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Adaptation of a comprehensive simplification method to the Adaptive Exponential Integrate and Fire Neuron and Its FPGA-based verification

Kapsamlı bir basitleştirme yönteminin Uyarlanabilir Üstel Entegre ve Ateşlemeli Nöronuna uyarlanması ve FPGA tabanlı doğrulaması

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Abstract

The preference of a comprehensive method usage is as important as less hardware usage on digital device-based implementations. The mathematical series expansions have a widespread usage in the transformation of expressions into simpler forms. The exponential, trigonometric, logarithmic, etc. functions are usually converted to simpler expressions for digital implementation easiness. In these implementations, it is an expected output that as the operands of the series increases, the revised model converges to the original one. However, the most appropriate number determination of these operands is important for hardware efficiency. In here, the exponential expression of the Adaptive Exponential Integrate and Fire (ADEX) neuron model is expanded up to the tenth operand of the Taylor series. Then, an optimum operand number is identified for getting both hardware utilization efficiency and neuronal meaningfulness. The differences between the original and revised models are compared with the error calculations and the neuronal observations. Lastly, the revised ADEX neuron model is realized by FPGA device to prove the efficiency of the proposed adaptation.

Keywords: Adaptive exponential integrate and fire (ADEX) neuron model, Field Programmable Gate Array (FPGA), Digital implementation, Neuromorphic engineering.

Öz

Dijital cihaz tabanlı gerçekleştirimlerde kapsamlı bir yöntemin kullanılmasının tercihi, az donanım kullanımı kadar önemlidir. Matematiksel seri açılımları, ifadelerin daha basit biçimlere dönüştürülmesinde yaygın bir kullanıma sahiptir. Üstel, trigonometrik, logaritmik vb. işlevler genellikle dijital uygulama kolaylığı için daha basit ifadelere dönüştürülür. Bu uygulamalarda, serinin işlenenleri arttıkça, revize edilmiş modelin orijinal modele yakınsaması beklenen bir çıktıdır. Bununla birlikte, bu işlenenlerin en uygun sayısını belirlenmesi donanım verimliliği için önemlidir. Burada, Uyarlanabilir Üstel Entegre ve Ateşlemeli (ADEX) nöron modelinin üstel ifadesi, Taylor serisinin onuncu işlenenine kadar genişletilmiştir. Daha sonra, hem donanım kullanım verimliliği hem de nöronal anlamlılığı elde etmek için optimum bir işlenen sayısı belirlenmiştir. Orijinal ve revize edilmiş modeller arasındaki farklar, hata hesaplamaları ve nöronal gözlemlerle karşılaştırılmıştır. Son olarak, revize edilmiş ADEX nöron modeli, önerilen adaptasyonun verimliliğini kanıtlamak için FPGA cihazı tarafından gerçekleştirilmiştir.

Anahtar kelimeler: Uyarlanabilir üstel entegre ve ateşlemeli (ADEX) nöron modeli, Alan Programlanabilir Kapı Dizisi (FPGA), Dijital gerçekleştirim, Nöromorfik mühendislik.

1 Introduction

The recent scientific studies focus on the brain functionality from enlightened of its neuronal behavior and structure. Its functionality and morphological structure are based on a collaborative working of the neurons. Thus, the investigation of a single neuron becomes important research issue for understanding the structure of the brain and nervous system. The dynamic behaviors of a real neuron are expressed by the various neuron model and these models are usually represented by the interrelated ordinary differential equations (ODEs) [1-5]. When any biological neuron model defines the response of a neuron, the scope of this model can be classified according to the reflection level of the real neurons' biophysical functionalities to the mathematical definitions. These model definitions can be generalized into three categories [6]: The models, which reflect the trans-inductance definitions such as Hodgkin-Huxley (HH) and Morris-Lecar (ML) models, are in the first category and they describe the details about the biophysical functionalities [4, 5]. The second category involves the connectionist models such as FitzHugh–Nagumo and Hindmarsh-Rose models [7, 8]. These type models focus on the production of the membrane potential rather than the biological functionalities. The last category is neural networks such as Wilson-Cowan model that take into account the synaptic connections between the neuronal populations [9]. This study dwells on the Adaptive

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Exponential Integrate and Fire (ADEX) neuron model. This model is a connectionist neuron model, and it is derived from the Integrate&Fire (IF) type model that is one of the basic neuron models [10]. After IF model, its spike-triggered adaptation integrate and fire mechanism version was introduced by Fuortes and Mantegazzini [11]. Subsequently, the leaky-IF model with quadratic descriptions has been introduced for spike-triggered adaptation and they were combined by Treves [12] and Latham et al. [13]. Then, the reset parameter has been added to these definitions as the second state variable Izhikevich [1] and Richardson [14]. Lastly, the parabolic expression in the Izhikevich model is changed by an exponential function in the ADEX neuron model and finally, the ADEX neuron model is presented to the literature [15].

