\kappa_\rho}, i_{m_\rho}\right)\varphi\left(\Lambda_\chi\left(i_{\kappa_\rho+1}, i_{m_\rho+1}\right)\right) \\
&\leq \psi\left(\Xi\left(i_{\kappa_\rho}, i_{m_\rho}\right)\right) \\
&\leq \varphi\left(\Xi\left(i_{\kappa_\rho}, i_{m_\rho}\right)\right),
\end{aligned}
$$

where

$$
\begin{aligned}
\Xi\left(i_{\kappa_\rho}, i_{m_\rho}\right) &= \Lambda_\chi\left(i_{\kappa_\rho}, i_{m_\rho}\right)^{\mu_1}.\Lambda_\chi\left(i_{\kappa_\rho}, \coprod i_{\kappa_\rho}\right)^{\mu_2}.\Lambda_\chi\left(i_{m_\rho}, \coprod i_{m_\rho}\right)^{1-\mu_1-\mu_2} \\
&= \Lambda_\chi\left(i_{\kappa_\rho}, i_{m_\rho}\right)^{\mu_1}.\Lambda_\chi\left(i_{\kappa_\rho}, i_{\kappa_\rho+1}\right)^{\mu_2}.\Lambda_\chi\left(i_{m_\rho}, i_{m_\rho+1}\right)^{1-\mu_1-\mu_2}.
\end{aligned}
$$

Taking the limit as $\rho \to \infty$ with (2.9), (2.10), (2.11) and (2.12), we have

$$0 \leq \varphi(\varepsilon) < \varphi(0) = 0 \quad \text{iff} \quad \varepsilon = 0.$$

This situation presents a contradiction, thereby establishing that the sequence $\{i_\kappa\}$ qualifies as a Cauchy sequence. Since $\mathcal{X}_\Lambda$ is complete non-Archimedean modular metric space, there exists $i^* \in \mathcal{X}_\Lambda$ such that $i_\kappa \to i^*$ as $\kappa \to \infty$. Based on the continuity of $\coprod$, it can be deduced that the sequence defined by $i_{\kappa+1} = \coprod i_\kappa \to \coprod i^*$ as $\kappa \to \infty$. By virtue of the uniqueness of limits, we conclude that, $i^* = \coprod i^*$, that is, i^* is a fixed point of $\coprod$. □

In the subsequent theorem, it is possible to dispense with the continuity of $\coprod$ by introducing an alternative condition.

Theorem 2.2. *Let $\mathcal{X}_\Lambda$ be a complete non-Archimedean modular metric space and $\coprod$ be a generalized interpolative $\alpha-(\psi, \varphi)_Z-$contractive mapping satisfying the following conditions:*

- (i) *$\coprod$ is a triangular $\alpha-$admissible mapping,*
- (ii) *there exists $i_0 \in \mathcal{X}_\Lambda$ such that $\alpha\left(i_0, \coprod i_0\right) \geq 1$,*
- (iii) *If $\{i_\kappa\}$ is a sequence in $\mathcal{X}_\Lambda$ such that $\alpha\left(i_\kappa, i_{\kappa+1}\right) \geq 1$ for all κ and $i_\kappa \to i \in S_\Lambda$ as $\kappa \to \infty$, then $\alpha\left(i_\kappa, i\right) \geq 1$ for all κ.*

Then, $\coprod$ admits a fixed point in $\mathcal{X}_\Lambda$.

Proof. In light of the proof of Theorem 2.1, we can conclude that $\{i_\kappa\}$ is a Cauchy sequence. Then, $i^* \in \mathcal{X}_\Lambda$ exits such that $i_{\kappa_\rho} \to i^*$ as $\rho \to \infty$. From (2.2) and the hypothesis (iii), we have

