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Initial Basic Feasible Solution Approach for Transportation Problem: Logarithmic Fermi Approximation

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Ulaştırma Problemi İçin Uygun Başlangıç Çözüm Yaklaşımı: Logaritmik Fermi Yaklaşımı

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Abstract

Transportation problems are considered as a fundamental topic in operations research. In large-scale and complex network structures, initial solutions provide cost effectiveness by influencing the quality and speed of algorithms. Transportation problems aim to balance supply-demand and minimize costs. Appropriate initial solutions save time by producing results close to the optimal solution. Using a single appropriate initial solution algorithm for balanced and unbalanced transportation problems can be more efficient. The new approach proposed in this study is called the Logarithmic Fermi Approach. To analyze the performance of the approach, P1 and P2 test problems with different cost ranges were generated. Analyses have shown that Logarithmic Fermi Method (LFM) and Karagül-Şahin Approximation Method (KSAM) stand out in balanced and unbalanced problems, while North-West Corner Method (NCW) demonstrates poor performance. In large-scale problems, Logarithmic Fermi Method (LFM) and Least Cost Method (LCM) algorithms have produced results close to the optimal solution. The proposed method has shown to have a competitive structure in the analyses.

Keywords: *Transportation problem, basic initial feasible solution, KSAM, LCM, NCW.*

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Öz

Yöneylem araştırmasında ulaştırma problemleri temel bir konu olarak ele alınır. Büyük ölçekli ve karmaşık ağ yapılarında, başlangıç çözümleri algoritmaların kalitesini ve hızını etkileyerek maliyet etkinliği sağlar. Ulaştırma problemlerinde arz-talep dengesi ve maliyetlerin minimize edilmesi amaçlanır. Uygun başlangıç çözümleri optimal çözüme yakın sonuçlar üreterek zaman tasarrufu sağlar. Dengeli ve dengesiz ulaştırma problemleri için tek bir uygun başlangıç çözüm algoritması kullanmak daha verimli olabilir. Bu çalışmada önerilen yeni yaklaşıma Logaritmik Fermi Yaklaşımı adı verilmiştir. Yaklaşımın performansını analiz etmek için farklı maliyet aralıklarına sahip P1 ve P2 test problemleri üretilmiştir. Analizler, dengeli ve dengesiz problemlerde Logaritmik Fermi Yaklaşımı (LFM) ve Karagül-Şahin Yaklaşım Metodu (KSAM)'ın öne çıktığını, Kuzey-Batı Köşe Yöntemi (NCW)'nin ise düşük performans gösterdiğini ortaya koymuştur. Büyük ölçekli problemlerde Logaritmik Fermi Yaklaşımı (LFM) ve En Küçük Maliyetli Göze Metodu (LCM) algoritmaları, optimal çözüme yakın sonuçlar üretmiştir. Önerilen yöntemin analizlerde rekabetçi bir yapısının olduğu görülmüştür.

Anahtar Kelimeler: *Ulaştırma problemi, uygun başlangıç çözüm, KSAM, en küçük maliyetli göze, kuzey-batı köşe.*

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1. Introduction

At the macroeconomic level, Operations Research techniques contribute to economic growth by providing an effective and efficient economic solution system in the optimal planning of resources and infrastructure in a country. At the microeconomic scale, the accumulation of efficiency and effectiveness obtained through Operations Research techniques used at the level of enterprises contributes to the country's economy at the macroeconomic level. This contribution is possible by optimizing the logistics costs of the production and supply process, which starts with raw materials and ends with the delivery of finished products to the customer. At this point, optimization of transportation costs in supply chain networks built from raw material sources to factories, from factories to warehouses, and from warehouses to customers emerges as a fundamental problem. In this context, in Operations Research, the necessary models are developed by deciding which of the different types of transportation problems developed to analyze these processes is appropriate. According to the type of transportation problem modelled, solutions that optimize transportation costs can be developed by using different optimization solutions and techniques.

In this study, the Transportation Problem, which is the most basic structure among logistics models, is discussed. For the Transportation Problem, either a two-stage solution or a solution is developed with a Linear Programming model (Winston, 2004). In the two-stage solution, a suitable initial solution is obtained in the first stage, and the second stage, optimal solution methods are used to carry this initial solution to the optimal solution.

Initial solutions are an important research topic for two-stage solution approaches (Hillier and Lieberman, 2020). Researchers have always made a concerted effort to obtain more efficient feasible initial solutions. The well-known North-West Corner Method, Least-Cost Method, and Vogel's Approximation Method, which are well-established in the literature, have been included in textbooks (Hillier and Lieberman, 2020; Taha, 2017). In this context, this study aims to contribute to the literature by introducing a new initial solution proposal and analyzing the relative performances of different initial solution methods.

The paper includes a literature review on the transportation problem and appropriate initial solutions in Section 2, the proposed initial solution method in Section 3, and the comparison and analysis of the proposed method with other methods in Section 4. In the last section, conclusions and recommendations are given.

2. Literature Review

In the analysis and evaluation of logistics processes in Supply Chain Management, the Transportation Problem is a very important technical tool in terms of resource allocation efficiency (Taha, 2017; Winston, 2004). Logistics studies that can be modelled as Transportation Problems are very useful optimization techniques for reducing costs and improving service quality (Pishvaee et. al., 2011). In the two-stage solution approach to Transportation Problems, heuristic approaches developed to generate appropriate initial solutions have an important place in the literature (Winston, 2004; Taha, 2017). The most important reason for this attention to initial solutions is the consensus in optimization approaches that initial solutions save time and cost if they are used for the optimal solution (ReVelle and Swain, 1970). Therefore, a summary of the literature on appropriate initial solution approaches will be given, which forms the basis of this paper.

The Transportation Problem has been officially introduced to the Operations Research literature by Hitchcock (1941). It has been stated that the Transportation Problem in its usual form is of complexity class P and can be solved in polynomial time with commercial Linear Programming software. However, when all or some of the variables of the Transportation Problem are defined as integers, the problem will be in the NP class, and therefore, its solution will become more difficult and will enter the field of combinatorial optimization (Adhikari and Thapa, 2014: 68).

Gass (1990) reviewed three different papers proposing new solution approaches for the Transportation Problem and criticized the lack of sufficient numerical analysis to test the solution approaches. All three papers reviewed by Gass are considered to be approaches that propose new feasible initial solutions. Babu et al. (2020) analyzed the optimality conditions and limitations under the existing solution methods of the Transportation Problem. By improving on the basic theorems related to the Transportation Problem, they have determined the necessary and sufficient conditions for an optimal solution. They also established that there always exists a feasible solution for unbalanced problems. The authors argued that reaching the optimal solution without the need for a suitable initial solution is a more important solution approach. Mallia et al. (2021) discussed the optimal solution approach for the Transportation Problem with the Modified Distribution approach, by obtaining suitable initial solutions with the North-West Corner and Vogel's Approximation Method. They also included a discussion on the Capacitated Transportation Problem. Ekanayake et al. (2021) compared a solution approach for the Transportation Problem based on Graph Theory with the most well-known methods in the literature and obtained very good solutions. The method gave very successful results for both balanced and unbalanced Transportation Problems. The method they proposed is a heuristic algorithm and they named it North-East Corner Rule With Adjustments (NEWA). Adhikari and Thapa (2014) consider the Transportation Problem as a two-step solution approach. In the first step, they use Vogel's Approximation Method for the optimal initial solution and in the second step, they use Stepping Stone and Modified Distribution Method to obtain the optimal solution.