On the other hand, several studies about the hardware emulator circuits of the biological structure, especially biological neuron models are available in literature [17–20]. It has been recently uttered that the bio-inspired structures take place in the information processing process and the control structure in the near future [21]. Thus, the cost-effective hardware implementations of the neurons in a network structure attract the attention. The neuronal structures, which are implemented by the analog [22-28] and the digital [16], [29-31] hardware, are introduced to literature. The processing capacities and the realization speeds of the used hardware, the achieving sufficient accuracy and the providing a comprehensive solution are their desired features. However, the specifying of hardware that offers all these features together is a hard problem. While it is preferred to usages of biological neuron model that has the detailed descriptions of the biological processes in the numerical simulations, two important key points should be considered in the electronic hardware realization process: i) There should be no change in the model dynamics in the realization process, and ii) the used hardware should supply a reasonable part of the mentioned features [32]. While the hardware capacity problems are encountered in the analog programmable platforms-based implementations, the realization complicacy of the non-linear expressions arises in the digital programmable platforms. The mentioned problem about analog platforms is solved by increasing the number of used elements. The mentioned problem with digital platforms is generally tried to be solved by simplifying the nonlinear expression in the model [33-36]. For example, in Ref [33], the parabolic expression in the Izhikevich neuron model has been transformed into piecewise-linear functions for realization easiness on the digital environments. After this transformation, stochastic methods have been used successfully to identify the optimum values of some parameters process in Ref [34]. In Ref [35], two coupled HR neurons have been modified for digital implementation expediency and a low-cost hardware realization. The mathematical definition of the chemical synaptic connection between HR neurons have been transport a approximate function in Ref [36] and the chemical synaptic connection between neurons have been able to embedded to the digital hardware. Additionally, the exponential expression of the ADEX Neuron model has been simplified by utilizing the successful approaches for digital implementation easiness in literature [37-39]. This paper presents also an alternative approach for digital implementation of the ADEX neuron model with the programmable digital hardware. The mathematical series expansion method offers a general solution for contractions of the nonlinear expressions on the digital platforms. The exponential, trigonometric, logarithmic etc. functions have been converted to the addition and multiplication operations. The number of multiplexer is limited in digital hardware, so it is generally preferable to use as little as possible. However, i) if the converting expression is consistent with the characteristic of the original model, ii) unless the model accuracy is compromised, iii) if it has been used a reasonable number of multiplexer, and iv) most importantly, if the proposed alternative approach has a comprehensive application field, it is considered that the discussing of this alternative approach makes significant contributions to literature. In here, the exponential expression of the ADEX neuron model has been re-constructed by utilizing from the Taylor series expansion for realization easiness on digital platform. As the number of operands of the series expansion increases, it is an expected output that the error between modified and original models decreases. Thus, the exponential expression is expanded up to the tenth operand in the Taylor series. As the order of Taylor series increases, it is required to use the more multiplexer. Thus, both the error calculations have been reported and the neuronal dynamics of the modified and original systems are observed with the numerical simulations. According to these error calculation and the numerical simulation results, the fourth order expansion is suitable for the simplification process of this exponential nonlinear expression. As a verification of efficiency of this conversion, the ADEX neuron model, which consists of four operands-Taylor series instead of the exponential function, is realized with Field Programmable Gate Array (FPGA) platform, successfully.

This paper is organized as follows: After the introducing of the ADEX definition and Taylor series expansion, the numerical simulation results, the error calculation and the correlation analyses of original and modified systems are presented in Section II. The hardware verification results of the proposed process are given by using FPGA-based implementations results in Section-III. The obtained outcomes are discussed in the last section.

2 Adaptive Exponential Integrated and Fire (ADEX) Neuron Model

The ADEX neuron model is in the connectionist neuron models category and it is based on the IF type model that is one of the simplest neuron models. The ADEX neuron is derived from the Izhikevich neuron model by changing the nonlinear expressions. The Izhikevich neuron model points out with its computational efficiency and modeling achievement in terms of ability to exhibit the rich neural dynamics. Similar to Izhikevich's one, the ADEX neuron model is a successful neuron model and it is defined by ODEs in Equation 1:

$$C\frac{dV}{dt} = -g_L(V - E_L) + g_L\Delta_T \exp\left(\frac{V - V_T}{\Delta_T}\right) + I_{inj} - \omega$$
$$\tau_\omega \frac{d\omega}{dt} = \alpha(V - E_L) - \omega \quad (1)$$

In Equation 1, the $\{V, C, \omega, \alpha, \tau_w, I, g_L, E_L, V_T, \Delta_T\}$ symbols represents the membrane potential and capacitance, the adaptation variable, adaptation coupling parameter and the adaptation time constant, the input current, leak conductance and the leak reversal potential, the threshold voltage and the slope factor, respectively. The working process in this neuron model is as follows: The external input current (I) is induced the membrane potential (V) by exceeding the threshold voltage (V_T). The exponential term provides positive feedback and it causes the occurrence of the spike behavior.

Additionally, the exponential term corresponds to the sodium channel and the sodium channel causes spike formation in the Hodgkin-Huxley-type neuron model. The positive feedback causes the membrane potential to go to infinity and the reset conditions in Equation 2 are applied in order to prevent infinitive definitions when the membrane potential reaches a finite value.

$$V > 0 \Longrightarrow V \rightarrow V_R \;\&\; \omega \rightarrow \omega_R = \omega + b \quad (2)$$

The reset value of the membrane potential is equalized to V_R and the ω reset value is determined by utilizing the constant b value. The substantial neuronal patterns form by the setting these variables to the optimum values [40]. The list of neuronal dynamics and the values of these parameters are given in Table 1 [41].

As mentioned in introduction part, the digital device-based implementations of the nonlinear expressions (as the exponential function in Equation 1) are difficult. The mathematical series expansion methods offer a general solution for contractions of these nonlinear expressions on the digital platforms. Not only the exponential expression in Equation 1, but many more functions (trigonometric, logarithmic, etc.) can be also converted to a combination of addition/subtraction and multiplication/division operations. Taylor series expansion method is the most well-known of them [42-44]. Taylor series expanses a mathematical function to the infinitive sums of the converted function that is calculated from the derivative at a single point of the original functions. When this single point is equal to zero, this series is named a Maclaurin series [44]. It is common practice to use a finite number of operands to extend a function to a series. The partial sum of the $n + 1$ operands of the Taylor series is an n^{th} -order polynomial. The approximate function converges to the original one at the higher n orders. The Taylor series is a power series (m) and its general expansion is as in Equation 3 [42, 43]:

$$f(m) + \frac{f'(m)}{1!}(x - m) + \frac{f''(m)}{2!}(x - m)^2 + \frac{f'''(m)}{3!}(-m)^3 + \ldots + \frac{f^n(m)}{n!}(x - m)^n \quad (3)$$

These expressions can be rewritten in a generalized form as in Equation 4:

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(m)}{n!}(x - m)^n, \quad (4)$$

The Euler's number-based exponential function is calculated as the sum of the infinite Taylor- Maclaurin series as in Equation 5 [44]:

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} \quad (5)$$

The error calculation between original and modified models is provided insight their compatibleness. Root mean square error (RMSE) is a common preferred error calculation method and represents the quadratic mean of the differences between these data. In fact, the RMSE is an accuracy measurement method for comparing error calculations of datasets (N) of different models [45]. The RMSE calculation always yields a positive result and its lower values means less erroneous. The calculated error values changes by depending on the number of the used data. RMS error is calculated by Equation 6 [46].

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(x_{i(original\_data)} - x_{i(modified\_data)}\right)^2} \quad (6)$$

The RMSE calculations usually provide information about the difference in the amplitudes of the datasets. However, when investigating the similarities of the original and modified biological neuron models, it is very important to determine their phase relationships. Thus, the calculations of the phase error between two neuronal dynamics are also recorded in the literature. One of the phase error calculation methods is given in Equation 7 [47]:

$$\emptyset(t) = arctan\frac{s_1(t) - s_1(0)}{s_2(t) - s_2(0)} \quad (7)$$

Table 1. The values of the parameters in the ADEX neuron model for substantial neural dynamics [41].