$$\alpha\left(i_{\kappa_\rho}, i^*\right) \geq 1, \tag{2.13}$$

for all ρ. From (2.1) and (2.13), we get

$$
\begin{aligned}
0 &\leq \zeta\left(\alpha\left(i_{\kappa_\rho}, i^*\right)\varphi\left(\Lambda_\chi\left(\coprod i_{\kappa_\rho}, \coprod i^*\right)\right), \psi\left(\Xi\left(i_{\kappa_\rho}, i^*\right)\right)\right) \\
&= \zeta\left(\alpha\left(i_{\kappa_\rho}, i^*\right)\varphi\left(\Lambda_\chi\left(i_{\kappa_\rho+1}, \coprod i^*\right)\right), \psi\left(\Xi\left(i_{\kappa_\rho}, i^*\right)\right)\right) \qquad (2.14) \\
&< \psi\left(\Xi\left(i_{\kappa_\rho}, i^*\right)\right) - \alpha\left(i_{\kappa_\rho}, i^*\right)\varphi\left(\Lambda_\chi\left(i_{\kappa_\rho+1}, \coprod i^*\right)\right)
\end{aligned}
$$

which is equivalent to

$$
\begin{aligned}
\varphi\left(\Lambda_\chi\left(i_{\kappa_\rho+1}, \coprod i^*\right)\right) &\leq \alpha\left(i_{\kappa_\rho}, i^*\right)\varphi\left(\Lambda_\chi\left(i_{\kappa_\rho+1}, \coprod i^*\right)\right) \\
&< \psi\left(\Xi\left(i_{\kappa_\rho}, i^*\right)\right) \qquad (2.15) \\
&< \varphi\left(\Xi\left(i_{\kappa_\rho}, i^*\right)\right),
\end{aligned}
$$

where

$$\begin{aligned}\Xi\left(i_{\kappa_\rho}, i^*\right) &= \Lambda_\chi\left(i_{\kappa_\rho}, i^*\right)^{\mu_1} . \Lambda_\chi\left(i_{\kappa_\rho}, \coprod i_{\kappa_\rho}\right)^{\mu_2} . \Lambda_\chi(i^*, \coprod i^*)^{1-\mu_1-\mu_2} \\ &= \Lambda_\chi\left(i_{\kappa_\rho}, i^*\right)^{\mu_1} . \Lambda_\chi\left(i_{\kappa_\rho}, i_{\kappa_\rho+1}\right)^{\mu_2} . \Lambda_\chi(i^*, \coprod i^*)^{1-\mu_1-\mu_2}.\end{aligned} \tag{2.16}$$

Now letting $\rho \to \infty$, from the property of φ, we get $\varphi\left(\Lambda_\chi\left(i^*, \coprod i^*\right)\right) = 0$ implying $\Lambda_\chi\left(i^*, \coprod i^*\right) = 0$. This can be concluded that i^* is a fixed point of $\coprod$. □

We suggest the following hypotheses for the uniqueness of the fixed point of $\coprod$.

(U) For all $i, j \in Fix\{\coprod\}$, we get $\alpha(i, j) \geq 1$.

Theorem 2.3. *Adding the condition (U) to the hypotheses of the Theorem 2.1 (resp. Theorem 2.2), we attain the uniqueness of the fixed point of $\coprod$.*

Proof. We assume that j^* is an another fixed point of $\coprod$, that is, $\Lambda_\chi\left(i^*, j^*\right) \neq 0$. From the condition (U), we get $\alpha\left(i^*, j^*\right) \geq 1$. Owing to $\coprod$ is a generalized interpolative $\alpha - (\psi, \varphi)_Z-$contractive mapping, we derive that

$$\begin{aligned}0 &\leq \zeta\left(\alpha\left(i^*, j^*\right) \varphi\left(\Lambda_\chi\left(\coprod i^*, \coprod j^*\right)\right), \psi\left(\Xi\left(i^*, j^*\right)\right)\right) \\ &= \zeta\left(\alpha\left(i^*, j^*\right) \varphi\left(\Lambda_\chi\left(i^*, j^*\right)\right), \psi\left(\Xi\left(i^*, j^*\right)\right)\right) \\ &< \psi\left(\Xi\left(i^*, j^*\right)\right) - \alpha\left(i^*, j^*\right) \varphi\left(\Lambda_\chi\left(i^*, j^*\right)\right),\end{aligned} \tag{2.17}$$

which is equivalent to

$$\begin{aligned}\varphi\left(\Lambda_\chi\left(i^*, j^*\right)\right) &\leq \alpha\left(i^*, j^*\right) \varphi\left(\Lambda_\chi\left(i^*, j^*\right)\right) \\ &< \psi\left(\Xi\left(i^*, j^*\right)\right) \\ &< \varphi\left(\Xi\left(i^*, j^*\right)\right),\end{aligned} \tag{2.18}$$

where

$$\Xi\left(i^*, j^*\right) = \Lambda_\chi(i^*, j^*)^{\mu_1} . \Lambda_\chi\left(i^*, \coprod i^*\right)^{\mu_2} . \Lambda_\chi\left(j^*, \coprod j^*\right)^{1-\mu_1-\mu_2} = 0. \tag{2.19}$$