In the case where the authors consider the Transportation Problem as a fuzzy Transportation Problem from a decision-making point of view, it refers to an approach based on the Pareto solution space. Garg and Rizk-Allah, (2021) treated the Transportation Problem as a fuzzy environment modelling and proposed a structure that generates alternative solutions for decision-makers to choose the best compromise alternatives, which is a good approach for industrial applications. Ahmed et al. (2015) addressed the problem of minimizing transportation times for the Transportation Problem. The authors proposed a suitable initial solution algorithm to minimize transportation times. The proposed initial solution algorithm was tested with numerical examples and its relative superiority was argued. Hussein et al. (2020) emphasized that North-West Corner, Minimum Cost, and Vogel's Approximation Methods are effective initial solution methods for the Transportation Problem. They proposed a method called A New Effective Revised VAM and stated that they reached near-optimal solutions and optimal solutions for some problems in their analysis. Veeramani et al. (2021) modelled the transportation problem with a fuzzy number mechanism to take into account conflicting objectives such as cost, time, and environmental and social concerns for the multi-objective transportation problem. They proposed solution approaches for the fuzzy-modelled multi-objective transportation problem. Karagül and Şahin (2020) proposed a new solution approach called Karagül-Şahin Approximation Method (KSAM) to obtain appropriate initial solutions to Balanced and Unbalanced Transportation Problems. The authors also analyzed the performance of the KSAM proposal on a specific problem set and demonstrated its relative superiority over other methods. Garg and Rizk-Allah (2021) state that if the Transportation Problem is considered a fuzzy Transportation Problem in terms of decision-making, it is handled with an approach based on the Pareto solution space. The authors consider the Transportation Problem as a fuzzy environment modelling and propose a structure that generates alternative solutions for decision-makers to choose the best compromise alternatives, providing an effective approach for industrial applications.

Amaliah et al. (2022) emphasized the importance of appropriate initial solution approaches that produce efficient solutions to reach the optimal solution in Transportation Problems. Therefore, in search of an efficient method, they proposed an approach called the Bilqis-Chastine-Erma method (BCE) as a suitable initial solution approach. They performed a comparative analysis of their proposed BCE approach with Vogel's Approximation Method (VAM), Total Difference Method 1 (TDM1), Total Opportunity Cost Matrix - Minimal Sum (TOCM-MT) and Juman & Hoque Method

(JHM) initial solution methods on a test set of 35 problems. The proposed initial solution approach was successful in reaching an optimal solution for 85% of the problems in the test problem set. Mutlu et al. (2022) proposed an approach called the Avoid Maximum Cost Method (AMCM) as an initial solution algorithm for transportation problems. They compared the relative superiority of the method with NWC (North-West Corner), LCM (Least Cost Method), RAM (Russell Approximation Method), VAM (Vogel's Approximation Method), RM (Row Minima), CM (Column Minima), TCM (Tuncay Can's Method), and TOCM-SUM (Total Opportunity Cost Matrix-SUM) methods based on the optimal solution on 35 test problems. The authors also presented literature on the initial solution approaches used in the comparison as well as other initial solutions. Sarhani et al. (2023) conducted an in-depth study of meta-heuristic approaches. They stated that systematic and comprehensive research in this field is missing in the literature. They also emphasized the importance of appropriate initial solutions for meta-heuristics as in Transportation Problems.

Shaikh et al. (2024) used for appropriate initial solutions of Transportation Problems is Vogel's Approximation Method. The authors proposed an approach that produces more efficient solutions compared to VAM by using some statistical adjustments for VAM. Since this approach is based on VAM, they called to revamp VAM and numerically demonstrated the superiority of the proposed method. Kalaivani and Visalakshidevi (2024) proposed a new solution approach for transportation problems by using game theory and graph theory from a different perspective than traditional methods. The authors supported their innovative and efficient approach with numerical analysis. Baranidharan and Mahapatra (2024) expressed the cost, demand and supply parameters of the transportation problem with heptagonal fuzzy numbers and developed appropriate solutions to provide reliable solutions to real-world decision-making problems. They numerically demonstrated the effectiveness of their proposed solutions with a case study for vegetable transportation from regional markets. Rathod and Pajgade (2024) addressed transportation problems from a more strategic perspective. In this framework, while addressing problems such as economic development and social welfare, traffic congestion, inadequate route planning, and environmental degradation, on the other hand, they emphasized the importance of solution methods such as linear programming and heuristic algorithms under the phenomena of route optimization and capacity planning and presented their contributions.

3. Method and Material

3.1. Transportation Problem

In the literature, the Transportation Problem is mathematically defined as follows (Hillier and Lieberman, 2020:317-318): Z is defined as the total distribution cost. If we consider x_{ij}'yi ($i = 1,2,\dots,m;\ ,j = 1,2,\dots,n$) as the number of units to be distributed from source to destination, the linear programming (LP) formulation of the problem is expressed in Equations (1)-(4). In addition, a well-known graph illustrating the classical transportation problem is presented in Figure 1:

$$Min\ Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} \tag{1}$$

This formulation is given with the following constraints:

$$\sum_{j=1}^{n} x_{ij} = s_i \text{, for i} = 1,2,\dots,\text{m} \tag{2}$$

$$\sum_{i=1}^{m} x_{ij} = d_j \text{ , for j} = 1,2, \dots, \text{n} \tag{3}$$

$$x_{ij} \geq 0 \text{ , } for \; \forall \, i \text{ and } j \tag{4}$$

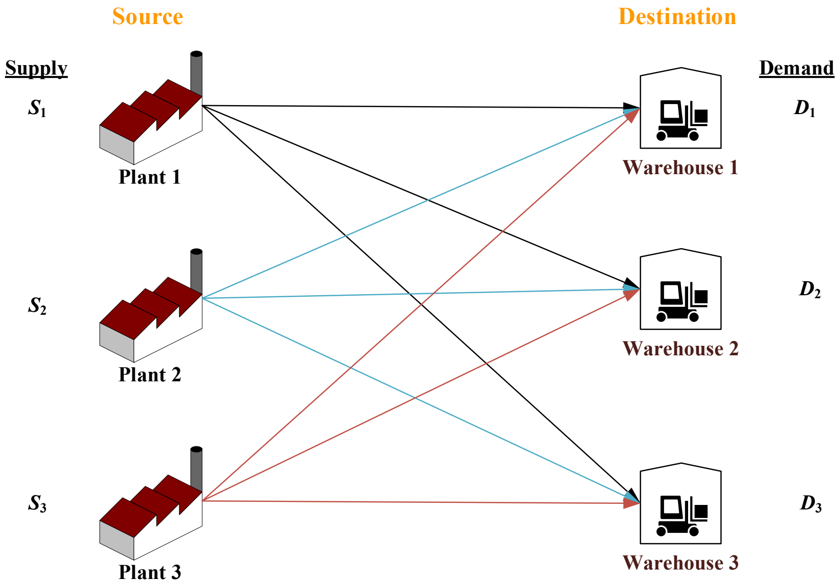

Figure 1. Transportation Problem representation

3.2. Proposed Algorithm

The proposed method, termed the Logarithmic Fermi Approximation (LFA), is an original approach developed in this study, inspired by the Italian physicist Enrico Fermi's techniques for solving estimation and decision-making problems under uncertainty using zeroth-order approximations (Fermi, 1962). Fermi problems are defined as an approach that generates approximate solutions by breaking down complex questions into simpler components, and the pedagogical and intuitive benefits of this method have been widely examined in mathematics education (Peter-Koop, 2009). Moreover, in the popular literature, the Fermi approach has been associated with the concept of "guesstimation" to enhance estimation skills and has been disseminated through various examples (Right Attitudes, 2017). The pseudo-code for the proposed method, Logarithmic Fermi Approximation for Transportation Problem, is given as Table 1.