Neural Dynamics	C (pF)	g_L (nS)	E_L (mV)	V_T (mV)	Δ_T (mV)	α (nS)	b (pA)	τ_w (ms)	V_R (mV)	I (pA)
Regular Spiking	200	10	-70	-50	2	2	0	30	-58	500
Tonic Spiking	200	12	-70	-50	2	2	60	300	-58	500
Intrinsic Bursting	130	18	-58	-50	2	4	120	150	-50	400
Bursting	200	10	-58	-50	2	2	30	120	-46	210
Inhibition Spiking	200	12	-70	-50	2	-10	0	120	-58	110
Inhibition Bursting	200	12	-70	-50	2	-10	30	300	-47	110

If the biological neuron model is identified as $\dot{x} = f(x)$, $s_1(t) = s_1(x, \dot{x}, \ddot{x} \dots)$ and $s_2(t) = s_2(x, \dot{x}, \ddot{x} \dots)$ are arbitrary combinations of the state variables. The phase synchronizations of two neuronal dynamics are provided in the $\emptyset_1(t) \cong \emptyset_2(t)$ situation. According to this situation, the phase differences between these dynamics can be represented by utilizing the Equation 8.

$$Phase\ Error = \sqrt{\frac{1}{N}\sum_{i=1}^{N} |\emptyset_1(i) - \emptyset_2(i)|} \tag{8}$$

On the other hand, the correlation analysis is another statistical method [48]. This method provides information about the direction and amount of the relationship between different datasets. The correlation coefficient ($\rho_{X,Y}$) is a measure of the linear relationship between two independent variables(X, Y) and this coefficient is between the $-1 \leq \rho_{X,Y} \leq 1$ range. When the correlation coefficient approaches zero, it indicates the existence of a weak relationship between these variables. If these variables increase or decrease together, there is a positive relationship. If one variable increases while the other decreases (or vice versa), there is a negative relationship. The correlation calculation is as in Equation 9 for two independent variables.

$$\rho_{X,Y} = \frac{(N\sum_{i=1}^{N} X_i Y_i) - ((\sum_{i=1}^{N} X_i)(\sum_{i=1}^{N} Y_i))}{(\sqrt{(N\sum_{i=1}^{N} X_i^2) - (\sum_{i=1}^{N} X_i)^2})(\sqrt{(N\sum_{i=1}^{N} Y_i^2) - (\sum_{i=1}^{N} Y_i)^2})} \tag{9}$$

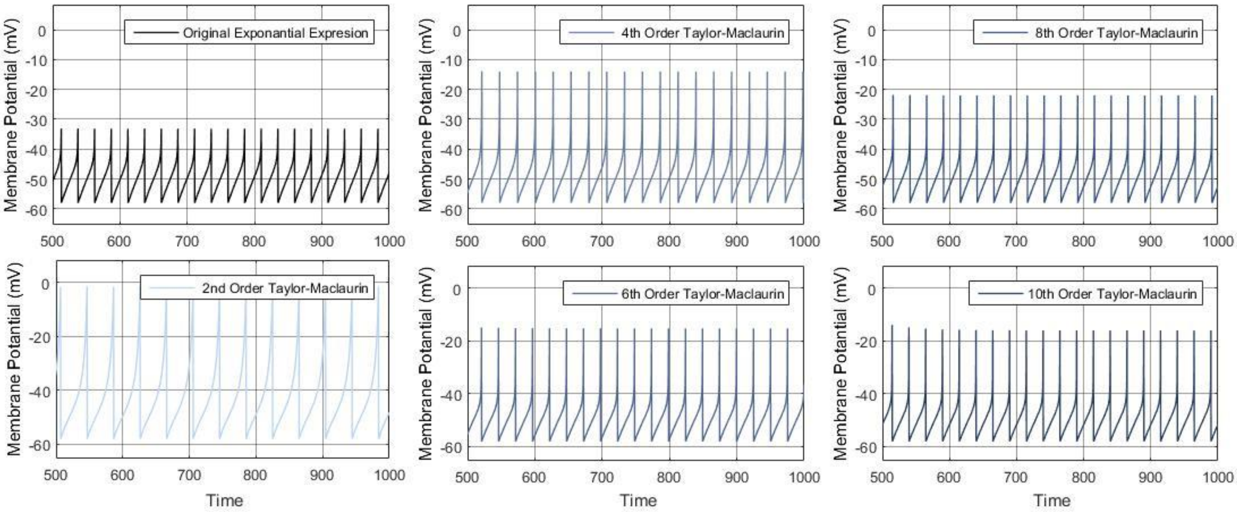
where, while the X_i independent variable represents the dataset of the original ADEX neuron model, the Y_i independent variable is the dataset of the modified neuron model.

These mentioned error calculations have been executed for determining the compatibility between the original and the modified ADEX models. In these calculations, the exponential expression in the modified neuron model is extended to the tenth operand of the Taylor series. In this process, the error calculation results, which are calculated for N=3000 term, are recorded to Table 2. This table includes both the RMSE and phase error results. Additionally, the correlation results, which are a measure of their linear relationships, have been also calculated and recorded in the same table.

Table 2. Error calculation results for Taylor- Maclaurin Serial Expansion based ADEX Neuron Model