This results in a contradiction. Hence, $\coprod$ has a unique fixed point in $\mathfrak{X}_\Lambda$. □

Example 2.1. Let $\mathfrak{X}_\Lambda = \mathbf{R}$, $\Lambda_\chi(i, j) = \frac{1}{\chi}|i - j|$, for all $i, j \in \mathfrak{X}_\Lambda, \chi > 0$ and $\coprod i = \frac{i}{2}$. Presume the mapping $\alpha : \mathfrak{X}_\Lambda \times \mathfrak{X}_\Lambda \to [0, \infty)$ is defined by

$$\alpha(i, j) = \begin{cases} 1, & i, j \in [0, 1] \\ 0, & \text{otherwise.} \end{cases}$$

Consider the mapping as $\zeta(t, s) = s - t$, thus we get

$$\alpha(i, j) \varphi\left(\Lambda_\chi(\coprod i, \coprod j)\right) \leq \psi\left(\Xi(i, j)\right). \tag{2.20}$$

Also, if we take $\varphi(t) = \frac{t}{5}$, $\psi(t) = \frac{t}{3}$, $\mu_1 = \frac{1}{2}$, $\mu_2 = \frac{1}{3}$, $\chi = 3$ and $(i, j) \in [0, 1]$, then we demonstrate as in the figure below that the left side of inequality is less than or equal to the right side. Thus, all the hypotheses of Theorem 2.1 are satisfied, and 0 is a unique fixed point of $\coprod$.

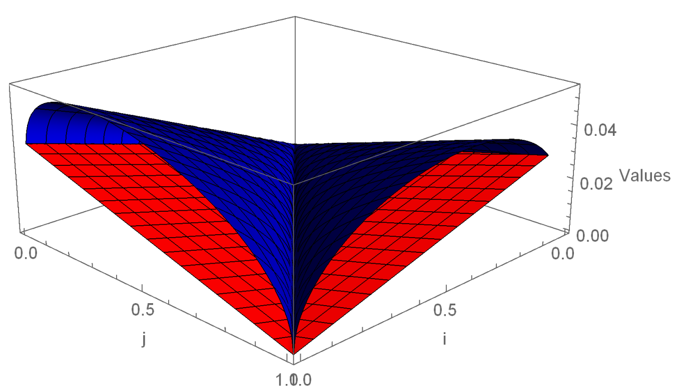

Figure 1. 3D representation of the inequality (2.20).

Corollary 2.1. *Consider $\mathcal{X}_\Lambda$ to be a complete non-Archimedean modular metric space. Presume $\coprod$ be a self mapping on $\mathcal{X}_\Lambda$ satisfying the following conditions:*

(i) $\coprod$ *is a triangular $\alpha-$admissible mapping,*

(ii) *there exists $i_0 \in \mathcal{X}_\Lambda$ such that $\alpha\left(i_0, \coprod i_0\right) \geq 1$,*

(iii) $\coprod$ *is a continuous mapping,*

(iv) *if there exist $\alpha : \mathcal{X}_\Lambda \times \mathcal{X}_\Lambda \to [0,\infty)$, $\psi \in \Psi$, $\varphi \in \Phi$ and $\mu_1, \mu_2 \in (0,1)$ such that $\varphi\left(t\right) > \psi\left(t\right),\ t > 0$ and $\mu_1 + \mu_2 < 1$ satisfying the inequality*

$$\alpha\left(i,j\right)\varphi\left(\Lambda_\chi(\coprod i, \coprod j)\right) \leq \psi\left(\Xi(i,j)\right) \tag{2.21}$$

for all $i, j \in \mathcal{X}_\Lambda$.

Then, $\coprod$ admits a unique fixed point in $\mathcal{X}_\Lambda$.