Table 1. Psuede-code for the Proposed Algorithm

```
ALGORITHM LogarithmicFermiApproximationTransportationSolver
INPUT: supply, demand, costs
OUTPUT: allocation: Matrix showing optimal transportation assignments
STRUCT TransportProblem: supply, demand, costs, balanced
FUNCTION CalculateLogarithmicFermiCosts(transportProblem):
  m = number of rows in costs, n = number of columns in costs, LogarithmicFermiCosts = empty matrix of size m x n
  FOR i = 1 to m:
    FOR j = 1 to n:
      LogarithmicFermiCosts[i,j] = ROUND(LOG(costs[i,j]) / LOG(SQRT(supply[i] * demand[j])), 2)
  RETURN LogarithmicFermiCosts
FUNCTION MakeAssignments(LogarithmicFermiCosts, transportProblem):
  m = number of rows in LogarithmicFermiCosts, n = number of columns in LogarithmicFermiCosts, allocation = zero matrix of size m x n
  remainingSupply = COPY(supply), remainingDemand = COPY(demand), workingCosts = COPY(LogarithmicFermiCosts)
  WHILE (SUM(remaining supply) > 0 AND SUM(remainingDemand) > 0):
    // Find cell with minimum cost
    (i, j) = FIND_MIN_INDEX(workingCosts)
    // Calculate assignment amount
    assignAmount = MIN(remainingSupply[i], remainingDemand[j])
    // Make assignment
    allocation[i,j] = assignAmount
    remainingSupply[i] = remainingSupply[i] – assignAmount; remainingDemand[j] = remainingDemand[j] - assignAmount
    // Update constraints
    IF remainingSupply[i] = 0:
      SET row i of workingCosts to INFINITY
    IF remainingDemand[j] = 0:
      SET column j of workingCosts to INFINITY
  RETURN allocation
FUNCTION CalculateTotalCost(allocation, costs):
  total = 0
  FOR each cell (i,j) in allocation:
    total = total + (allocation[i,j] * costs[i,j])
  RETURN total
MAIN ALGORITHM:
  1. Create transportProblem instance with input data
  2. LogarithmicFermiCosts = CalculateLogarithmicFermiCosts (transportProblem)
  3. allocation = MakeAssignments(LogarithmicFermiCosts, transportProblem)
  4. totalCost = CalculateTotalCost(allocation, costs)
  5. RETURN allocation, totalCost
```

The features of the Logarithmic Fermi approach are as follows:

- Calculates new costs logarithmically
- Makes assignments based on the lowest-cost cells
- Calculates total cost

Key features of the Logarithmic Fermi Algorithm:

- Normalizes costs using logarithmic transformation
- Prioritizes the lowest-cost cells with a greedy approach
- Maintains supply and demand constraints

The mathematical relationships of the Logarithmic Fermi approximation and the properties of the resulting transformation can be explained as follows:

- Logarithmic conversion reduces the impact of large cost differences
- Normalizes costs by taking into account the magnitudes of supply and demand
- Considers source and target dimensions while maintaining original cost relationships

3.3. Properties of the Logarithmic Fermi Transformations

1. Domain Constraints:

$$s_i > 0 \quad \forall i, d_j > 0 \quad \forall j, c_{ij} > 0 \quad \forall i,j \tag{5}$$

2. Transformation Characteristics:

- Normalizes costs based on supply and demand magnitudes
- Reduces the impact of large cost variations
- Maintains relative cost relationships while considering source and destination sizes

3. Mathematical Transformation Process for Logarithmic Fermi transformations:

The transformed costs, Logarithmic Fermi Costs (c_{ij}^t), are typically rounded to 2 decimal places:

$$c_{ij}^t = \frac{\ln(c_{ij})}{\ln\left(\sqrt{s_i \cdot d_j}\right)} \tag{6}$$

4. Special Cases:

Average Converted Cost: If the product of resource supply (s_i) and target demand (d_j) is equal to 1, the transformed cost c_{ij}^t is simply the natural logarithm of the original cost c_{ij}.

$$\text{If } s_i \times d_j = 1 \implies c_{ij}^t = \ln(cij) \tag{7}$$

This means that when resource supply and destination demand are exactly balanced for a given route, the converted cost is directly proportional to the original cost.

Insignificant Converted Cost: As the product of resource supply (s_i) and target demand (d_j) approaches infinity, the converted cost (c_{ij}^t) approaches zero.

$$s_i \times d_j \to \infty \implies c_{ij}^t \to 0 \tag{8}$$

This means that when resource supply and target demand are extremely high compared to the original cost, the converted cost becomes irrelevant, i.e. the original cost is ignored.

Extremely High Converted Cost: As the original transportation cost (c_{ij}) approaches infinity, the transformed cost (c_{ij}^t) also approaches infinity.

$$c_{ij} \rightarrow \infty \text{ ise}, c_{ij}^t \rightarrow \infty \tag{9}$$

This means that routes with extremely high original costs will become less desirable in the optimization process.

4. Computational Analysis

Comparative analyses were made with the proposed method, Logarithmic Fermi Approximation, traditional initial solution approaches, North-West Corner (NCW), Least Cost (LCM) methods and the algorithms referred to as TimeMinAlg and KSAM from the literature. The algorithm referred to as TimeMinAlg was proposed by Ahmed et al. (2015). The KSAM initial solution algorithm was proposed by Karagül and Şahin (2020). All methods used for the analyses are coded in Julia scientific computing language. In addition, the test problems were coded with Julia JuMP and their optimal solutions were obtained using the HiGHS open-source solver. The test problems were randomly generated in two different main groups, P1 and P2. For test problems, P1 and P2, the demand and supply generation parameters are the same, but the unit cost parameters are set differently. The ranges of the parameters are (50, 100), (30, 80), (100, 200) for P1 and (50, 100), (30, 80), (1, 20) for P2 respectively. The numerical distribution of the test problem set is 12 balanced and 12 unbalanced in group P1 and 12 balanced and 12 unbalanced in group P2. In addition, an equal number of small, medium and large problems were planned in terms of size in each group test problem set. In the experimental design, two different unit transportation cost ranges, P1 (100-200) and P2 (1-20), are determined to test the cost sensitivity and robustness of the proposed algorithm, aiming to systematically evaluate the algorithm's consistent performance across a wide cost spectrum.

The results of the analysis are given in Table 2 for the P1 Balanced group. Figure 2 shows the graphs of small, medium and large solution comparisons for the P1 balanced group. Table 3 shows the relative comparison table for the performance of the initial solutions concerning the optimal solution. In this table, the reference value for optimal solutions is taken as 1 and other methods are compared according to 1. Positive divergences at 1 indicate a worsening of the solution's success. Figure 3 shows a graph of the comparison between the optimal solution approach and the initial solution approaches for all P1 balanced group problems.

Table 2. Initial Solution Results for Group P1 Balanced Test Set

Pr. No	Problem Name	Type	JuMP/HiGHS	NCW	LCM	Fermi Method	KSAM	TimeMinAlg
1	smallB-TP_3x4_P1	Balanced	32.049	35.070	32.049	32.069	33.729	32.049
2	smallB-TP_4x5_P1	Balanced	34.797	40.874	36.510	35.035	35.194	36.510
3	smallB-TP_5x6_P1	Balanced	53.073	63.576	56.098	54.598	56.346	56.098
4	smallB-TP_6x10_P1	Balanced	59.678	73.897	64.588	59.991	63.159	64.588
5	mediumB-TP_20x30_P1	Balanced	184.254	256.100	191.020	191.707	207.593	191.020
6	mediumB-TP_30x40_P1	Balanced	233.769	339.870	246.655	250.451	247.546	246.655
7	medium_TP_40x100_P1	Balanced	594.967	779.571	659.843	615.011	652.129	659.843
8	medium_TP_70x140_P1	Balanced	824.201	1.172.880	904.591	848.679	892.829	904.591
9	largeB-TP_150x250_P1	Balanced	1.452.470	2.104.357	1.568.876	1.495.254	1.561.707	1.568.876
10	largeB-TP_250x400_P1	Balanced	2.251.739	3.321.759	2.388.783	2.305.744	2.377.121	2.388.783
11	largeB-TP_400x800_P1	Balanced	4.605.344	6.527.251	5.079.886	4.696.009	4.889.163	5.079.886
12	largeB-TP_800x1000_P1	Balanced	6.090.739	9.083.304	6.377.481	6.190.980	6.256.698	6.377.481
		Averages	**1.368.090**	**1.983.209**	**1.467.198**	**1.397.961**	**1.439.435**	**1.467.198**

Similarly, tables and graphs for the P1 unbalanced problem group are given in Table 4, Table 5, Figure 3 and Figure 4. The tables and graphs of P2 group problems, balanced and unbalanced, are given in Table 5, Table 6, Table 7, Table 8, Figure 6, Figure 7, Figure 8 and Figure 9.