Characteristic Dynamic	2nd Order Taylor-Maclaurin Series Expansion			3rd Order Taylor-Maclaurin Series Expansion			4th Order Taylor-Maclaurin Series Expansion		
	RMS Error	Correlation	Phase Error	RMS Error	Correlation	Phase Error	RMS Error	Correlation	Phase Error
Regular Spiking	14.8123	0.0031	0.4903	9.8007	0.0054	0.3071	8.5729	-0.0022	0.2732
Tonic Spiking	8.2946	0.022	0.1001	NaN*	NaN*	NaN*	3.5627	0.0527	0.0587
Intrinsically Bursting	5.9038	0.0278	0.1307	NaN*	NaN*	NaN*	3.1829	0.4271	0.0825
Bursting	12.5658	0.0036	0.2956	8.3172	0.037	0.3039	6.0215	0.0741	0.2809
Inhibition Spiking	11.6437	0.1014	0.29	NaN*	NaN*	NaN*	6.3721	0.1221	0.1869
Inhibition Bursting	8.253	0.351	0.3741	NaN*	NaN*	NaN*	4.5509	0.2711	0.4002
Irregular Spiking	9.236	-0.0012	0.3037	5.0977	-0.0101	0.2765	3.9316	0.033	0.2854
Characteristic Dynamic	**5th Order Taylor-Maclaurin Series Expansion**			**6th Order Taylor-Maclaurin Series Expansion**			**7th Order Taylor-Maclaurin Series Expansion**		
	RMS Error	Correlation	Phase Error	RMS Error	Correlation	Phase Error	RMS Error	Correlation	Phase Error
Regular Spiking	7.6211	0.0097	0.243	7.8061	0.0286	0.2441	7.4223	0.0321	0.237
Tonic Spiking	NaN*	NaN*	NaN*	1.7611	0.8759	0.0372	3.0816	0.6367	0.0439
Intrinsically Bursting	2.5757	0.602	0.0522	1.7762	0.7857	0.0289	1.7723	0.7933	0.0224
Bursting	4.1496	0.5419	0.2092	0.1756	0.7233	0.1428	2.9733	0.7589	0.105
Inhibition Spiking	NaN*	NaN*	NaN*	5.7537	0.1735	0.1646	NaN*	NaN*	NaN*
Inhibition Bursting	NaN*	NaN*	NaN*	3.5862	0.4993	0.3401	NaN*	NaN*	NaN*
Irregular Spiking	3.6423	0.353	0.2944	3.4187	0.0606	0.2957	3.2985	0.0668	0.2897
Characteristic Dynamic	**8th Order Taylor-Maclaurin Series Expansion**			**9th Order Taylor-Maclaurin Series Expansion**			**10th Order Taylor-Maclaurin Series Expansion**		
	RMS Error	Correlation	Phase Error	RMS Error	Correlation	Phase Error	RMS Error	Correlation	Phase Error
Regular Spiking	7.4141	0.0245	0.2347	7.3945	0.0247	0.2347	7.5104	0.0536	0.2315
Tonic Spiking	2.0661	0.8388	0.0251	2.3628	0.7887	0.0265	1.3587	0.9263	0.0147
Intrinsically Bursting	1.7032	0.8129	0.02	0.1126	0.9994	0.0049	0.0562	0.9999	0.0034
Bursting	2.4044	0.8267	0.0668	2.2663	0.8493	0.0506	0.2353	0.9985	0.0101
Inhibition Spiking	6.3542	-0.0342	0.1748	NaN*	NaN*	NaN*	5.0804	0.3368	0.1248
Inhibition Bursting	2.5547	0.7503	0.2696	NaN*	NaN*	NaN*	1.5395	0.9122	0.1906
Irregular Spiking	3.24	0.0533	0.2939	3.2104	0.0966	0.276	3.0803	0.1197	0.2648

***Not a Number**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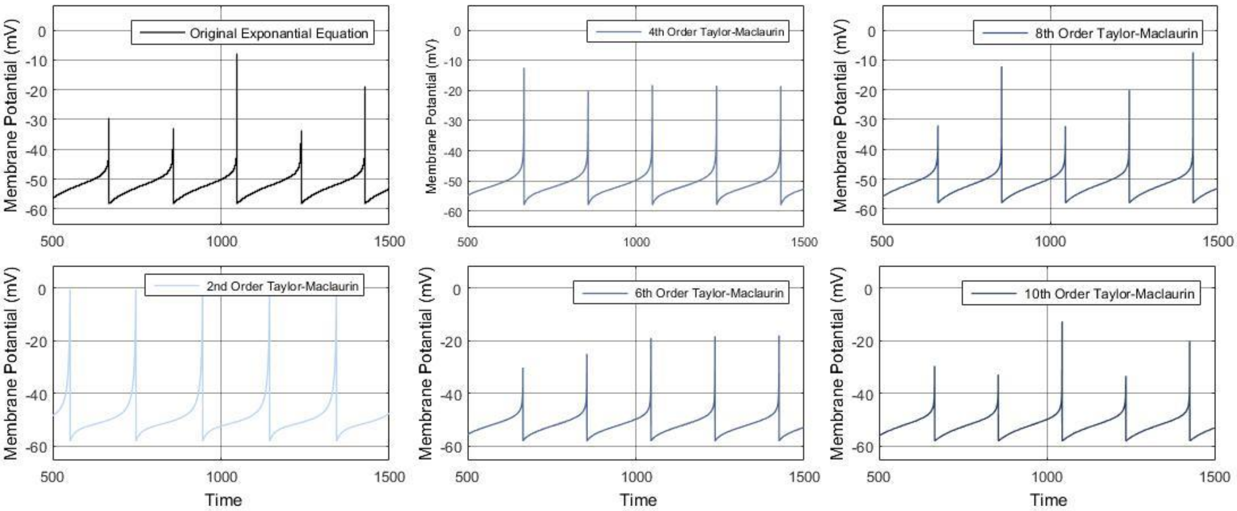
2nd Order Taylor-Maclaurin
6th Order Taylor-Maclaurin
10th Order Taylor-Maclaurin
Membrane Potantial (mV)
Time

(a)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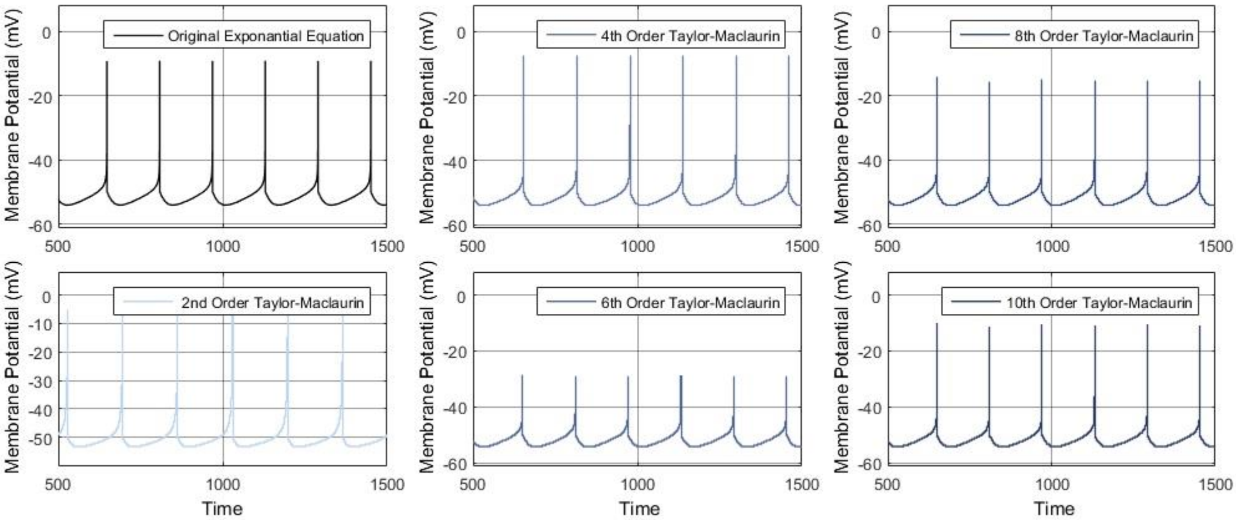
Membrane Potantial (mV)
Time