Corollary 2.2. *Let $\coprod$ be a self-mapping on a complete non-Archimedean modular metric space $\mathcal{X}_\Lambda$. If there exist $\psi \in \Psi$ and $\mu_1, \mu_2 \in (0,1)$ such that $\varphi\left(t\right) > \psi\left(t\right),\ t > 0$ and $\mu_1 + \mu_2 < 1$ satisfying the inequality*

$$\Lambda_\chi(\coprod i, \coprod j) \leq \psi\left(\Xi(i,j)\right) \tag{2.22}$$

for all $i, j \in \mathcal{X}_\Lambda$. Then, $\coprod$ admits a unique fixed point in $\mathcal{X}_\Lambda$.

3. An application to a nonlinear Fredholm integral equation

In this part, we investigate the nonlinear Fredholm integral equation in the setting of a non-Archimedean modular metric space. Let $\mathcal{X} = C[\tau, \upsilon]$ be a set of all real continuous function on $[\tau, \upsilon]$ with a non-Archimedean modular metric $\Lambda_\chi\left(\gamma, \delta\right) = \frac{1}{\chi}\left|\gamma - \delta\right| = \frac{1}{\chi}\max_{t\in[\tau,\upsilon]}\left|\gamma - \delta\right|$, for all $\gamma, \delta \in C[\tau, \upsilon]$ and $\chi \in (0,1)$. Then $\mathcal{X}_\Lambda$ is a non-Archimedean modular metric space. Now, we consider the nonlinear Fredholm integral equation

$$\iota\left(a\right) = u\left(a\right) + \frac{1}{\upsilon - \tau}\int_\tau^\upsilon K\left(a, b, \iota\left(b\right)\right)db, \tag{3.1}$$

where $a, b \in [\tau, \upsilon]$. Assume that $K : [\tau, \upsilon] \times [\tau, \upsilon] \times \mathcal{X} \to R$ and $u : [\tau, \upsilon] \to R$ continuous where $u(a)$ is a given function in $\mathcal{X}$.

Theorem 3.1. *Suppose $\mathcal{X}_\Lambda$ be a complete non-Archimedean modular metric space with*

$$\Lambda_\chi\left(\gamma, \delta\right) = \frac{1}{\chi}\left|\gamma - \delta\right| = \frac{1}{\chi}\max_{t\in[\tau,\upsilon]}\left|\gamma - \delta\right|,$$

for all $\gamma, \delta \in C[\tau, \upsilon]$, $\chi \in (0,1)$ and $\coprod : \mathcal{X}_\Lambda \to \mathcal{X}_\Lambda$ be an operator defined by

$$\coprod \iota\left(a\right) = u\left(a\right) + \frac{1}{\upsilon - \tau}\int_\tau^\upsilon K\left(a, b, \iota\left(b\right)\right)db. \tag{3.2}$$

If there exist $\wp \in [0,1)$, $\mu_1, \mu_2 \in (0,1)$ with $\mu_1 + \mu_2 < 1$ such that for all $\iota, \jmath \in \mathcal{X}_\Lambda$, $a, b \in [\tau, \upsilon]$ satisfying the following inequality

$$\begin{gathered}\left|K\left(a, b, \iota\left(a\right)\right) - K\left(a, b, \jmath\left(a\right)\right)\right| \leq \wp\Xi\left(\iota\left(a\right), \jmath\left(a\right)\right), \\ \Xi\left(\iota\left(a\right), \jmath\left(a\right)\right) = \left|\iota\left(a\right) - \jmath\left(a\right)\right|^{\mu_1}.\left|\iota\left(a\right) - \coprod\iota\left(a\right)\right|^{\mu_2}.\left|\jmath\left(a\right) - \coprod\jmath\left(a\right)\right|^{1-\mu_1-\mu_2}.\end{gathered} \tag{3.3}$$

Then, the integral equation (3.1) has a unique solution in $\mathcal{X}_\Lambda$.