Table 1 shows that the Logarithmic Fermi Method produces the lowest cost solutions, followed by KSAM, LCM and TimeMinAlg, while NCW performs the worst. For large-scale problems (150×250 and above), it is observed that the Logarithmic Fermi Method produces results closer to the optimal solution, while LCM and TimeMinAlg produce solutions with equal cost. This performance gap becomes more pronounced as problem size increases.

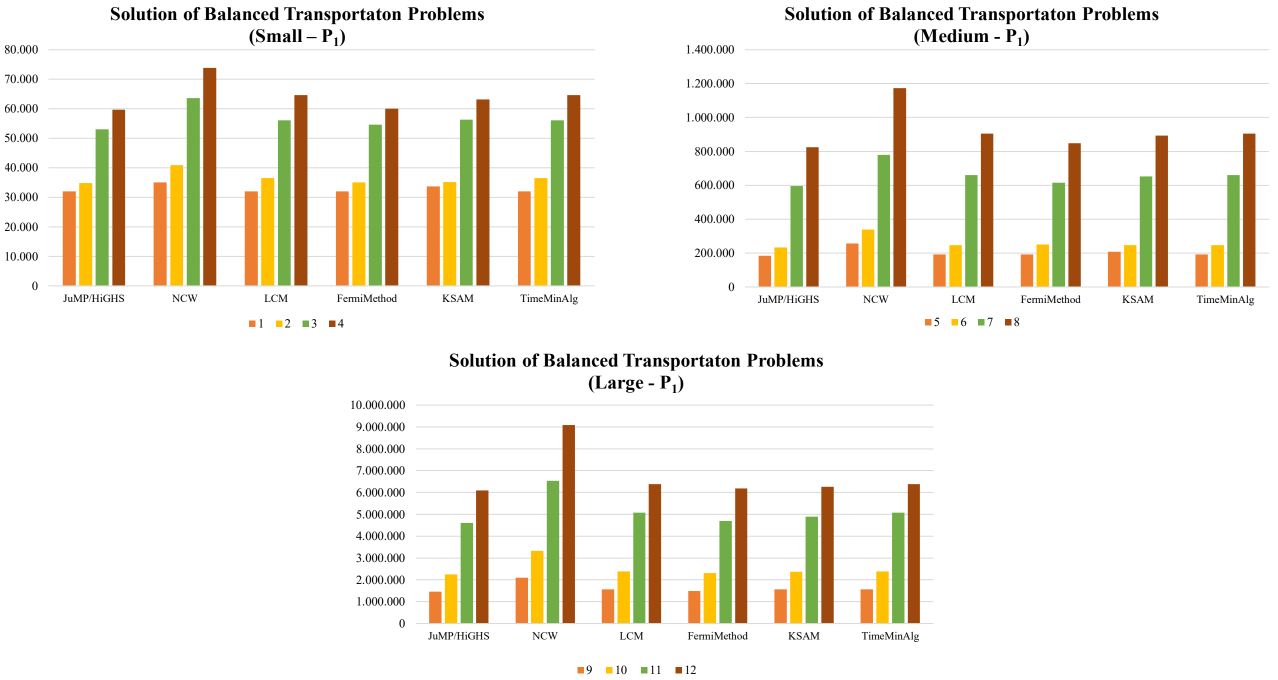

Figure 2. Solution Comparisons of P1 Group Balanced Test Problems

Table 3. Relative Comparison Table of Solution Results for Group P1 Balanced Test Set

Pr. No	Problem Name	Balance Of Type	JuMP/HiGHS	NCW	LCM	Fermi Method	KSAM	TimeMinAlg
1	smallB-TP_3x4_P1	Balanced	1.00	1.09	1.00	1.00	1.05	1.00
2	smallB-TP_4x5_P1	Balanced	1.00	1.17	1.05	1.01	1.01	1.05
3	smallB-TP_5x6_P1	Balanced	1.00	1.20	1.06	1.03	1.06	1.06
4	smallB-TP_6x10_P1	Balanced	1.00	1.24	1.08	1.01	1.06	1.08
5	mediumB-TP_20x30_P1	Balanced	1.00	1.39	1.04	1.04	1.13	1.04
6	mediumB-TP_30x40_P1	Balanced	1.00	1.45	1.06	1.07	1.06	1.06
7	mediumB-TP_40x100_P1	Balanced	1.00	1.31	1.11	1.03	1.10	1.11
8	mediumB-TP_70x140_P1	Balanced	1.00	1.42	1.10	1.03	1.08	1.10
9	largeB-TP_150x250_P1	Balanced	1.00	1.45	1.08	1.03	1.08	1.08
10	largeB-TP_250x400_P1	Balanced	1.00	1.48	1.06	1.02	1.06	1.06
11	largeB-TP_400x800_P1	Balanced	1.00	1.42	1.10	1.02	1.06	1.10
12	largeB-TP_800x1000_P1	Balanced	1.00	1.49	1.05	1.02	1.03	1.05
		Averages	**1.00**	**1.34**	**1.06**	**1.03**	**1.06**	**1.06**

When Table 3 is analyzed, The Logarithmic Fermi Method produced better results than the other heuristics in all problem sizes and obtained the values closest to the optimal solution. While the performances of the algorithms are close to each other for small-scale problems, the difference becomes remarkable for medium and large-scale problems. TimeMinAlg and LCM algorithms produced the same results for all problem sizes and showed a moderate performance. The KSAM algorithm, on the other hand, performed better than TimeMinAlg and LCM in general, but worse than Fermi, and produced consistent results for large-scale problems. NCW performed the worst for all problem sizes. As the size of the balanced test problem set increases from small to large, the performance differences between the heuristic algorithms increase significantly. For the largest problem size of 800x1000, the performance difference between the heuristic algorithms reached the highest level, which demonstrates the scalability properties of the algorithms. It can be observed that Figure 1 and Figure 2 also support the analysis from Table 2 and Table 3.