(b)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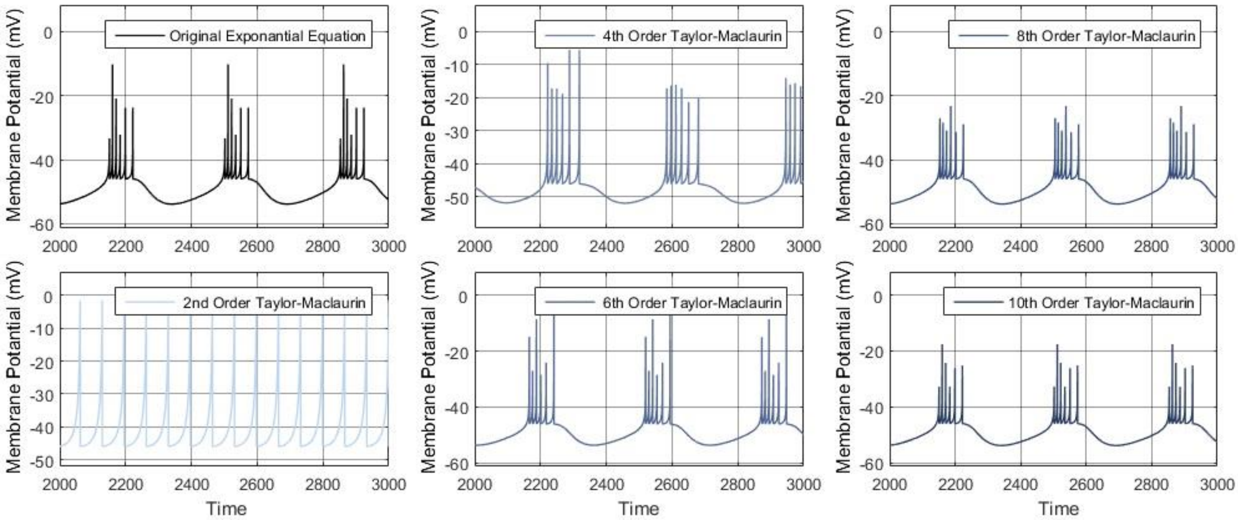
Time

(c)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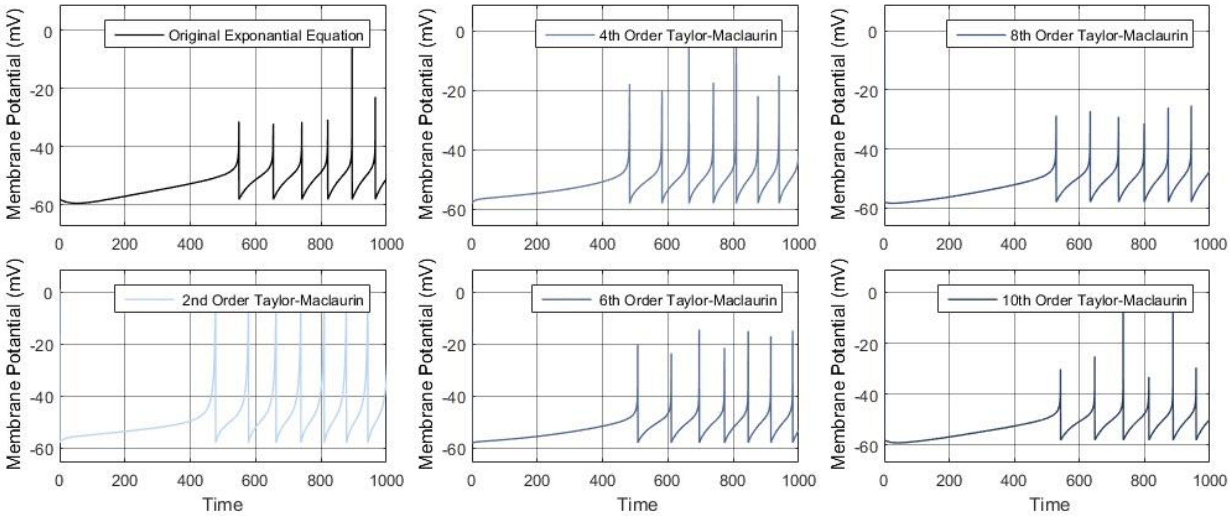
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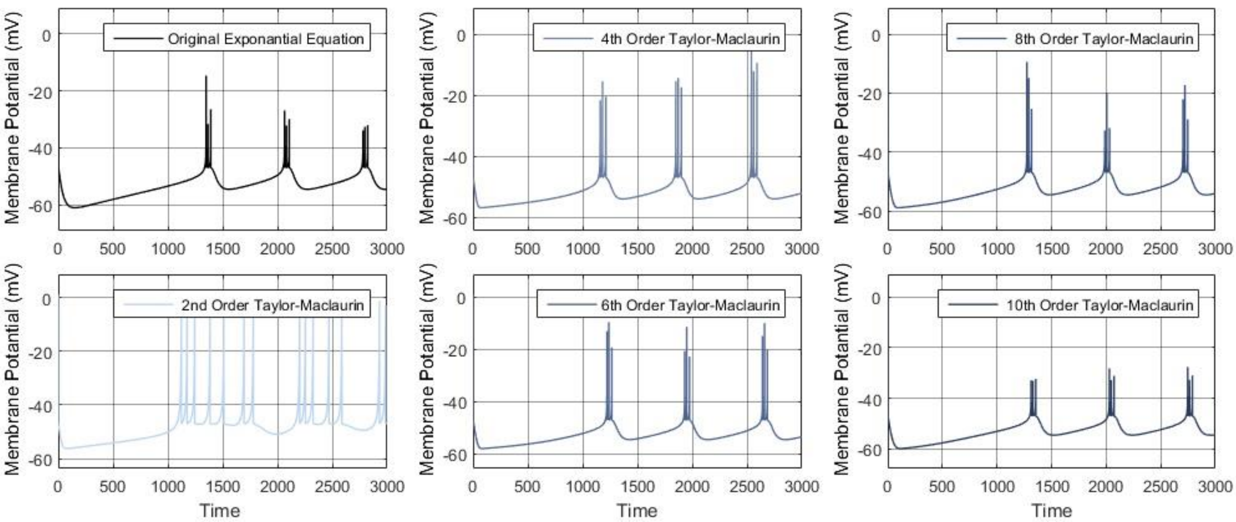
Membrane Potantial (mV)
Time