Proof. From (3.1) and (3.2), we have

$$
\begin{aligned}
\left|\coprod \iota(a)-\coprod \jmath(a)\right| &\leq \frac{1}{|\upsilon-\tau|}\left|\int_{\tau}^{\upsilon} K(a,b,\iota(a))\,db-\int_{\tau}^{\upsilon} K(a,b,\jmath(a))\,db\right| \\
&\leq \frac{1}{|\upsilon-\tau|}\int_{\tau}^{\upsilon}\left|K(a,b,\iota(a))-K(a,b,\jmath(a))\right|db \\
&\leq \frac{\wp}{|\upsilon-\tau|}\int_{\tau}^{\upsilon}\Xi(\iota(a),\jmath(a))\,db \\
&\leq \frac{\wp}{|\upsilon-\tau|}\int_{\tau}^{\upsilon}|\iota(a)-\jmath(a)|^{\mu_1}.|\iota(a)-\coprod\iota(a)|^{\mu_2}.|\jmath(a)-\coprod\jmath(a)|^{1-\mu_1-\mu_2}db.
\end{aligned} \tag{3.4}
$$

Taking maximum on both sides for all $a\in[\tau,\upsilon]$, we get

$$
\begin{aligned}
\Lambda_{\chi}\left(\coprod\iota,\coprod\jmath\right) &= \frac{1}{\chi}\max_{a\in[0,1]}\left|\coprod\iota(a)-\coprod\jmath(a)\right| \\
&\leq \frac{\wp}{|\upsilon-\tau|}\max_{a\in[\tau,\upsilon]}\int_{\tau}^{\upsilon}\frac{1}{\chi}|\iota(a)-\jmath(a)|^{\mu_1}.\frac{1}{\chi}|\iota(a)-\coprod\iota(a)|^{\mu_2}.\frac{1}{\chi}|\jmath(a)-\coprod\jmath(a)|^{1-\mu_1-\mu_2}db \\
&\leq \frac{\wp}{|\upsilon-\tau|}\max_{a\in[\tau,\upsilon]}\left[\frac{1}{\chi}|\iota(a)-\jmath(a)|^{\mu_1}.\frac{1}{\chi}|\iota(a)-\coprod\iota(a)|^{\mu_2}.\frac{1}{\chi}|\jmath(a)-\coprod\jmath(a)|^{1-\mu_1-\mu_2}\right]\int_{\tau}^{\upsilon}db \\
&= \wp\left[\Lambda_{\chi}(\iota,\jmath)^{\mu_1}.\Lambda_{\chi}(\iota,\coprod\iota)^{\mu_2}.\Lambda_{\chi}(\jmath,\coprod\jmath)^{1-\mu_1-\mu_2}\right] \\
&= \wp\Xi(\iota,\jmath).
\end{aligned} \tag{3.5}
$$

Thus, all the conditions of Corollary 2.2 are satisfied by setting $\psi(t)=\wp t$ for all $t>0$, where $\wp\in[0,1)$ and hence the integral equation (3.2) has a unique solution in $\mathfrak{X}_{\Lambda}$. □

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References

[1] Khojasteh, F., Shukla, S., Radenovic, S.: *A new approach to the study of fixed point theorems via simulation functions*.Filomat. **29** (6), 1189-1194 (2015).

[2] Radenovic, S., Chandok, S.: *Simulation type functions and coincidence point results*. Filomat. **32** (1), 141-147 (2018).

[3] Samet, B.: *Best proximity point results in partially ordered metric spaces via simulation functions*. Fixed Point Theory and Applications. **2015** (232), (2015).

[4] Tchier, F., Vetro, C., Vetro, F.: *Best approximation and variational inequality problems involving a simulation function*. Fixed Point Theory and Applications. **2016** (26), (2016).

[5] Sawangsup, K., Sintunavarat, W.: *Fixed point results for orthogonal $Z-$contraction mappings in $O-$complete metric spaces*. International Journal of Applied Physic and Mathematics. **10** (1), 33-40 (2020).

[6] Joonaghany, G.H., Farajzadeh, A., Azhini, M., Khojasteh, F.: *A new common fixed point theorem for Suzuki type contraction via generalized $\Psi-$simulation functions*. Sahand Communications in Mathematical Analysis. **16** (1), 129-148 (2019).

[7] Joonaghany, G. H., Karapınar, E., Khojasteh, F., Radenovic, S.: *Study of $\Gamma-$simulation functions, $Z_{\Gamma}-$contractions and revisiting the $L-$contractions*. Filomat. **35** (1), 201-224 (2021).

[8] Karapınar, E.: *Revisiting the Kannan type contractions via interpolation*. Advances in the Theory of Nonlinear Analysis and its Application. **2**, 85–87 (2018).