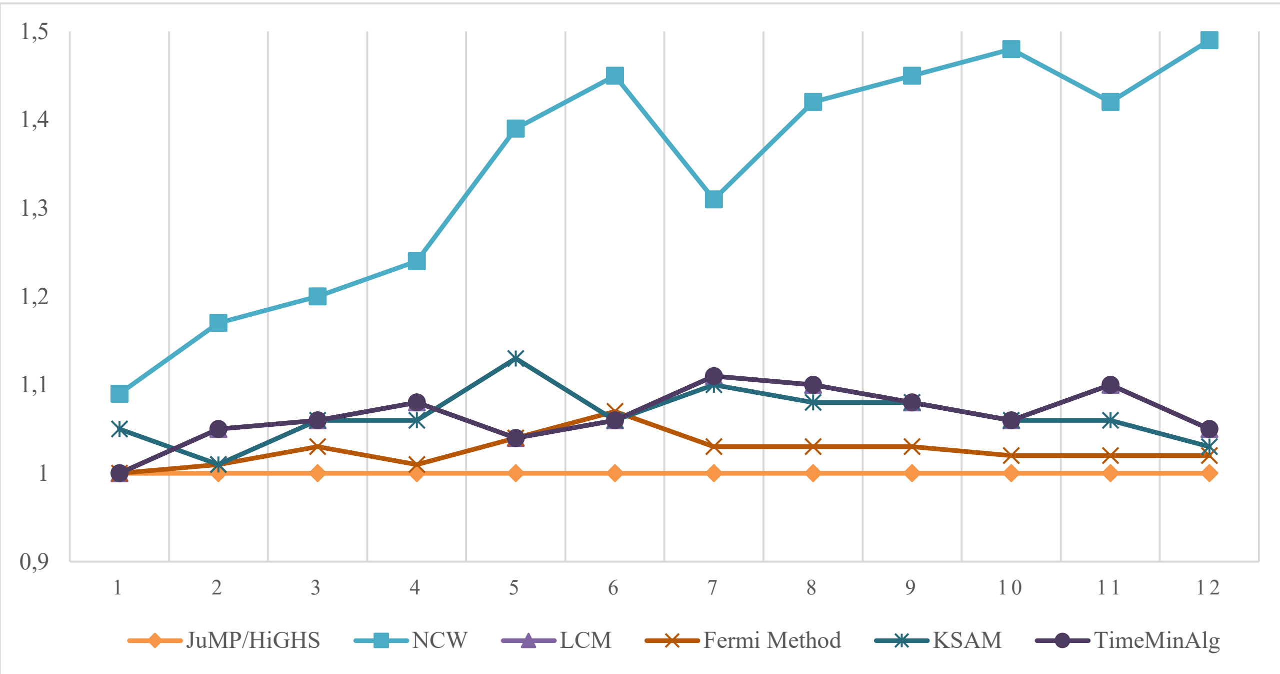

Figure 3. Relative Quality of Solutions (Balanced - P1)

Table 4. Solution Results for Group P1 Unbalanced Test Set

Pr. No	Problem Name	Type	JuMP/HiGHS	NCW	LCM	Fermi Method	KSAM	TimeMinAlg
1	smallUB-TP_3x4_P1	Unbalanced	29.519	35.450	29.897	30.335	32.325	30.373
2	smallUB-TP_4x5_P1	Unbalanced	28.968	36.776	29.566	29.790	29.024	32.237
3	smallUB-TP_5x6_P1	Unbalanced	40.028	55.327	40.223	41.975	41.437	42.208
4	smallUB-TP_6x10_P1	Unbalanced	49.379	60.080	49.902	53.196	50.173	57.082
5	mediumUB-TP_20x30_P1	Unbalanced	157.249	216.839	159.575	164.575	167.832	162.718
6	mediumUB-TP_30x40_P1	Unbalanced	231.552	333.235	235.936	243.967	240.993	242.155
7	mediumUB-TP_40x100_P1	Unbalanced	298.527	414.489	299.210	310.198	315.076	314.072
8	mediumUB-TP_70x140_P1	Unbalanced	511.035	760.637	513.190	528.240	533.765	528.829
9	largeUB-TP_150x250_P1	Unbalanced	1.146.345	1.690.333	1.149.820	1.171.978	1.181.011	1.167.443
10	largeUB-TP_250x400_P1	Unbalanced	1.902.570	2.820.232	1.907.099	1.941.419	1.948.516	1.923.232
11	largeUB-TP_400x800_P1	Unbalanced	2.955.066	4.449.286	2.957.386	3.005.109	3.017.207	2.979.585
12	largeUB-TP_800x1000_P1	Unbalanced	5.544.882	8.398.534	5.553.947	5.622.359	5.635.543	5.571.990
		Averages	**1.074.593**	**1.605.935**	**1.077.146**	**1.095.262**	**1.099.409**	**1.087.660**

When Table 4 is analyzed, the average solution cost of the optimal solution for solving imbalanced transportation problems is 1074593 JuMP/HiGHS, while LCM produces the lowest cost solutions with an average of 1077146 among the initial solution algorithms, followed by TimeMinAlg 1087660, Logarithmic Fermi Method 1095262 and KSAM 1099409, respectively. While the performance differences of the algorithms are more significant in small-scale problems, in large-scale problems, the solutions produced by all algorithms except NCW show similar costs.

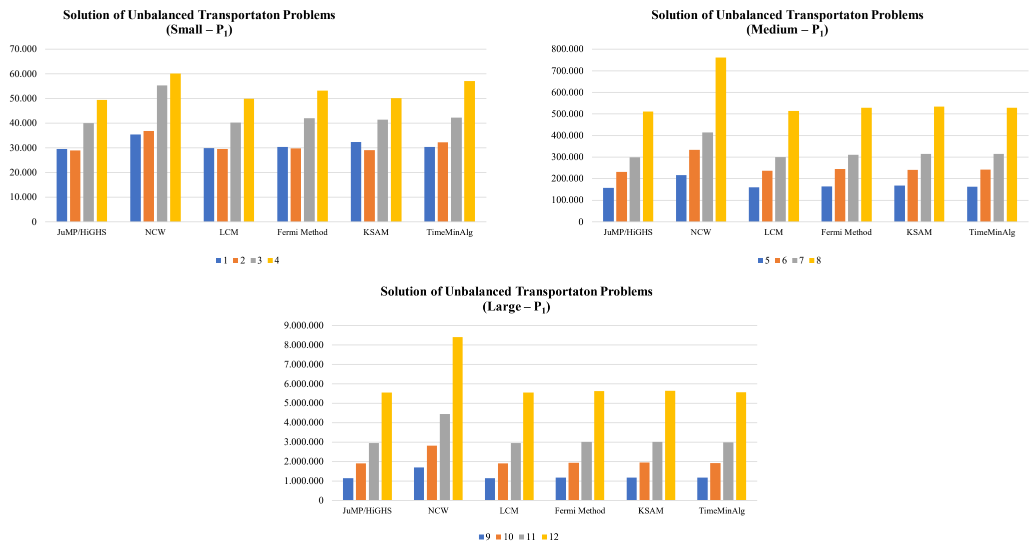

Figure 4. Solution Comparisons of P1 Group Unbalanced Test Problems

Table 5. Relative comparison table of solution approaches for P1 Group Unbalanced Test Problems

Pr. No	Problem Name	Balance Of Type	JuMP/HiGHS	NCW	LCM	Fermi Method	KSAM	TimeMinAlg
1	smallUB-TP_3x4_P1	Unbalanced	1.00	1.20	1.01	1.03	1.10	1.03
2	smallUB-TP_4x5_P1	Unbalanced	1.00	1.27	1.02	1.03	1.00	1.11
3	smallUB-TP_5x6_P1	Unbalanced	1.00	1.38	1.00	1.05	1.04	1.05
4	smallUB-TP_6x10_P1	Unbalanced	1.00	1.22	1.01	1.08	1.02	1.16
5	mediumUB-TP_20x30_P1	Unbalanced	1.00	1.38	1.01	1.05	1.07	1.03
6	mediumUB-TP_30x40_P1	Unbalanced	1.00	1.44	1.02	1.05	1.04	1.05
7	mediumUB-TP_40x100_P1	Unbalanced	1.00	1.39	1.00	1.04	1.06	1.05
8	mediumUB-TP_70x140_P1	Unbalanced	1.00	1.49	1.00	1.03	1.04	1.03
9	largeUB-TP_150x250_P1	Unbalanced	1.00	1.47	1.00	1.02	1.03	1.02
10	largeUB-TP_250x400_P1	Unbalanced	1.00	1.48	1.00	1.02	1.02	1.01
11	largeUB-TP_400x800_P1	Unbalanced	1.00	1.51	1.00	1.02	1.02	1.01
12	largeUB-TP_800x1000_P1	Unbalanced	1.00	1.51	1.00	1.01	1.02	1.00
		Averages Rates	**1.00**	**1.40**	**1.01**	**1.04**	**1.04**	**1.05**

The relative comparison data for imbalanced transportation problems in Table 5 provides important information about the performance of the heuristic algorithms. It can be seen that the LCM algorithm performs the best with an average deviation of 1.01. Fermi Method and KSAM algorithms perform second best with an average deviation of 1.04, followed by TimeMinAlg with an average deviation of 1.05. The NCW algorithm performed worse than the other algorithms with an average deviation rate of 1.40. As the problem size increases, the performance of all algorithms except NCW improves. In large-scale problems, with dimensions of 150x250 and above, LCM reached the optimal solution, while the Logarithmic Fermi Method, KSAM and TimeMinAlg algorithms produced results very close to the optimal solution. While the difference in performance between the algorithms is more pronounced for small-scale problems, as the problem size increases, the performances of all algorithms except NCW converge with each other. The fact that TimeMinAlg can find the optimal solution for the largest problem size 800x1000 and the other algorithms produce very close results shows that these algorithms can produce very effective results in large-scale imbalanced transportation problems. It can be observed that Figure 4 and Figure 5 also support the analysis from Table 4 and Table 5.