(e)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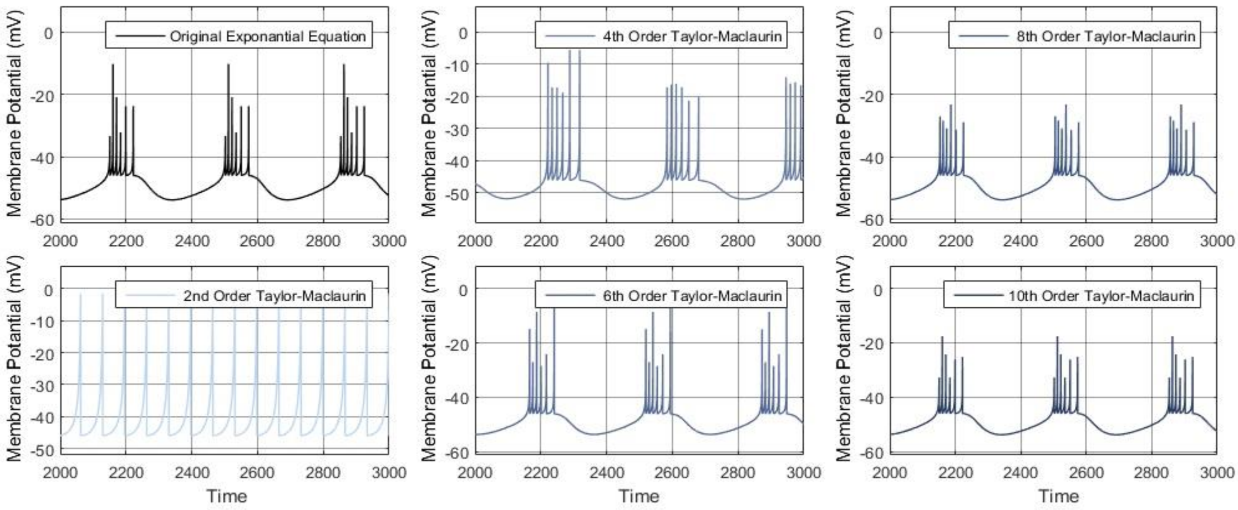
Membrane Potantial (mV)
Time

(f)



(g)

Figure 1. The numerical simulation results, which are includes the membrane potentials in millivolts versus time in milliseconds of the ADEX neuron models, for a) the regular spiking, b) the tonic spiking, c) the intrinsic bursting, d) the bursting, e) the inhibition spiking, f) the inhibition bursting, and g) the irregular spiking neuronal behaviors.

From Table 2, as the operands of the series expansion increases, the error values decrease. This result is in line with the expected output. However, some uncalculated results (Not a Numbers-NaNs) are founded in the odd-order operands of the Taylor series and this expression occurs when a mathematically undefined or invalid operation is performed in the calculations. Accordingly, it is an incomplete practice to evaluate the results by calculating just the error values. For this reason, the numerical simulations studies have also been made in here. Their results allow the observation of both the modified ADEX neuron model and the original one, simultaneously. In these simulations, the original and modified ADEX neuron models are discretized by using the Euler method for $\Delta h = 0.1$ step size. After that, the initial conditions of both models are set to the same values. Their parameters are set to the values in Table 1. The exponential function of the original ADEX neuron model are converted to second, fourth, sixth, eighth and tenth-order Taylor expansions and the numerical simulations are executed these modified structures. The numerical simulation results in MATLAB™ Simulink are presented in Figures 1.a-e for the regular spiking, the tonic spiking, the intrinsic bursting, the bursting, the inhibition spiking, the inhibition bursting, and the irregular spiking neuronal behaviors, respectively.

According to both these error calculations in Table 2 and the numerical simulation results in Figure 1, the fourth order expansion of the Taylor series is suitable for the simplification process of this exponential nonlinear expression. As an efficiency verification of this conversion, the ADEX neuron model, which consists of four operands-Taylor series instead of the exponential function, is implemented with Field Programmable Gate Array (FPGA) platform.

3 The digital implementation of the fourth-order taylor expansion-based ADEX neuron model with FPGA device

An electronic signal has a spatiotemporal quantity and it is continuous (analog) or discrete (digital) characteristic. Although the real world has an analog nature, most of today's information processing technologies are based on digital systems. The membrane potentials measured from real biological neurons are also in analog form. However, the usage of digital equipment is also common when transferring the biological patterns to bio-inspired systems. In this transfer process, it is not always possible to directly perform some continuous form defined nonlinear functions with digital equipment. Thus, it is tried to make these nonlinear terms suitable for digital systems. As mentioned before, the usage of the comprehensive conversions such as the Taylor series is a source of inspiration for further studies.

The modifying process of the exponential function in the ADEX with the Taylor series has been discussed in the previous section. The FPGA-based realization details of the ADEX neuron model defined by the Taylor series element are presented in this section. The FPGA device is a prominent hardware with the following features in the literature: i) ability to process in parallel, ii) re-programmability, iii) reaching high frequencies, iv) enable hardware update with software, and v) resource usage efficiency, etc. Here, the ADEX neuron model is realized by using the SPARTAN-3AN board produced by XILINX™ Company [49]. The System Generator for DSP tool, which allows programming visually, has been used and the calculations have been executed by the fixed-point arithmetic Q (32, 18). A configuration for the ADEX neuron model built on this tool is shown in Figure 2.