[9] Karapınar, E., Agarwal, R. P., Aydi, H.: *Interpolative Reich–Rus–Ćirić type contractions on partial metric spaces*. Mathematics. **6**, 256 (2018).

[10] Kesik, D., Büyükkaya, A., Öztürk, M.: *On modified interpolative almost $E-$type contraction in partial modular $b-$metric spaces*. Axioms. **12** (7), 669 (2023).

[11] Karapınar, E., Fulga, A., Roldán López de Hierro, A. F.: *Fixed point theory in the setting of $(\alpha, \beta, \phi, \psi)$interpolative contractions*. Advances in Difference Equations. **2021**, 339 (2021).

[12] Karapınar, E., Aydi, H., Mitrovic, D.: *On interpolative Boyd–Wong and Matkowski type contractions*. Canadian Mathematical Bulletin. **11**(2), 204–212 (2020).

[13] Karapınar, E., Fulga, A., Yesilkaya, S.S.: *New results on Perov-interpolative contractions of Suzuki type mappings*. Journal of Function Spaces. **2021**, Article ID: 9587604.

[14] Benterki, A.: *Some data dependences results from using $C-$class functions in partial metric spaces*. Universal Journal of Matematics and Applications. **7** (4), 152-162 (2024).

[15] Duman, O.: *Controllability analysis of fractional order delay differential equations via contraction principle*. Journal of Matematical Sciences and Modelling. **7**(3), 121-127 (2024).

[16] Saleem, N., Ahmad, H., Aydi, H., Gaba, Y.U.: *On some coincidence best proximity point results*. Journal of Mathematics. **2021**, Article ID: 8005469.

[17] Saleem, N., Işık, H., Khaleeq, S., Park, C.: *Interpolative Ciri ´c-Reich-Rus-type best proximity point results with applications*. AIMS Mathematics. **7** (6), 9731–9747 (2022).

[18] Bashir, S. Saleem, N., Husnine, S.M.: *Fixed point results of a generalized reversed F-contraction mapping and itsapplication*. AIMS Mathematics. **6** (8), 8728–8741 (2021).

[19] Latif, A., Saleem, N., Abbas, M.: *α-optimal best proximity point result involving proximal contractionmappings in fuzzy metric space*. Journal of Nonlinear Sciences and Applications. (10), 92–103 (2017).

[20] Chistyakov, V.V.: *Modular metric spaces, I: Basic concepts*. Nonlinear Analysis. **72**, 1-14 (2010).

[21] Chistyakov, V.V.: *Modular metric spaces, II: Application to superposition operators*. Nonlinear Analysis. **72**, 15-30 (2010).

[22] Girgin, E., Büyükkaya, A., Kuru, N. K., Younis, M., Öztürk, M.: *Analysis of Caputo-type non-linear fractional differential equations and their Ulam–Hyers stability*. Fractal and Fractional. **8** (10), 558 (2024).

[23] Girgin, E., Büyükkaya, A., Kuru, N.K., Öztürk, M.: *On the impact of some fixed point theorems on dynamic programming and RLC circuit models in R-modular b-metric-like spaces*. Axioms. **13**(7), 441 (2024).

[24] Büyükkaya, A., Öztürk, M.: *Multivalued Sehgal-Proinov type contraction mappings involving rational terms in modular metric spaces*. Filomat. **38** (10), 3563-3576 (2024).

[25] Büyükkaya, A., Öztürk, M.: *On Suzuki-Proinov type contractions in modular b-metric spaces with an application*. Communications in Advanced Mathematical Sciences. **7** (1), 27-41 (2024).

[26] Khan, M. S., Swaleh M., Sessa, S.: *Fixed point theorems by altering distances between the point*. Bulletin of the Australian Mathematical Society. **30**, 1-9 (1984).

[27] Berinde, V.: *Generalized contractions in quasimetric spaces*. Seminar on fixed point theory, Babeş-Bolyai University. **3**, 3-9 (1993).

[28] Samet, B., Vetro, C., Vetro, P.: *Fixed point theorems for $\alpha-\psi-$contractive type mappings*. Nonlinear Analysis. **75**, 2154-2165 (2012).

[29] Karapınar, E., Kumam, P., Salimi, P.: *On a $\alpha-\psi-$Meir-Keeler contractive mappings*. Fixed Point Theory and Applications. **2013**, 94 (2013).

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