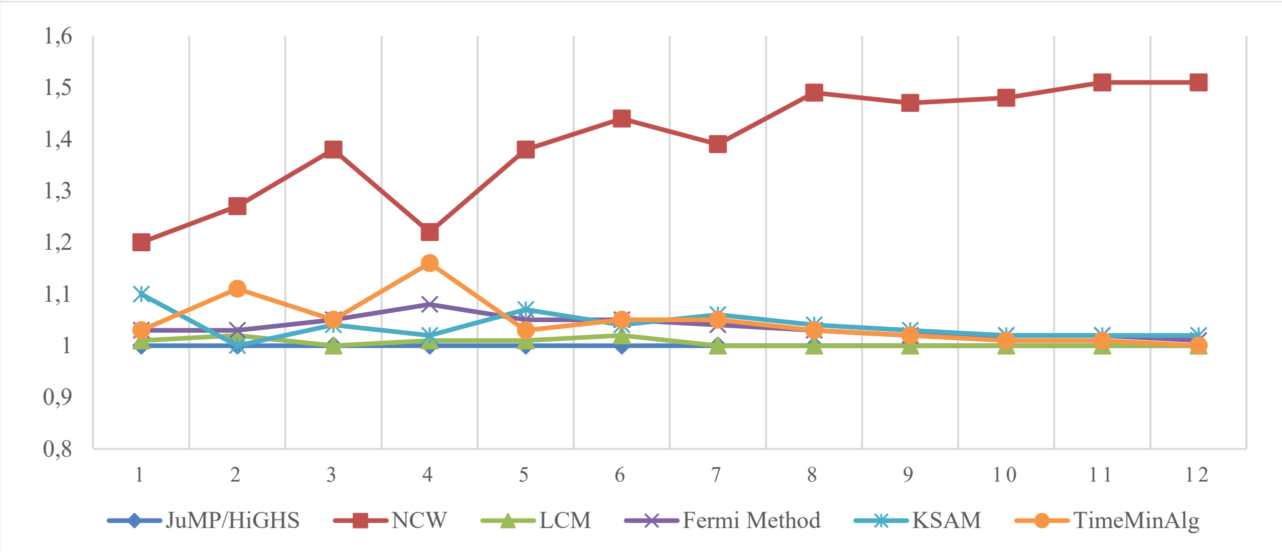

Figure 5. Relative Quality of Solutions (Unbalanced - P1)

Table 6. Solution Results for Group P2 Balanced Test Set

Pr. No	Problem Name	Balance Of Type	JuMP/HiGHS	NCW	LCM	Fermi Method	KSAM	TimeMinAlg
1	smallB-TP_3x4_P2	Balanced	2,504	2,744	2,504	2,504	2,504	2,504
2	smallB-TP_4x5_P2	Balanced	1,337	2,716	1,884	1,682	1,884	1,884
3	smallB-TP_5x6_P2	Balanced	1,650	2,398	2,117	2,117	2,117	2,117
4	smallB-TP_6x10_P2	Balanced	2,183	5,680	2,632	2,668	2,557	2,632
5	mediumB-TP_20x30_P2	Balanced	3,720	17,031	4,422	4,791	4,831	4,422
6	mediumB-TP_30x40_P2	Balanced	4,686	24,905	5,908	6,287	5,743	5,908
7	mediumB-TP_40x100_P2	Balanced	20,197	62,137	36,263	35,847	24,195	36,263
8	mediumB-TP_70x140_P2	Balanced	13,611	72,668	24,687	24,703	17,046	24,687
9	largeB-TP_150x250_P2	Balanced	20,910	148,631	44,971	43,856	24,821	44,971
10	largeB-TP_250x400_P2	Balanced	25,168	221,483	47,982	47,992	30,010	47,982
11	largeB-TP_400x800_P2	Balanced	82,319	452,845	175,293	175,119	91,139	17,5293
12	largeB-TP_800x1000_P2	Balanced	63.312	630.338	105.484	104.595	68.012	105,484
		Averages	**20.133**	**136.965**	**37.846**	**37.680**	**22.905**	**37.846**

The solutions for the P2 balanced transportation problem set are given in Table 6. When Table 6 is analyzed, among the feasible initial solution algorithms, KSAM produces the lowest cost solutions, followed by the logarithmic Fermi Method, followed by LCM and TimeMinAlg, which produce equal results, while NCW performs the worst. For small-scale problems, all algorithms produced similar results except NCW, but for large-scale problems, KSAM performed significantly better than the other heuristic algorithms.