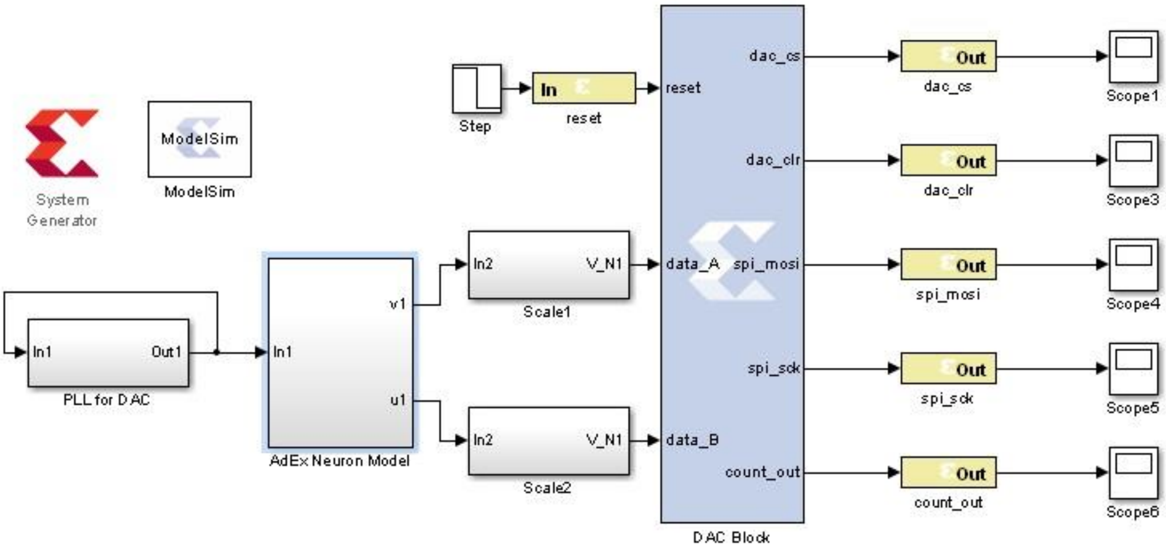


Figure 2. A configuration for the ADEX neuron model built on the System Generator for DSP tool.

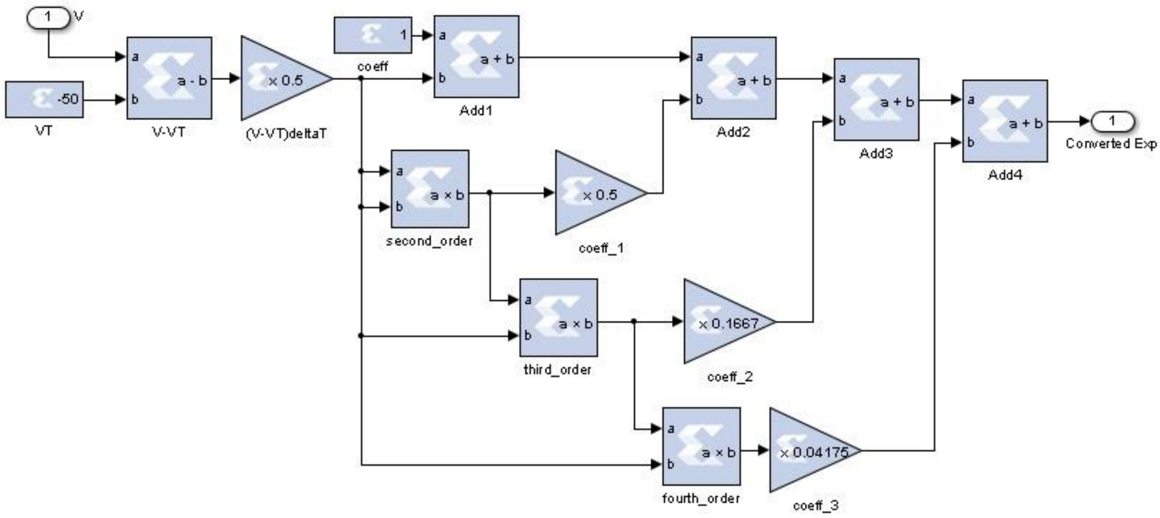
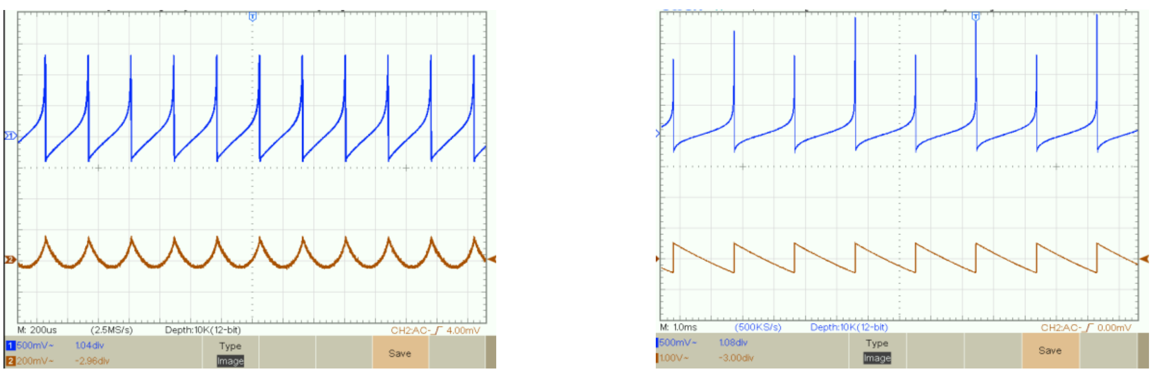
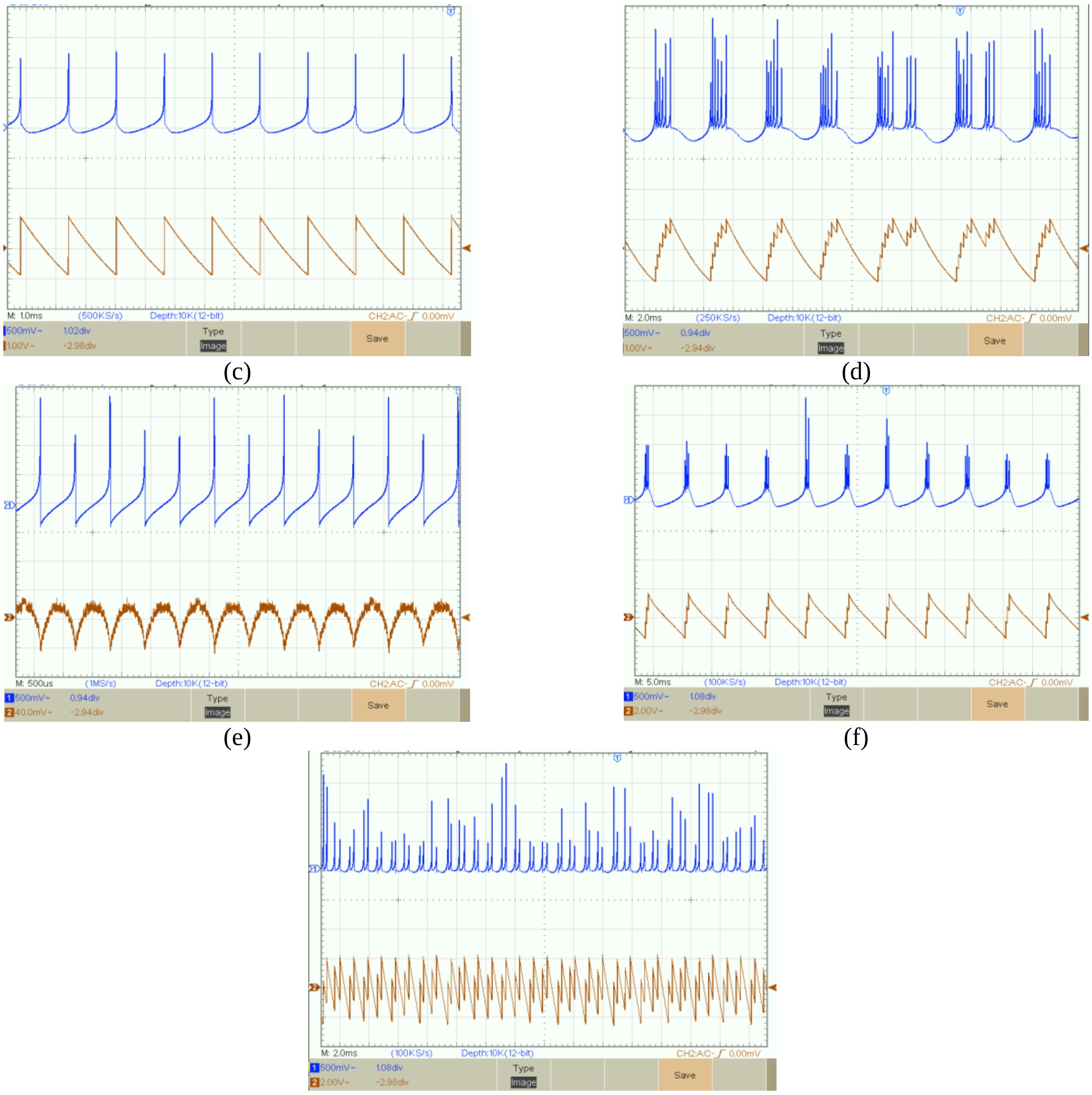