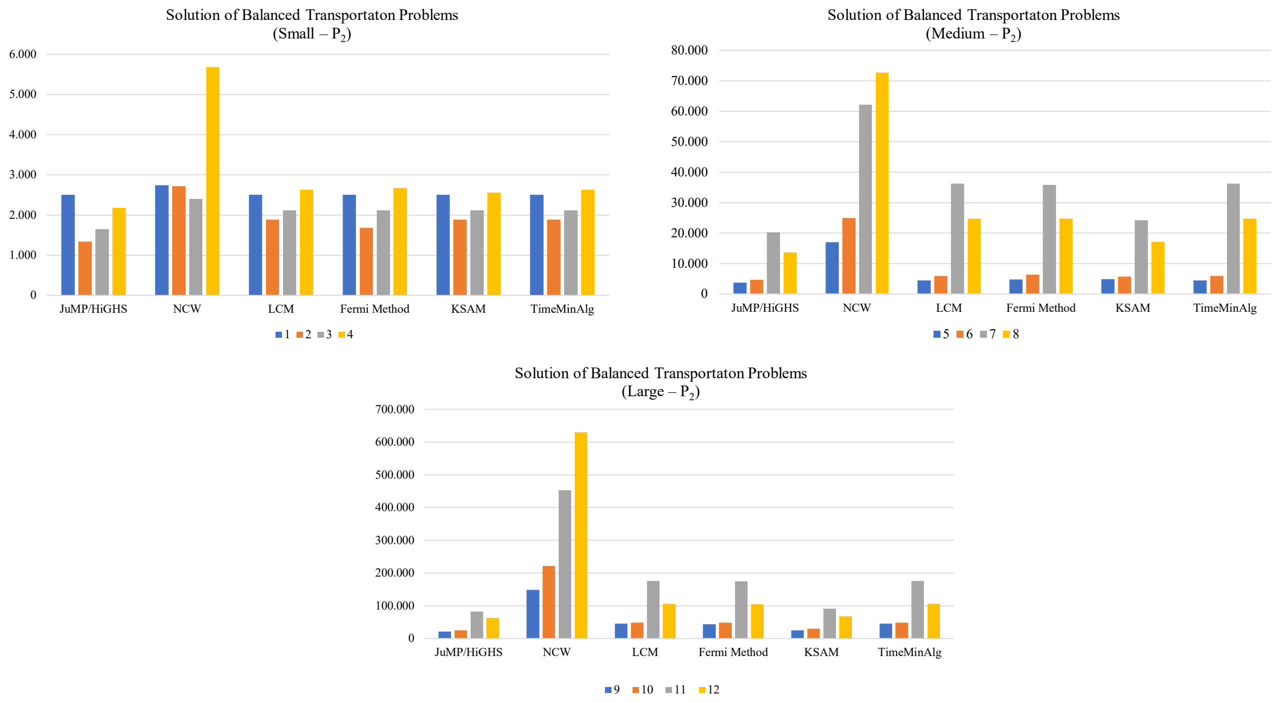

Figure 6. Solution Comparisons of P2 Group Balanced Test Problems

The relative comparison analysis of P2 balanced transport problems based on the optimal solution is given in Table 6. When the relative deviation ratios in Table 6 are analyzed, it is observed that KSAM shows the best performance with an average deviation ratio of 1.20, logarithmic Fermi Method 1.56, LCM and TimeMinAlg with equal deviation ratios of 1.57, and NCW shows the worst performance with a very high deviation ratio of 4.74 compared to the optimal solution. It is observed that as the problem size increases, especially the deviation rate of KSAM decreases, the deviation rates of the other algorithms fluctuate, but the deviation rate of NCW increases significantly.

It can be observed that Figure 6 and Figure 7 also support the analysis from Table 6 and Table 7.

Table 7. Relative comparison table of solution results for Group P2 Balanced test set

Pr. No	Problem Name	Balance Of Type	JuMP/HiGHS	NCW	LCM	Fermi Method	KSAM	TimeMinAlg
1	smallB-TP_3x4_P2	Balanced	1.00	1.10	1.00	1.00	1.00	1.00
2	smallB-TP_4x5_P2	Balanced	1.00	2.03	1.41	1.26	1.41	1.41
3	smallB-TP_5x6_P2	Balanced	1.00	1.45	1.28	1.28	1.28	1.28
4	smallB-TP_6x10_P2	Balanced	1.00	2.60	1.21	1.22	1.17	1.21
5	mediumB-TP_20x30_P2	Balanced	1.00	4.58	1.19	1.29	1.30	1.19
6	mediumB-TP_30x40_P2	Balanced	1.00	5.31	1.26	1.34	1.23	1.26
7	mediumB-TP_40x100_P2	Balanced	1.00	3.08	1.80	1.77	1.20	1.80
8	mediumB-TP_70x140_P2	Balanced	1.00	5.34	1.81	1.81	1.25	1.81
9	largeB-TP_150x250_P2	Balanced	1.00	7.11	2.15	2.10	1.19	2.15
10	largeB-TP_250x400_P2	Balanced	1.00	8.80	1.91	1.91	1.19	1.91
11	largeB-TP_400x800_P2	Balanced	1.00	5.50	2.13	2.13	1.11	2.13
12	largeB-TP_800x1000_P2	Balanced	1.00	9.96	1.67	1.65	1.07	1.67
		Averages	**1.00**	**4.74**	**1.57**	**1.56**	**1.20**	**1.57**

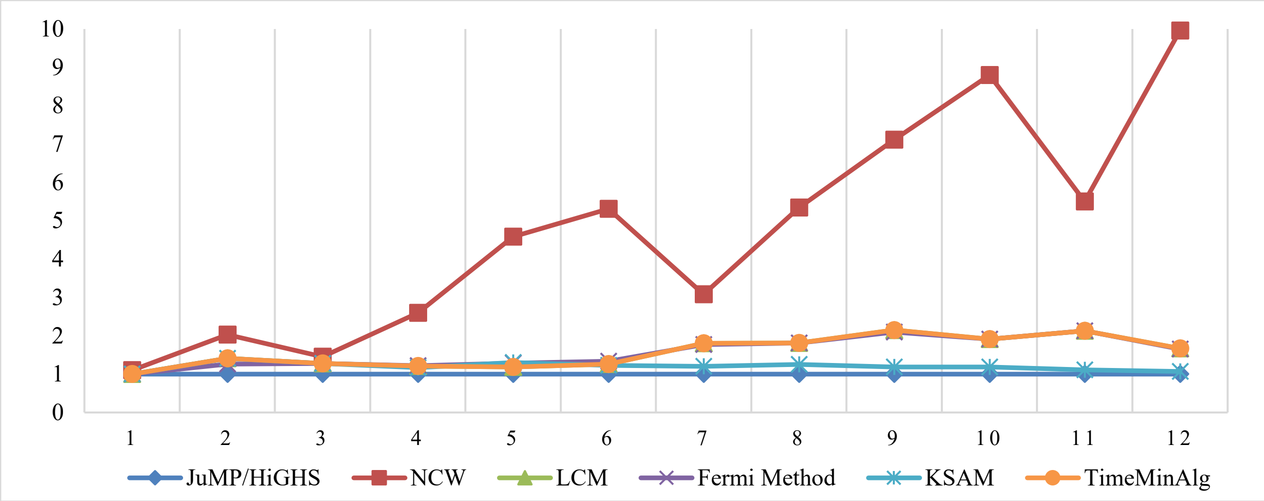

Figure 7. Relative Quality of Solution (Balanced - P2)

Table 8. Solution Results for Group P2 Unbalanced Test Set

Pr. No	Problem Name	Balance Of Type	JuMP/HiGHS	NCW	LCM	Fermi Method	KSAM	TimeMinAlg
1	smallUB-TP_3x4_P2	Unbalanced	1.528	2.121	1.528	1.528	1.528	1.528
2	smallUB-TP_4x5_P2	Unbalanced	1.512	3.483	2.086	2.086	2.086	2.099
3	smallUB-TP_5x6_P2	Unbalanced	1.357	3.203	1.363	1.363	1.363	1.381
4	smallUB-TP_6x10_P2	Unbalanced	2.211	4.811	2.710	2.736	2.536	2.926
5	mediumUB-TP_20x30_P2	Unbalanced	2.665	15.777	3.230	3.124	3.170	4.103
6	mediumUB-TP_30x40_P2	Unbalanced	4.123	20.693	5.064	5.008	4.970	5.710
7	mediumUB-TP_40x100_P2	Unbalanced	3.858	30.699	3.981	3.981	4.058	7.002
8	mediumUB-TP_70x140_P2	Unbalanced	5.417	50.080	5.697	5.697	5.964	8.254
9	largeUB-TP_150x250_P2	Unbalanced	11.267	119.960	11.957	11.914	11.984	14.988
10	largeUB-TP_250x400_P2	Unbalanced	18.195	189.461	18.526	18.526	18.687	22.254
11	largeUB-TP_400x800_P2	Unbalanced	30.084	317.900	30.084	30.084	30.084	33.620
12	largeUB-TP_800x1000_P2	Unbalanced	54.935	565.382	55.267	55.267	55.218	57.988
		Averages	**11.429**	**110.298**	**11.791**	**11.776**	**11.804**	**13.488**

Table 7 shows the solutions for the P2 group of imbalanced transport problems. The Logarithmic Fermi Method produced the lowest cost solutions, followed by LCM and KSAM with very close values. TimeMinAlg and NCW performed poorly, producing higher cost solutions. For small-scale problems, TimeMinAlg's performance was worse than the other heuristic algorithms, while for large-scale problems, all algorithms except NCW produced very close results.

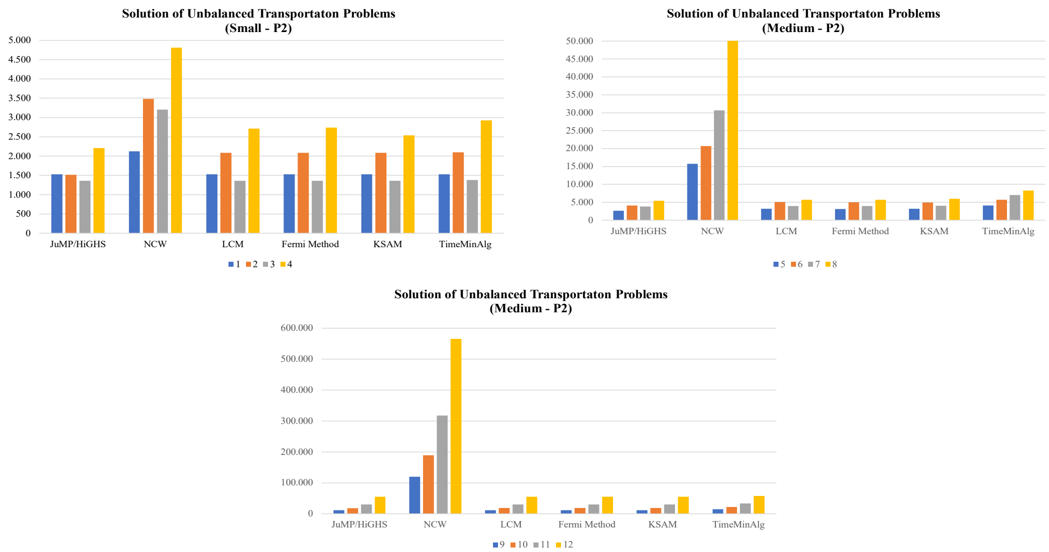

Figure 8. Solution Comparisons of P2 Group Unbalanced Test Problems

Table 9. Relative Comparison of Solution Results for Group P2 Unbalanced Test Set

Pr. No	Problem Name	Balance Of Type	JuMP/HiGHS	NCW	LCM	Fermi Method	KSAM	TimeMinAlg
1	smallUB-TP_3x4_P2	Unbalanced	1.00	1.39	1.00	1.00	1.00	1.00
2	smallUB-TP_4x5_P2	Unbalanced	1.00	2.30	1.38	1.38	1.38	1.39
3	smallUB-TP_5x6_P2	Unbalanced	1.00	2.36	1.00	1.00	1.00	1.02
4	smallUB-TP_6x10_P2	Unbalanced	1.00	2.18	1.23	1.24	1.15	1.32
5	mediumUB-TP_20x30_P2	Unbalanced	1.00	5.92	1.21	1.17	1.19	1.54
6	mediumUB-TP_30x40_P2	Unbalanced	1.00	5.02	1.23	1.21	1.21	1.38
7	mediumUB-TP_40x100_P2	Unbalanced	1.00	7.96	1.03	1.03	1.05	1.81
8	mediumUB-TP_70x140_P2	Unbalanced	1.00	9.24	1.05	1.05	1.10	1.52
9	largeUB-TP_150x250_P2	Unbalanced	1.00	10.65	1.06	1.06	1.06	1.33
10	largeUB-TP_250x400_P2	Unbalanced	1.00	10.41	1.02	1.02	1.03	1.22
11	largeUB-TP_400x800_P2	Unbalanced	1.00	10.57	1.00	1.00	1.00	1.12
12	largeUB-TP_800x1000_P2	Unbalanced	1.00	10.29	1.01	1.01	1.01	1.06
	Averages Rates		**1.00**	**6.52**	**1.10**	**1.10**	**1.10**	**1.31**

Table 9 shows the relative comparison table for the algorithm solutions. When JuMP/HiGHS is taken as a reference for all problems, the logarithmic Fermi Method, LCM and KSAM perform the closest to JuMP/HiGHS among the suitable initial solution algorithms. NCW appears to be the most inefficient method for large-size problems.

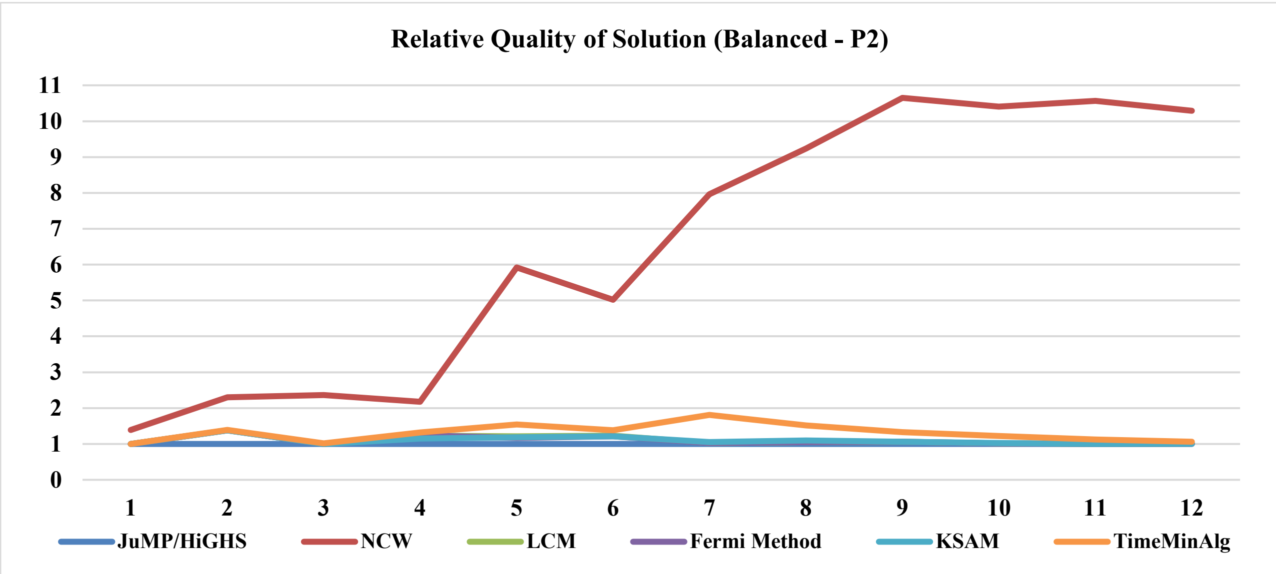

Figure 9. Solution Comparisons of P2 Group Unbalanced Test Problems

It can be observed that Figure 8 and Figure 9 also support the analysis from Table 8 and Table 9.

5. Conclusions

In terms of logistics processes, transport problems and methods for appropriate initial solutions for these problems are very important. They represent a large area of research in the literature. Therefore, in this study, a new initial solution approach is proposed and performance comparisons are made with other methods in the literature.

The performance of different feasible initial solution algorithms on transport test problems with the same supply and demand parameters but different cost ranges are analyzed numerically. The results of the analyses show that the cost range differences have a significant effect on the performance of the algorithms. This difference in cost distribution between the P1 and P2 groups reveals the Logarithmic Fermi Method and KSAM algorithms as effective and efficient optimal initial solution algorithms, especially for large-scale unbalanced problems.

For test problem set P1, the Logarithmic Fermi Method and LCM produced highly efficient results for large-scale balanced and unbalanced problems.

For the P2 test problem set, the KSAM algorithm showed that it is a competitive approach with effective solutions in a narrow cost range.

The NCW Algorithm showed overall poor performance, producing the highest cost solutions in both problem sets. This analysis shows how the choice of algorithm according to different problem characteristics affects the solution efficiency and is important in showing the competitive advantages of the Logarithmic Fermi Method, LCM and KSAM algorithms, especially for large-scale unbalanced transport problems.

The results obtained in this study can form a basis for future research in the development of algorithms for transport problems. Improving the solution quality with hybrid and adaptive algorithms, investigating the effects of different cost structures and degrees of imbalance on algorithm performance, and investigating parallel and memory-efficient methods for large-scale problems can be suggested. Developing algorithms for more complex transportation models in uncertain environments, addressing multi-objective transportation problems and parameter optimization with machine learning can also be considered as studies that will increase solution efficiency. Performance analyses related to speed and accuracy balance can be suggested as studies that will contribute to increasing the usability of algorithms in practical applications at the industrial level.

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