Figure 3. The used predefined-blocks in the System Generator for DSP tool for conversion the exponential function to the fourth-order Taylor series expansion.



(a) (b)



(c) (d) (e) (f) (g)

Figure 4. The realization results of the ADEX on FPGA, which is constructed the fourth-order Taylor series, for a) the regular spiking, b) the tonic spiking, c) the intrinsic bursting, d) the bursting, e) the inhibition spiking, f) the inhibition bursting, and g) the irregular spiking neuronal behaviors.

The predefined-blocks, which are used to perform the exponential function conversion to the fourth-order Taylor series expansion, are presented in Figure 3. After the constructed model in Figure 2 has been converted to VHDL, this model has been transferred to the FPGA board. The obtained outputs have been measured through a digital-analog convertor (DAC) module. The FPGA-based implementation results of the modified ADEX neuron model are recorded by an oscilloscope and they are shown in Figure 4. The experimental implementation results are given in Figures 4.a-e for the regular spiking, the tonic spiking, the intrinsic bursting, the bursting, the inhibition spiking, the inhibition bursting, and the irregular spiking neuronal behaviors, respectively. In Figures 4.a-e, the top pattern is the (V) potential and the bottom pattern is the (ω) adaptation variable.

The exponential expression of the ADEX neuron model has been built by utilizing from the Taylor series expansion for realization easiness on digital platform. As a verification of efficiency of this conversion, the ADEX neuron model, which consists of four operands-Taylor series instead of the exponential function, is realized with FPGA platform. As seen from the results in Figure .4, this process has been completed successfully.

4 Conclusion

This study focuses on the digital device based-implementation of the ADEX neuron model. There are several studies on the implementation of this neuron model in the literature. However, the proposed approach in here constitutes an inspiration for the further studies in terms of providing a comprehensive approach. In fact, the nonlinear exponential term in the definition of the ADEX has been expanded to the Taylor- Maclaurin series and this serial expansion can be applied most of any other nonlinear systems. In this context, the serial expansions of the nonlinear term of the ADEX have been continued until the tenth term. In this process, the amplitude error, the phase error and the correlation analysis between the original and the modified models have been calculated and the calculation results have been recorded to tables. Moreover, the numerical simulation responses of the original and the modified models have been gotten for seven characteristic dynamic patterns of this neuron model. Excess term usage in the serial expansion means the excessive hardware usage. For this reason, the optimum serial expansion degree has been determined as fourth expansion by depending on the numerical simulation observations as well as the calculated errors and correlation results. In the last part of this study, the FPGA-based implementations of the modified ADEX neuron model have been performed for determining the efficiency of this conversion process in a digital system and the fourth-order Taylor expansion have been used in these implementations. The results of the obtained real-time signals have been presented by using an oscilloscope. The element consumption results and the map report are as follows in these FPGA-based implementations: While 1399 of 11776 the 4-LUT inputs are used (11%), 695 of 5888 the SLICEs are occupied (16%). Additionally, 12 of 20 the MULT18X18SIO multipliers are used (60%) and the maximum delay in the system is 1.058 nanoseconds.

Conflict of interest: The authors declare that they have no conflict of interest.

Similarity rate (iThenticate): 18%

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